

MATH 119: Some Practice Final Problems

Here are problems that cover the last two weeks of our class.

The final is cumulative; you should look at Practice Midterm 1+2 and Midterm 1+2 as well.

1. Answer the following:

(a) Find a hyperbola with foci $(\pm 5, 0)$ and vertices $(\pm 3, 0)$, if possible.

opens left/right

foci: $(\pm c, 0) \rightarrow c = 5$ so $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

vertices: $(\pm a, 0) \rightarrow a = 3$

For b : $c^2 = a^2 + b^2$

$5^2 = 3^2 + b^2$

$b^2 = 25 - 9 = 16$ $b = 4$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

(b) Find an ellipse with eccentricity 0.5 and vertices $(\pm 4, 0)$, if possible.

wide ellipse

Vertices: $(\pm a, 0) \rightarrow a = 4$ For b : $c^2 = a^2 - b^2$

eccentricity: $\frac{c}{a} = \frac{1}{2}$

$c = \frac{a}{2} = \frac{4}{2} = 2$

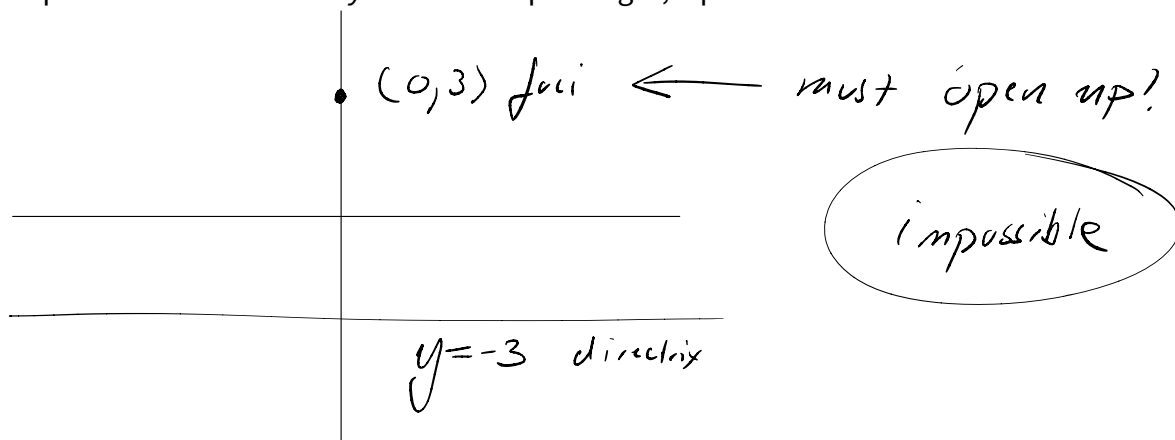
$b^2 = a^2 - c^2 = 4^2 - 2^2 = 12$

$b = \pm \sqrt{12}$

So $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow$

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

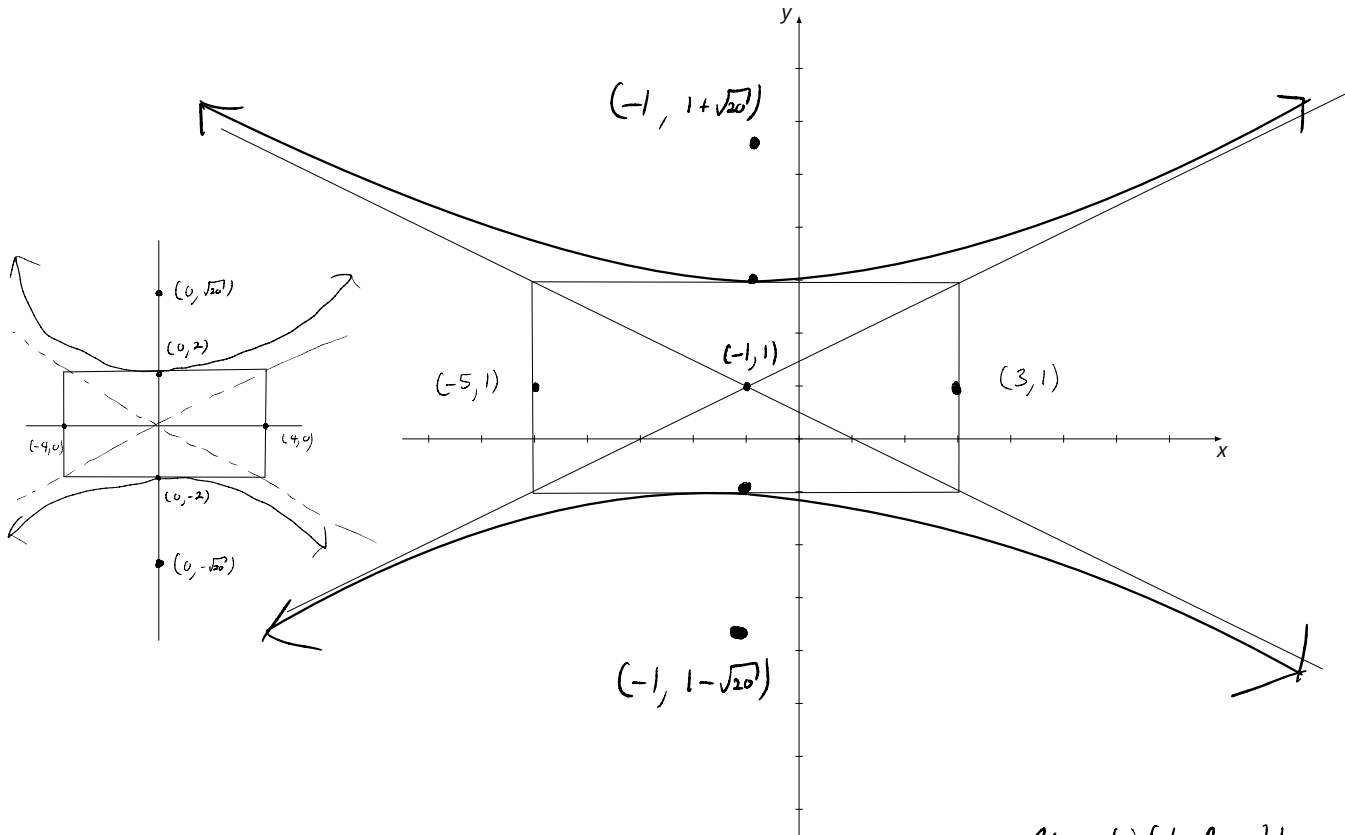
(c) Find a parabola with directrix $y = -3$ and opens right, if possible.



2. Consider the hyperbola

$$4(y-1)^2 - (x+1)^2 = 16$$

Find the foci, vertices, and sketch an accurate graph.



$$\frac{4(y-1)^2}{16} - \frac{(x+1)^2}{16} = \frac{16}{16}$$

$$\frac{4(y-1)^2}{16} - \frac{(x+1)^2}{16} = 1$$

$$\frac{(y-1)^2}{4} - \frac{(x+1)^2}{16} = 1$$

shift up 1, left 1

unshifted #'s:

