MATH 118: Midterm 1

Name: Key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

(a) Suppose you write

$$(x+y)^2z^2 = x^2 + y^2z^2$$

What are the two errors you made?

① x and y are terms. Cannot use laws of exponents on terms.

② $(x+y)^2$ generates three terms $x^2 + 2xy + y^2$ so z^2 needs to be distributed to each term by the distributive law, not just y^2

(b) True or false: We can simplify

$$\frac{(x+1)(x-2)+(x-2)(x+3)}{x-1}$$

by crossing out the $x-1$'s to become $\frac{(x-2)+(x-2)(x+3)}{1}$.

No because $(x+1)$ is a part of a term in the numerator not a factor in the context of the entire numerator.

Also $x+1$ and $x-1$ are different factors

(c) If $f(x) = x^2 - x$, evaluate $f(-x+h)$ and expand.

do not forget

$$f(-x+h) = (-x+h)^2 - (-x+h)$$

$$= \boxed{x^2 - 2xh + h^2 + x - h}$$

(d) If $i^2 = -1$, what is i^{444}?

$$i^{444} = (i^2)^{222} = (-1)^{222} = \boxed{1}$$

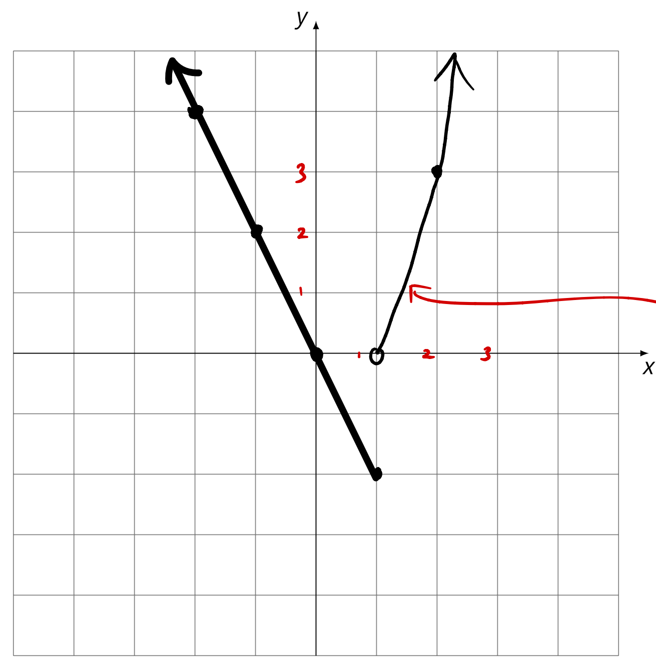
2. Suppose

$$f(x) = \begin{cases} x^2 - 1 & x > 1 \\ -2x & x \le 1 \end{cases}$$

this means graph x^2-1 when x is greater than 1

(a) Sketch a graph of $f(x)$.

x	$f(x)$
-2	4
-1	2
0	0
1	-2
2	3



many of you forgot $x>1$ also includes x-values between 1 and 2

(b) What is $f(1)$?

$$f(1) = -2 \cdot 1 = \boxed{-2}$$

3. Fully simplify or factor the following using relevant properties and laws.

(a) $\left(\frac{x+1}{(x-1)}\right)^2 \cdot \left(\frac{(x-1)(x+1)}{x+2}\right)^{-2}$

1 mistake: not stopping and looking for terms. You can't use LoE on terms. No Law out of the seven we know has any terms involved.

