# MATH 119: Midterm 2

Name: key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

| Problem | Score | Points |
|---|---|---|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 10 |
| | | **50** |

conceptual understanding

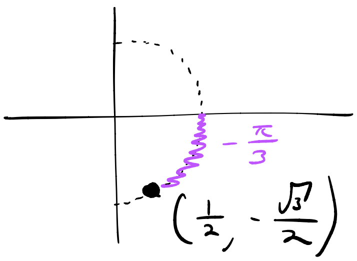
execution

common pitfalls

1. Evaluate the following:

   (a) $\tan^{-1}\left(-\sqrt{3}\right) = \boxed{-\frac{\pi}{3}}$

Range of $\tan^{-1}(x)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$




   (b) $\cos^2 15° - \sin^2 15° = \cos\left(2 \cdot 15°\right) = \cos\left(30°\right)$ $\frac{\pi}{6}$ rad

   $= \boxed{\frac{\sqrt{3}}{2}}$

   (c) $2 \sin 22.5° \cos 22.5° = \sin\left(2 \cdot 22.5°\right)$

   $= \sin\left(45°\right)$ $\frac{\pi}{4}$ rad

   $= \boxed{\frac{\sqrt{2}}{2}}$

(d) $\cos 18^\circ \cos 27^\circ - \sin 18^\circ \sin 27^\circ = \cos(18^\circ + 27^\circ)$

$$= \cos(45^\circ)$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

$$\frac{100\pi}{3} = \frac{99\pi + \pi}{3} = 33\pi + \frac{\pi}{3}$$

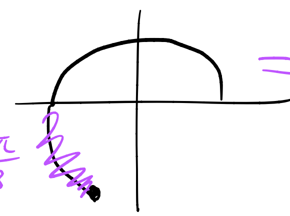
$$= 3\cancel{2}\pi + \pi + \frac{\pi}{3}$$

(e) $\sin^{-1}\left(\sin\left(\frac{100\pi}{3}\right)\right)$

$$= \frac{4\pi}{3}$$

$$= \sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$$

$t = \frac{\pi}{3}$



$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{-\frac{\pi}{3}}$$

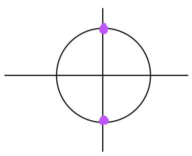
range of $\sin^{-1}(x)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

2. Solve the equation for $\theta$. Check your work if necessary.

(a) $\sin\theta\cos\theta + \cos\theta = 0$

$$\cos\theta \cdot (\sin\theta + 1) = 0$$

$\cos\theta = 0$



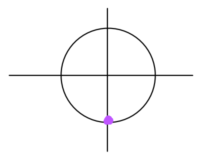
$\theta = \frac{\pi}{2} + 2k\pi$

$\theta = \frac{3\pi}{2} + 2k\pi$ same

$k \in \mathbb{Z}$

$\sin\theta + 1 = 0$

$\sin\theta = -1$



$\theta = \frac{3\pi}{2} + 2k\pi$

$$\boxed{\begin{gathered}\theta = \frac{\pi}{2} + 2k\pi \\ \theta = \frac{3\pi}{2} + 2k\pi \\ k \in \mathbb{Z}\end{gathered}}$$

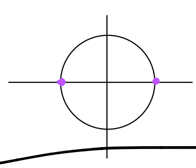
(b) $4\sin^3\theta = \sin\theta$

Do not divide by $\sin\theta$. You lose equation $\sin\theta = 0$

$$4\sin^3\theta - \sin\theta = 0$$

$$\sin\theta\,(4\sin^2\theta - 1) = 0$$

$\sin\theta = 0$



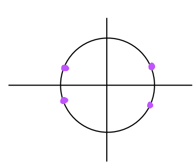
$$\boxed{\begin{gathered}\theta = 0 + 2k\pi \\ \theta = \pi + 2k\pi \\ k \in \mathbb{Z}\end{gathered}}$$

$4\sin^2\theta - 1 = 0$

$\sin^2\theta = \frac{1}{4}$

$\sqrt{\sin^2\theta} = \pm\sqrt{\frac{1}{4}}$

$\sin\theta = \pm\frac{1}{2}$



$$\boxed{\begin{gathered}\theta = \frac{\pi}{6} + 2k\pi \\ \theta = \frac{5\pi}{6} + 2k\pi \\ \theta = \frac{7\pi}{6} + 2k\pi \\ \theta = \frac{11\pi}{6} + 2k\pi \\ k \in \mathbb{Z}\end{gathered}}$$

