MATH 119: Midterm 1

Name: Key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60

work

conceptual understanding

1. Short answer questions:

   (a) Suppose you write

   $$(x+y)^2z^2 = x^2 + y^2z^2$$

   What are the two errors you made?

   ① x and y are terms. can only manipulate exponents (exponent laws) across factors.

   ② everything to the left of z^2 should be encapsulated in parentheses since you are multiplying z^2 into ≥ 2 terms

   (b) True or false: We can simplify

   $$\frac{(x+1)(x-2)+(x-2)(x+3)}{x+1}$$

   by crossing out the $x+1$.

   False because $(x+1)$ is only a local factor in the context of the term $(x+1)(x-2)$.

   (c) Bob has a function $f(x)$. It is not one-to-one. However, he goes ahead and finds the inverse f^{-1}. **What** is the problem with f^{-1} and **why**?

   $f^{-1}(x)$ is not a function. One input gives two outputs.

   (d) Suppose $f(x) = \sin(x)$. Do

   $$g(x) = \sin(x+\pi) \qquad h(x) = \sin(2x+\pi) = \sin\left(2\left(x+\frac{\pi}{2}\right)\right)$$

   have the same horizontal shift? If not, what are both $g(x)$ and $h(x)$'s horizontal shift?

   No. $g(x)$ has a horizontal shift to the left π units.

   $h(x)$ has a horizontal shift to the left $\frac{\pi}{2}$ units.

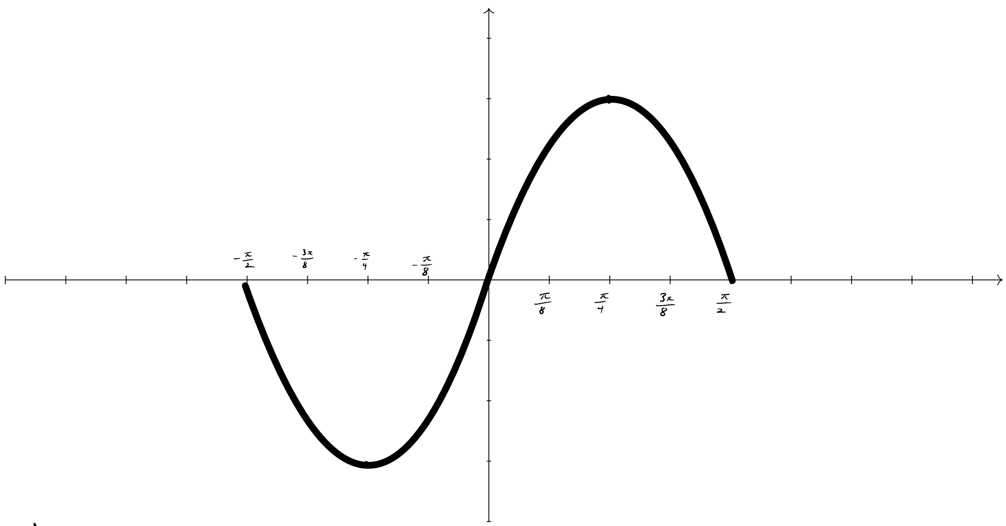
2. Consider

$$f(x) = -3\sin(2x+\pi) = -3\sin\left(2\left(x+\frac{\pi}{2}\right)\right)$$

Do two things:

(a) Graph one period of $f(x)$ using transformations. Label the x-axis tick marks you are using.

(b) Write out the **algebraic list** of transformations **in the order they are performed**.



