MATH 141: Quiz 3

Name: key

Directions:

* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

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1. Find the limit or state it does not exist:

$$\lim_{x\to 2} \frac{\sqrt{4x+1}-3}{x-2}$$

By limit laws $\lim_{x\to 2} \frac{\sqrt{4x+1}-3}{x-2} = \frac{\sqrt{4\lim_{x\to 2} x + \lim_{x\to 2} 1} - \lim_{x\to 2} 3}{\lim_{x\to 2} x - \lim_{x\to 2} 2}$

$$= \frac{\sqrt{4\cdot 2 + 1} - 3}{2-2}$$

$$= \frac{\sqrt{9}-3}{0}$$

$$= \frac{0}{0}$$

1 pt

Now remove $(x-2)$ by rationalizing numerator:

$$\lim_{x\to 2} \frac{\overbrace{\sqrt{4x+1}}^{A} - \overbrace{3}^{B}}{x-2} \cdot \frac{\overbrace{\sqrt{4x+1}}^{A} + \overbrace{3}^{B}}{\sqrt{4x+1}+3} = \lim_{x\to 2} \frac{4x+1-3^2}{(x-2)(\sqrt{4x+1}+3)}$$

$$= \lim_{x\to 2} \frac{4x-8}{(x-2)(\sqrt{4x+1}+3)}$$

$$= \lim_{x\to 2} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{4x+1}+3)}$$

$$= \lim_{x\to 2} \frac{4}{\sqrt{4x+1}+3}$$

lim laws $= \frac{4}{\sqrt{4\cdot 2+1}+3}$

$$= \frac{4}{\sqrt{9}+3}$$

$$= \frac{4}{3+3} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

2. Given

$$f(x) = \begin{cases} x^2 & x \leq 1 \\ x - 1 & x > 1 \end{cases}$$

use the **three-part definition of continuity** to determine whether $f(x)$ is continuous at $x = 1$.

① $f(1) = 1^2 = 1$

② for $\lim_{x \to 1} f(x)$:

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x-1) = 1-1 = 0$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1^2 = 1$$

violates condition 2 because $\lim_{x \to 1} f(x)$ DNE.

$\therefore$, $f(x)$ is not continuous at $x = 1$.

3. How would you define $f(1)$ in the function

$$f(x) = \frac{x^2 - 1}{x - 1}$$

in order to make $f(x)$ continuous at $x = 1$?

$\lim_{x \to 1} f(x) = f(1)$ to be continuous at $x = 1$.

So $f(1) = \lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} \longrightarrow \frac{0}{0}$, cancel out $x-1$ from $\lim_{x \to 1}$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} (x+1) = 1 + 1 = \boxed{2}$$

$\therefore \boxed{f(1) = 2}$