3. Prove these identities algebraically:

(a) $\sin(\theta) \cdot \sec(\theta) = \tan(\theta)$

$$\begin{aligned} LHS &= \sin\theta \sec\theta \\ &= \sin\theta \frac{1}{\cos\theta} \\ &= \tan\theta \\ &= RHS \quad \blacksquare \end{aligned}$$

(b) $\dfrac{\cos x}{\sec x \sin x} = \csc x - \sin x$

$$\begin{aligned} LHS &= \frac{\cos x}{\frac{1}{\cos x} \cdot \sin x} \\ &= \frac{\cos x}{\frac{\sin x}{\cos x}} \\ &= \cos x \cdot \frac{\cos x}{\sin x} \\ &= \frac{\cos^2 x}{\sin x} \\ &= \frac{1 - \sin^2 x}{\sin x} \end{aligned}$$

two terms! RHS is two terms. On right track.

$$\begin{aligned} &= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} \\ &= \csc x - \sin x \\ &= RHS \quad \blacksquare \end{aligned}$$

(c) $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$$LHS = \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} = \csc\theta\sec\theta = RHS \quad \blacksquare$$

(d) $\dfrac{\cos x}{1-\sin x} = \dfrac{\sin x - \csc x}{\cos x - \cot x}$

$$RHS = \frac{\sin x - \frac{1}{\sin x}}{\cos x - \frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\sin x}{\sin x}\sin x - \frac{1}{\sin x}}{\frac{\sin x}{\sin x}\cos x - \frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\sin^2 x - 1}{\sin x}}{\frac{\sin x\cos x - \cos x}{\sin x}}$$

$A^2 - B^2$

$$= \frac{\sin^2 x - 1}{\cancel{\sin x}}\cdot\frac{\cancel{\sin x}}{\sin x\cos x - \cos x}$$

$$= \frac{\cancel{(\sin x - 1)}(\sin x + 1)}{\cos x\,\cancel{(\sin x - 1)}}$$

$$= \frac{\sin x + 1}{\cos x}$$

need $1-\sin x$

getting ready for $A^2 - B^2$

$$= \frac{1 + \sin x}{\cos x}\cdot\frac{1-\sin x}{1-\sin x}$$

$$= \frac{1-\sin^2 x}{\cos x(1-\sin x)}$$

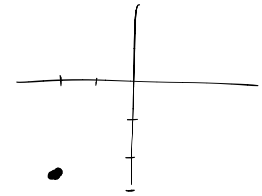
$\cos^2 x = 1 - \sin^2 x$

$$= \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}\,(1-\sin x)}$$

$$= \frac{\cos x}{1-\sin x}$$

$$= LHS \quad \blacksquare$$

Graph the point!



4. Answer the following:

(a) Convert $(-2, -2\sqrt{3})$ into polar coordinates.

$-2$ and $\sqrt{3}$ are factors

exponent law.

For $r$: $r^2 = x^2 + y^2 = (-2)^2 + (-2\sqrt{3})^2$

$= 4 + (-2)^2(\sqrt{3})^2$

$= 4 + 4 \cdot 3$

$= 16$

$r = \pm 4$

For $\theta$: $\tan\theta = \frac{y}{x} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

So $\theta = \frac{\pi}{3}$, $\theta = \frac{4\pi}{3}$

From graph:

$\left(4, \frac{4\pi}{3}\right)$ or $\left(-4, \frac{\pi}{3}\right)$

(b) Convert $r = \dfrac{1}{1 - \sin\theta}$ into rectangular form.

$(1-\sin\theta) \cdot r = \frac{1}{1-\sin\theta} \cdot (1-\sin\theta)$

$r - r\sin\theta = 1$

$r - y = 1$

$r = 1 + y$

$r^2 = (1+y)^2$

$x^2 + y^2 = (1+y)^2$

(c) Convert $r^3 = r$ into rectangular form.

