MATH 119: Midterm 1

Name: key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

Conceptual understanding

execution

Common pitfalls

1. Short answer questions:

(a) Suppose you write

$$(x+1)^2 = x^2 + 1^2$$

Using concepts from terms/factors and exponents, explain why this is false.

① The exponent 2 is describing multiplication, meaning the exponent only interacts with factors.

② The 'x' and '1' are terms. Because a mathematical expression is either a term or factor, exponent 2 cannot be distributed to terms.

(b) True or false: We can simplify

$$\frac{(2x+1)^2(x+2)-(x^2+1)^2}{2x+1}$$

by crossing out the $2x+1$.

False, Fraction Law 5 says $\frac{a \cdot \cancel{c}}{b \cdot \cancel{c}} = \frac{a}{b}$

The 'c' is a global factor. In our problem $(2x+1)$ is a local factor in the numerator.

(c) Suppose $f(x)$ is a one-to-one function. If you draw a horizontal line anywhere on the graph, what is the maximum number of times the line will intersect the graph?

Once.

(d) Suppose $f(x) = \tan(x)$. Do

$$g(x) = \tan(\pi - x) \qquad h(x) = \tan(x - \pi)$$

h = shift right π.

have the same horizontal shift? If not, what are both $g(x)$ and $h(x)$'s horizontal shift?

Yes. $g(x) = \tan(\pi - x) = \tan(-x+\pi)$

$= \tan(-(x-\pi))$

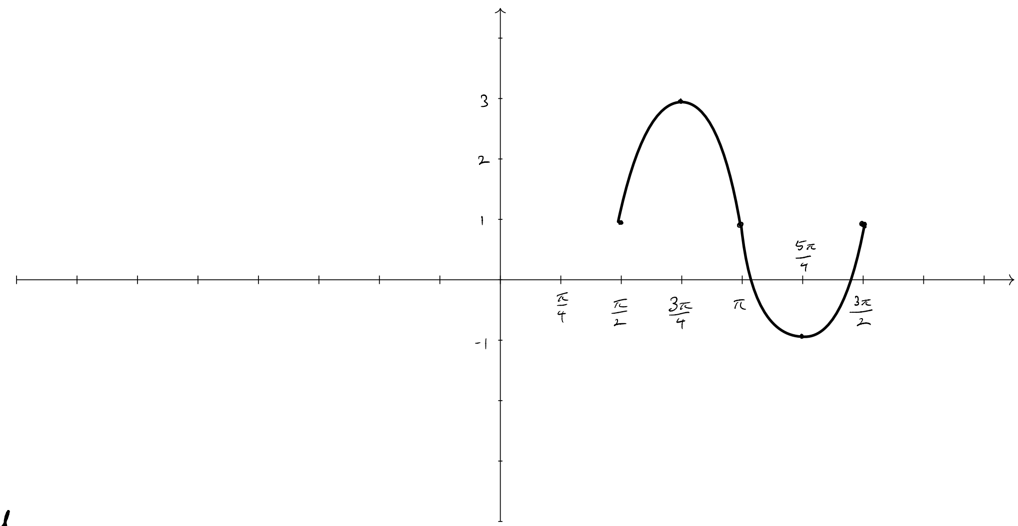
h-shift right π

2. Consider

v shift up 1   v stretch by   h shrink by $\frac{1}{2}$ h shift right $\frac{\pi}{2}$

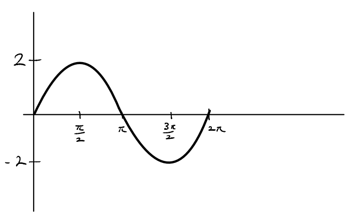
$$f(x) = 1 + 2\sin(2x - \pi) = 1 + 2\sin\left(2\left(x - \frac{\pi}{2}\right)\right)$$

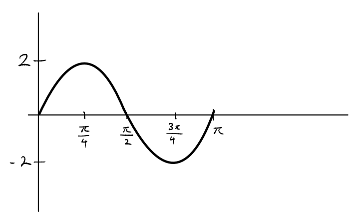
(a) Graph one period of $f(x)$ using transformations. Label the x-axis tick marks you are using.

