EE 779 Advanced Topics in Signal Processing
Assignment 4
Assigned: 25/10/12, Due: 5/11/12
Indian Institute of Technology Bombay

Note

- Most of these problems are from Chapter 6 in [1].
- Submit **only** the simulations.
- For the simulation problems, you can use the Matlab functions provided by S & M [1] or Hayes [2]. A copy of the files is in moodle.

Problems

1. [$*$]Consider a ULA comprising M sensors, with inter-element spacing equal to d. Let λ denote the wavelength of the signals impinging on the array. The spatial-frequency resolution of the beamforming used with this ULA is given by

$$\Delta\omega_s = \frac{2\pi}{M} \quad \Longleftrightarrow \quad \Delta f_s = \frac{1}{M}.$$

   (a) Use this to show that the direction-of-arrival (DOA) resolution of beamforming for signals coming from *broadside* is

$$\Delta\theta \simeq \sin^{-1}(1/L)$$

   where L is the array's length measured in wavelengths:

$$L = \frac{(M-1)d}{\lambda}.$$

   Explain how the DOA resolution approximately reduces to

$$\Delta\theta \simeq \frac{\lambda}{L}.$$

   (b) Next, show that, for signals impinging from an *arbitrary direction* θ, the DOA resolution of beamforming is approximately

$$\Delta\theta \simeq \frac{1}{L|\cos\theta|}.$$

   Hence, for signals coming from nearly end-fire ($\pm\pi/2$), the DOA resolution is much worse.

2. [$*$]Consider an $M-$element array, with M odd, shaped as an "L" with element spacing d. Thus, the array elements are located at points $(0,0),(0,d),\ldots,(0,d(M-1)/2)$ and $(d,0),(2d,0),\ldots,(d(M-1)/2,0)$. Determine the array's beampattern ($a^H(\theta_0)a(\theta)$) at angle θ_0, by assuming the sensor at $(0,0)$ as the reference.

3. In words, MUSIC (for both frequency and DOA estimation) says that the direction vectors $\{a(\theta_k)\}$ belong to the subspace spanned by the columns of $\mathbf{V}_s$ (signal space eigenvectors). Therefore, we can think of estimating the DOAs by choosing θ (a generic DOA variable) so that the distance between $a(\theta)$ and the closest vector in the span of $\hat{\mathbf{V}}_s$ is minimized, i.e.,

$$\min_{\beta,\theta} \|a(\theta) - \widehat{\mathbf{V}}_s\beta\|^2,$$

   where $\|.\|$ denotes the Euclidean vector norm. Note that the dummy vector variable β is defined in such a way that $\hat{\mathbf{V}}_s\beta$ is closest to $a(\theta)$ in Euclidean norm. Show that the DOA estimation method derived from the subspace-fitting criterion is the same as MUSIC.

   Hint: Show that

$$\|a(\theta) - \widehat{\mathbf{V}}_s\beta\|^2 = \|\beta - \widehat{\mathbf{V}}_s a(\theta)\|^2 + a^H(\theta)\widehat{\mathbf{V}}_n\widehat{\mathbf{V}}_n^H a(\theta).$$

4. In ESPRIT technique, the signal space eigenvector matrices $\mathbf{S}_1$ and $\mathbf{S}_2$ are obtained as $\mathbf{S}_1 = [\mathbf{I}_{\tilde{M}} \quad \mathbf{0}]\mathbf{V}_s$ and $\mathbf{S}_2 = [\mathbf{0} \quad \mathbf{I}_{\tilde{M}}]\mathbf{V}_s$. The intermediate matrix $\mathbf{\Phi}$, whose eigenvalues will be used to estimate the source directions, can be obtained by solving $\widehat{\mathbf{S}}_2 \simeq \widehat{\mathbf{S}}_1\mathbf{\Phi}$. Obtain the least-squares solution for $\mathbf{\Phi}$.

Simulations

1. Problem C6.15 in S & M [1].

Reference

1. Petre Stoica and Randolph Moses, "Spectral analysis of signals", Prentice Hall, 2005. (Indian edition available)

2. Monson H. Hayes, "Statistical signal processing and modeling", Wiley India Pvt. Ltd., 2002. (Indian edition available)