MATH 161: Quiz 5

Name: _key_

Directions:

* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

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1. A function $f(x)$ is continuous at $x = -1$. Using the mathematical definition of continuity, state the three conditions that must be true.

① $f(-1)$ is defined

② $\lim\limits_{x \to -1} f(x)$ exists

③ $\lim\limits_{x \to -1} f(x) = f(-1)$

2. **Using the definition of continuity,** determine whether the following function is continuous at $x = -2$:

$$f(x) = \begin{cases} (x+3)^2 & x < -2 \\ -1 & x = -2 \\ (x+1)^2 & x > -2 \end{cases}$$

① $f(-2) = -1$

② $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} (x+1)^2 \overset{\text{limit laws}}{=} \left(\lim\limits_{x \to -2^+} x + \lim\limits_{x \to -2^+} 1 \right)^2 = (-2+1)^2 = (-1)^2 = 1$

$\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} (x+3)^2 \overset{\text{limit laws}}{=} \left(\lim\limits_{x \to -2^-} x + \lim\limits_{x \to -2^-} 3 \right)^2 = (-2+3)^2 = 1^2 = 1$

$\therefore \lim\limits_{x \to -2} f(x) = 1$

③ $\lim\limits_{x \to -2} f(x) = 1 \neq -1 = f(-2)$

$\boxed{\therefore f(x) \text{ is not continuous at } x = -2}$

3. State in interval notation where this function is continuous:

$$f(x) = \frac{\sin(x)\cos(x)}{3x^2 - 7x - 6} - \sqrt{3x - 1}$$

Find domain:

① Problems:

ⓐ division by 0.

$$3 \quad 2$$
$$1 \quad -3$$

$$3x^2 - 7x - 6 = 0$$
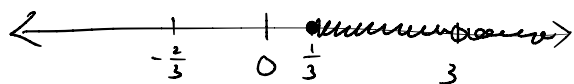
$$(3x + 2)(x - 3) = 0$$

$$3x + 2 = 0 \qquad x - 3 = 0$$

$$\boxed{x = -\frac{2}{3} \qquad x = 3}$$

ⓑ Square root of negative.

$$3x - 1 < 0$$

$$\boxed{x < \frac{1}{3}}$$

② Remove problems from $\mathbb{R}$

Domain: $\left[\frac{1}{3}, 3\right) \cup (3, \infty)$

Because this function is continuous on its domain,

$f(x)$ is continuous on $\left[\frac{1}{3}, 3\right) \cup (3, \infty)$

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