$$a^2 = 4 \rightarrow a = 2$$

$$b^2 = 16 \rightarrow b = 4$$

$$c^2 = a^2 + b^2 = 2^2 + 4^2 = 20$$

$$c = \sqrt{20}$$

foci are $(0, \pm\sqrt{20})$

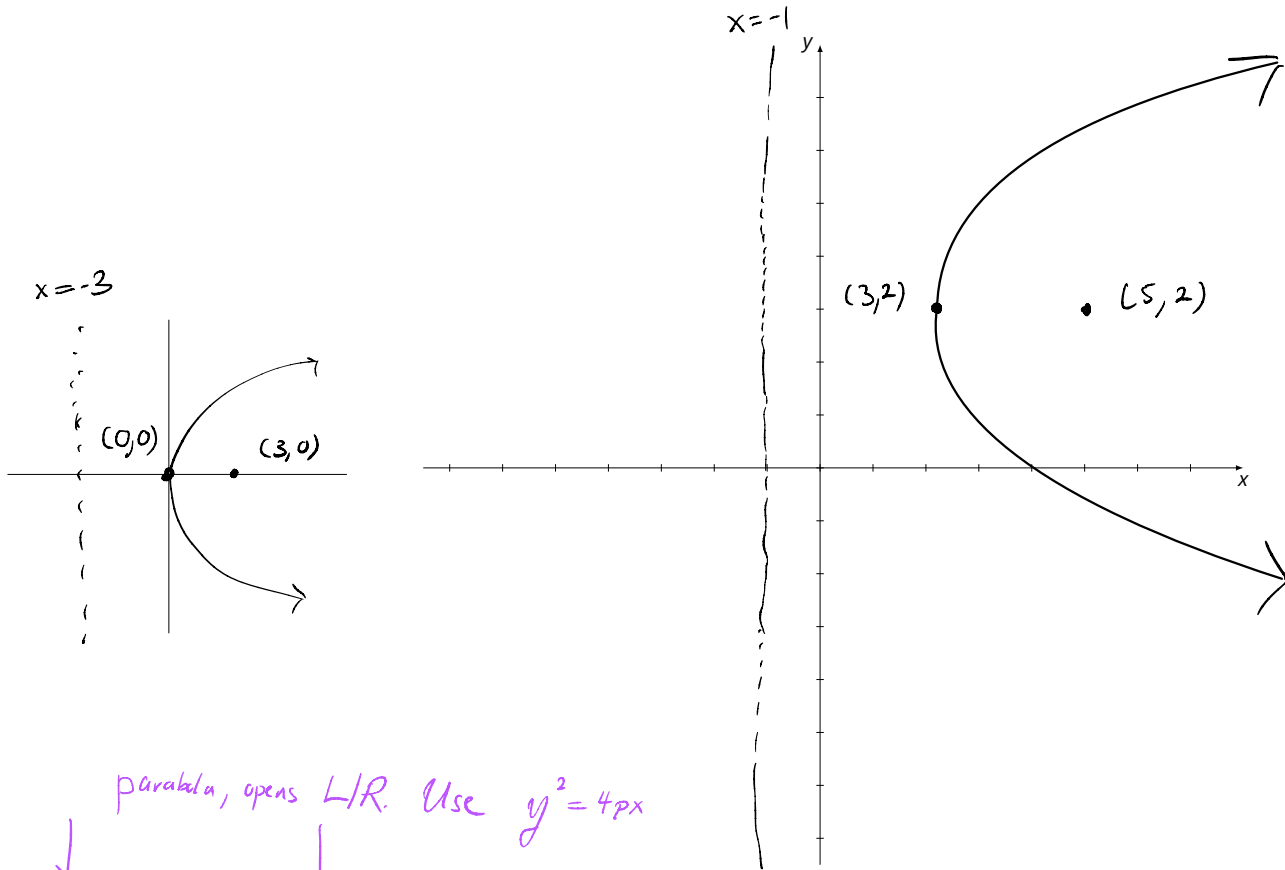
shifted foci are

$$(0-1, 1 \pm \sqrt{20}) = (-1, 1 \pm \sqrt{20})$$

3. Consider the parabola

$$y^2 - 6y - 12x + 33 = 0$$

Find the foci and sketch an accurate graph.



