How to factor $ax^2 + bx + c$

I'm going to introduce an intuitive way to think about factoring $ax^2 + bx + c$. Here is the general setup with an example.

Example: Consider $x^2 - x - 6 = (x-3)(x+2)$. If we focus on their coefficients and arrange those coefficients where each row is one factor, we get the following figure:

$$\begin{array}{cc} 1 & -3 \\ \\ 1 & 2 \end{array}$$

Notice the first row is $x - 3$ and the second row is $x + 2$. We now make the following observations:

1. **Observation 1** Multiplying columnwise gets us back the coefficient of x^2 and -6:

   $$\begin{array}{cc} 1 & -3 \\ \vert & \vert \\ 1 & 2 \end{array}$$

   We get $1 \times 1 = 1$ which is the coefficient for x^2 and $-3 \times 2 = -6$ which is the last number, or -6.

2. **Observation 2** Cross-multiplying and adding up the numbers gives us the coefficient on x, or the coefficient of the middle term:

   $$\begin{array}{ccc} 1 & & -3 \\ & \times & \\ 1 & & 2 \end{array}$$

   We get $1 \times 2 + 1 \times -3 = -1$ which is the coefficient for $-x$.

We have found a new way to factor by relating the coefficient of the factors with the coefficient of the original trinomial!

In summary, when given a general trinomial $ax^2 + bx + c$:

* Multiplying columnwise will gives us a and c. First column is a, second column is c.
* Cross-multiplying then adding up gives us b.

Here are more examples to get familiar with the setup:

* $x^2 + 6x + 8$

$$\begin{array}{ccc} 1 & & 4 \\ | & \times & | \\ 1 & & 2 \end{array}$$

  ∞ Coefficient for x^2 is 1, which is $1 \times 1 = 1$ in the figure

  ∞ Coefficient for 8 is 8, which is $4 \times 2 = 8$ in the figure

  ∞ Coefficient for $6x$ is 6, which is $1 \times 2 + 1 \times 4 = 6$ in the figure

  Then according to the figure, the factors are $(1x + 4)(1x + 2) = (x + 4)(x + 2)$.

* $x^2 - 2x - 35$

$$\begin{array}{ccc} 1 & & \text{-7} \\ | & \times & | \\ 1 & & 5 \end{array}$$

  ∞ Coefficient for x^2 is 1, which is $1 \times 1 = 1$ in the figure

  ∞ Coefficient for -35 is -35, which is $-7 \times 5 = -35$ in the figure

  ∞ Coefficient for $-2x$ is -2, which is $1 \times 5 + 1 \times -7 = -2$ in the figure

  Then according to the figure, the factors are $(1x - 7)(1x + 5) = (x - 7)(x + 5)$.

* $6x^2 + 19x + 10$

$$\begin{array}{ccc} 3 & & 2 \\ | & \times & | \\ 2 & & 5 \end{array}$$

  ∞ Coefficient for $6x^2$ is 6, which is $3 \times 2 = 6$ in the figure

  ∞ Coefficient for 10 is 10, which is 2×5 in the figure

  ∞ Coefficient for $19x$ is 19, which is $3 \times 5 + 2 \times 2 = 19$ in the figure

  Then according to the figure, the factors are $(3x + 2)(2x + 5)$.

This method is much more intuitive because it shows you how the coefficients for x in each factor must multiply to the coefficient in x^2. Same for the lone coefficient c. Lastly, it's showing you exactly what you need to undo in your expansion process to get your coefficient for bx.