MATH 161: Midterm 2

Name: key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Draw **one single graph** of a function which satisfies the following:

   (a) $f(-1) = 1$

   (b) $f(1) = -1$

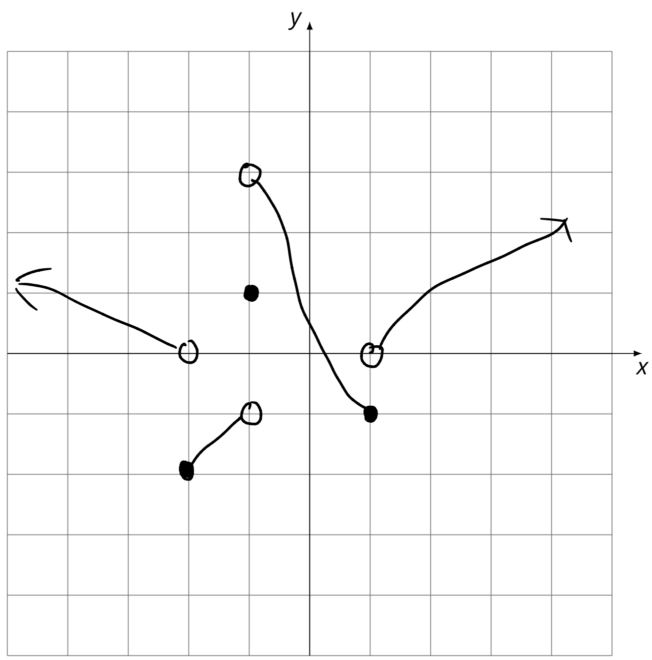
   (c) Continuous from the left at $x = 1$, but discontinuous at $x = 1$.

   (d) $\lim_{x \to -1^-} f(x) = -1$

   (e) $\lim_{x \to -1^+} f(x) = 3$

   (f) Jump discontinuity at $x = -2$ but continuous from the right at $x = -2$

answers may vary.



2. Answer the following:

   (a) Given a function $f(x)$, if

$$\boxed{\lim_{x \to a}} f(x) = \frac{0}{0}$$

   what global factor do you need to manifest in the numerator and denominator and why?

$$(x-a)$$

$\frac{0}{0}$ if try lim laws.

   (b) Find

$$\lim_{x \to 0} \left[\frac{1}{x\sqrt{1+x}} - \frac{1}{x} \right]$$

needs factor of $\sqrt{1+x}$

   Hint: Subtract to get one fraction first.

$$\lim_{x\to 0}\left[\frac{1}{x\sqrt{1+x}} - \frac{1}{x}\cdot\frac{\sqrt{1+x}}{\sqrt{1+x}}\right] = \lim_{x\to 0}\left[\frac{1}{x\sqrt{1+x}} - \frac{\sqrt{1+x}}{x\sqrt{1+x}}\right]$$

rationalize to create glob. factor of x.

$$= \lim_{x\to 0}\frac{1-\sqrt{1+x}}{x\sqrt{1+x}}\cdot\frac{1+\sqrt{1+x}}{1+\sqrt{1+x}}$$

$$= \lim_{x\to 0}\frac{1^2-(\sqrt{1+x})^2}{x\sqrt{1+x}\,(1+\sqrt{1+x})}$$

$$= \lim_{x\to 0}\frac{1-(1+x)}{x\sqrt{1+x}\,(1+\sqrt{1+x})}$$

$$= \lim_{x\to 0}\frac{-\cancel{x}}{\cancel{x}\sqrt{1+x}\,(1+\sqrt{1+x})}$$

$$= \lim_{x\to 0}\frac{-1}{\sqrt{1+x}\,(1+\sqrt{1+x})}$$

$$= \frac{-1}{\sqrt{1+0}\,(1+\sqrt{1+0})}$$

continuity

$$= \boxed{-\frac{1}{2}}$$

(c) Find

$\frac{0}{0}$ if try lim laws.

