MATH 161: Some Practice Final Problems

Here are problems that cover the last two weeks of our class.

Remember the final is cumulative; you should look at Practice Midterm 1+2 and Midterm 1+2 as well.

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1. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mph passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.

2. Find the following derivatives. You are allowed to use the Differentiation Rules.

   (a) $f(x) = 300$

   (b) $f(x) = 5x^4 - x^2 + 3x$

   (c) $f(x) = \dfrac{\sin^2(x)}{x^2}$

(d) $g(x) = x^2 \cos(x^2)$

(e) $f(x) = \left(\dfrac{x^2 - 1}{x^2 + 3} \right)^4$

3. The following three equations are in implicit form. Find $\dfrac{dy}{dx}$.

(a) $3x^2 + 2y = 2x^4 + 3y^2$

(b) $x^2 - 2xy + y^2 = 5$

(c) $\cos(xy) = 1 + \sin y$

4. Find the absolute minimum and maximum value, if any, of

$$f(x) = \frac{1}{8}x^2 - 4\sqrt{x} \qquad [0, 9]$$

5. Find the derivative of the following functions:

(a) $f(x) = \left(x^2 + \dfrac{1}{e^{-x}}\right)^{3/2}$

(b) $f(x) = (\ln(x))^3$

(c) $f(x) = x^{\sin x}$

6. Find the 4th degree Taylor polynomial of

$$f(x) = \cos(x)$$

at $a = 0$.

7. Short answer questions:

   (a) What do critical numbers tell us when finding absolute minima/maxima?

   (b) Suppose $f(x)$ is continuous on $\mathbb{R}$ and you find $f(1) = -1$ and $f(2) = -3$. Must $f(1)$ be an absolute maximum? Explain your answer with at least one reason.

   (c) What is the 100th degree Taylor polynomial of $f(x) = x$?