MATH 141: Quiz 4

Name: key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. **Using the limit definition of the derivative,** find the derivative of $g(t) = t^2 + 1$.

   You must use the limit definition, i.e.

   $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   Shortcuts will receive zero credit.

$$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h}$$

$$= \lim_{h \to 0} \frac{(t+h)^2 + 1 - (t^2 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{t^2} + 2th + h^2 \cancel{+1} - \cancel{t^2} + 1}{h}$$

$$= \lim_{h \to 0} \frac{2th + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2t + h)}{\cancel{h}} = \lim_{h \to 0} (2t + h) \overset{\text{continuity}}{=} 2t + 0$$

$$= \boxed{2t}$$

1 pt

2. **Using the limit definition of the derivative,** find the derivative of $f(x) = \frac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

LCD of $\frac{1}{x+h}$ and $\frac{1}{x}$ is $x(x+h)$.

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

frac law 1

$$= \lim_{h \to 0} \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right) \cdot x(x+h)}{h \cdot x \cdot (x+h)}$$

dist

$$= \lim_{h \to 0} \frac{\frac{1}{\cancel{x+h}} \cdot x\cancel{(x+h)} - \frac{1}{\cancel{x}} \cdot \cancel{x}(x+h)}{hx(x+h)}$$

frac law 5

$$= \lim_{h \to 0} \frac{x - (x+h)}{hx(x+h)}$$

dist law

$$= \lim_{h \to 0} \frac{x - x - h}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}x(x+h)}$$

frac law 5

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

continuity

$$= \frac{-1}{x(x+0)}$$

$$= \boxed{-\frac{1}{x^2}}$$

$\boxed{\text{Hw}}$ grade #1

$f(1) = 1^4 + 1 - 3 = -1$

1 pt

$f(2) = 2^4 + 2 - 3 = 15$

1 pt

Because $f(x)$ is continuous, by IVT there must be a $c \in (1,2)$ where $f(c) = 0$ i.e. a root.