MATH 118: Midterm 1

Name: key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

$\boxed{\text{Legend}}$

— my work

— or — my thoughts while I work

— Common mistakes made to avoid

1. Short answer questions:

   (a) True or False: We are allowed to use exponent laws in the following way:

$$\left(\frac{x^3y^2}{z^3}\right)^2 = \frac{x^5y^4}{z^5}$$

False. By LoE we have

$$\left(\frac{x^3y^2}{z^3}\right)^2 \overset{5}{=} \frac{(\overset{\text{factors}}{x^3 y^2})^2}{(z^3)^2} \overset{4}{\underset{3}{=}} \frac{(x^3)^2\cdot(y^2)^2}{z^6} \overset{3}{=} \boxed{\frac{x^6y^4}{z^6}}$$

   (b) True or false: We can simplify

$$\frac{3(x-2)^2(x+3) - 4(x+2)(x-3)^2}{5x(x-3)^2(x-2) - 4(x+3)}$$

   by crossing out the $x + 3$.

False, $(x+3)$ is only a factor in a single term context, not global numerator/denominator.

   (c) True or False: $x = 3$ is a solution of the equation

$$\frac{1}{4-x} + \frac{1}{2} = \frac{3}{2}$$

True. $\frac{1}{4-3} + \frac{1}{2} = \frac{3}{2}$

$1 + \frac{1}{2} = \frac{3}{2} \rightarrow \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

$\frac{3}{2} = \frac{3}{2}$ ✓

   (d) A student tries to simplify $x^2 - (x-3)$ by doing the following:

$$x^2 - (x-3) = x^2 - x - 3$$

   What mistake did they make?

$x^2 - (x-3)$ is really $x^2 + (-1)(x-3)$. The factor of (-1) needs to be distributed to both x and -3.

2. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

   (a) Factor and simplify: $\underbrace{(x+3)^2(x-2)}_{①} + \underbrace{(x+3)(x-2)^2}_{②}$

Two term factoring problem. Try GCF first.

Breaking each term into factors:

① : $(x+3)^2(x-2) = (x+3)(x+3)(x-2)$

② : $(x+3)(x-2)^2 = (x+3)(x-2)(x-2)$

Factor out $(x+3)(x-2)$ from each term.

$$(x+3)^2(x-2) + (x+3)(x-2)^2 \overset{GCF}{=} (x+3)(x-2)\overbrace{\big((x+3) + (x-2)\big)}^{\text{Simplify}}$$

$$= \boxed{(x+3)(x+2)(2x+1)}$$

   (b) Subtract: $\dfrac{1}{\boxed{x+1}} - \dfrac{x}{\boxed{x-1}}$

Subtraction of fractions. Use fraction law 3

$$\frac{a}{\boxed{c}} + \frac{b}{\boxed{c}} = \frac{a+b}{c}$$

c needs to be made up of factors!

$(x+1)$ ← missing factor of $(x-1)$

$(x-1)$ ← missing factor of $(x+1)$

$$\frac{1}{x+1} - \frac{x}{x-1} \overset{\text{introduce missing factors}}{=} \frac{(x-1)}{(x-1)} \cdot \frac{1}{(x+1)} - \frac{x}{(x-1)} \cdot \frac{(x+1)}{(x+1)}$$

$$\overset{\text{fraction law ①}}{=} \frac{x-1}{(x-1)(x+1)} - \frac{x(x+1)}{(x-1)(x+1)}$$

Common mistake: $-\dfrac{x(x+1)}{(x-1)(x+1)}$ is not $\dfrac{\boxed{-x^2+x}}{(x-1)(x+1)}$ wrong!

Look at the solution; both − and x need to be distributed.

$$\overset{\text{fraction law ③}}{=} \frac{x-1-x(x+1)}{(x-1)(x+1)}$$

$$\overset{\text{dist}}{=} \frac{x-1-x^2-x}{(x-1)(x+1)}$$

$$= \boxed{\frac{-x^2-1}{(x-1)(x+1)}}$$

(c) Simplify: $\dfrac{2+\frac{1}{x}}{3-\frac{1}{x^2}}$

Compound fraction structure. "Simplify" means to remove nested fraction denominators.

LCD between $\frac{1}{x}$ and $\frac{1}{x^2}$ is x^2. Introduce x^2 by multiplying by 1.

$$\frac{2+\frac{1}{x}}{3-\frac{1}{x^2}} \cdot \frac{x^2}{x^2} \underset{\text{law ①}}{\overset{\text{frac}}{=}} \frac{\left(2+\frac{1}{x}\right)\cdot x^2}{\left(3-\frac{1}{x^2}\right)\cdot x^2}$$

$$\overset{\text{dist}}{=} \frac{2x^2+\frac{1}{x}\cdot x^2}{3x^2-\frac{1}{x^2}\cdot x^2}$$

$$\underset{\text{① thru ⑤}}{\overset{\text{frac law}}{=}} \frac{2x^2+x}{3x^2-1}$$

$$\overset{\text{GCF}}{=} \boxed{\frac{x(2x+1)}{3x^2-1}}$$

(d) Expand: $(x^3+6)(2x+1)-(x^2+x-2)(3x^2)$

Convert to terms, no parenthesis.

