MATH 161: Midterm 2

Name: key

Directions: No calculators. **Simplify all expressions + show all logical steps for full credit**. If you have a question, ask me. Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) True or False: $\dfrac{d}{dx}[f(x) \cdot g(x)] = \dfrac{d}{dx}[f(x)] \cdot \dfrac{d}{dx}[g(x)]$

   False; you need to use the product rule.

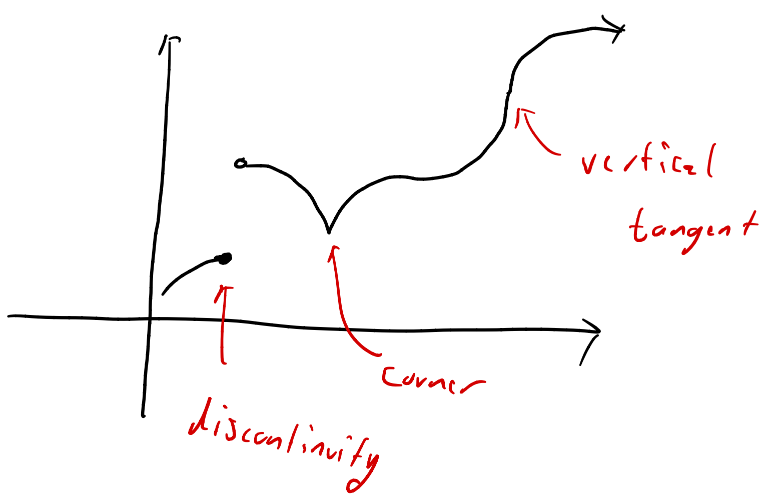
   (b) What is the derivative of $g(x) = 2\sqrt{x}$?

   $g(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$

   $g'(x) = 2 \cdot \frac{1}{2} x^{\frac{1}{2}-1} = x^{-\frac{1}{2}} = \boxed{\dfrac{1}{\sqrt{x}}}$

   (c) Draw one graph with all of the cases where a function fails to be differentiable.

   means draw one function



2. **Using the limit definition of a derivative,** find the derivative of

$$f(x) = 5x - 9x^2$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{5(x+h) - 9(x+h)^2 - (5x - 9x^2)}{h}$$

$$= \lim_{h\to 0} \frac{5x + 5h - 9(x^2 + 2xh + h^2) - 5x + 9x^2}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{5x} + 5h \boxed{-9x^2} - 18xh - 9h^2 \boxed{-5x} \boxed{+9x^2}}{h}$$

$$= \lim_{h\to 0} \frac{5h - 18xh - 9h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5 - 18x - 9h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 5 - 18x - 9h$$

$$= 5 - 18x - 9\cdot 0$$

$$= \boxed{5 - 18x}$$

3. Find the derivatives of the following functions:

(a) $y = \dfrac{\sin x}{x^2}$

$$y' = \frac{x^2 \frac{d}{dx}\sin(x) - \sin(x)\frac{d}{dx}x^2}{(x^2)^2}$$

$$= \frac{x^2\cos(x) - 2x\sin(x)}{x^4}$$

$$= \frac{\cancel{x}(x\cos(x) - 2\sin(x))}{x^{\cancel{4}\,3}}$$

$$= \boxed{\frac{x\cos(x) - 2\sin(x)}{x^3}}$$

(b) $y = e^{-2x}\cos(5x)$ use product rule

$$y' = \cos(5x)\frac{d}{dx}[e^{-2x}] + e^{-2x}\frac{d}{dx}[\cos(5x)]$$

chain rule   chain rule

$$= \cos(5x)\cdot e^{-2x}\cdot(-2) + e^{-2x}\cdot(-\sin(5x)\cdot 5)$$

$$= \boxed{-2\cos(5x)e^{-2x} - 5\sin(5x)e^{-2x}}$$

4. Find the derivatives of the following:

(a) $y = x^x$

$$\ln y = \ln x^x$$ ← bring down with laws of logarithms

$$\ln y = x \cdot \ln x$$ now implicitly differentiate

$$\frac{d}{dx} \ln y = \frac{d}{dx}[x \cdot \ln x]$$

$$\frac{1}{y} \cdot y' = \ln(x) \cdot 1 + x \cdot \frac{1}{x}$$

$$y' = y(\ln(x) + 1) = \boxed{x^x(\ln(x) + 1)}$$

(b) $4\cos(x)\sin(y) = 1$ implicitly differentiate

$$\frac{d}{dx}[4\cos(x)\sin(y)] = \frac{d}{dx} 1$$

$$\frac{dy}{dx} = \boxed{\tan(x)\tan(y)}$$

$$4 \frac{d}{dx}[\cos(x)\sin(y)] = 0$$ ↑ product rule

$$4\left(\sin(y) \cdot \frac{d}{dx}[\cos(x)] + \cos(x)\frac{d}{dx}[\sin y]\right) = 0$$

$$4\left(\sin(y)(-\sin(x)) + \cos(x)\cos(y)\frac{dy}{dx}\right) = 0$$

$$-4\sin(y)\sin(x) + 4\cos(x)\cos(y)\frac{dy}{dx} = 0$$

$$\frac{\cancel{4\cos(x)\cos(y)}\frac{dy}{dx}}{\cancel{4\cos(x)\cos(y)}} = \frac{\cancel{4}\,\boxed{\sin(x)}\boxed{\sin(y)}}{\cancel{4}\,\boxed{\cos(x)}\boxed{\cos(y)}}$$

5. Find the second-degree Taylor polynomial for the function $y = \sin x$ at $x = \frac{\pi}{2}$.

$f(x) = \sin(x)$ $\quad$ $f(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

$f'(x) = \cos(x)$ $\quad$ $f'(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) = 0$

$f''(x) = -\sin(x)$ $\quad$ $f''(\frac{\pi}{2}) = -\sin(\frac{\pi}{2}) = -1$

$T_2(x)$ at $\frac{\pi}{2}$ for $\sin(x)$ is

$$T_2(x) = f(\tfrac{\pi}{2}) + f'(\tfrac{\pi}{2})(x - \tfrac{\pi}{2}) + \frac{f''(\frac{\pi}{2})}{2!}(x - \tfrac{\pi}{2})^2$$

$$= 1 + 0(x - \tfrac{\pi}{2}) + \frac{-1}{2}(x - \tfrac{\pi}{2})^2$$

$$= \boxed{1 - \frac{1}{2}(x - \tfrac{\pi}{2})^2}$$