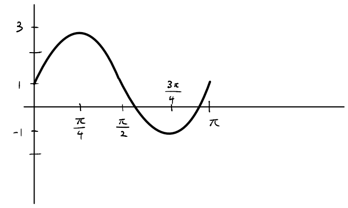


Parent function and

① v stretch by 2 $\rightarrow$ ② h shrink by $\frac{1}{2}$ $\rightarrow$ ③ v shift up 1 $\rightarrow$ ④ h shift right $\frac{\pi}{2}$







$0 + \frac{\pi}{2} = \frac{\pi}{2}$

$\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$

$\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$\frac{3\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4} + \frac{2\pi}{4} = \frac{5\pi}{4}$

$\pi + \frac{\pi}{2} = \frac{2\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$

(b) Evaluate $f(0)$.

$$f(0) = 1 + 2\sin(2 \cdot 0 - \pi)$$
$$= 1 + 2\sin(-\pi)$$
$$= 1 + 2 \cdot 0$$
$$= \boxed{1}$$

3. Consider

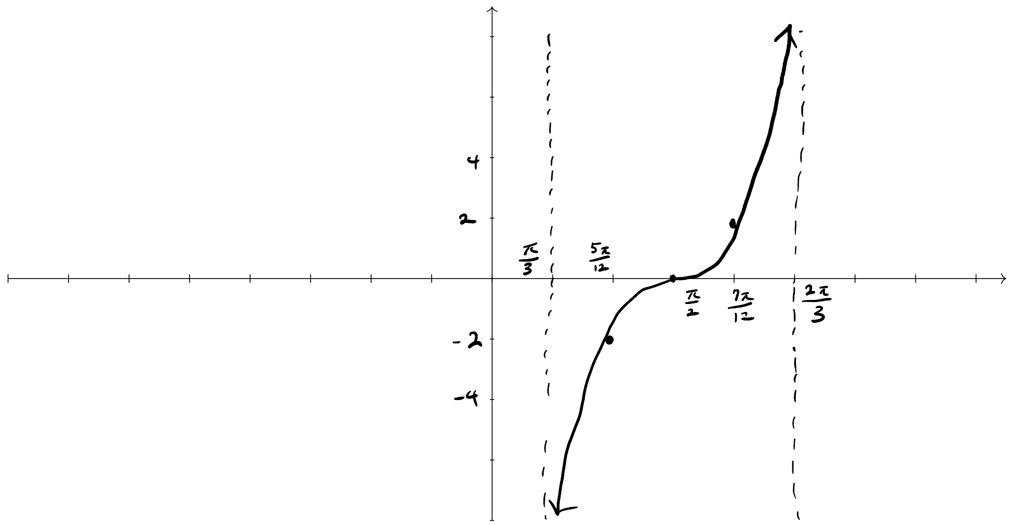
$$f(x) = 2\tan\left(3x - \frac{3\pi}{2}\right) = 2\tan\left(3\left(x - \frac{\pi}{2}\right)\right)$$

v stretch by 2

h shrink by $\frac{1}{3}$

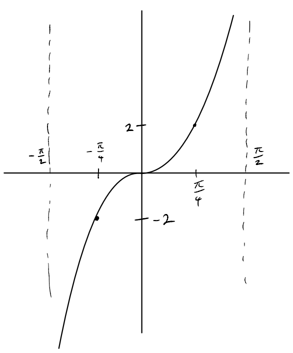
h shift right $\frac{\pi}{2}$

(a) Graph one period of $f(x)$ using transformations. Label the x-axis tick marks you are using.

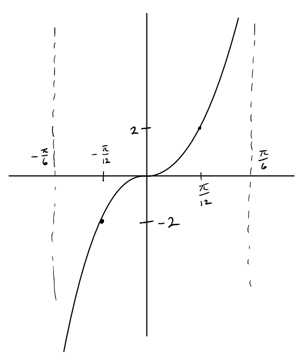


Parent function and

① v stretch by 2



② h shrink by $\frac{1}{3}$



③ h shift right $\frac{\pi}{2}$

$-\frac{\pi}{6} + \frac{\pi}{2} = \frac{-\pi}{6} + \frac{3\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

$-\frac{\pi}{12} + \frac{\pi}{2} = -\frac{\pi}{12} + \frac{6\pi}{12} = \frac{5\pi}{12}$

$0 + \frac{\pi}{2} = \frac{\pi}{2}$

$\frac{\pi}{12} + \frac{\pi}{2} = \frac{\pi}{12} + \frac{6\pi}{12} = \frac{7\pi}{12}$

$\frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi}{6} + \frac{3\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

(b) Evaluate $f\left(\frac{\pi}{2}\right)$.