$r \cdot r^3 = r \cdot r$

$r^4 = r^2$

$(r^2)^2 = r^2$

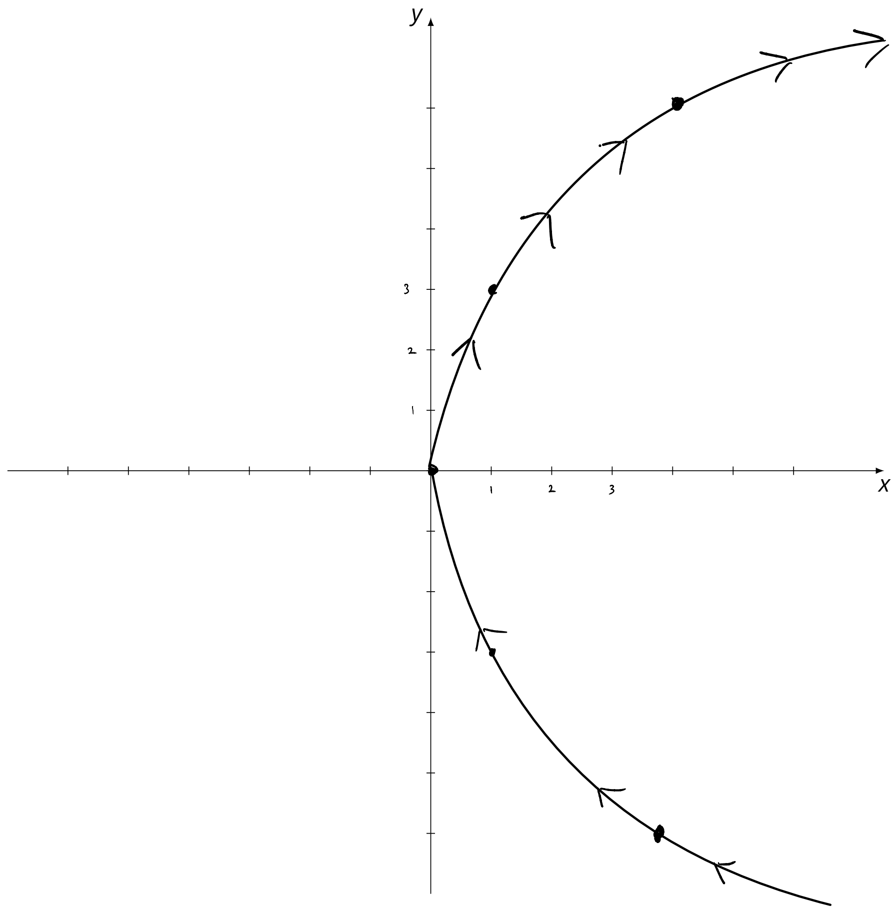
$(x^2+y^2)^2 = x^2 + y^2$ or $x^2 + y^2 = 1$

5. Here is a pair of parametric equations

$$x = t^2 \qquad y = 3t$$

(a) Sketch the curve represented by the equations.

| $t$ | $x$ | $y$ |
|---|---|---|
| -2 | 4 | -6 |
| -1 | 1 | -3 |
| 0 | 0 | 0 |
| 1 | 1 | 3 |
| 2 | 4 | 6 |




(b) Find a rectangular coordinate equation for the curve by eliminating the parameter.

$y = 3t$ so $t = \frac{y}{3}$.

Substituting: $x = \left(\frac{y}{3}\right)^2$

$$\boxed{x = \frac{y^2}{9}}$$

*next page...*

Some people did

$x = t^2$ so $\sqrt{x} = t$

Wrong because forgot $\pm$.

(c) This equation is a parabola. Find the focus and directrix of this equation.

$x = \frac{y^2}{9}$ so $y^2 = 9x$

$y^2 = 4 \cdot \frac{1}{4} \cdot 9x$

$y^2 = \underbrace{4 \cdot \frac{9}{4} \cdot x}_{4\ p\ x}$

$$\boxed{\begin{array}{l} \text{Focus } (p, 0) = \left(\frac{9}{4}, 0\right) \\ \text{Directrix } x = -p = -\frac{9}{4} \end{array}}$$

# Chapter 7 Trigonometric Formulas

### Addition and Subtraction Formulas

$$\sin(s+t) = \sin s \cos t + \cos s \sin t$$
$$\sin(s-t) = \sin s \cos t - \cos s \sin t$$
$$\cos(s+t) = \cos s \cos t - \sin s \sin t$$
$$\cos(s-t) = \cos s \cos t + \sin s \sin t$$
$$\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$
$$\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

### Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$
$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

### Half-Angle Formulas

The $+$ or $-$ sign depends on the quadrant $\frac{u}{2}$ is in.

$$\sin \frac{u}{2} = \pm\sqrt{\frac{1 - \cos u}{2}}$$
$$\cos \frac{u}{2} = \pm\sqrt{\frac{1 + \cos u}{2}}$$
$$\begin{aligned} \tan \frac{u}{2} &= \frac{1 - \cos u}{\sin u} \\ &= \frac{\sin u}{1 + \cos u} \end{aligned}$$

### Product-to-Sum Formulas

$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$
$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$
$$\cos u \cos v = \frac{1}{2}\left[\cos(u+v) + \cos(u-v)\right]$$
$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$

### Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$
$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$