

# MATH 118: Midterm 1 Key

Name: \_\_\_\_\_

Directions:

- \* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- \* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- \* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		<b>50</b>

1. Short answer questions:

(a) True or False: We are allowed to use exponent laws in the following way:

$$\left(\frac{x-2}{x^2y}\right)^2 = \frac{x^2-2^2}{x^4y^2}$$

**False.** The  $x$  and  $2$  in the numerator are terms. Exponents = multiplication = **factors**. Properties for factors do not interact with terms.

There is **only one property** where terms and factors interact. It is the distributive property.

(b) True or false: We can simplify

$$\frac{\overbrace{(x-1)^2x}^{\text{Term}} - \overbrace{1}^{\text{Term}}}{(x-1)}$$

by crossing out the  $(x-1)$  (the one in parentheses).

**False.** The numerator has global terms. Therefore you cannot have global factors. You can only cancel global factors.

(c) Suppose

$$f = x^2 - x \quad g = x(x-1)$$

Expand and simplify  $f - g$ .

Don't forget to distribute the negative to **both terms**.

$$\begin{aligned} f - g &= x^2 - x - x(x-1) \\ &= x^2 - x - x^2 + x \\ &= \boxed{0} \end{aligned}$$

2. Factor and simplify:

Factor means to convert **terms** into **factors**. Do not expand.

(a)  $x^2 - 1$

Two term factoring problem. Can't use GCF.

Using  $A^2 - B^2$  where  $A = x$  and  $B = 1$ , we have

$$\overset{A^2}{x^2} - \overset{B^2}{1} = \boxed{(x - 1)(x + 1)}$$

(b)  $x^3 - 3x^2 - x + 3$

Four term factoring problem. Can't use GCF.

Using grouping:

$$\begin{aligned} x^3 - 3x^2 - x + 3 &= x^2(x - 3) - (x - 3) \\ &= (x - 3)(x^2 - 1) \\ &= \boxed{(x - 3)(x - 1)(x + 1)} \end{aligned}$$

(c)  $\frac{4(2x + 1)(x - 2)^2}{\text{Term}} + \frac{2(x - 2)(2x + 1)^2}{\text{Term}}$

Two term factoring problem. Can use GCF, common factor is  $2(2x + 1)(x - 2)$ . Undo distributive law:

$$\begin{aligned} 4(2x + 1)(x - 2)^2 + 2(x - 2)(2x + 1)^2 &= 2(2x + 1)(x - 2)[2(x - 2) + (2x + 1)] \\ &= 2(2x + 1)(x - 2)(2x - 4 + 2x + 1) \\ &= \boxed{2(2x + 1)(x - 2)(4x - 3)} \end{aligned}$$

3. Expand and simplify:

(a)  $(x + h) - 1 - (x - 1)$

Note that  $(x + h) - 1 = (x + h) + (-1)$ . Do not distribute negatives backwards; the  $-1$  is a different term.

$$\begin{aligned}(x + h) - 1 - (x - 1) &= x + h - 1 - x + 1 \\ &= \boxed{h}\end{aligned}$$

(b)  $3(2x + 1) - 2(x + 1)^2x$

I will expand  $(x + 1)^2 = x^2 + 2x + 1$  first. Remember the parentheses.

$$\begin{aligned}3(2x + 1) - 2(x + 1)^2x &= 6x + 3 - 2(x^2 + 2x + 1)x \\ &= 6x + 3 - 2x^3 - 4x^2 - 2x \\ &= \boxed{-2x^3 - 4x^2 + 4x + 3}\end{aligned}$$

4. Simplify:

(a)  $\frac{x}{2x-1} - \frac{x}{x-1}$

Left fraction missing  $(x-1)$  as a factor, right missing  $(2x-1)$  as a factor. Introduce and don't forget parentheses due to multiplying 2 terms:

$$\begin{aligned}\frac{x}{2x-1} - \frac{x}{x-1} &= \frac{(x-1)}{(x-1)} \cdot \frac{x}{2x-1} - \frac{x}{x-1} \cdot \frac{(2x-1)}{(2x-1)} \\ &= \frac{(x-1)x}{(x-1)(2x-1)} - \frac{x(2x-1)}{(x-1)(2x-1)} \\ &= \frac{(x-1)x - x(2x-1)}{(x-1)(2x-1)} \\ &= \frac{x^2 - x - 2x^2 + x}{(x-1)(2x-1)} \\ &= \frac{-x^2}{(x-1)(2x-1)} \\ &= \boxed{-\frac{x^2}{(x-1)(2x-1)}}\end{aligned}$$

