MATH 119: Midterm 2

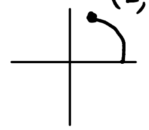
Name: Key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60

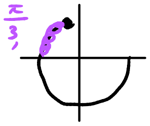
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

1. Simplify these expressions:

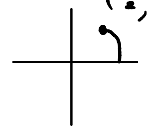
$t = \frac{\pi}{3}$, cos - in III



$\cos\left(-\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$

$= -\frac{1}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



$\tan\left(\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

* $\sin^2\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{-4\pi}{3}\right) + 3\tan\left(\frac{\pi}{4}\right)$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2\cdot\left(-\frac{1}{2}\right) + 3\cdot 1$$

LoE #5, frac law #1

$$= \frac{(\sqrt{3})^2}{2^2} - \frac{2}{2} + 3$$

$$= \frac{3}{4} - 1 + 3$$

$$= \frac{3}{4} + 2$$

$$= \frac{3}{4} + \frac{8}{4}$$

$$= \frac{3+8}{4}$$

$$= \boxed{\frac{11}{4}}$$

* $\underbrace{\frac{1}{1+\sin\theta}} + \frac{1}{1-\sin\theta}$

Form: frac + frac

use frac law #3 : common denominator.

$$\frac{1-\sin\theta}{1-\sin\theta}\cdot\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta} = \frac{1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} + \frac{1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$$

$(A-B)\cdot(A+B) = A^2 - B^2$

$(1-\sin\theta)(1+\sin\theta) = 1-\sin^2\theta$

$$= \frac{1-\sin\theta + 1+\sin\theta}{1-\sin^2\theta}$$

$\sin^2\theta + \cos^2\theta = 1$

$\downarrow$

$\cos^2\theta = 1-\sin^2\theta$

$$= \frac{2}{\cos^2\theta}$$

5.2 identity

$$= \boxed{2\sec^2\theta}$$

2. Short answer questions.

⚠ **Justify each answer with formulas or facts for full credit; do not just write "yes" or "no"** ⚠.

(a) Given $f(x) = \sin(x)$, does there exist $x \in \mathbb{R}$ such that $f(x) = 2$? Why or why not?

No, the maximum y-coordinate on the unit circle is 1, no matter where you walk to.

(b) If a mass attached to a spring is moving in simple harmonic motion, can we use the function

$$d(t) = a\tan(\omega t)$$

to model it's displacement? Why or why not?

No, simple harmonic motion is modeled by $a\sin(\omega t)$ or $a\cos(\omega t)$.

tangent is inappropriate because $\tan(x) \to \infty$ as $x \to \frac{\pi}{2}$ from the left so a spring would have to stretch infinitely.

(c) Is it possible for linear speed to be less than angular speed? Why or why not?

Yes, angular speed is $\omega = \frac{\theta}{t}$

linear speed is $v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = r \cdot \omega$

If $r < 1$ then linear speed is less than angular speed.

(d) When proving a trig identity, are we allowed to square both sides? Why or why not?

No, you need to either

1. start with one side, perform steps to reach the other or
2. simplify both sides and "meet in the middle"

3. Prove these identities:

* $\frac{1}{\sin x} - \sin x = \cot x \cdot \cos x$

$$LHS = \frac{1}{\sin x} - \sin x \cdot \frac{\sin x}{\sin x} = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$\sin^2 x + \cos^2 x = 1$
$\downarrow$
$\cos^2 x = 1 - \sin^2 x$

$$= \frac{\cos^2 x}{\sin x}$$

$$\overset{\text{frac law \#1}}{=} \frac{\cos x}{\sin x} \cdot \cos x \overset{5.2}{=} \cot x \cdot \cos x = RHS \;\blacksquare$$

$1-\sin^2\alpha \quad - \quad (1-\cos^2\beta)$

* $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$

Start w/ LHS; looks like addition identity.

$$LHS = \cos(\alpha+\beta)\cos(\alpha-\beta) \overset{7.2}{=} (\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$(A \quad - \quad B) \cdot (A \quad + \quad B)$

$$\overset{A^2-B^2}{=} \cos^2\alpha\cos^2\beta - \underbrace{\sin^2\alpha}\,\underbrace{\sin^2\beta}$$

convert to it's $1-\cos^2$ version

$$= \cos^2\alpha\cos^2\beta - (1-\cos^2\alpha)(1-\cos^2\beta)$$

$$\overset{\text{expand dist law}}{=} \cos^2\alpha\cos^2\beta - \left[(1-\cos^2\alpha)\cdot 1 - (1-\cos^2\alpha)\cdot\cos^2\beta\right]$$

