

Estimation of Interference Temperature

Swrangsar Basumatary

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1 Introduction

The concept of interference temperature was introduced by the FCC as a new metric for quantifying and managing interference. Using this model, cognitive radios (CRs) operating in licensed frequency bands would be able to measure their current interference environment and adjust their transmission characteristics so as not to raise the interference temperature over a regulatory limit.

As of now it is hard to predict if interference temperature is going to be practical because no one has been able to come up with a solution that is agreed upon by everyone. The research on interference temperature has mostly been abandoned but there is some hope of a comeback.

The purpose of the metric is to remove the subjective context that has been the basis of interference analysis up to now[3]. The development of an interference metric is critical to making efficient use of the spectrum.

The current regulatory definition of interference is not precise. The process used to estimate interference by the regulatory bodies takes a lot of time. It is specific to particular deployment geometry and cannot be automated. They usually do a very detailed analysis of the surroundings.

With the progress in technology, new devices have come up that are capable of waveform modification and changing frequency of operation very quickly. These new devices have the potential to make more efficient use of the spectrum. But without a clear definition of interference, we would not be able to utilize these new technologies completely. Performing detailed analysis for every combination of waveforms and receivers is not possible.

Relying on worst case analysis leaves much of the spectrum unused. Thus there is a need for some real-time interference metric so that we can make

efficient use of the spectral resources that are available.

2 Driving forces for interference metric

Spectrum management revolves around interference management[3]. The design and operation of all RF equipments is based on reducing electromagnetic interference. Interference limits the usable range of wireless signals.

Changing landscape

Technological advances have led to increased diversity of spectrum-based consumer applications. This has resulted in an increased demand for spectral resources. So, static interference management cannot meet the demands anymore.

Increased density and mobility

With more people owning cellular devices and with people owning more cellular devices, density of mobile devices has increased. Increased density has made interference management more difficult.

3 Interference Temperature model

Interference temperature, T_I is defined[3, 1] as

$$T_I(f_c, B) = \frac{P_I(f_c, B)}{kB}$$

where $P_I(f_c, B)$ is the average interference power centered at frequency, f_c , and covering bandwidth B . Here, k is the Boltzmann's constant $k = 1.38 * 10^{-23}$ joules per degree Kelvin. In SI system, the unit of interference temperature turns out to be *degrees Kelvin*.

The Interference Temperature model[1] is not clear cut. There is ambiguity regarding whether to represent T_I in terms of the transmitter's parameters (f_c and B) or the ones from the i 'th receiver (f_i and B_i). This results into two models[1]:

1. Ideal model
2. Generalized model

3.1 Ideal model

In the ideal model we try to guarantee that

$$T_I(f_i, B_i) + \frac{M_i P}{k B_i} \leq T_L(f_i) \quad \forall \quad 1 \leq i \leq n \quad (1)$$

where,

$T_L(f_i) \longrightarrow$ the interference temperature limit of the i 'th receiver
at center frequency f_i

$T_I(f_i, B_i) \longrightarrow$ the interference temperature of the i 'th receiver
with center frequency f_i and bandwidth B_i

$P \longrightarrow$ power of the unlicensed transmitter

$M_i \longrightarrow$ a constant between 0 and 1 representing the attenuation
of the transmitted power P at the i 'th receiver.

In words, equation (1) means we try to keep the sum of the interference at the receiver (due to all other factors except the unlicensed transmitter) and the interference caused by the unlicensed transmitter's signal below the temperature limit T_L . Usually, a unified factor M is used instead of M_i 's because we do not know the distance of every receiver. The factor M is fixed by the regulators.

Challenges in implementing the ideal model

- It's hard to differentiate between licensed and unlicensed signals unless we already know the coexisting transmitters in the surrounding environment.
- It's difficult to measure the interference temperature T_I in the presence of a licensed signal. We can determine the noise floor only when there is no signal power. For that we have to know when the signal power is going to be zero if it does. If the signal periodically goes to zero then we can measure the T_I during the interval when it goes to zero. The other way is to take an average of the noise floor just outside the licensed signal band but we need to have information about the licensed signal's properties, like center frequency and bandwidth, to do that.