(b)  $\frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h}$

Expand numerator because like terms are created.

$$\begin{aligned}\frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h} &= \frac{2(x^2 + 2xh + h^2) - 3 - 2x^2 + 3}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\ &= \frac{4xh + 2h^2}{h} \\ &= \frac{h(4x + 2h)}{h} \\ &= 4x + 2h \\ &= \boxed{2(2x + h)}\end{aligned}$$

$$(c) \frac{\frac{1}{x-1} - \frac{1}{x+1}}{x+1}$$

Compound fraction. Imagine the numerator is a separate problem.

Left missing (x+1), right missing (x-1).

$$\begin{aligned} \frac{\frac{1}{x-1} - \frac{1}{x+1}}{x+1} &= \frac{\frac{x+1}{x+1} \cdot \frac{1}{x-1} - \frac{1}{x+1} \cdot \frac{x-1}{x-1}}{x+1} \\ &= \frac{\frac{x+1}{(x+1)(x-1)} - \frac{x-1}{(x+1)(x-1)}}{x+1} \\ &= \frac{\frac{x+1 - (x-1)}{(x+1)(x-1)}}{x+1} \\ &= \frac{\frac{x-1 - x+1}{(x+1)(x-1)}}{x+1} \\ &= \frac{\frac{2}{(x+1)(x-1)}}{x+1} \\ &= \frac{2}{(x+1)(x-1)} \cdot \frac{1}{x+1} \\ &= \boxed{\frac{2}{(x+1)^2(x-1)}} \end{aligned}$$

5. Perform the given instruction:

(a) Simplify  $-8^{\frac{2}{3}}$

Memorize the **Compendium**. Follow the definitions of negative and fractional exponent:

$$\begin{aligned}-8^{\frac{2}{3}} &= (-1) \cdot 8^{\frac{2}{3}} \\ &= (-1) \cdot \sqrt[3]{8^2} \\ &= (-1) \cdot \sqrt[3]{(2^3)^2} \\ &= (-1) \cdot \sqrt[3]{32^6} \\ &= (-1) \cdot 2^{\frac{6}{3}} \\ &= (-1) \cdot 2^2 \\ &= \boxed{-4}\end{aligned}$$

(b) Solve the equation

$$8x^2 - 10x = 7$$

Presence of  $x^2$  is a **quadratic equation**  $ax^2 + bx + c = 0$ :

$$\begin{aligned}8x^2 - 10x &= 7 \\ 8x^2 - 10x - 7 &= 0\end{aligned}$$

$a = 8, b = -10$  and  $c = -7$ .

One diagonal product  $2 \cdot -7 = -14$  which is close to  $b$  so try it

$$\begin{array}{cc} 4 & 7 \\ | & \times & | \\ -2 & & 1 \end{array}$$

which works, cross-product and sum is  $-14 + 4 = -10 = b$ . So:

$$\begin{aligned}8x^2 - 10x &= 7 \\ (4x - 7)(2x + 1) &= 0 \\ 4x - 7 &= 0 & 2x + 1 &= 0 \\ 4x &= 7 & 2x &= -1 \\ \boxed{x = \frac{7}{4}} & & \boxed{x = -\frac{1}{2}}\end{aligned}$$

(c) Isolate the variable  $y$ :

$$3xy - 2(xy + y) - x^2 = y$$

Follow the four steps.

$$3xy - 2(xy + y) - x^2 = y$$

$$3xy - 2xy - 2y - x^2 = y$$

Step 1: Expand everything

$$3xy - 2xy - 2y - y = x^2$$

Step 2: Terms with  $y$  on one side

$$xy - 3y = x^2$$

Combine like terms

$$y(x - 3) = x^2$$

Step 3: Factor out  $y$

$$y = \frac{x^2}{x - 3}$$

Step 4: Divide by factor attached to  $y$

Solution:  $y = \frac{x^2}{x - 3}$