$$f\left(\frac{\pi}{2}\right) = 2\tan\left(3\cdot\frac{\pi}{2} - \frac{3\pi}{2}\right)$$

$$= 2\tan\left(\frac{3\pi}{2} - \frac{3\pi}{2}\right)$$

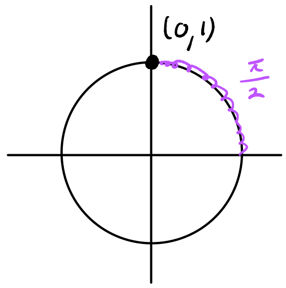
$$= 2\tan(0)$$

$$= 2\cdot 0$$

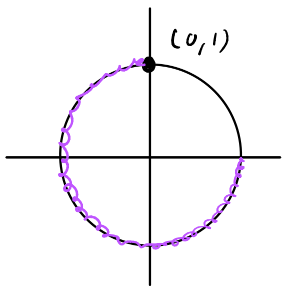
$$= \boxed{0}$$

4. Evaluate the following trigonometric functions:

(a) $\cos\left(\frac{2\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = \boxed{0}$



(b) $\sec\left(\frac{-9\pi}{6}\right) = \sec\left(-\frac{3\pi}{2}\right) = \frac{1}{0}$ $\boxed{\text{undefined}}$



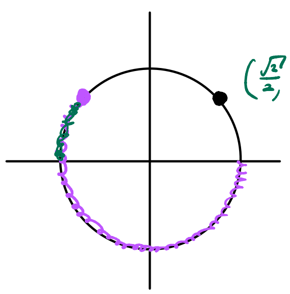
common mistake! can't just ignore.

(c) $\cot\left(\frac{-37\pi}{4}\right) = \cot\left(\frac{-36\pi - \pi}{4}\right) = \cot\left(-\frac{36\pi}{4} - \frac{\pi}{4}\right)$

$= \cot\left(-9\pi - \frac{\pi}{4}\right)$

this you can now ignore

$= \cot\left(-8\pi - \pi - \frac{\pi}{4}\right)$



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, $t = \frac{\pi}{4}$

① $\bar{t} = \frac{\pi}{4}$

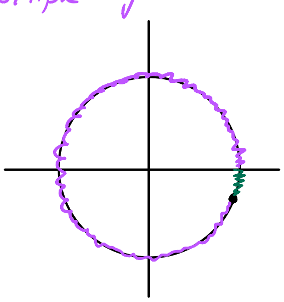
② cot − in II

$= -\cot\left(\frac{\pi}{4}\right)$

$= -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{-1}$

(d) $\sin\left(-123456\pi + \frac{11\pi}{6}\right)$

ignore because multiple of 2π



① $\bar{t} = \frac{\pi}{6}$

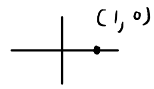
② sin − in IV

$= -\sin\left(\frac{\pi}{6}\right)$

$= \boxed{-\frac{1}{2}}$

5. Let

$$f(t) = \sin(t) \qquad g(t) = \cos(t) \qquad h(t) = \sin^{-1}(t) \qquad k(t) = \cos^{-1}(t)$$

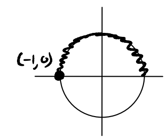


Find the following:

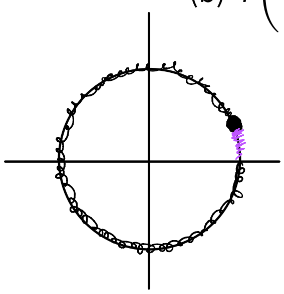
(a) $f(\pi \cdot g(0)) = f(\pi \cdot \cos(0)) = f(\pi \cdot 1)$

$= \sin(\pi)$

$= \boxed{0}$

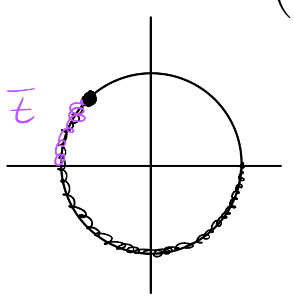


(b) $f\left(\frac{-11\pi}{6}\right) = \sin\left(-\frac{11\pi}{6}\right) = +\sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$



$\bar{t} = \frac{\pi}{6}$, sin + in I

(c) $g\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$



$\bar{t} = \frac{\pi}{4}$, cos − in III

(d) If $f(t) = -\dfrac{4}{5}$ and the terminal point of t is in Quadrant IV, what is $g(t)$?

Using $\sin^2(t) + \cos^2(t) = 1$

$\left(-\frac{4}{5}\right)^2 + \cos^2(t) = 1$

$\frac{16}{25} + \cos^2(t) = 1$

$\cos^2(t) = 1 - \frac{16}{25}$

$\cos^2(t) = \frac{25}{25} - \frac{16}{25}$

$\cos^2(t) = \frac{9}{25}$

$\cos(t) = \pm\sqrt{\frac{9}{25}} = \pm\frac{\sqrt{9}}{\sqrt{25}} = \pm\frac{3}{5}$

cos(t) is + in IV

so $\boxed{\cos(t) = \frac{3}{5}}$