MATH 141: Quiz 6

Name: _key_

Directions:

* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

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1. The length of a rectangle is increasing at a rate of 10 m/s and its width is increasing at a rate of 4 m/s. When the length is 30 meters and the width is 8 meters, how fast is the area of the rectangle increasing?

①
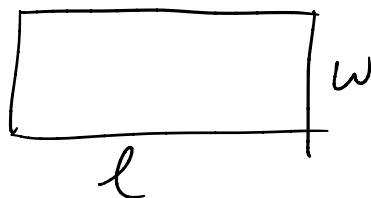
$\boxed{\ell, w}$

take derivative of these and treat as functions of time

② Given

$$\ell = 30m \qquad w = 8m \qquad \frac{d\ell}{dt} = 10 \, m/s \qquad \frac{dw}{dt} = 4m/s$$

Want to solve for

$$\frac{dA}{dt}$$

③ $A = \ell \cdot w$

look at step ①. Use product rule

④ $\frac{d}{dt}[A] = \frac{d}{dt}[\ell \cdot w]$

$$\frac{dA}{dt} = \ell \cdot \frac{d}{dt}[w] + w \cdot \frac{d}{dt}[\ell] = \ell \cdot \frac{dw}{dt} + w \cdot \frac{d\ell}{dt}$$

step ⑥ $= 30 \cdot 4 + 8 \cdot 10 = \boxed{200 \, m^2/s}$

2. Find the absolute maximum and minimum for the function

$$f(x) = x^3 - 6x^2 + 5 \qquad [-3, 5]$$

① values at critical #'s

$$f'(x) = 3x^2 - 12x = 3x(x-4)$$

ⓐ solve $f'(x) = 0$

$$3x(x-4) = 0$$

$$3x = 0 \quad, \quad x - 4 = 0$$

$$x = 0, \quad x = 4$$

ⓑ find where $f'(x)$ DNE.

not applicable, $f'(x)$ is a polynomial; domain IR

Now $f(0) = 0^3 - 6 \cdot 0^2 + 5 = 5$

$$f(4) = 4^3 - 6 \cdot 4^2 + 5 = 64 - 96 + 5 = -27$$

② Values at endpoints:

$$f(-3) = (-3)^3 - 6 \cdot (-3)^2 + 5 = -27 - 54 + 5 = -76$$

$$f(5) = 5^3 - 6 \cdot 5^2 + 5 = 125 - 150 + 5 = -20$$

Since

$$f(0) = 5$$
$$f(4) = -27$$
$$f(-3) = -76$$
$$f(5) = -20$$

Absolute maximum of $f(0) = 5$

Absolute minimum of $f(-3) = 76$