$g(x) = \sin(x)$

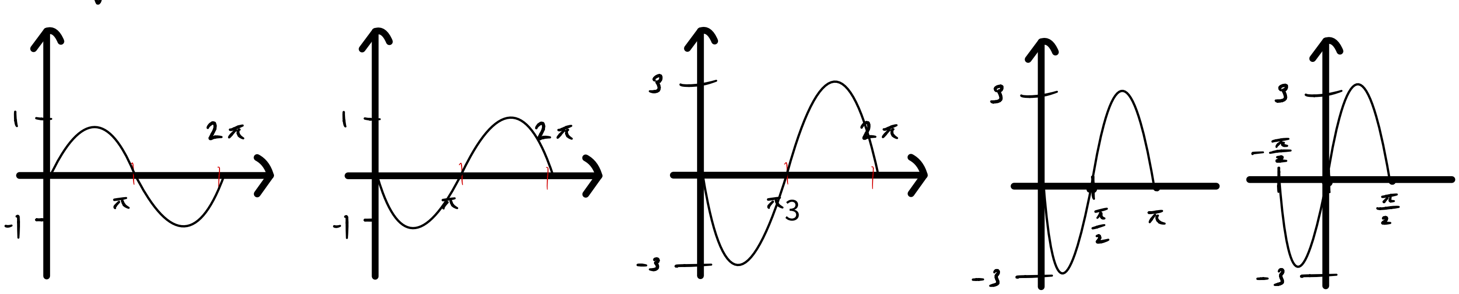
$h(x) = -g(x) \quad = -\sin(x)$ reflection around x-axis

$j(x) = 3h(x) \quad = -3\sin(x)$ v. stretch 3 units

$k(x) = j(2x) \quad = -3\sin(2x)$ h. shrink factor of 1/2

$f(x) = k\left(x+\frac{\pi}{2}\right) \quad = -3\sin\left(2\left(x+\frac{\pi}{2}\right)\right)$ h. shift $\frac{\pi}{2}$ left

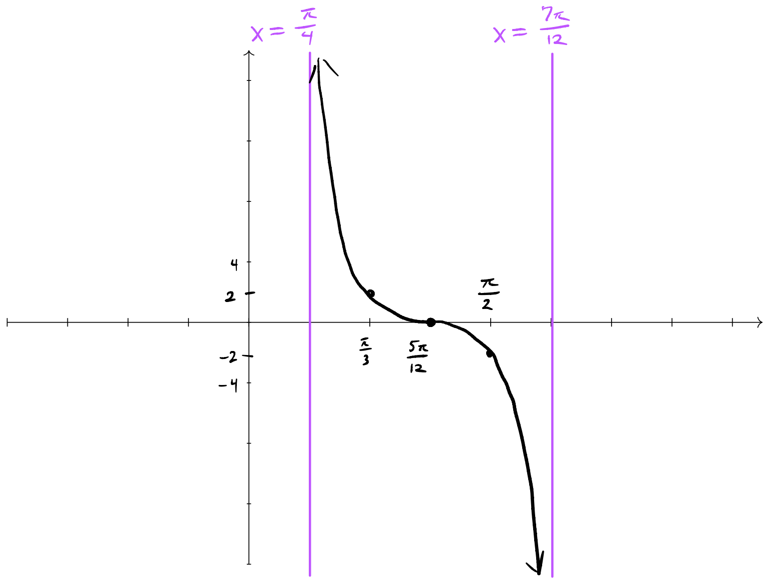
$g(x) \longrightarrow h(x) \longrightarrow j(x) \longrightarrow k(x) \longrightarrow f(x)$



v stretch h shrink h shift

3. Consider
$$f(x) = 2\cot\left(3x - \frac{3\pi}{4}\right) = 2\cot\left(3\left(x - \frac{\pi}{4}\right)\right)$$

Graph one period of $f(x)$ using transformations. Label the x-axis tick marks you are using.



See in class notes.

4. Evaluate the following trigonometric functions:

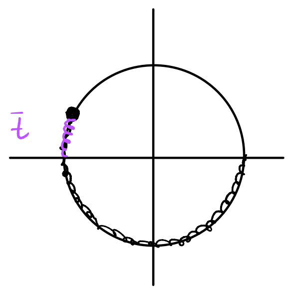
(a) $\sin\left(\frac{5\pi}{4}\right)$ ① $\bar{t} = \frac{\pi}{4}$

② sin negative in III

$$\sin\left(\frac{5\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

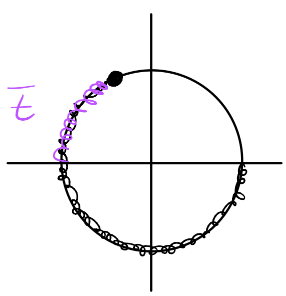
(b) $\cos\left(\frac{-7\pi}{6}\right)$ ① $\bar{t} = \frac{\pi}{6}$