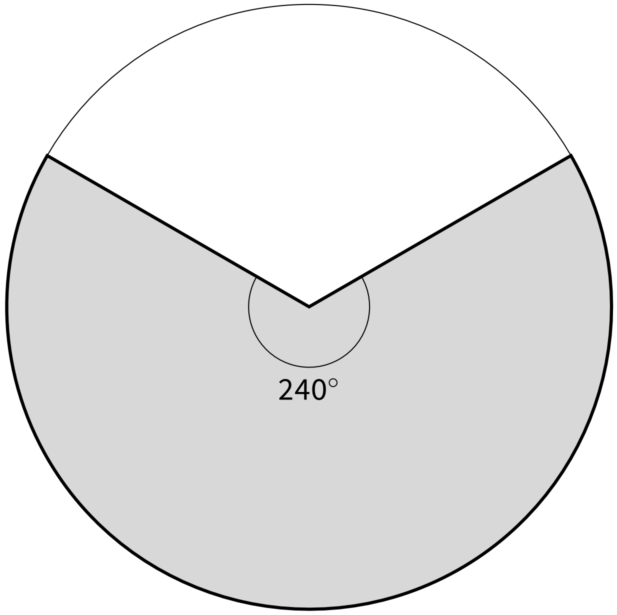
$$= \cos^2\alpha\cos^2\beta - (1 - \cos^2\alpha - \cos^2\beta + \cos^2\alpha\cos^2\beta)$$

$$= \cancel{\cos^2\alpha\cos^2\beta} - 1 + \cos^2\alpha + \cos^2\beta \cancel{- \cos^2\alpha\cos^2\beta}$$

$$\overset{\text{factor out }-1}{=} \cos^2\alpha - (1-\cos^2\beta)$$

$$\overset{5.2}{=} \cos^2\alpha - \sin^2\beta = RHS \;\blacksquare$$

4. Suppose the shaded region is 6π in^2. Find the radius of the circle; your answer should be an integer.



area of sector:

$$A = \frac{1}{2} r^2 \theta \quad , \quad \theta \text{ in rad.}$$

$$6\pi = \frac{1}{2} r^2 \cdot \cancel{240}^{\,4} \cdot \frac{\pi}{\cancel{180}_{\,3}}$$

$$6\pi = \frac{\cancel{4}^{\,2}\pi}{\cancel{2}\cdot 3} \cdot r^2$$

$$\frac{3\pi}{\cancel{2}_{\,1}} \cdot \cancel{6}^{\,3}\pi = \cancel{\frac{2\pi}{3}} \cdot r^2 \cdot \cancel{\frac{3\pi}{2}}$$

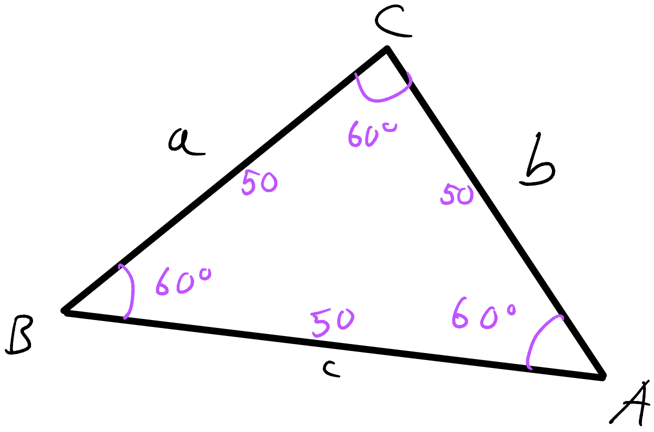
$$9\pi^2 = r^2$$

$$\sqrt{r^2} = \pm\sqrt{9\pi}$$ ← + version as r is not negative

$$r = +\sqrt{9\pi} = (9\pi)^{\frac{1}{2}} = 9^{\frac{1}{2}}\pi^{\frac{1}{2}} = \sqrt{9}\cdot\sqrt{\pi} = 3\sqrt{\pi}$$

$$\boxed{r = 3\sqrt{\pi}}$$

5. Suppose a triangle has $a = 50, b = 50, \angle A = 60^\circ$. Solve the triangle.



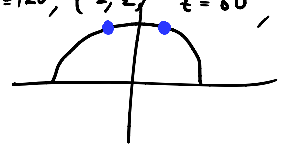
Solve for $\angle B$

$$\cancel{50} \cdot \frac{\sin 60^\circ}{\cancel{50}} = \frac{\sin B}{\cancel{50}} \cdot \cancel{50}$$

$$\sin B = \sin 60^\circ$$

$$\sin B = \frac{\sqrt{3}}{2}$$

$t = 120^\circ, \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\quad t = 60^\circ, \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



