MATH 118: Midterm 2 Key

Name: ______________________________

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

(a) Given the function

$$F(x) = \sqrt{x^2 - 2}$$

find two functions f, g where $f \circ g = F$. You are not allowed to choose $f(x) = x$ or $g(x) = x$.

$\boxed{f(x) = \sqrt{x} \text{ and } g(x) = x^2 - 2}$

(b) Determine whether $x = 1$ is a solution to the equation

$$\frac{2}{x} - \frac{1}{2x - 1} = 1$$

using calculations.

A few of you solved the equation. To check solutions, just plug in $x = 1$ and see if LHS = RHS.

$$\text{LHS} = \frac{2}{1} - \frac{1}{2 \cdot 1 - 1} = 2 - \frac{1}{1} = 2 - 1 = 1 = \text{RHS}$$

$\boxed{x = 1 \text{ is a solution.}}$

(c) If $x = -2$ is a x-intercept of $P(x)$, what must be a factor of $P(x)$?

$\boxed{(x - (-2)) = (x + 2)}$

(d) True or False: The function $g(x) = \sqrt{2x + 2}$ is shifted to the right two units from $f(x) = \sqrt{x}$.

$\boxed{\text{False.}}$

We have $g(x) = \sqrt{2x + 2} = \sqrt{2(x + 1)}$.

The correct shift is 1 to the left.

2. Solve the following equations and inequalities:

   (a) $-2x - 3 \leq 5$

   Don't forget to flip the inequality when dividing by a negative.

$$\begin{aligned} -2x - 3 &\leq 5 \\ -2x &\leq 8 \\ x &\geq -4 \end{aligned}$$

$\boxed{x \geq -4}$

   (b) $\sqrt{8x-1} = 3$

   Root is isolated. Square both sides and treat like a linear equation.

$$\begin{aligned} \sqrt{8x-1} &= 3 \\ \left(\sqrt{8x-1}\right)^2 &= 3^2 \\ 8x - 1 &= 9 \\ 8x &= 10 \\ x &= \frac{10}{8} = \frac{5}{4} \end{aligned}$$

$\boxed{x = \frac{5}{4}}$

   (c) $\frac{1}{x} - \frac{1}{x-1} = 4$

   Multiply both sides by the LCD $x(x-1)$ to rescue x from denominator.

$$\begin{aligned} x(x-1) \cdot \left(\frac{1}{x} - \frac{1}{x-1}\right) &= 4 \cdot x(x-1) \\ \cancel{x}(x-1) \cdot \frac{1}{\cancel{x}} - x\cancel{(x-1)} \cdot \frac{1}{\cancel{x-1}} &= 4x^2 - 4x && \text{Distribute } x(x-1) \text{ and cancel} \\ x - 1 - x &= 4x^2 - 4x \\ 0 &= 4x^2 - 4x + 1 \\ 0 &= (2x-1)^2 && A^2 - 2AB + B^2 \text{ with } A = 2x, B = 1 \\ 0 &= 2x - 1 && \text{Take roots of both sides} \end{aligned}$$

$$\boxed{x = \frac{1}{2}}$$

3. Perform the given instruction.

(a) Determine the end behavior for the polynomial $P(x) = -x^3(2x-3)^2(x-2)^5$.

Find the leading **term**. A few of you picked $-x^3$ which is a **factor**, not a term.

The leading term is $-x^3 \cdot (2x)^2 \cdot x^5 = -x^3 \cdot 4x^2 \cdot x^5 = -4x^{10}$.

Leading coefficient is $-1 < 0$, with even degree. So the end behavior is

$$\boxed{y \to -\infty \text{ as } x \to \infty \text{ and } y \to -\infty \text{ as } x \to -\infty}$$

(b) Complete the square for the quadratic function $f(x) = 2x^2 - 8x - 3$.