② cos negative in II



$$\cos\left(-\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$$

(c) $\tan\left(\frac{-40\pi}{3}\right) = \tan\left(-\frac{39\pi}{3} - \frac{\pi}{3}\right)$ ① $\bar{t} = \frac{\pi}{3}$



$$= \tan\left(-13\pi - \frac{\pi}{3}\right)$$

② tan negative in II

$$\underbrace{6\cdot(-2\pi)}_{\text{ignore.}} - \pi$$

$$\tan\left(-\frac{40\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right)$$

$$= -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1}$$

$$= \boxed{-\sqrt{3}}$$

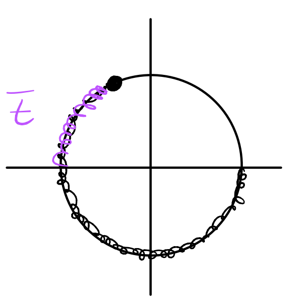
(d) $\csc\left(10000000000000\pi - \frac{4\pi}{3}\right)$

ignore

$$= \csc\left(-\frac{4\pi}{3}\right)$$

① $\bar{t} = \frac{\pi}{3}$

② csc positive in II since sin positive in II



$$\csc\left(-\frac{4\pi}{3}\right) = \csc\left(\frac{\pi}{3}\right) = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \boxed{\frac{2\sqrt{3}}{3}}$$

5. Evaluate the following expressions:

(a) $\tan^{-1}(1) = \boxed{\frac{\pi}{4}}$

because $\tan\left(\frac{\pi}{4}\right) = 1$ and $\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(b) $\tan\left(\sin^{-1} \frac{\sqrt{2}}{2}\right) = \tan\left(\frac{\pi}{4}\right)$

because $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

and $\frac{\pi}{4} \in \left[\frac{\pi}{2}, \frac{\pi}{2}\right)$

$= \boxed{1}$

(c) $\sin^{-1}(2)$ $\boxed{\text{Undefined}}$ because $2 \notin [-1, 1]$

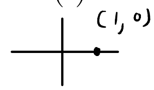
(d) $\sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{\pi}{3}}$

because $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6. Let

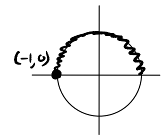
$$f(t) = \sin(t) \qquad g(t) = \cos(t)$$



Find the following:

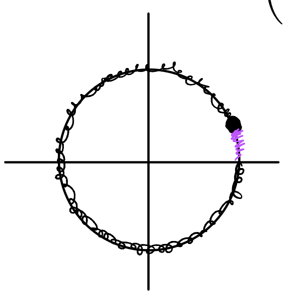
(a) $f(\pi \cdot g(0)) = f\left(\pi \cdot \cos(0)\right) = f(\pi \cdot 1)$

$= \sin(\pi)$



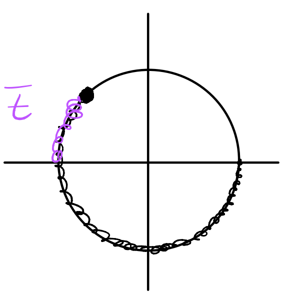
$= \boxed{0}$

(b) $f\left(\frac{-11\pi}{6}\right) = \sin\left(-\frac{11\pi}{6}\right) = +\sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$



$\bar{t} = \frac{\pi}{6}$, sin + in I

(c) $g\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$



$\bar{t} = \frac{\pi}{4}$, cos − in III

(d) If $f(t) = -\frac{4}{5}$ and the terminal point of t is in Quadrant IV, what is $g(t)$?

Using $\sin^2(t) + \cos^2(t) = 1$

$\left(-\frac{4}{5}\right)^2 + \cos^2(t) = 1$

$\frac{16}{25} + \cos^2(t) = 1$

$\cos^2(t) = 1 - \frac{16}{25}$

$\cos^2(t) = \frac{25}{25} - \frac{16}{25}$

$\cos^2(t) = \frac{9}{25}$

$\cos(t) = \pm\sqrt{\frac{9}{25}} = \pm\frac{\sqrt{9}}{\sqrt{25}} = \pm\frac{3}{5}$

cos(t) is + in IV

so $\boxed{\cos(t) = \frac{3}{5}}$