

MATH 118: Midterm 1

Name: key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60

1. Short answer questions:

(a) Suppose you write

$$(x+y)^2 z^2 = x^2 + y^2 z^2$$

What are the two errors you made?

① x and y are terms. can only manipulate exponents (exponent laws) across factors.

② everything to the left of z^2 should be encapsulated in parentheses since you are multiplying z^2 into ≥ 2 terms

(b) True or false: We can simplify $\frac{x^2 + x - 2}{x - 1}$ by crossing out the x 's to become $\frac{x^2 - 2}{-1}$. If not, properly simplify the expression.

False; x is both a term in the context of the entire numerator and denominator.

$$\frac{x^2 + x - 2}{x - 1} = \frac{(x-1) \cdot (x+2)}{(x-1)} = x + 2$$

(c) If $f(x) = \frac{x}{1-x}$, find $f(x^2 - 1)$.

$$f(x^2 - 1) = \frac{x^2 - 1}{1 - (x^2 - 1)} = \frac{x^2 - 1}{x^2 + 2}$$

(d) If $i^2 = -1$, what is i^{531} ?

$$i^{531} = i^{530 + 1}$$

$$= i^{530} \cdot i^1$$

$$= (i^2)^{265} \cdot i$$

$$= (-1)^{265} \cdot i$$

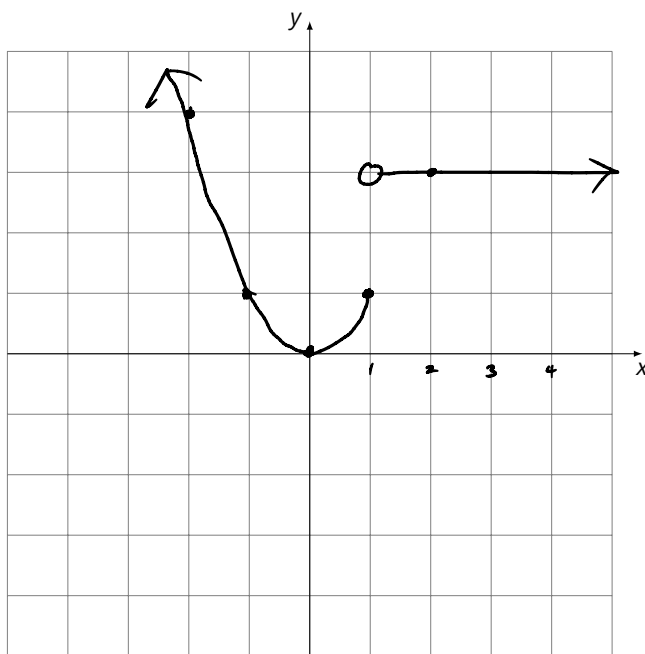
$$= -i$$

2. Suppose

$$f(x) = \begin{cases} 3 & x > 1 \\ x^2 & x \leq 1 \end{cases}$$

(a) Sketch a graph of $f(x)$.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	3



(b) What is $f(1)$?

$$f(1) = 1^2 = 1$$

3. Fully simplify the following using relevant properties and laws.

$$\begin{aligned}
 \text{(a)} \quad \left(\frac{4x^2y}{5z^{-1}} \right)^2 \cdot \frac{1}{x^2z^2} &= \left(\frac{4x^2yz}{5} \right)^2 \cdot \frac{1}{x^2z^2} \\
 &= \frac{(4x^2yz)^2}{5^2} \cdot \frac{1}{x^2z^2} \\
 &= \frac{4^2 \cdot (x^2)^2 \cdot y^2 \cdot z^2}{25} \cdot \frac{1}{x^2z^2} \\
 &= \frac{16 \cancel{x^4}^{\cancel{x^2}} y^2 \cancel{z^2}}{25 \cancel{x^2}^{\cancel{x^2}} \cancel{z^2}} = \boxed{\frac{16x^2y^2}{25}}
 \end{aligned}$$

$A^2 - B^2$, *terms*

$$\text{(b)} \quad \left(\frac{1}{x^2-1} - \frac{2}{x-1} \right)^2$$

*subtraction of fractional expressions.
Find LCD. New factors.*

factors.

$$= \left(\frac{1}{(x-1)(x+1)} - \frac{2}{x-1} \right)^2$$

$$= \left(\frac{1}{(x-1)(x+1)} - \frac{2}{x-1} \cdot \frac{(x+1)}{(x+1)} \right)^2$$

$$= \left(\frac{1}{(x-1)(x+1)} - \frac{2(x+1)}{(x-1)(x+1)} \right)^2$$

$$= \left(\frac{1 - 2x - 2}{(x-1)(x+1)} \right)^2$$

$$= \left(\frac{1 - 2x}{(x-1)(x+1)} \right)^2$$

$$= \frac{(1-2x)^2}{((x-1)(x+1))^2} = \boxed{\frac{(1-2x)^2}{(x-1)^2(x+1)^2}}$$

4. Given $ax - bx(c + d) - ex = gx$, isolate x .

$$ax - bcx - bdx - ex = gx$$

$$ax - bcx - bdx - ex - gx = 0$$

$$\frac{x \cdot (a - bc - bd - e - g)}{a - bc - bd - e - g} = \frac{0}{a - bc - bd - e - g}$$

$$\boxed{x = 0}$$

5. Solve for x. Check your work if necessary.

$$x+1 = \sqrt{5-x}$$

$(A+B)^2$ → $(x+1)^2 = (\sqrt{5-x})^2$

$$x^2 + 2x + 1 = 5 - x$$

$\begin{matrix} 1 & 4 \\ 1 & -1 \end{matrix}$ → $x^2 + 3x - 4 = 0$

$$(x+4)(x-1) = 0$$

$$x+4=0, \quad x-1=0$$

$$x = -4, \quad x = 1$$

check $x = -4$

$$-4+1 = \sqrt{5-(-4)}$$

$$-3 = \sqrt{9}$$

$$-3 = 3$$

extraneous

check $x = 1$

$$1+1 = \sqrt{5-1}$$

$$2 = \sqrt{4}$$

$$2 = 2 \quad \checkmark$$

Solution

$$x = 1$$

6. Fully factor and simplify

$$(x^3 + x^2 + x + 1)^2 - 2(x^3 + x^2 + x + 1) + 1$$

Let $y = x^3 + x^2 + x + 1$. Substituting:

$$y^2 - 2y + 1 \longrightarrow \begin{array}{cc} 1 & -1 \\ 1 & -1 \end{array}$$

$$= (y - 1)^2$$

$$= (x^3 + x^2 + x + 1 - 1)^2$$

$$= \boxed{(x^3 + x^2 + x)^2}$$