3.2 Generalized model

The generalized model assumes that we have no a priori knowledge about the signal environment. Thus the constraint in this model has to be written in terms of the unlicensed transmitter's parameters.

$$T_I(f_c, B) + \frac{MP}{kB} \leq T_L(f_c) \quad (2)$$

where,

$T_L(f_c) \longrightarrow$ the interference temperature limit of the unlicensed transmitter

$T_I(f_c, B) \longrightarrow$ the interference temperature of the unlicensed transmitter

$P \longrightarrow$ power of the unlicensed transmitter

$M \longrightarrow$ a constant between 0 and 1 representing the factor by which the transmitted power gets attenuated

If we constrain the transmitted power P to be less than the one in the ideal model, from the equations (1) and (2) we get the following constraint,

$$B(T_L - T_I(f_c, B)) \leq B_i(T_L - T_I(f_i, B_i)) \quad \forall \quad 1 \leq i \leq n \quad (3)$$

Suppose, every receiver receives a power P_i and suppose the noise floor for all the receivers is the thermal noise temperature T_N , then we can rewrite the above equation (3) as,

$$\begin{aligned} BT_L - BT_I(f_c, B) &\leq B_i T_L - B_i T_I(f_i, B_i) \\ BT_L - B_i T_L &\leq BT_I(f_c, B) - B_i T_I(f_i, B_i) \\ T_L(B - B_i) &\leq BT_I(f_c, B) - B_i T_I(f_i, B_i) \\ T_L(B - B_i) + B_i T_I(f_i, B_i) &\leq BT_I(f_c, B) \\ kBT_L(B - B_i) + kBB_i T_I(f_i, B_i) &\leq kB^2 T_I(f_c, B) \\ kBT_L(f_c)(B - B_i) + kBT_N \sum_{j=1}^N B_j &\leq \sum_{j=1}^N B_j P_j \quad \forall \quad 1 \leq i \leq n \end{aligned} \quad (4)$$

Though $T_I(f_c, B)$ can be measured easily, inclusion of all signals in its measurement gives rise to a more complex interference environment.

4 Properties of the Interference Temperature

The problem with Interference Temperature model is that it is too simple. It tries to quantify interference assuming that interference behaves like noise. But interference doesn't actually behave like noise because interference includes all other signals other than the one from the licensed transmitter. We are defining interference to be that way. So, *Interference Temperature* can't really model interference well enough.

The difference between the regulated temperature limit and the measured temperature is the amount of temperature that we are allowed to generate at the receiver. This temperature difference determines how much power our unlicensed transmitter can transmit to comply with the regulations.

The interference temperature T_I can be written in terms of bandwidth B as

$$\begin{aligned} T_I(f_c, B) &= \frac{1}{kB} P_I(f_c, B) \\ &= \frac{1}{kB} \left(\frac{1}{B} \int_{f_c-B/2}^{f_c+B/2} S(f) df \right) \\ &= \frac{1}{B^2 k} \int_{f_c-B/2}^{f_c+B/2} S(f) df \end{aligned} \quad (5)$$

The transmit power P satisfying the general model can be found as

$$\begin{aligned} T_I(f_c, B) + \frac{MP}{kB} &\leq T_L(f_c) \\ \frac{MP}{kB} &\leq T_L(f_c) - T_I(f_c, B) \\ P &\leq \frac{kB}{M} T_L(f_c) - \frac{kB}{M} T_I(f_c, B) \\ P &\leq \frac{kB}{M} T_L(f_c) - \frac{1}{BM} \int_{f_c-B/2}^{f_c+B/2} S(f) df \end{aligned} \quad (6)$$

We cannot find an explicit expression for B satisfying the constraint. But if we consider the special case of $S(f) = c$ where c is some constant, then

the inequality (6) can be rewritten as,

$$\begin{aligned} P &\leq \frac{kB}{M} T_L(f_c) - \frac{c}{M} \\ B &\geq \frac{PM + c}{kT_L(f_c)} \end{aligned} \quad (7)$$

Thus now we have an expression for a minimum transmission bandwidth. Also the maximum transmit power turns out to be $kBT_L(f_c)$. As we increase B the upper bound for transmit power increases. This implies that as we use more bandwidth we can produce more interference.