$\overset{6}{=} \left(\frac{x+1}{x-1}\right)^2 \cdot \left(\frac{x+2}{(x-1)(x+1)}\right)^2$

$\overset{5}{=} \frac{(x+1)^2}{(x-1)^2} \cdot \frac{(x+2)^2}{\left(\underbrace{(x-1)(x+1)}_{\text{factors}}\right)^2}$

$\overset{4}{=} \frac{(x+1)^2}{(x-1)^2} \cdot \frac{(x+2)^2}{(x-1)^2(x+1)^2} \underset{\text{law (1)}}{\overset{\text{frac}}{=}} \frac{\cancel{(x+1)}^2 (x+2)^2}{(x-1)^2(x-1)^2\cancel{(x+1)}^2} \underset{\text{frac law (5)}}{\overset{\text{LoE (1)}}{=}} \boxed{\frac{(x+2)^2}{(x-1)^4}}$

(b) Expand: $(x-2)^2(x+3)+(x-3)(x+2)$

$(A-B)^2$

$\underset{\text{dist law}}{\overset{(A-B)^2}{=}} (x^2-4x+4)(x+3) + (x-3)x + (x-3)2$

$\underset{\text{law}}{\overset{\text{dist}}{=}} (x^2-4x+4)x + (x^2-4x+4)3 + x^2-3x+2x-6$

$\underset{\text{law}}{\overset{\text{dist}}{=}} x^3-4x^2+4x+3x^2-12x+12+x^2-x-6$

$= \boxed{x^3-9x+6}$

(c) $\left(\boxed{\dfrac{1}{x^2+4x+4}}-\boxed{\dfrac{2}{(x-2)(x+2)}}\right)^2$

#1 mistake: These are terms. Do not distribute exponents.

Instead, the problem is $\boxed{\text{subtraction of fractions.}}$ Use fraction law 3: get a common denominator.

So we need $\boxed{\text{factors}}$ in the denominator.

$$\left(\frac{1}{\underline{x^2}+\underline{4x}+\underline{4}}-\frac{2}{(x-2)(x+2)}\right)^2 \underset{\text{method}}{\overset{\text{new X}}{=}} \left(\frac{1}{\underbrace{(x+2)(x+2)}_{\text{factors, missing } x-2}}-\frac{2}{\underbrace{(x-2)(x+2)}_{\text{factors, missing } x+2}}\right)^2$$

terms, convert to factors.

1 2

1 2

$$\underset{\text{by 1 for LCD}}{\overset{\text{multiply}}{=}} \left(\frac{x-2}{x-2}\cdot\frac{1}{(x+2)(x+2)}-\frac{2}{(x-2)(x+2)}\cdot\frac{x+2}{x+2}\right)^2$$

$$\underset{1}{\overset{\text{frac law}}{=}} \left(\frac{\overset{A}{\overline{x-2}}}{(x-2)(x+2)^2}-\frac{\overset{B}{\overline{2(x+2)}}}{(x+2)^2(x-2)}\right)^2$$

frac law 3 says all of $2(x+2)$ MUST be subtracted.

$$\underset{3}{\overset{\text{frac law}}{=}} \left(\frac{x-2-2(x+2)}{(x-2)(x+2)^2}\right)^2$$

$$\underset{\text{law}}{\overset{\text{dist}}{=}} \left(\frac{x-2-2x-4}{(x-2)(x+2)^2}\right)^2$$

$=$

$$\underset{\text{like terms}}{\overset{\text{combine}}{=}} \left(\frac{-x-6}{(x-2)(x+2)^2} \right)^2$$

$$\underset{⑤}{\overset{\text{LoE}}{=}} \frac{(-x-6)^2}{\left((x-2)(x+2)^2\right)^2}$$

$$\underset{④}{\overset{\text{LoE}}{=}} \frac{(-x-6)^2}{(x-2)^2\left((x+2)^2\right)^2}$$

$$\underset{③}{\overset{\text{LoE}}{=}} \boxed{\frac{(-x-6)^2}{(x-2)^2(x+2)^4}}$$

goal : $x = \cdots$

4. Find all real-valued solutions for

$$\underbrace{\frac{1}{x-1}} + \underbrace{\frac{1}{x+2}} = \frac{5}{4}$$

Check your work if necessary.

get x out of denominator.

