MATH 118: Quiz 7

Name: Key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

① Find and remove holes

② Then find asymptotes

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1. Find all vertical and horizontal asymptotes. Make sure to describe potential holes.

   (a) $\dfrac{x^2-1}{x^2-x-2}$

$= \dfrac{(x-1)(x+1)}{(x-2)(x+1)}$ hole

$= \dfrac{x-1}{x-2}, \quad x \neq -1$

vertical asymptote: $x=2$

horizontal asymptote: $y = \frac{1}{1} = 1$

   (b) $\dfrac{x^5-x^3}{x^7+x^4}$

On next page (was not graded)

   (c) $\dfrac{x^3+3x^2+3x+1}{x+1}$, hole at $x=-1$

$= \dfrac{(x+1)(x^2+2x+1)}{(x+1)}$

$x+1 \;\overline{)\; x^3+3x^2+3x+1}$

$-x^3 - x^2$

$2x^2+3x$

$-2x^2-2x$

$x+1$

$x+1$

0

$= x^2+2x+1, \quad x \neq -1$ quadratic, no asymptotes

$$\frac{x^5 - x^3}{x^7 + x^4}$$

$$= \frac{x^3(x^2 - 1)}{x^4(x^3 + 1)}$$

Need to factor.

$P(x) = x^3 + 1$

$P(-1) = (-1)^3 + 1 = -1 + 1 = 0$

$\Leftrightarrow$

factor theorem says $x - (-1) = x + 1$ is a factor

$$= \frac{x^3(x-1)(x+1)}{x^4(x+1)(x^2 - x + 1)}$$

hole

$$\begin{array}{r} x^2 - x + 1 \\ x+1 \overline{)\, x^3 + 0x^2 + 0x + 1} \\ -x^3 + x^2 \\ \hline 0 - x^2 + 0x \\ -x^2 - x \\ \hline 0 + x + 1 \\ x + 1 \\ \hline 0 \end{array}$$

$b^2 - 4ac = 1 - 4 < 0$, no real solutions so no v. asymptotes for this quadratic

$$= \frac{x-1}{x(x^2 - x + 1)} \qquad x \neq 0, -1$$

$$= \frac{x-1}{x^3 - x^2 + x} \qquad x \neq 0, -1$$

horizontal asymptotes: none

vertical: $x = 0$

2. If $f(x) = 4^x$, find $f(0)$, $f(2)$ and $f(1.5)$. Show your work for full credit.

$f(0) = 4^0 = 1$ 1 pt

$f(2) = 4^2 = 16$ 1 pt

$f(1.5) = 4^{1.5} = 4^{\frac{3}{2}} = \sqrt{4^3} = \sqrt{64} = 8$ 1 pt

3. With a base function of $f(x) = e^x$, describe what transformations you need to perform to create $g(x) = -1 - e^{-2(x-3)}$.

Shifts must be last.

$f(x) = e^x$

$a(x) = -f(x) = -e^x$

$b(x) = a(-x) = -e^{-x}$

$c(x) = b(2x) = -e^{-2x}$

$d(x) = c(x-3) = -e^{-2(x-3)}$

$f(x) = -1 + d(x) = -1 - e^{-2(x-3)}$

(1) reflection around x-axis

(2) reflection around y-axis

(3) horizontal shrink $\frac{1}{2}$

(4) horizontal shift 3 units to the right

(5) vertical shift 1 unit down