MATH 118: Midterm 2

Name: Key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) Given three functions

$$f(x) = \sqrt{x-2}, \qquad g(x) = \frac{2x^2}{3x+1}, \qquad h(x) = x+1$$

   find the composition $g \circ f \circ h$ and simplify.

$$(g \circ f \circ h)(x) = g(f(h(x))) = g(f(x+1)) = g\left(\sqrt{x+1-2}\right)$$

$$= g\left(\sqrt{x-1}\right) = \frac{2\left(\sqrt{x-1}\right)^2}{3\sqrt{x-1}+1} = \frac{2(x-1)}{3\sqrt{x-1}+1}$$

   (b) Given a degree n polynomial, how many real and complex zeros must it have?

   n real and complex zeros.

   (c) Find a degree four polynomial with zeros $x = 2$ and $x = 3$.

   Answers may vary. The factors $(x-2)(x-3)$ must be present. For example

$$P(x) = (x-2)^2(x-3)^2$$

   (d) Given a base function $f(x) = \frac{1}{x}$ and two transformed functions

$$g(x) = \frac{1}{x+2} \qquad h(x) = \frac{1}{2x+2}$$

   do both $g(x)$ and $h(x)$ have the same horizontal shift from $f(x)$? If not, state both of the horizontal shifts of $g(x)$ and $h(x)$.

   No. $h(x) = \frac{1}{2(x+1)}$

   $g(x)$ is horizontally shifted two units to the left from $f(x)$.

   $h(x)$ is horizontally shifted one unit to the left from $f(x)$.

2. Suppose $f(x) = x - x^2$.

(a) What is the domain of $f(x)$?

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(b) Find a complete factorization of $f(x)$.

$f(x) = x(1-x)$

(c) Calculate and **fully expand + simplify** the expression $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)-(x+h)^2-(x-x^2)}{h}$$

$$= \frac{x+h-(x^2+2xh+h^2)-x+x^2}{h}$$

$$= \frac{x+h-x^2-2xh-h^2-x+x^2}{h}$$

$$= \frac{h-2xh-h^2}{h}$$

$$\overset{\text{GCF}}{=} \frac{\cancel{h}(1-2x-h)}{\cancel{h}}$$

$$\overset{\text{frac law}}{=} \boxed{1-2x-h}$$

5

3. Given the polynomial $P(x) = x^4 - x^2 - 2x + 2$:

(a) What is the average rate of change of $P(x)$ on $[\underset{a}{0}, \underset{b}{1}]$?

this generates ≥ 2 terms, do not forget

$$\frac{P(1) - P(0)}{1-0} = \frac{1^4 - 1^2 - 2\cdot 1 + 2 - (0^4 - 0^2 - 2\cdot 0 + 2)}{1-0}$$

$$= \frac{0-2}{1} = \boxed{-2}$$

(b) Is $P(x)$ even, odd, or neither? Full credit requires using the definition of even/odd.

$$P(-x) = (-x)^4 - (-x^2) - 2(-x) + 2$$

$$= x^4 - x^2 + 2x + 2$$

only one term changed sign; either all terms have to change sign or none.

$\boxed{\text{neither}}$

(c) $x = 1$ is a zero of multiplicity two for $P(x)$. Use this information to find a complete factorization of $P(x)$.

$(x-1)^2 = x^2 - 2x + 1$ is a factor of $P(x)$. Use division alg.

$$\begin{array}{r|l} & x^2 + 2x + 2 \\ \hline x^2 - 2x + 1 & x^4 + 0x^3 - x^2 - 2x + 2 \\ & -(x^4 - 2x^3 + x^2) \\ \hline & 0 + 2x^3 - 2x^2 - 2x \\ & -(2x^3 - 4x^2 + 2x) \\ \hline & 0 + 2x^2 - 4x + 2 \\ & -(2x^2 - 4x + 2) \\ \hline & 0 \end{array}$$

So $P(x) = (x-1)^2 \underline{(x^2 + 2x + 2)}$

use quadratic formula.

$a = 1,\ b = 2,\ c = 2.$

$$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 2}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm i\sqrt{4}}{2} = \frac{-2 \pm 2i}{2} \overset{GCF}{=} \frac{\cancel{2}(-1 \pm i)}{\cancel{2}}$$

frac law #5

$$= -1 \pm i$$

$x = -1 \pm i$ are zeros $\Rightarrow (x - (-1+i))(x - (-1-i))$ are factors

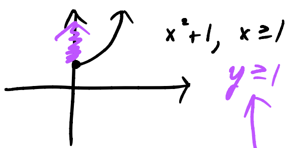
Simplifying (using dist law):

$$\boxed{P(x) = (x-1)^2 (x+1-i)(x+1+i)}$$

4. Given the function

$$f(x) = x^2 + 1, \ x \geq 0$$

(a) Calculate the inverse f^{-1} algebraically.

