MATH 141: Quiz 5

Name: key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Find the derivatives of the following functions:

1. $x^3 + 4x^2 + \sqrt{x} - 1$ $= x^3 + 4x^2 + x^{\frac{1}{2}} - 1$

$$f'(x) = \frac{d}{dx} x^3 + 4 \frac{d}{dx} x^2 + \frac{d}{dx} x^{\frac{1}{2}} - \frac{d}{dx} 1$$

$$= 3x^2 + 4 \cdot 2x + \frac{1}{2} x^{-\frac{1}{2}} - 0$$

$$= 3x^2 + 8x + \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \boxed{3x^2 + 8x + \frac{1}{2\sqrt{x}}}$$

2. $\sin(x) \cdot \cos(x)$

$$\frac{d}{dx}\Big[\overbrace{\sin(x)}^{f(x)} \cdot \overbrace{\cos(x)}^{g(x)}\Big] \overset{\text{product rule}}{=} \overbrace{\sin(x) \cdot \frac{d}{dx}[\cos(x)]}^{f(x) \cdot g'(x)} + \overbrace{\cos(x) \cdot \frac{d}{dx}[\sin(x)]}^{g(x) \cdot f'(x)}$$

$$= \sin(x) \cdot (-\sin(x)) + \cos(x) \cdot \cos(x)$$

$$= \boxed{\cos^2(x) - \sin^2(x)}$$

3. $\dfrac{\boxed{x^2+1}}{\boxed{x^2-1}}$ $f(x)$ / $g(x)$ Quotient Rule.

$g(x) \cdot f'(x) - f(x) \cdot g'(x)$

$$\frac{d}{dx}\left[\frac{x^2+1}{x^2-1}\right] = \frac{(x^2-1)\cdot\frac{d}{dx}\left[x^2+1\right] - (x^2+1)\cdot\frac{d}{dx}\left[x^2-1\right]}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\cdot 2x - (x^2+1)\,2x}{(x^2-1)^2}$$

dist

$$= \frac{\cancel{2x^3} - 2x \;\; \cancel{-2x^3} - 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2} = \boxed{-\frac{4x}{(x^2-1)^2}}$$

4. $\sqrt[2]{1+\cos x}$

$= (1+\cos(x))^{\frac{1}{2}}$ Chain Rule

$$\frac{d}{dx}\left[\left(1+\cos(x)\right)^{\frac{1}{2}}\right] = \frac{1}{2}\left(1+\cos(x)\right)^{\frac{1}{2}-1} \cdot \frac{d}{dx}\left[1+\cos(x)\right]$$

$$= \frac{1}{2}\left(1+\cos(x)\right)^{-\frac{1}{2}} \cdot \left(0-\sin(x)\right)$$

$$= \frac{1}{2}\left(-\sin(x)\right)\cdot\left(1+\cos(x)\right)^{-\frac{1}{2}}$$

frac law #1, $a^{-n} = \frac{1}{a^n}$

$$= \boxed{-\frac{\sin(x)}{2\sqrt{1+\cos(x)}}}$$

$\boxed{hw}$ grade #3

$$g(r) = \sqrt{r} + \sqrt[3]{r} = r^{\frac{1}{2}} + r^{\frac{1}{3}}$$

$$g'(r) = \boxed{\frac{1}{2} r^{-\frac{1}{2}} + \frac{1}{3} r^{-\frac{2}{3}}}$$ 1pt

$$g''(r) = \frac{1}{2} \cdot \left(-\frac{1}{2}\right) r^{-\frac{3}{2}} + \frac{1}{3} \left(-\frac{2}{3}\right) \cdot r^{-\frac{5}{3}}$$

$$= \boxed{-\frac{1}{4} r^{-\frac{3}{2}} - \frac{2}{9} r^{-\frac{5}{3}}}$$ 1pt