parabola, opens L/R. Use $y^2 = 4px$

$$y^2 - 6y - 12x + 33 = 0$$

$$b = -6, \left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$$

$$(y^2 - 6y + 9) - 9 = 12x - 33$$

$$(y - 3)^2 = 12x - 33 + 9$$

$$(y - 3)^2 = 12x - 24$$

$$(y - 3)^2 = 12(x - 2)$$

$$(y - 3)^2 = 4 \cdot 3 \cdot (x - 2)$$

shift up 3

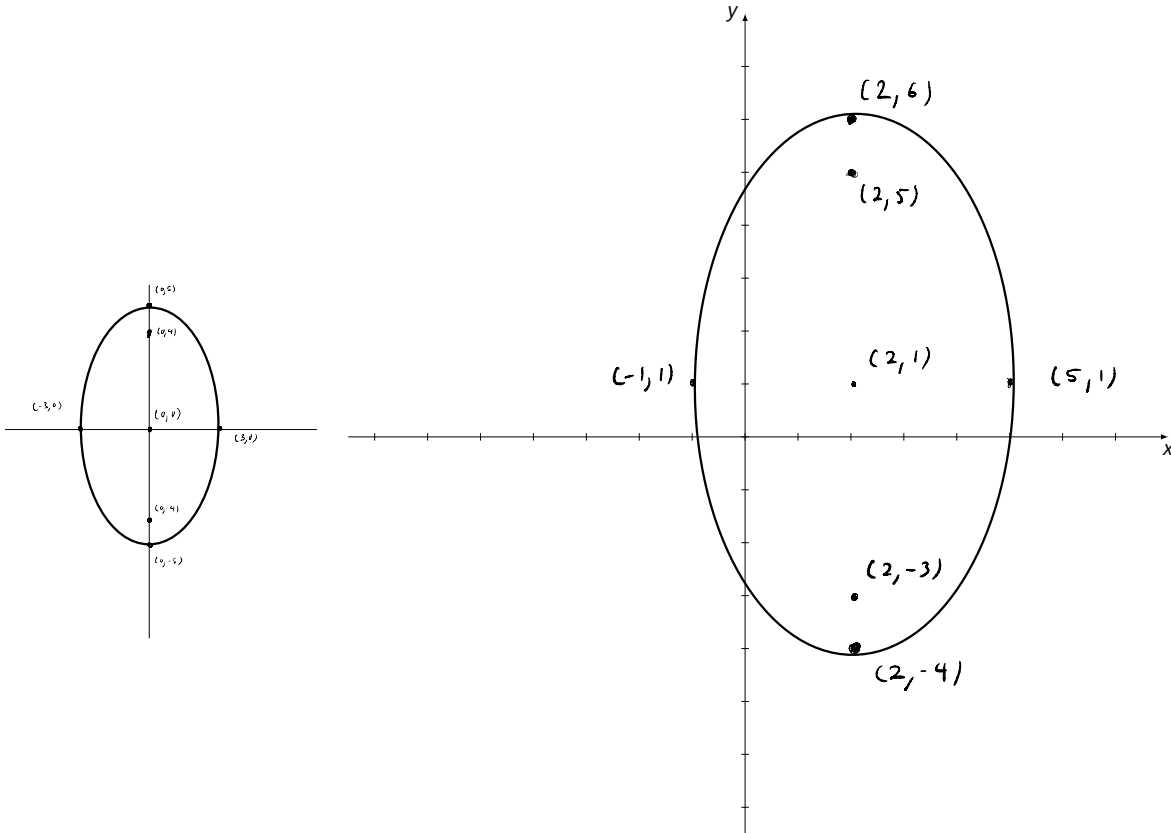
shift right 2

$$p = 3, \text{ foci: } (p, 0) = (3, 0)$$

4. Consider the ellipse

$$25(x-1)^2 + 9(y-2)^2 = 25 \cdot 9$$

Find the foci, vertices and sketch an accurate graph.



$$\frac{25(x-1)^2}{25 \cdot 9} + \frac{9(y-2)^2}{25 \cdot 9} = \frac{25 \cdot 9}{25 \cdot 9}$$

$$\frac{\cancel{25}(x-1)^2}{\cancel{25} \cdot 9} + \frac{\cancel{9}(y-2)^2}{\cancel{25} \cdot \cancel{9}} = 1$$

$$\frac{(x-1)^2}{9} + \frac{(y-2)^2}{25} = 1$$

Shift right 2

Shift up 1

unshifted info:

$$a^2 = 25 \rightarrow a = 5$$

$$b^2 = 9 \rightarrow b = 3$$

$$\text{vertices: } (0, \pm a) = (0, \pm 5)$$

$$\text{foci: } (0, \pm c) = (0, \pm 4)$$

$$c^2 = a^2 - b^2 = 25 - 9 = 16$$

$$c = 4$$