We need $x^2 + bx$ where the coefficient of x^2 is 1. Currently, it is 2. So:

$$\begin{aligned}
2x^2 - 8x - 3 &= 2(x^2 \overset{b}{-4}x) - 3 \\
&= 2(x^2 - 4x + 4 - 4) - 3 && b = -4 \text{ so } \left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4 \\
&= 2\left[(x^2 - 4x + 4) - 4\right] - 3 \\
&= 2\left[(x-2)^2 - 4\right] - 3 && A^2 - 2AB + B^2 \text{ with } A = x, B = 2 \\
&= 2(x-2)^2 - 8 - 3 && \text{Dist. Law} \\
&= \boxed{2(x-2)^2 - 11}
\end{aligned}$$

(c) Suppose $g(x) = 1 + \sqrt{-2x-2}$. Write the order of transformations you would use to transform $f(x) = \sqrt{x}$ into $g(x)$.

Do not graph.

Put into $A + B \cdot f\big(C(x+D)\big)$ form by factoring out -2, getting $-2x - 2 = -2(x+1)$.

We have $g(x) = 1 + \sqrt{-2x-2} = \underset{3}{1} + \sqrt{\underset{1}{-}\underset{2}{2}(\underset{4}{x+1})}$ with transformations

1. Reflection around y-axis
2. Horizontal shrink by a factor of $\frac{1}{2}$
3. Vertical shift up 1 unit
4. Horizontal shift left 1 unit

(d) Find the inverse of the function $f(x) = \dfrac{x-1}{3x-2}$.

You may use the fact that $f(x)$ is one-to-one.

1 One-to-one. Inverse exists.

2 Write $y = \dfrac{x-1}{3x-2}$.

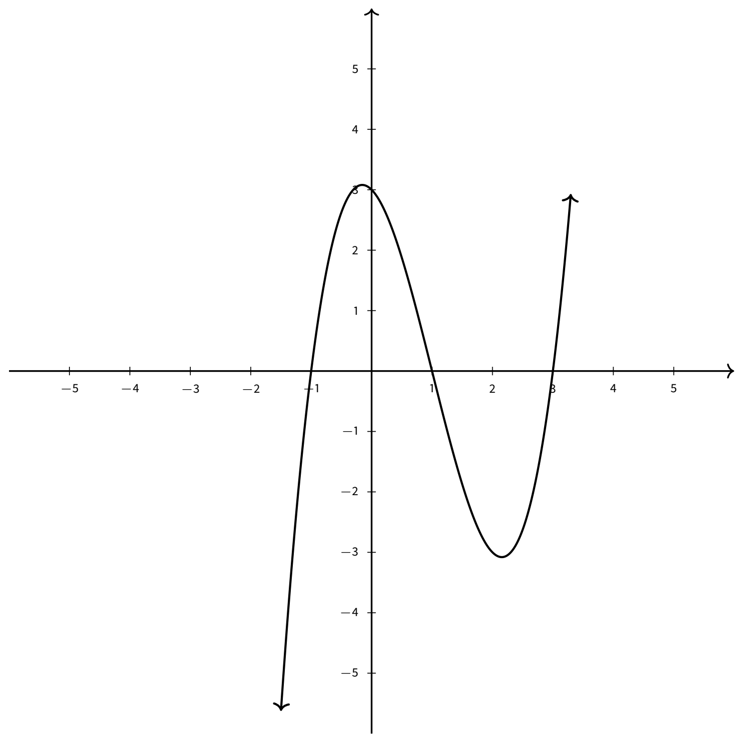
3 Solve for x. Follow 4 steps in Section 1.4.

$$
\begin{aligned}
y &= \frac{x-1}{3x-2} \\
(3x-2)\cdot y &= \frac{x-1}{\cancel{3x-2}}\cdot \cancel{(3x-2)} \\
3xy - 2y &= x - 1 && \text{Global terms} \\
3xy - x &= 2y - 1 && \text{Get } x \text{ on one side} \\
x(3y-1) &= 2y - 1 && \text{Factor } x \text{ out} \\
x &= \frac{2y-1}{3y-1} && \text{Isolated}
\end{aligned}
$$

4 Swap. $y = \dfrac{2x-1}{3x-1}$.

Result: $\boxed{f^{-1}(x) = \dfrac{2x-1}{3x-1}}$

4. Suppose $P(x) = x^3 - 3x^2 - x + 3$. Sketch a graph of $P(x)$ using the four step process.



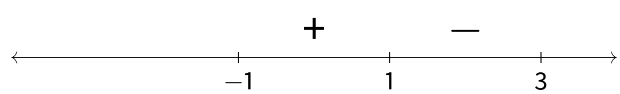
1 x-intercepts, Solve $P(x) = 0$.

