MATH 161: Midterm 1

Name: key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

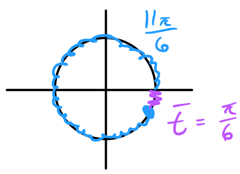
1. If

$$f(x) = 1 - x^2 \qquad g(x) = \ln(x) \qquad h(x) = \cos(x) \qquad j(x) = e^x$$

Evaluate, expand, and/or simplify the following:

(a) $h\left(\frac{11\pi}{6}\right) = \cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

cos is + in IV



(b) $g(j(-1)) = g\left(e^{-1}\right) = \ln\left(e^{-1}\right) = \boxed{-1}$ by inverse function property

look! $f(x)$ is 2 terms and you're multiplying into 2 terms

(c) $f(-1) \cdot f(x)$

$= \left(1-(-1)^2\right) \cdot \left(1-x^2\right)$

$= (1-1) \cdot \left(1-x^2\right)$

$= 0 \cdot \left(1-x^2\right) = \boxed{0}$

(d) $f(x+h) - f(x)$

again!!! You are subtracting into ≥ 2 terms!

Subtraction is adding a factor of (-1). This is multiplication.

$= 1-(x+h)^2 - \left(1-x^2\right)$

$= 1-\left(x^2+2xh+h^2\right) - 1 + x^2$ dist law

$= 1 - x^2 - 2xh - h^2 - 1 + x^2$

$= -2xh - h^2$

$= \boxed{h(-2x-h)}$

2. Short answer questions:

   (a) State the mathematical definition of "**a sequence** a_n **which converges to** 3".

$$\lim_{n \to \infty} a_n = 3$$

   (b) True or false: We can simplify

$$\frac{(x+1)(x-2)-(x-1)(x+2)}{(x+1)^2(x-2)-(x-1)(x+2)}$$

   by crossing out the $x+1$.

False; $(x+1)$ is not a global factor.

   (c) If $f(x) = 2x^2$, evaluate $f(x+h)$ and fully expand + simplify.

$$f(x+h) = 2(x+h)^2 = 2(x^2 + 2xh + h^2)$$
$$= \boxed{2x^2 + 4xh + 2h^2}$$

   (d) If $F(x) = \sqrt[4]{\ln(\sin(x))}$ find three functions f, g, h where $f \circ g \circ h = F$.

$f(x) = \sqrt[4]{x}$

$g(x) = \ln(x)$

$h(x) = \sin(x)$

3. Suppose

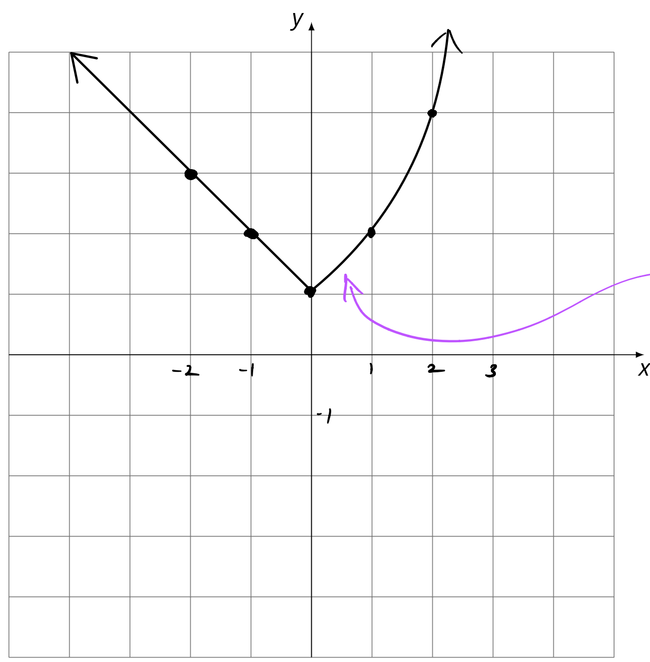
$$f(x) = \begin{cases} -x+1 & x \leq 0 \\ 2^x & x > 0 \end{cases}$$

(a) What is $f(0)$?