compound fraction.

$$\lim_{h\to 0}\frac{\frac{1}{(x+h+1)}-\frac{1}{(x+1)}}{h}$$

$$\lim_{h\to 0}\frac{\frac{(x+1)}{(x+1)}\cdot\frac{1}{(x+h+1)}-\frac{1}{(x+1)}\cdot\frac{(x+h+1)}{(x+h+1)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x+1-(x+h+1)}{(x+1)(x+h+1)}}{h}$$

frac law 1 then 3

$$=\lim_{h\to 0}\frac{\frac{x+1-x-h-1}{(x+1)(x+h+1)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{-h}{(x+1)(x+h+1)}}{h}$$

$$=\lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}(x+1)(x+h+1)}$$

:) frac law 2 then 1

$$=\lim_{h\to 0}\frac{-1}{(x+1)(x+h+1)}$$

$$=\frac{-1}{(x+1)(x+0+1)}$$

continuity

$$=\boxed{-\frac{1}{(x+1)^2}}$$

3. Answer the following:

(a) Use **the mathematical definition of continuity** to determine if the function

$$f(x) = \begin{cases} 4 - 8x & x < 1 \\ -4 & x = 1 \\ x^2 - 5 & x > 1 \end{cases}$$

is continuous at the number $x = 1$.

① $f(1) = -4$ ✓

② $\lim_{x \to 1} f(x) = -4$ because

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2 - 5) = 1^2 - 5 = -4$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (4 - 8x) = 4 - 8 \cdot 1 = -4$

③ Since $\lim_{x \to 1} f(x) = -4 = f(1)$

by the definition of continuity $f(x)$ is continuous at $x = 1$.

~~(b) What type of discontinuity is this called?~~

4. Suppose $f(x) = 3x^2 - x$.

(a) What does the expression $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ represent?

The derivative of $f(x)$, or the slope of the tangent line.

(b) Find

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for the given function $f(x)$. You must use this limit definition to receive credit.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - x - h - 3x^2 + x}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} 6x + 3h - 1$$

$$= 6x + 0 - 1$$

$$= \boxed{6x - 1}$$

(c) Find the equation of the tangent line of $f(x)$ at the point $(1, 2)$.

$$f'(1) = 6 \cdot 1 - 1 = 5$$

$$y - 2 = 5 \cdot (x - 1)$$

$$y = 5x - 5 + 2$$

$$\boxed{y = 5x - 3}$$

5. Find the following derivatives. You are allowed to use the Differentiation Rules.

(a) $f(x) = \sqrt{e}$ ← this is a number.

$$\boxed{f'(x) = 0}$$

(b) $g(x) = 4x^3 - 2e^x + 3\sin x$

$$g'(x) = 4 \cdot 3x^2 - 2e^x + 3\cos(x)$$

$$= \boxed{12x^2 - 2e^x + 3\cos(x)}$$

Simplify

(c) $g(x) = (\sqrt{x} - 4)x^2$

$$= x^{\frac{1}{2}} \cdot x^2 - 4x^2$$

$$= x^{\frac{5}{2}} - 4x^2$$ ← use power rule.

$$g'(x) = \frac{5}{2}x^{\frac{5}{2}-1} - 4 \cdot 2x^{2-1}$$

$$= \frac{5}{2}x^{\frac{3}{2}} - 8x$$

$$= \boxed{\frac{5}{2}\sqrt{x^3} - 8x}$$

Simplify

(d) $f(\theta) = \dfrac{\theta^3 + \theta^2 - \theta}{\theta}$

$$= \frac{\theta^3}{\theta} + \frac{\theta^2}{\theta} - \frac{\theta}{\theta}$$ frac law 1

$$= \theta^2 + \theta - 1$$ frac law 5

$$f'(\theta) = \frac{d}{d\theta}[\theta^2] + \frac{d}{d\theta}[\theta] - \frac{d}{d\theta}[1]$$

$$= \boxed{2\theta + 1}$$