MATH 118: Midterm 1 Key

Name: _____

Directions:

* ⋆ Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* ⋆ If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* ⋆ Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

    (a) True or False: We are allowed to use exponent laws in the following way:

    $$\left(\frac{x-2}{x^2 y}\right)^2 = \frac{x^2 - 2^2}{x^4 y^2}$$

    False. The x and 2 in the numerator are terms. Exponents = multiplication = **factors**. Properties for factors do not interact with terms.

    There is **only one property** where terms and factors interact. It is the distributive property.

    (b) True or false: We can simplify

    $$\frac{\overbrace{(x-1)^2 x}^{\text{Term}} - \overbrace{1}^{\text{Term}}}{(x-1)}$$

    by crossing out the $(x-1)$ (the one in parentheses).

    False. The numerator has global terms. Therefore you cannot have global factors. You can only cancel global factors.

    (c) Suppose

    $$f = x^2 - x \qquad g = x(x-1)$$

    Expand and simplify $f - g$.

    Don't forget to distribute the negative to **both terms.**

    $$f - g = x^2 - x - x(x-1)$$
    $$= x^2 - x - x^2 + x$$
    $$= \boxed{0}$$

2. Factor and simplify:

Factor means to convert **terms** into **factors**. Do not expand.

(a) $x^2 - 1$

Two term factoring problem. Can't use GCF.

Using $A^2 - B^2$ where $A = x$ and $B = 1$, we have

$$\overset{A^2}{x^2} - \overset{B^2}{1} = \boxed{(x-1)(x+1)}$$

(b) $x^3 - 3x^2 - x + 3$

Four term factoring problem. Can't use GCF.

Using grouping:

$$\begin{aligned}
x^3 - 3x^2 - x + 3 &= x^2(x-3) - (x-3) \\
&= (x-3)(x^2-1) \\
&= \boxed{(x-3)(x-1)(x+1)}
\end{aligned}$$

(c) $\underbrace{4(2x+1)(x-2)^2}_{\text{Term}} + \underbrace{2(x-2)(2x+1)^2}_{\text{Term}}$

Four term factoring problem. Can use GCF, common factor is $2(2x+1)(x-2)$. Undo distributive law:

$$\begin{aligned}
4(2x+1)(x-2)^2 + 2(x-2)(2x+1)^2 &= 2(2x+1)(x-2)\big[2(x-2) + (2x+1)\big] \\
&= 2(2x+1)(x-2)(2x-4+2x+1) \\
&= \boxed{2(2x+1)(x-2)(4x-3)}
\end{aligned}$$

3. Expand and simplify:

(a) $(x + h) - 1 - (x - 1)$

Note that $(x + h) - 1 = (x + h) + (-1)$. Do not distribute negatives backwards; the -1 is a different term.

$$(x + h) - 1 - (x - 1) = x + h - 1 - x + 1$$
$$= \boxed{h}$$

(b) $3(2x + 1) - 2(x + 1)^2 x$

I will expand $(x + 1)^2 = x^2 + 2x + 1$ first. Remember the parentheses.

$$3(2x + 1) - 2(x + 1)^2 x = 6x + 3 - 2(x^2 + 2x + 1)x$$
$$= 6x + 3 - 2x^3 - 4x^2 - 2x$$
$$= \boxed{-2x^3 - 4x^2 + 4x + 3}$$

4. Simplify:

(a) $\dfrac{x}{2x-1} - \dfrac{x}{x-1}$

Left fraction missing $(x-1)$ as a factor, right missing $(2x-1)$ as a factor. Introduce and don't forget parentheses due to multiplying 2 terms:

$$\begin{aligned}
\frac{x}{2x-1} - \frac{x}{x-1} &= \frac{(x-1)}{(x-1)} \cdot \frac{x}{2x-1} - \frac{x}{x-1} \frac{(2x-1)}{(2x-1)} \\
&= \frac{(x-1)x}{(x-1)(2x-1)} - \frac{x(2x-1)}{(x-1)(2x-1)} \\
&= \frac{(x-1)x - x(2x-1)}{(x-1)(2x-1)} \\
&= \frac{x^2 - x - 2x^2 + x}{(x-1)(2x-1)} \\
&= \frac{-x^2}{(x-1)(2x-1)} \\
&= \boxed{-\frac{x^2}{(x-1)(2x-1)}}
\end{aligned}$$

