MATH 161: Quiz 3

Name: key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Find the following limits:

1. $\lim_{n\to\infty} 5^{-n} = \lim_{n\to\infty} \frac{1^n}{5^n} = \lim_{n\to\infty} \left(\frac{1}{5}\right)^n = 0$

2. $\lim_{n\to\infty} \frac{e^{2n} - e^{n}}{e^{3n} + 1} = \lim_{n\to\infty} \frac{\frac{e^{2n}}{e^{3n}} - \frac{e^{n}}{e^{3n}}}{\frac{e^{3n}}{e^{3n}} + \frac{1}{e^{3n}}}$

$= \lim_{n\to\infty} \frac{e^{-n} - e^{-2n}}{1 + \frac{1^n}{(e^3)^n}}$ Laws of Exponents

$= \lim_{n\to\infty} \frac{\frac{1^n}{e^n} - \frac{1^n}{(e^2)^n}}{1 + \left(\frac{1}{e^3}\right)^n}$ Laws of Exponents

$= \lim_{n\to\infty} \frac{\left(\frac{1}{e}\right)^n + \left(\frac{1}{e^2}\right)^n}{1 + \left(\frac{1}{e^3}\right)^n}$ Laws of Exponents

$= \frac{\lim_{n\to\infty} \left(\frac{1}{e}\right)^n + \lim_{n\to\infty} \left(\frac{1}{e^2}\right)^n}{\lim_{n\to\infty} 1 + \lim_{n\to\infty} \left(\frac{1}{e^3}\right)^n}$ Limit Laws

$= \frac{0 + 0}{1 + 0}$

$= \boxed{0}$

$\frac{1}{e} \approx \frac{1}{2.7} < 1$

$\frac{1}{e^2} \approx \frac{1}{(2.7)^2} < 1$

$\frac{1}{e^3} \approx \frac{1}{(2.7)^3} < 1$

3. $\lim_{x\to\infty} [x^3 - x^2 + x] = \lim_{x\to\infty} \left[x^2(x-1) + x\right]$

$$= \left[\lim_{x\to\infty} x^2\right] \cdot \left[\lim_{x\to\infty} x-1\right] + \left[\lim_{x\to\infty} x\right]$$ Limit Laws

$$= \underbrace{\infty \cdot \infty}_{\downarrow} + \infty$$

$$= \infty + \infty = \boxed{\infty}$$

4. $\lim_{x\to\infty} \dfrac{1-3^x}{e^x - 2^{-x}}$

$$= \lim_{x\to\infty} \frac{\frac{1}{e^x} - \frac{3^x}{e^x}}{\frac{e^x}{e^x} - \frac{2^{-x}}{e^x}}$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{e}\right)^x - \left(\frac{3}{e}\right)^x}{1 - \frac{1}{2^x}\cdot\frac{1}{e^x}}$$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{e}\right)^x - \left(\frac{3}{e}\right)^x}{1 - \left(\frac{1}{2}\right)^x \cdot \left(\frac{1}{e}\right)^x}$$

$$= \frac{\lim_{x\to\infty} \left(\frac{1}{e}\right)^x - \lim_{x\to\infty} \left(\frac{3}{e}\right)^x}{\lim_{x\to\infty} 1 - \left[\lim_{x\to\infty} \left(\frac{1}{2}\right)^x\right]\left[\lim_{x\to\infty} \left(\frac{1}{e}\right)^x\right]}$$

$$= \frac{0 - \infty}{1 - 0}$$

$$= \boxed{-\infty}$$