Many of you got this problem incorrect. When solving "polynomial = 0", convert to factors and set each factor to 0. Since we have 4 terms and GCF doesn't work, we have to group.

$$x^3 - 3x^2 - x + 3 = 0$$
$$x^2(x-3) - (x-3) = 0$$
$$(x-3)(x^2-1) = 0$$
$$x - 3 = 0 \qquad x^2 - 1 = 0$$
$$\boxed{x = 3} \qquad x^2 = 1$$
$$\boxed{x = \pm\sqrt{1} = \pm 1}$$

So $x = 3, 1, -1$ are the intercepts.

2 Test points for sign. Use factored form $P(x) = (x-3)(x^2-1)$ for quick computation of signs.



$P(0) = (0-3)(0^2-1) = - \cdot - = +$

$P(2) = (2-3)(2^2-1) = - \cdot + = -$

3 End behavior. Leading term is x^3, odd degree, leading coefficient $1 > 0$. So end behavior is $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$.

4 Graph!

5. Perform the given instruction.

(a) Find the domain for each of the following functions:

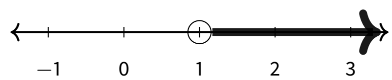
i. $h(x) = \dfrac{1}{\sqrt{x-1}}$

1 Problems.

A. Solve $\sqrt{x-1} = 0$. Squaring both sides, $x - 1 = 0$ so $x = 1$.

B. Solve $x - 1 < 0$. Adding one, $x < 1$.

2 Remove problems $x = 1$ and $x < 1$.



Domain: $\boxed{(1, \infty)}$

ii. $f(x) = \dfrac{1}{x^3 - 4x}$

1 Problems.

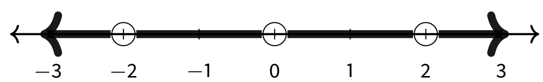
A. Solve $x^3 - 4x = 0$. We have

$$
\begin{aligned}
x^3 - 4x &= 0 \\
x(x^2 - 4) &= 0 \\
x = 0 \qquad x^2 - 4 &= 0 \\
x^2 &= 4 \\
x &= \pm\sqrt{4} = \pm 2
\end{aligned}
$$

so $x = 0, \pm 2$.

B. No root, N/A.

2 Remove problems $x = 0$, -2 and 2.



Domain:

$\boxed{(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)}$

(a) Consider $f(x) = x^2 - 2$ and $g(x) = -2x + 1$. Expand **and simplify** the following:

i. $g \circ f$

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= g(x^2 - 2) \\
&= -2(x^2 - 2) + 1 \\
&= -2x^2 + 4 + 1 \\
&= \boxed{-2x^2 + 5}
\end{aligned}
$$

ii. $f(x) - 3g(x)$

$$
\begin{aligned}
f(x) - 3g(x) &= x^2 - 2 - 3(-2x + 1) \\
&= x^2 - 2 + 6x - 3 \\
&= \boxed{x^2 + 6x - 5}
\end{aligned}
$$

iii. $f(x)g(x)$[1]

$$
\begin{aligned}
f(x)g(x) &= (x^2 - 2)(-2x + 1) \\
&= -2x(x^2 - 2) + (x^2 - 2) \\
&= -2x^3 + 4x + x^2 - 2 \\
&= \boxed{-2x^3 + x^2 + 4x - 2}
\end{aligned}
$$

[1] This problem should have said to only expand. You got full credit for correct expansion :)