① 

$y \geq 1$ certainly 1-1

② $y = x^2 + 1, \quad y \geq 1$ since x^2+1 is increasing on $(0, \infty)$

③ $y - 1 = x^2, \quad y \geq 1$

$\pm\sqrt{y-1} = \sqrt{x^2}, \quad y \geq 1$

$+\sqrt{y-1} = x, \quad y \geq 1$

because of and $x \geq 0$ originally.

④ $\boxed{f^{-1}(x) = y = \sqrt{x-1}, \quad x \geq 1}$

(b) Use the Inverse Function Property to verify your result of f^{-1} is actually the inverse of $f(x)$.

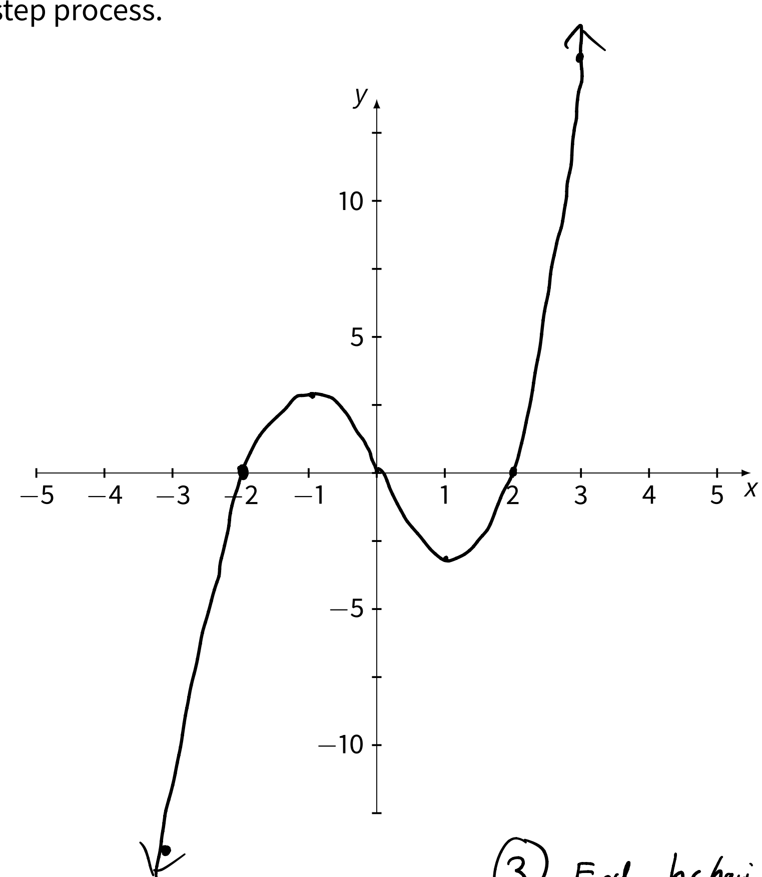
Show $(f^{-1} \circ f)(x) = x$ or $(f \circ f^{-1})(x) = x$

$$\begin{aligned}(f \circ f^{-1})(x) &= f(f^{-1}(x)) \\ &= f(\sqrt{x-1}) \\ &= (\sqrt{x-1})^2 + 1 \\ &= x - 1 + 1 \\ &= x \ \checkmark\end{aligned}$$

5. Sketch an accurate graph of the polynomial

$$P(x) = x^3 - 4x$$

using the four step process.



① Solve $P(x) = 0$

$0 = P(x) = x^3 - 4x$

GCF

$= x(x^2 - 4)$

$A^2 - B^2$

$= x(x-2)(x+2)$

By zero product property

$x = 0, \quad x - 2 = 0, \quad x + 2 = 0$

$x = 0, 2, -2$

② Use IVT

$-$ $+$ $-$ $+$

-2 0 2

$P(-3) = -3(-3-2)(-3+2)$

$= -3(-5)(-1)$

$= -15$

$P(1) = 1(1-2)(1+2)$

$= 1 \cdot (-1)(3)$

$= -3$

$P(3) = 3(3-2)(3+2)$

$= 3(1)5$

$= 15$

$P(-1) = -1(-1-2)(-1+2) = -1(-3)(1) = 3$

③ End behavior: leading term x^3

$y \to \infty$ as $x \to \infty$

$y \to -\infty$ as $x \to -\infty$

④ Graph

x	P(x)
-3	-15
-2	0
-1	3
0	0
1	-3
2	0
3	15