$$(x-1)(x+2)\left(\frac{1}{x-1} + \frac{1}{x+2}\right) = \frac{5}{4}(x-1)(x+2)$$

dist law, expand

$$\cancel{(x-1)}(x+2)\cdot\frac{1}{\cancel{x-1}} + (x-1)\cancel{(x+2)}\cdot\frac{1}{\cancel{x+2}} = \frac{5}{4}(x^2+x-2)$$

frac law ① then ⑤

$$4\cdot(x+2 + x-1) = \frac{5}{4}(x^2+x-2)\cdot 4$$

≥ 2 terms!

mult both sides by 4

$$4(2x+1) = 5(x^2+x-2)$$

dist law

$$8x+4 = 5x^2 + 5x - 10$$

this tells me get into $ax^2+bx+c=0$

$$-8x-4 \qquad\qquad -8x-4$$

$$0 = 5x^2 - 3x - 14$$

$5 \quad 7$

$1 \quad -2$

$\rightarrow -10 + 7 = -3$ ✓

$$(5x+7)\cdot(x-2) = 0$$

$$5x + 7 = 0 \quad , \quad x - 2 = 0$$

$$\downarrow$$

$$\boxed{x = -\frac{7}{5} \quad , \quad x = 2}$$

Check $x = -\frac{7}{5}$

$$LHS = \frac{1}{-\frac{7}{5} - 1} + \frac{1}{-\frac{7}{5} + 2}$$

$$\overset{\text{LCD}}{=} \frac{1}{-\frac{7}{5} - \frac{5}{5}} + \frac{1}{-\frac{7}{5} + \frac{10}{5}}$$

$$\underset{\text{law } 3}{\overset{\text{frac}}{=}} \frac{1}{\frac{-7-5}{5}} + \frac{1}{\frac{-7+10}{5}}$$

$$= \frac{1}{-\frac{12}{5}} + \frac{1}{\frac{3}{5}}$$

$$\overset{\frac{a}{b} = a \cdot \frac{1}{b}}{=} -\frac{5}{12} + \frac{5}{3} \cdot \frac{4}{4}$$

$$\overset{\text{LCD}}{=} -\frac{5}{12} + \frac{20}{12}$$

$$\underset{\text{law } 3}{\overset{\text{frac}}{=}} \frac{-5 + 20}{12}$$

$$= \frac{15}{12}$$

$$= \frac{5 \cdot \cancel{3}}{4 \cdot \cancel{3}} \underset{\text{law } 5}{\overset{\text{frac}}{=}} \frac{5}{4} = RHS \checkmark$$

Check $x = 2$

$$LHS = \frac{1}{2-1} + \frac{1}{2+2}$$

$$= \frac{1}{1} + \frac{1}{4}$$

$$= \frac{4}{4} \cdot 1 + \frac{1}{4}$$

$$\overset{\text{LCD}}{=} \frac{4}{4} + \frac{1}{4}$$

$$\underset{3}{\overset{\text{frac law}}{=}} \frac{4+1}{4}$$

$$= \frac{5}{4} = RHS \checkmark$$

Both $x = -\frac{7}{5}$, $x = 2$ are solutions

Strategy: isolate root and square both sides.

5. Solve for x. Check your work if necessary.

$$\sqrt{2x+1}+1=x$$

$$\sqrt{2x+1} = x-1$$

$$\Big\downarrow \text{square}$$

$$\left(\sqrt{2x+1}\right)^2 = (x-1)^2$$

don't distribute 2 to x and 1! They are terms!!

$$\Big\downarrow (A-B)^2$$

$$2x+1 = x^2-2x+1$$

$$\Big\downarrow ax^2+bx+c=0$$

$$0 = x^2-4x$$

$$\Big\downarrow \text{GCF}$$

$$0 = x\cdot(x-4)$$

$$\Big\downarrow \text{zero product property}$$

$$x=0, \quad x-4=0$$

$$\boxed{x=0, \; x=4}$$

$x=4$ only solution.

$x=0$ extraneous

Check $x=0$

$$\sqrt{2\cdot 0+1}+1=0$$

$$\sqrt{1}+1=0$$

not true.

Check $x=4$

$$\sqrt{2\cdot 4+1}+1=4$$

$$\sqrt{9}+1=4$$

$$3+1=4$$

true!

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