

**How to factor  $ax^2 + bx + c$** 

I'm going to introduce an intuitive way to think about factoring  $ax^2 + bx + c$ . Here is the general setup with an example.

**Example:** Consider  $x^2 - x - 6 = (x - 3)(x + 2)$ . If we focus on their coefficients and arrange those coefficients where each row is one factor, we get the following figure:

$$\begin{array}{cc} 1 & -3 \\ | & | \\ 1 & 2 \end{array}$$

Notice the first row is  $x - 3$  and the second row is  $x + 2$ . We now make the following observations:

1. **Observation 1** Multiplying columnwise gets us back the coefficient of  $x^2$  and  $-6$ :

$$\begin{array}{cc} 1 & -3 \\ | & | \\ 1 & 2 \end{array}$$

We get  $1 \times 1 = 1$  which is the coefficient for  $x^2$  and  $-3 \times 2 = -6$  which is the last number, or  $-6$ .

2. **Observation 2** Cross-multiplying and adding up the numbers gives us the coefficient on  $x$ , or the coefficient of the middle term:

$$\begin{array}{cc} 1 & -3 \\ \times & | \\ 1 & 2 \end{array}$$

We get  $1 \times 2 + 1 \times -3 = -1$  which is the coefficient for  $-x$ .

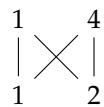
We have found a new way to factor by relating the coefficient of the factors with the coefficient of the original trinomial!

In summary, when given a general trinomial  $ax^2 + bx + c$ :

- \* Multiplying columnwise will give us  $a$  and  $c$ . First column is  $a$ , second column is  $c$ .
- \* Cross-multiplying then adding up gives us  $b$ .

Here are more examples to get familiar with the setup:

\*  $x^2 + 6x + 8$



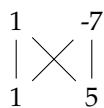
∞ Coefficient for  $x^2$  is 1, which is  $1 \times 1 = 1$  in the figure

∞ Coefficient for 8 is 8, which is  $4 \times 2 = 8$  in the figure

∞ Coefficient for  $6x$  is 6, which is  $1 \times 2 + 1 \times 4 = 6$  in the figure

Then according to the figure, the factors are  $(1x + 4)(1x + 2) = (x + 4)(x + 2)$ .

\*  $x^2 - 2x - 35$



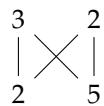
∞ Coefficient for  $x^2$  is 1, which is  $1 \times 1 = 1$  in the figure

∞ Coefficient for  $-35$  is  $-35$ , which is  $-7 \times 5 = -35$  in the figure

∞ Coefficient for  $-2x$  is  $-2$ , which is  $1 \times 5 + 1 \times -7 = -2$  in the figure

Then according to the figure, the factors are  $(1x - 7)(1x + 5) = (x - 7)(x + 5)$ .

\*  $6x^2 + 19x + 10$



∞ Coefficient for  $6x^2$  is 6, which is  $3 \times 2 = 6$  in the figure

∞ Coefficient for 10 is 10, which is  $2 \times 5$  in the figure

∞ Coefficient for  $19x$  is 19, which is  $3 \times 5 + 2 \times 2 = 19$  in the figure

Then according to the figure, the factors are  $(3x + 2)(2x + 5)$ .

This method is much more intuitive because it shows you how the coefficients for  $x$  in each factor must multiply to the coefficient in  $x^2$ . Same for the lone coefficient  $c$ . Lastly, it's showing you exactly what you need to undo in your expansion process to get your coefficient for  $bx$ .