MATH 161: Midterm 1

Name: ______________________________

Directions: No calculators. **Simplify all expressions + show all logical steps for full credit**. If you have a question, ask me. Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

(a) Find a formula for a_n for the following sequence: $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots$

$$a_n = (-1)^{n+1} \cdot \frac{1}{3^{n-1}}$$

n starts at 1 unless otherwise stated

(b) Suppose

$$\lim_{x\to 0+} = 0 \qquad \text{and} \qquad \lim_{x\to 0-} = 0.0001$$

Does $\lim_{x\to 0} f(x)$ exist? Why or why not?

no; $\lim_{x\to 0^+} f(x)$ must be equal to $\lim_{x\to 0^-} f(x)$ for $\lim_{x\to 0} f(x)$ to exist.

(c) What is the domain of $f(x) = \sqrt{x}$?

$[0, \infty)$

(d) Simplify the following: $\left(\frac{4^{-3}(xy^2)^3}{(3x^{-2}y)^4}\right)^3 = \left(\frac{x^3 y^6}{4^3 3^4 x^{-8} y^4}\right)^3$

$$= \left(\frac{x^{11} y^2}{4^3 3^4}\right)^3$$

$$= \boxed{\frac{x^{33} y^6}{4^9 3^{12}}}$$

2. Solve the following equations for x:

(a) $e^{2x} - 3e^{x} + 2 = 0$

$e^{x} \cdot e^{x}$

Let $y = e^{x}$. Then substituting:

$y^2 - 3y + 2 = 0$

$1 \quad -2$
$1 \quad -1$

$(y-2)(y-1) = 0$

$y = 2, \quad y = 1$

$e^{x} = 2 \quad e^{x} = 1$

$\ln e^{x} = \ln 2, \quad \ln e^{x} = \ln 1$

$\boxed{x = \ln 2}, \quad \boxed{x = \ln 1 = 0}$

(b) $\ln(3x - 10) = 2$

$e^{\ln(3x-10)} = e^{2}$

$3x - 10 = e^{2}$

$3x = e^{2} + 10$

$$\boxed{x = \frac{e^{2} + 10}{3}}$$

3. Determine whether the sequence is convergent or divergent. If it is convergent, find what the limit converges to.

$$a_n = \frac{5^n}{5+5^n}$$

naively using limit laws:

$$\lim_{n\to\infty} a_n = \frac{\lim_{n\to\infty} 5^n}{\lim_{n\to\infty} 5 + \boxed{\lim_{n\to\infty} 5^n}} = \frac{\infty}{5+\infty} = \frac{\infty}{\infty}$$

divide by largest infinity to get rid of this

So,

$$\lim_{n\to\infty} \frac{5^n}{(5+5^n)} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{\frac{5^n}{5^n}}{\frac{5}{5^n} + \frac{5^n}{5^n}} = \lim_{n\to\infty} \frac{1}{5 \cdot \frac{1}{5^n} + 1}$$

don't forget parenthesis due to 2+ terms

$$= \frac{\lim_{n\to\infty} 1}{5 \lim_{n\to\infty} \frac{1}{5^n} + \lim_{n\to\infty} 1}$$

after preprocessing we now apply limit laws

$$= \frac{1}{5 \cdot 0 + 1} = \boxed{1}$$

4. Find the following limits:

(a) $\lim_{x\to\infty}\left[\sqrt{4x^2+1}-2x\right]$

Multiply by conjugate radical because using limit laws gives $\infty - \infty$.

$$\lim_{x\to\infty}\left(\sqrt{4x^2+1}-2x\right)\cdot\frac{\sqrt{4x^2+1}+2x}{\sqrt{4x^2+1}+2x} = \lim_{x\to\infty}\frac{4x^2+1-4x^2}{\sqrt{4x^2+1}+2x}$$

$$= \lim_{x\to\infty}\frac{1}{\sqrt{4x^2+1}+2x}$$

$$= \frac{1}{\sqrt{4\lim_{x\to\infty}x^2+\lim_{x\to\infty}1}+2\lim_{x\to\infty}x} = \frac{1}{\infty+\infty} = \frac{1}{\infty} = \boxed{0}$$

adding is fine. $\infty - \infty$ is not

(b) $\lim_{x\to\infty}\frac{1-e^x}{1+2\boxed{e^x}}$

divide by largest ∞ under the limit.

$$\lim_{x\to\infty}\frac{(1-e^x)}{(1+2e^x)}\cdot\frac{\frac{1}{e^x}}{\frac{1}{e^x}} = \lim_{x\to\infty}\frac{\frac{1}{e^x}-1}{\frac{1}{e^x}+2}$$

$$= \lim_{x\to\infty}\frac{\frac{1^x}{e^x}-1}{\frac{1^x}{e^x}+2} = \frac{\lim_{x\to\infty}\left(\frac{1}{e}\right)^x - \lim_{x\to\infty}1}{\lim_{x\to\infty}\left(\frac{1}{e}\right)^x+\lim_{x\to\infty}2}$$

$$= \frac{0-1}{0+2} = \boxed{-\frac{1}{2}}$$

5. Draw a graph that satisifies the following:

$\lim_{x \to 3^+} f(x) = 4$ $\lim_{x \to 3^-} f(x) = 2$ $\lim_{x \to -2} f(x) = 2$ $f(3) = 3$ $f(-2) = 1$