

MATH 141: Midterm 1

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Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

Legend

— my work

— or my thoughts while I work

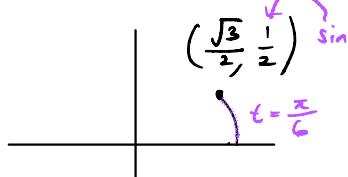
— Common mistakes made to avoid

1. If

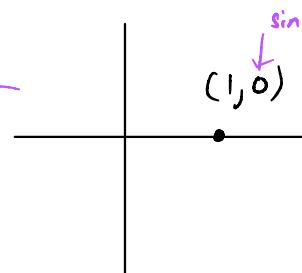
$$f(x) = x^2 - x \quad g(x) = 3x^2 - x + 1 \quad h(x) = \sin(x) \quad j(x) = 2^x$$

Evaluate, expand, and/or simplify the following:

$$(a) h\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$$



$$(b) j(1) \cdot h(0) = 2^1 \cdot \sin(0)$$



$$= 2 \cdot 0$$

$$= \boxed{0}$$

$$(c) f(x) \cdot g(x)$$

two term three term

Don't forget parenthesis when multiplying into ≥ 2 terms!

$$f(x) \cdot g(x) = (x^2 - x)(3x^2 - x + 1) \stackrel{\text{dist law}}{=} x^2(3x^2 - x + 1) + (-x)(3x^2 - x + 1)$$

$$(d) f(x+h) - f(x)$$

$$\begin{aligned} & \stackrel{\text{dist law}}{=} 3x^4 \boxed{-x^3} + \boxed{x^2} \boxed{-3x^3} + \boxed{x^2} - x \\ & = \boxed{3x^4 - 4x^3 + 2x^2 - x} \end{aligned}$$

Since $f(x) = x^2 - x$

look! $x+h$ replaces the "x" visually! Now do it!

$$f(\boxed{x+h}) - f(x) = \underbrace{(x+h)^2 - (x+h)}_{f(x+h)} - \underbrace{(x^2 - x)}_{f(x)}$$

Common mistake:
forgot the parenthesis!

$$\begin{aligned} & \stackrel{\text{expand}}{=} \boxed{x^2} + 2xh + h^2 \boxed{-x} - h \boxed{-x} + x \\ & = 2xh + h^2 - h \end{aligned}$$

$$\begin{aligned} & \stackrel{\text{GCF}}{=} h \boxed{(2x + h - 1)^2} \end{aligned}$$

2. Short answer questions:

- (a) Explain in English the intuition behind the symbols $\lim_{x \rightarrow a} f(x) = L$.

$f(x)$ is the height of the function at an x -value.

So as the x -values approach a but never a itself,
the heights $f(x)$ will get closer and closer to the length of L .

- (b) True or false: We can simplify

$$\frac{3(x-2)^2(x+3) - 4(x+2)(x-3)^2}{5x(x-3)^2(x-2) - 4(x+3)}$$

by crossing out the $x+3$.

No. $(x+3)$ is not a factor in the global context of both the numerator and denominator. The context in which it's a factor is underlined in purple above.

- (c) If $f(x) = x - x^2$, evaluate $f(x+h)$ and fully expand + simplify.

$$f(x+h) = (x+h) - (x+h)^2$$

this replaces x !

Compare the notations:

$$f(x+h)$$

$$f(x)$$

$$= x+h - (x^2 + 2xh + h^2) \stackrel{\text{dist}}{=} x+h - x^2 - 2xh - h^2$$

- (d) If $F(x) = e^{\sin(x^2)}$ find three functions f, g, h where $f \circ g \circ h = F$.

look how $x+h$ took the place of x .