$$(x^3+6)(2x+1)-(x^2+x-2)(3x^2) \overset{\text{dist}}{=} (x^3+6)2x+(x^3+6)\cdot 1-3x^4-3x^3+6x^2$$

$$\overset{\text{dist}}{=} 2x^4+12x+x^3+6-3x^4-3x^3+6x^2$$

$$= \boxed{-x^4-2x^2+6x^2+12x+6}$$

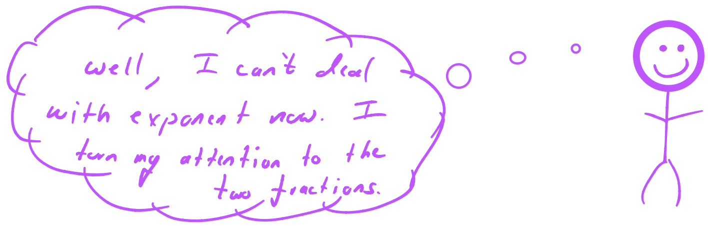
Common mistake: forgetting to distribute the factor of (-1)

3. Simplify (meaning write as one fraction and try to convert to all factors if you can) the following expression:

$$\left(\frac{1}{x^2-4}+\frac{x}{x+2}\right)^2$$

Problem structure is $\left(\frac{\text{fraction}}{\text{term}} - \frac{\text{fraction}}{\text{term}}\right)^2$. Exponents do not interact with terms. Do not distribute exponents to terms.

Instead, think



$$\left(\frac{1}{\underset{\uparrow\ \text{convert to factors}}{x^2-4}}+\frac{x}{x+2}\right)^2 \underset{A^2-B^2}{=} \left(\frac{1}{(x-2)(x+2)}+\overset{\text{missing }(x-2)}{\frac{x}{(x+2)}}\right)^2$$

$$= \left(\frac{1}{(x-2)(x+2)}+\frac{x}{(x+2)}\cdot\frac{(x-2)}{(x-2)}\right)^2$$

$$= \left(\frac{1}{(x-2)(x+2)}+\frac{x(x-2)}{(x+2)(x-2)}\right)^2$$

$$\overset{\text{frac law ③}}{=} \left(\frac{1+x(x-2)}{(x-2)(x+2)}\right)^2$$

$$\overset{\text{dist}}{=} \left(\frac{1+x^2-2x}{(x-2)(x+2)}\right)^2$$

$$\overset{\text{Commutative property of +}}{=} \left(\frac{\overset{A^2\ -\ 2AB\ +\ B^2}{x^2-2x+1}}{(x-2)(x+2)}\right)^2 \qquad A=x,\ B=1$$

$$\overset{(A-B)^2}{=} \left(\frac{(x-1)^2}{(x-2)(x+2)}\right)^2$$

$$\overset{\text{LoE ⑤}}{=} \frac{\left((x-1)^2\right)^2}{\left((x-2)(x+2)\right)^2}$$

$$\overset{\text{LoE ③ and ④}}{=} \boxed{\frac{(x-1)^4}{(x-2)^2(x+2)^2}}$$

4. Simplify the following:

$$\frac{(x^2+x-2)^2}{x+2}\cdot\frac{x^2+1}{x^3+4x^2+x+4}\cdot\boxed{\left(\frac{(x+1)(x-2)(x+3)}{x^5-x^3-x}\right)^0}$$

$a^0 = 1$

when fractions are present means convert to factors and cancel.

common mistake: cannot cancel $(x+2)$ because $((x+2)(x-1))^2 = (x+2)(x-1)(x+2)(x-1)$

1 2

1 −1

quadratic

$$\frac{(x^2+x-2)^2}{(x+2)}\cdot\frac{x^2+1}{x^3+4x^2+x+4}\cdot 1 \overset{\text{GCF}}{=} \frac{((x+2)(x-1))^2}{(x+2)}\cdot\frac{x^2+1}{x^2(x+4)+(x+4)}$$

grouping

$$\overset{\text{LoE (4)}}{\underset{\text{GCF}}{=}} \frac{\cancel{(x+2)^2}^{\,(x+2)}(x-1)^2}{\cancel{(x+2)}}\cdot\frac{\cancel{(x^2+1)}}{\cancel{(x^2+1)}(x+4)}$$

$$\overset{\text{frac law}}{\underset{5}{=}} \frac{(x+2)(x-1)^2}{1}\cdot\frac{1}{(x+4)}$$

$$\overset{\text{frac}}{\underset{\text{law 1}}{=}} \boxed{\frac{(x+2)(x-1)^2}{(x+4)}}$$

5. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

(a) Expand and simplify: $\dfrac{3(x+h)^2 - 1 - (3x^2 - 1)}{h}$

common mistake:

$3(x+h)^2 \neq (3x+3h)^2$

because $3(x+h)^2 = 3\cdot(x+h)\cdot(x+h)$

You can only distribute the 3 to one factor of $(x+h)$.

3 multiplies into 3 terms.

Don't forget parenthesis.

$$\frac{3\overbrace{(x+h)^2}^{(A+B)^2} - 1 - (3x^2-1)}{h} \overset{dist}{=} \frac{3(x^2+2xh+h^2) \cancel{-1} - 3x^2 \cancel{+1}}{h}$$

$$\overset{dist}{=} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-3x^2}}{h}$$

$$= \frac{6xh + 3h^2}{h} \overset{GCF}{=} \frac{\cancel{h}(6x+3h)}{\cancel{h}} \underset{law\ 5}{\overset{frac}{=}} \boxed{6x + 3h}$$

(b) Rationalize the numerator (remember to simplify): $\dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

the $\boxed{\text{only}}$ technique to square two terms.

$(A - B)\cdot(A+B) = A^2 - B^2$

$$\frac{\overbrace{\sqrt{x+h}}^{A} - \overbrace{\sqrt{x}}^{B}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\underset{law\ 5}{\overset{frac}{=}} \boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}}}$$

mistake #1

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)^2 \neq \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h^2}$$

because the numerator are terms and exponents don't interact with terms.

mistake #2

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \sqrt{x+h} + \sqrt{x}$$

forgot denominator $\sqrt{x+h} + \sqrt{x}$