Chapter 7 Trigonometric Formulas

Addition and Subtraction Formulas

$$\begin{aligned}
\sin(s+t) &= \sin s\cos t + \cos s\sin t\\
\sin(s-t) &= \sin s\cos t - \cos s\sin t\\
\cos(s+t) &= \cos s\cos t - \sin s\sin t\\
\cos(s-t) &= \cos s\cos t + \sin s\sin t\\
\tan(s+t) &= \frac{\tan s + \tan t}{1-\tan s\tan t}\\
\tan(s-t) &= \frac{\tan s - \tan t}{1+\tan s\tan t}
\end{aligned}$$

Double-Angle Formulas

$$\begin{aligned}
\sin 2x &= 2\sin x\cos x\\
\cos 2x &= \cos^2 x - \sin^2 x\\
&= 1 - 2\sin^2 x\\
&= 2\cos^2 x - 1\\
\tan 2x &= \frac{2\tan x}{1-\tan^2 x}
\end{aligned}$$

Half-Angle Formulas

The $+$ or $-$ sign depends on the quadrant $\frac{u}{2}$ is in.

$$\begin{aligned}
\sin\frac{u}{2} &= \pm\sqrt{\frac{1-\cos u}{2}}\\
\cos\frac{u}{2} &= \pm\sqrt{\frac{1+\cos u}{2}}\\
\tan\frac{u}{2} &= \frac{1-\cos u}{\sin u}\\
&= \frac{\sin u}{1+\cos u}
\end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned}
\sin u\cos v &= \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]\\
\cos u\sin v &= \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]\\
\cos u\cos v &= \frac{1}{2}\left[\cos(u+v) + \cos(u-v)\right]\\
\sin u\sin v &= \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]
\end{aligned}$$

Sum-to-Product Formulas

$$\begin{aligned}
\sin x + \sin y &= 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}\\
\sin x - \sin y &= 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\\
\cos x + \cos y &= 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\\
\cos x - \cos y &= -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}
\end{aligned}$$

Strategies

1. Because
$$\sin(2x) = 2\sin x \cos x$$
you can replace all the x with $2x$ to get
$$\sin(4x) = \sin(2 \cdot 2x) = 2\sin 2x \cos 2x$$
Can do this for even coefficients of x, for example
$$\sin(4x) \qquad \sin(6x) \qquad \sin(8x)$$

2. For odd coefficients of x in $\sin$, for example
$$\sin(3x) \qquad \sin(5x) \qquad \sin(7x)$$
Consider rewriting as
$$\sin(3x) = \sin(2x + x) = \cdots$$
and using an addition identity. This will get you a factor of $\sin(2x)$ which has even coefficient, which then you can use **Strategy 1**.

3. If you have terms of sines and cosines, consider trying the **Sum-to-Product** formulas first.