(b) $\dfrac{2(x+h)^2 - 3 - (2x^2 - 3)}{h}$

Expand numerator because like terms are created.

$$\begin{aligned}
\frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h} &= \frac{2(x^2 + 2xh + h^2) - 3 - 2x^2 + 3}{h} \\
&= \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\
&= \frac{4xh + 2h^2}{h} \\
&= \frac{\cancel{h}(4x + 2h)}{\cancel{h}} \\
&= 4x + 2h \\
&= \boxed{2(2x + h)}
\end{aligned}$$

(c) $\dfrac{\dfrac{1}{x-1} - \dfrac{1}{x+1}}{x+1}$

$$\frac{\dfrac{1}{x-1} - \dfrac{1}{x+1}}{x+1} = \frac{\dfrac{x+1}{x+1} \cdot \dfrac{1}{x-1} - \dfrac{1}{x+1} \cdot \dfrac{x-1}{x-1}}{x+1}$$

$$= \frac{\dfrac{x-1}{(x+1)(x-1)} - \dfrac{x+1}{(x+1)(x-1)}}{x+1}$$

$$= \frac{\dfrac{x-1-(x+1)}{(x+1)(x-1)}}{x+1}$$

$$= \frac{\dfrac{x-1-x-1}{(x+1)(x-1)}}{x+1}$$

$$= \frac{\dfrac{-2}{(x+1)(x-1)}}{x+1}$$

$$= \frac{-2}{(x+1)(x-1)} \cdot \frac{1}{x+1}$$

$$= \boxed{-\frac{2}{(x+1)^2(x-1)}}$$

5. Perform the given instruction:

   (a) Simplify $-8^{\frac{2}{3}}$

   Follow the definitions of negative and fractional exponent:

   $$
   \begin{aligned}
   -8^{\frac{2}{3}} &= (-1) \cdot 8^{\frac{2}{3}} \\
   &= (-1) \cdot \sqrt[3]{8^2} \\
   &= (-1) \cdot \sqrt[3]{(2^3)^2} \\
   &= (-1) \cdot \sqrt{32^6} \\
   &= (-1) \cdot 2^{\frac{6}{3}} \\
   &= (-1) \cdot 2^2 \\
   &= \boxed{-4}
   \end{aligned}
   $$

   (b) Solve the equation
   $$8x^2 - 10x = 7$$

   Presence of x^2 is a **quadratic equation** $ax^2 + bx + c = 0$:

   $$
   \begin{aligned}
   8x^2 - 10x &= 7 \\
   8x^2 - 10x - 7 &= 0
   \end{aligned}
   $$

   $a = 8, b = -10$ and $c = -7$.

   One diagonal product $2 \cdot -7 = -14$ which is close to b so try it

   $$
   \begin{array}{ccc}
   4 & & 7 \\
   & \times & \\
   -2 & & 1
   \end{array}
   $$

   which works, cross-product and sum is $-14 + 4 = -10 = b$. So:

   $$
   \begin{aligned}
   8x^2 - 10x &= 7 \\
   (4x - 7)(2x + 1) &= 0
   \end{aligned}
   $$

   $$
   \begin{array}{ll}
   4x - 7 = 0 & \qquad 2x + 1 = 0 \\
   4x = 7 & \qquad 2x = -1 \\
   \boxed{x = \dfrac{7}{4}} & \qquad \boxed{x = -\dfrac{1}{2}}
   \end{array}
   $$

(c) Isolate the variable y:

$$3xy - 2(xy + y) - x^2 = y$$

$$3xy - 2(xy + y) - x^2 = y$$

$$3xy - 2xy - 2y - x^2 = y \qquad \text{Step 1: Expand everything}$$

$$3xy - 2xy - 2y - y = x^2 \qquad \text{Step 2: Terms with y on one side}$$

$$xy - 3y = x^2 \qquad \text{Combine like terms}$$

$$y(x - 3) = x^2 \qquad \text{Step 3: Factor out y}$$

$$y = \frac{x^2}{x - 3} \qquad \text{Step 4: Divide by factor attached to y}$$

Solution: $\boxed{y = \dfrac{x^2}{x - 3}}$