So $\angle B = 60^\circ$ or $\angle B = 120^\circ$, possibly two solution case. But if $\angle B$ were 120°,

then $180^\circ = \angle A + \angle B + \angle C$

$$180^\circ = 60^\circ + 120^\circ + \angle C$$

$\angle C = 0^\circ$ impossible!

So one solution case.

$$\boxed{\angle B = 60^\circ}$$

Solve for $\angle C$:

$$180^\circ = \angle A + \angle B + \angle C$$

$$180^\circ = 60^\circ + 60^\circ + \angle C$$

$$\boxed{\angle C = 60^\circ}$$

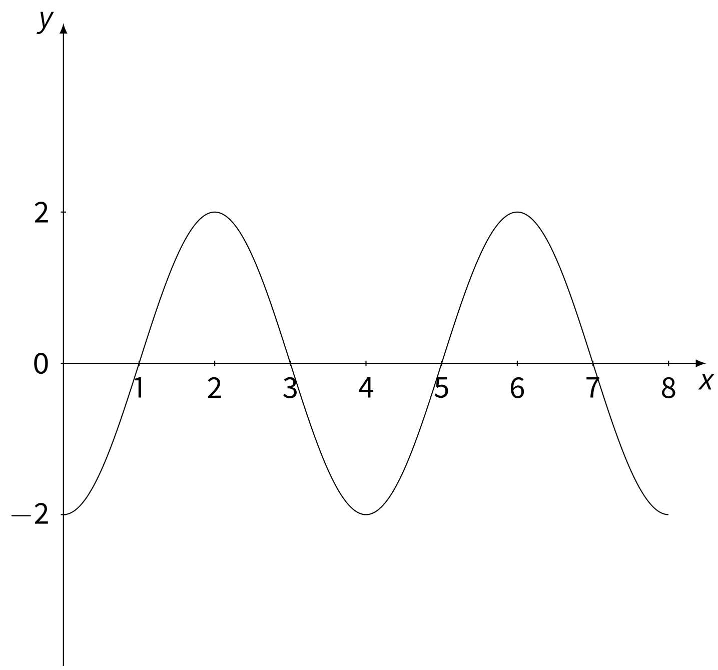
Solve for side c:

$$\frac{\sin 60^\circ}{50} = \frac{\sin 60^\circ}{c}$$

$$c = \cancel{\sin 60^\circ} \cdot \frac{50}{\cancel{\sin 60^\circ}}$$

$$\boxed{c = 50}$$

6. Suppose a mass attached to a spring is moving in simple harmonic motion. The displacement $f(t)$ is shown in the following graph.



Here, t is measured in seconds and $f(t)$ is measured in centimeters.

(a) Find a function $f(t)$ describing the displacement.

$y = a\cos\omega t$ since cos starts at 1, we just transform it.

$\boxed{a=2}$. period is 4. solve for ω. so $\frac{2\pi}{\omega} = 4 \longrightarrow \omega = \frac{2\pi}{4} = \frac{\pi}{2}$.

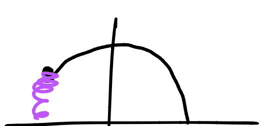
cosine is reflected around x-axis.

$$\boxed{y = -2\cos\left(\frac{\pi}{2}t\right)}$$

(b) How many centimeters is the mass displaced at time $t = \frac{3}{2}$?

$$y = -2\cos\left(\frac{\pi}{2}\cdot\frac{3}{2}\right)$$

$$= -2\cos\left(\frac{3\pi}{4}\right)$$



$$= -2\left(-\cos\left(\frac{\pi}{4}\right)\right)$$

$\bar{t} = \frac{\pi}{4}$, cos − in II

$$= 2\cdot\frac{\sqrt{2}}{2} = \boxed{\sqrt{2}\text{ cm}}$$