$f(x) = e^x$
$g(x) = \sin(x)$
$h(x) = x^2$

Verifying:

$$(f \circ g \circ h)(x) = f(g(h(x)))$$

deal w/ $h(x)$ first, we know what it is.

$$= f(g(x^2))$$

x^2 takes the place of x in $g(x)$

$$= f(\sin(x^2))$$

$\sin(x^2)$ replaces x in $f(x)$

$$= e^{\sin(x^2)} = F(x) \checkmark$$

Common mistake #1: plugging in 2 into $-x^2 + 1$ is negative law #1.

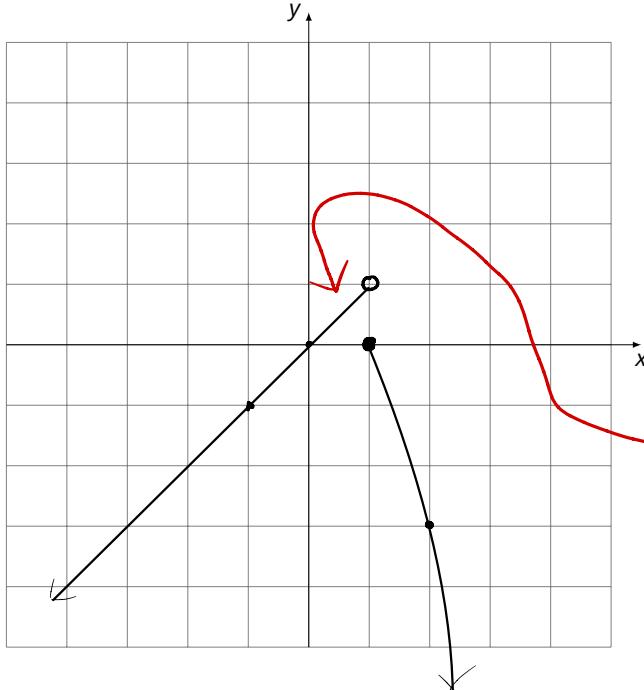
$$\begin{aligned} -2^2 + 1 &= (-1) \cdot 2^2 + 1 \\ &= -4 + 1 \\ &= \boxed{-3} \end{aligned}$$

3. Suppose

$$f(x) = \begin{cases} x & x < 1 \\ -x^2 + 1 & x \geq 1 \end{cases} \quad \text{Common mistake #2: } x < 1 \text{ is}$$

(a) Sketch a graph of $f(x)$.

x	$f(x)$
-1	-1
0	0
1	$-1^2 + 1 = 0$
2	$-2^2 + 1 = -3$
3	$-3^2 + 1 = -8$



(b) What is $f(1)$?

$$f(1) = -1^2 + 1 \stackrel{\text{negative law}}{=} (-1) \cdot 1^2 + 1 = -1 + 1 = \boxed{0}$$

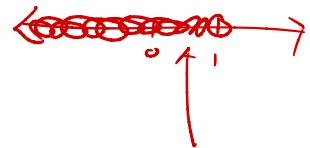
(c) Does $\lim_{x \rightarrow 1} f(x)$ exist? If it does, find the limit. If not, explain why.

No, because looking @ the graph above:

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x = \boxed{1}$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} -x^2 + 1 = \boxed{0}$$

Since $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$, $\lim_{x \rightarrow 1} f(x)$ DNE.



lots of people forgot to take this branch on the interval $[0, 1]$. Which means this part was forgotten.

4. Perform the given instruction. Remember to use the relevant laws/properties and **fully simplify**.

(a) Expand and simplify: $\frac{3(x+h)^2 - 1 - (3x^2 - 1)}{h}$

common mistake:

$$3(x+h)^2 \neq (3x + 3h)^2$$

because $3(x+h)^2 = 3 \cdot (x+h) \cdot (x+h)$

You can only distribute the 3 to one factor of $(x+h)$.