5 Spectral analysis of radio signals

The stimuli generated by radio emitters are nonstationary signals that depend on both time and space. But statistical analysis of nonstationary signals is complicated and cannot be implemented efficiently in practice. So, usually the incoming RF stimuli are sectioned into a continuous sequence of successive bursts, with each burst being short enough to justify pseudo-stationarity and yet long enough to produce an accurate spectral estimate[2].

A nonparametric spectral estimation method is required for this purpose and *multitaper spectral estimation*[6, 4] turns out to be the best choice. Multitaper spectral estimation has a nice property of allowing both reduced bias and reduced variance at the same time.

Multitaper spectral estimation

This method uses *multiple orthonormal tapers*[6] to give different estimates of the spectrum. These estimates are then averaged to give the final estimate. This final estimate turns out to have both reduced bias and reduced variance[6, 4].

For a time series $\{x_t\}_{t=1}^N$, the multitaper spectral estimation procedure determines two things.

1. K orthonormal slepian tapers denoted by $\{w_t^{(k)}\}_{t=1}^N$
2. The respective eigenspectra for each of the K tapers

$$Y_k(f) = \sum_{t=1}^N w_t^{(k)} x(t) e^{-j2\pi ft}, \quad k = 0, 1, \dots, K-1 \quad (8)$$

Most of the energy of the eigenspectra remain confined inside the *resolution bandwidth*, denoted by $2W$ i.e. between $f - W$ and $f + W$. The *time-bandwidth product*

$$p = 2NW \quad (9)$$

defines the *degrees of freedom* available for controlling the variance of the spectral estimator. A spectral estimate based on the first few eigenspectra with the least sidelobe leakage, is given by

$$\hat{S}(f) = \frac{\sum_{k=0}^{K-1} \lambda_k(f) |Y_k(f)|^2}{\sum_{k=0}^{K-1} \lambda_k(f)} \quad (10)$$

where λ_k is the eigenvalue for the k th eigenspectrum. We can improve the spectral estimate by giving low weightage to the eigenspectra with more sidelobe leakage i.e. by choosing smaller λ_k 's for the higher k 's. This is called “adaptive weighting”. The higher-order eigenspectra contain higher sidelobe leakage so we can weight them less.

6 Estimation of interference temperature

To get a reliable spectral estimate of the interference temperature, we do two things:

1. Use the multitaper method to estimate the interference at the receiver and
2. Use a lot of sensors to get a good approximation of the space dependent radio environment at the receiver. In case of mobile phones etc, we might have to stick to just one sensor.

Suppose we have M sensors then using K different slepian tapers for each sensor we may form the M -by- K matrix $\mathbf{A}(f)$, where the $\{w_m\}_{m=1}^M$ represent the weights attributed to the sensors.

$$\mathbf{A}(f) = \begin{bmatrix} w_1 Y_1^{(1)}(f) & w_1 Y_2^{(1)}(f) & \dots & w_1 Y_K^{(1)}(f) \\ w_2 Y_1^{(2)}(f) & w_2 Y_2^{(2)}(f) & \dots & w_2 Y_K^{(2)}(f) \\ \vdots & \vdots & & \vdots \\ w_M Y_1^{(M)}(f) & w_M Y_2^{(M)}(f) & \dots & w_M Y_K^{(M)}(f) \end{bmatrix} \quad (11)$$

Each element in $\mathbf{A}(f)$ has contributions from both the additive internal noise and the incoming signal. We may get rid of the noise by using *singular value decomposition*[5] to decompose $\mathbf{A}(f)$.

$$\mathbf{A}(f) = \sum_{k=0}^{K-1} \sigma_k(f) \mathbf{u}_k(f) \mathbf{v}_k^\dagger(f) \quad (12)$$

We then take the left singular vectors $\mathbf{u}_k(f)$ and the right singular vectors $\mathbf{v}_k(f)$ corresponding to the first few largest eigenvalues $|\sigma_k(f)|^2$. The first few largest eigenvalues $|\sigma_k(f)|^2$ provide a pretty good estimate of the interference temperature.

References

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