

# MATH 161: Midterm 2

Name: \_\_\_\_\_

Directions:

- \* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- \* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- \* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		<b>60</b>

1. Draw **one single graph** of a function which satisfies the following:

(a)  $f(0) = 1$

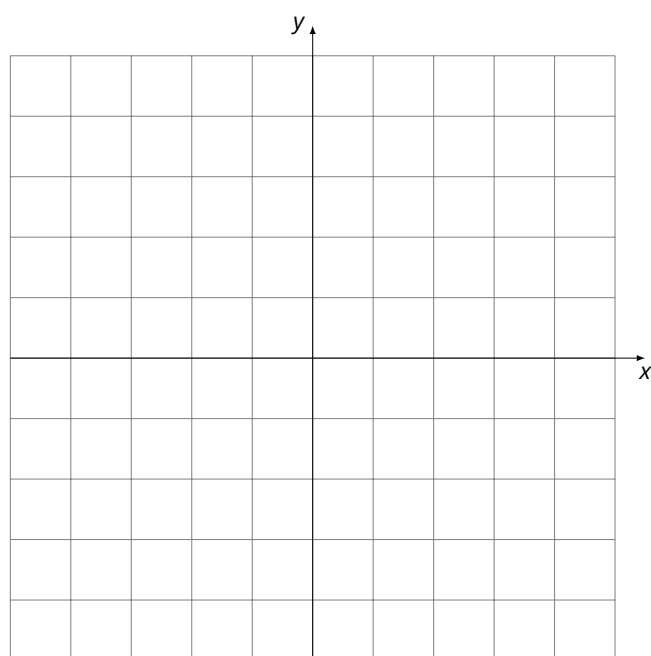
(b)  $f(2) = 1$

(c)  $\lim_{x \rightarrow 0} f(x) = 1$

(d)  $\lim_{x \rightarrow 2^-} f(x) = 0$

(e)  $\lim_{x \rightarrow 2^+} f(x) = 2$

(f)  $\lim_{x \rightarrow -2} f(x) = -\infty$



2. Consider this limit:

$$\lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

(a) Try using Limit Laws to find the limit. What ends up happening?

(b) Now find the actual limit.

3. Use **the mathematical definition of continuity** to prove the function

$$f(x) = \begin{cases} x(x-1) & x < 1 \\ 0 & x = 1 \\ \sqrt{x-1} & x > 1 \end{cases}$$

is continuous at the number  $x = 1$ .

4. Suppose  $f(x) = \sqrt{x}$ .

(a) What the expression  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  represent?

(b) Find the limit

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

for the given function  $f(x)$ . You must use this limit definition to receive credit.

(c) Find the equation of the tangent line of  $f(x) = \sqrt{x}$  at the point  $(1, 1)$ .

5. Short answer questions:

- (a) If a function  $f(x)$  is continuous at  $x = a$ , must it be differentiable at  $x = a$ ?  
If not, draw a graph of a function that is continuous but not differentiable.

- (b) True or False:

$$f(x) = \sin(x) + \frac{x}{x+1}$$

is continuous on  $\mathbb{R}$ .

- (c) Given  $f(x) = x$ , find  $f'(x)$ .

6. Find the following derivatives. You are allowed to use the Differentiation Rules.

(a)  $f(x) = \pi^2$

(b)  $f(\theta) = \theta^3 - e^\theta + 2 \sin \theta$

(c)  $g(x) = (x - 1)(x + 1)$