

MATH 141: Quiz 7

Name: key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

1. Given the function

$$f(x) = x^3 - x$$

(a) Find all local minima and maxima.

We use the first derivative test.

(1) crit #'s

$$f'(x) = 3x^2 - 1$$

(a) solve $f'(x) = 0$

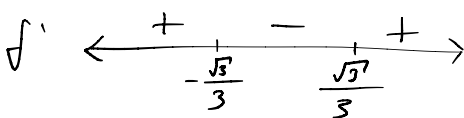
$$3x^2 - 1 = 0$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

(2) sign diagram of $f'(x)$



$$f'(-10) = 3 \cdot (-10)^2 - 1 = +$$

$$f'(0) = 3 \cdot 0^2 - 1 = -$$

$$f'(10) = 3 \cdot 10^2 - 1 = +$$

continued on back...

(b) $f'(x)$ DNE

N/A, $f'(x)$ is a polynomial.

$$\begin{aligned} \therefore \text{local maximum of} \\ f\left(-\frac{\sqrt{3}}{3}\right) &= \left(-\frac{\sqrt{3}}{3}\right)^3 - \left(-\frac{\sqrt{3}}{3}\right) \\ &= -\frac{3\sqrt{3}}{27} + \frac{\sqrt{3}}{3} \cdot \frac{3}{3} \\ &= -\frac{\sqrt{3}}{9} + \frac{3\sqrt{3}}{9} \\ &= \frac{3\sqrt{3} - \sqrt{3}}{9} = \boxed{\frac{2\sqrt{3}}{9}} \end{aligned}$$

$$\begin{aligned} \text{local minimum of} \\ f\left(\frac{\sqrt{3}}{3}\right) &= \left(\frac{\sqrt{3}}{3}\right)^3 - \frac{\sqrt{3}}{3} \\ &= \frac{3\sqrt{3}}{27} - \frac{\sqrt{3}}{3} \cdot \frac{3}{3} \\ &= \frac{\sqrt{3} - 3\sqrt{3}}{9} = \boxed{-\frac{2\sqrt{3}}{9}} \end{aligned}$$

(b) Find the intervals of concavity.

① inflection points

$$f''(x) = 6x$$

② solve $f''(x) = 0$

$$6x = 0 \rightarrow x = 0$$

③ $f'''(x)$ DNE
N/A

② sign diagram of f''

$$f'' \leftarrow \begin{array}{c} - \quad | \quad + \\ 0 \end{array} \rightarrow$$

$$f''(-1) = 6 \cdot (-1) = -$$

$$f''(1) = 6 \cdot (1) = +$$

\therefore concave down on $(-\infty, 0)$
concave up on $(0, \infty)$

2. Use the second derivative test to show $g(x) = x^2$ has a local minimum at $x = 0$.

① Solve $g'(x) = 0$

② test crit # $x = 0$

$$g'(x) = 2x$$

$$g''(x) = 2$$

$$2x = 0$$

$$g''(0) = 2 > 0$$

$$x = 0$$

By the second derivative test
 $f(0) = 0^2 = 0$ is a local minimum