3 multiplies into 3 terms.
Don't forget parenthesis.

$$\frac{\cancel{3} \frac{(x+h)^2}{(x+h)^2} - 1 - (3x^2 - 1)}{h} \stackrel{\text{dist}}{=} \frac{3(x^2 + 2xh + h^2) - 1 - 3x^2 + 1}{h}$$

$$\stackrel{\text{dist}}{=} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \frac{6xh + 3h^2}{h} \stackrel{\text{GCF}}{=} \frac{h(6x + 3h)}{h} \stackrel{\text{law 5}}{=} \boxed{6x + 3h}$$

(b) Rationalize the numerator (remember to simplify): $\frac{\sqrt{x+h} - \sqrt{x}}{h}$

the only technique to square two terms.

$$\frac{\cancel{(A - B)} \cdot (A + B)}{\cancel{\sqrt{x+h} - \sqrt{x}}} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{A^2 - B^2}{h(\sqrt{x+h} + \sqrt{x})} = \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\stackrel{\text{frac}}{=} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

mistake #1

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right)^2 \neq \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h^2}$$

because the numerators are terms and exponents don't interact with terms.

mistake #2

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

forgot denominator
 $\sqrt{x+h} + \sqrt{x}$

Compound fraction.
Technique: get rid of nested denominators by multiplying by LCD of nested denominators.

We begin:

(c) Simplify: $\frac{\frac{2}{x^2+x} - \frac{3}{\sqrt{x}}}{\sqrt{x} + \frac{1}{x}}$

two terms.

$\frac{\frac{2}{x^2+x} - \frac{3}{\sqrt{x}}}{\sqrt{x} + \frac{1}{x}} \cdot \frac{x\sqrt{x}(x+1)}{x\sqrt{x}(x+1)}$

law 1. $\frac{\left(\frac{2}{x(x+1)} - \frac{3}{\sqrt{x}}\right)x\sqrt{x}(x+1)}{\left(\sqrt{x} + \frac{1}{x}\right)x\sqrt{x}(x+1)}$

two terms

distributive law $= \frac{\frac{2}{x(x+1)} \cdot x\sqrt{x}(x+1) - \frac{3}{\sqrt{x}} \cdot x\sqrt{x}(x+1)}{x(\sqrt{x})^2(x+1) + \frac{1}{x} \cdot x\sqrt{x}(x+1)}$

fraction law $\stackrel{1, \text{ then } 5}{=} \frac{2\sqrt{x} - 3x(x+1)}{x^2(x+1) + \sqrt{x^3}(x+1)}$

$= \frac{2\sqrt{x} - 3x^2 - 3x}{x^3 + x^2 + \sqrt{x^3} + \sqrt{x}}$

$x^{\frac{1}{2}} \cdot (x+1) = x^{\frac{1}{2}} \cdot x + x^{\frac{1}{2}}$
 $= x^{\frac{3}{2}} + x^{\frac{1}{2}}$
 $= \sqrt{x^3} + \sqrt{x}$

You could have also factored out $x^{\frac{1}{2}}$ from numerator and denominator, then cancelled.

(d) Expand: $(x^3 + 6)(2x + 1) - (x^2 + x - 2)(3x^2)$

Convert to terms, no parenthesis.

$(x^3 + 6)(2x + 1) - (x^2 + x - 2)(3x^2)$

dist $= (x^3 + 6)2x + (x^3 + 6) \cdot 1 - 3x^4 - 3x^3 + 6x^2$

dist $= 2x^4 + 12x + x^3 + 6 - 3x^4 - 3x^3 + 6x^2$

$= -x^4 - 2x^2 + 6x^2 + 12x + 6$

Common mistake: forgetting to distribute the factor of (-1)

you must pass the Vertical Line Test.

5. Draw the graph of a function which satisfies the following:

- (a) $f(0) = 1$
- (b) $f(2) = 1$
- (c) $\lim_{x \rightarrow 0} f(x) = 1$
- (d) $\lim_{x \rightarrow 2^-} f(x) = 0$
- (e) $\lim_{x \rightarrow 2^+} f(x) = 2$
- (f) $\lim_{x \rightarrow -2} f(x) = -\infty$