$f(0) = -0 + 1 = \boxed{1}$

(b) Sketch a graph of $f(x)$.

x	$f(x)$
-2	$-(-2)+1=3$
-1	$-(-1)+1=2$
0	$-0+1=1$
1	$2^1=2$
2	$2^2=4$



plug in 0 into 2^x to get $(0, 2^0) = (0,1)$

open circle at $(0,1)$

Showing where the graph of 2^x "starts"

4. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

   (a) Expand and simplify: $\dfrac{2(x+h)^2 - 2x^2}{h}$

$$= \frac{2(x+h)(x+h) - 2x^2}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 2x^2}{h} \qquad (A+B)^2$$

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-2x^2}}{h} \qquad \text{dist law}$$

$$= \frac{4xh + 2h^2}{h}$$

$$= \frac{\cancel{h}(4x + 2h)}{\cancel{h}} \qquad \text{GCF}$$

$$= \boxed{4x + 2h} \qquad \text{fraction law 5}$$

   (b) Rationalize the numerator (remember to fully simplify): $\dfrac{3-\sqrt{x}}{9x - x^2}$

recall: in the context of a fraction, "simplify" means to cancel all common factors. Which means you need to factor every factor!

$$\frac{\overset{A}{3} - \overset{B}{\sqrt{x}}}{9x - x^2} \cdot \frac{\overset{A}{3} + \overset{B}{\sqrt{x}}}{3 + \sqrt{x}} = \frac{9 - x}{(9x - x^2)(3 + \sqrt{x})}$$

Stop forgetting parentheses. 9x and x^2 are two terms. These two terms are being multiplied.

$$= \frac{\cancel{(9 - x)}}{x\cancel{(9 - x)}(3 + \sqrt{x})} \qquad \text{GCF}$$

$$= \boxed{\frac{1}{x(3 + \sqrt{x})}}$$

(c) Simplify: $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$ ← deal with numerator as a subproblem!

missing factor of x missing factor of $(x+h)$

Beware!

$$\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h} = \frac{\frac{x}{x}\cdot\frac{1}{(x+h)}-\frac{1}{x}\cdot\frac{x+h}{x+h}}{h}$$

this does NOT simplify as $-x+h$.

Again, subtracting ≥ 2 terms necessitates parentheses.

$$= \frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}$$

fraction law 1

$$= \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

fraction law 3

$$= \frac{\frac{x-x-h}{x(x+h)}}{h} = \frac{\frac{-h}{x(x+h)}}{h} = -\frac{\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}} = \boxed{-\frac{1}{x(x+h)}}$$

(d) Expand: $\underbrace{2(x+1)^2}-\underbrace{(x+2)(x-3)3x}$

Two global terms, each of which are subproblems.

$(A+B)^2$ commutative law

$$2(x+1)^2-(x+2)(x-3)3x = 2(x^2+2x+1) - 3x(x^2-x-6)$$

$$= 2x^2+4x+2-3x^3+3x^2+18x$$

dist law

$$= \boxed{-3x^3+5x^2+22x+2}$$

5. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

   (a) Solve for x:

$$\ln\left(x^2 - 1\right) = 0$$

exponentiate both sides

$$e^{\ln(x^2-1)} = e^0$$

$$x^2 - 1 = 1$$

inverse function property

$$x^2 = 2$$

$$\sqrt{x^2} = \pm\sqrt{2}$$

$$\boxed{x = \pm\sqrt{2}}$$

   (b) Determine if the functions

$$f(x) = (e^x)^2 \qquad g(x) = \ln(\sqrt{x})$$

   are inverses of each other.

$$f(g(x)) = f(\ln\sqrt{x}) = \left(e^{\ln\sqrt{x}}\right)^2$$

$$= \left(\sqrt{x}\right)^2$$

$$= x$$

By the inverse function property $f(x)$ and $g(x)$ are inverses.