MATH 118: Midterm 2

Name: key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) Given the function

   $$F(x) = \sqrt{x^2 + 1}$$

   find two functions f, g where $f \circ g = F$. You are not allowed to choose $f(x) = x$ or $g(x) = x$.

   $f(x) = \sqrt{x}$

   $g(x) = x^2 + 1$

   check: $(f \circ g)(x) = f(g(x)) = f(x^2+1)$

   $= \sqrt{x^2+1}$

   $= F(x)$ ✓

   (b) True or False: Whenever you see a negative square root, such as $\sqrt{-r}$, you should immediately pull out the $-$ and write $i \cdot \sqrt{r}$.

   True.

   (c) True or False: If $f(1) = 2$ and $f(1) = 3$, then f is considered a function.

   False, the input "1" is sent to two different inputs.

   (d) True or False: If $f(x) = x^2$, then $f(x + h) = x^2 + h$. If not, what should it be instead?

   False. The $x+h$ replaces the x in $f(x)$ (literally).

So doing that:

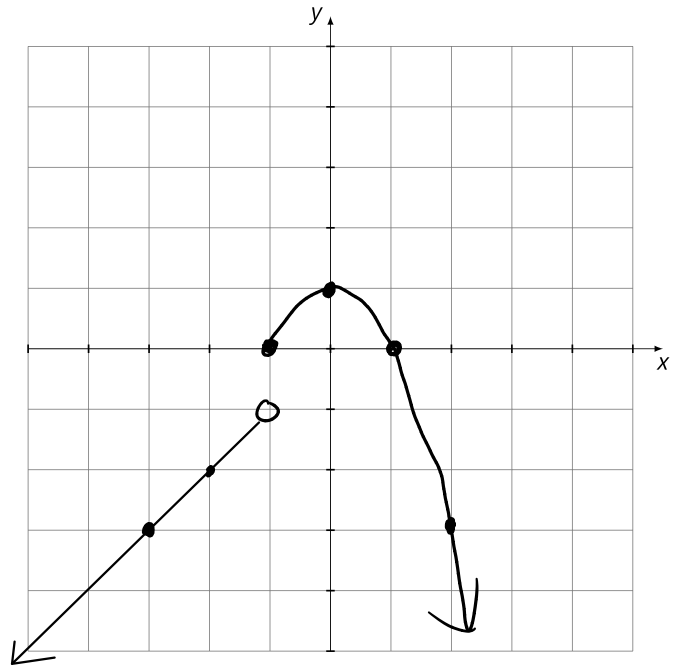
$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

2. Suppose

$$f(x) = \begin{cases} x & x < -1 \\ -x^2 + 1 & x \geq -1 \end{cases}$$

(a) Sketch a graph of $f(x)$.

x	$f(x)$
-3	-3
-2	-2
-1	$-(-1)^2 + 1 = -1 + 1 = 0$
0	$-0^2 + 1 = 1$
1	$-1^2 + 1 = 0$



(b) What is $f(-1)$?

$$f(-1) = -(-1)^2 + 1$$

$$= (-1) \cdot (-1)^2 + 1$$

$$= -1 + 1$$

$$= 0$$

remember to use negative law #1 to see how many negatives you have.

3. Perform the given instruction.

(a) Isolate $x + 4 = 3x - 8,$ for x

$$x + 4 = 3x - 8$$ terms w/ x on same side

$$4 + 8 = 3x - x$$

$$12 = 2x$$ remove factor in front of x

$$\boxed{6 = x}$$

(b) Isolate $4xy - 2(3x - 4xy) = 3x + 1,$ for x

① collect terms with x on one side; need to expand

$$4xy - 6x + 8xy = 3x + 1$$

$$12xy - 6x - 3x = 1$$

$$12xy - 9x = 1$$ now convert x into a factor

$$x(12y - 9) = 1$$ divide by factor attached to x.

$$\boxed{x = \frac{1}{12y - 9}}$$

(c) Solve the equation

$$\sqrt{3+x} = \sqrt{x^2+1}$$

for x.

Square both sides to rescue x from root.

$$\left(\sqrt{3+x}\right)^2 = \left(\sqrt{x^2+1}\right)^2$$

$$3 + x = x^2 + 1$$

quadratic, put into form $ax^2+bx+c=0$

$$x^2 - x + 1 - 3 = 0$$

$$x^2 - x - 2 = 0$$

1 -2

1 1

$$(x-2)(x+1) = 0$$

$$x - 2 = 0 \qquad x + 1 = 0$$

$$\boxed{x = 2, \quad x = -1}$$

Now check:

$\underline{x=2}$

$$\sqrt{3+2} = \sqrt{2^2+1}$$

$$\sqrt{5} = \sqrt{5} \checkmark$$

$\underline{x=-1}$

$$\sqrt{3-1} = \sqrt{(-1)^2+1}$$

$$\sqrt{2} = \sqrt{1+1}$$

$$\sqrt{2} = \sqrt{2} \checkmark$$

$$\boxed{\therefore \text{ solutions are } x = 2, \ x = -1}$$

4. Perform the given instruction. $g(x) = -2\sqrt{-3(x-2)}$

(a) Suppose $g(x) = -2\sqrt{-3x+6}$. Write out either the blueprint of transformations or state in English the order of transformations you would use to transform $f(x) = \sqrt{x}$ into $g(x)$.

Formula	Description
$f(x) = \sqrt{x}$	parent
$h(x) = 2 \cdot f(x) = 2\sqrt{x}$	v. stretch by 2 units
$i(x) = -h(x) = -2\sqrt{x}$	reflection around x-axis
$j(x) = i(-x) = -2\sqrt{-x}$	reflection around y-axis
$k(x) = j(3x) = -2\sqrt{-3x}$	horizontal shrink by 1/3 units
$f(x) = k(x-2) = -2\sqrt{-3(x-2)} = -2\sqrt{-3x+6}$	horizontal shift to the right 2 units.

Order: only shifts need to be last.

(b) Determine if the function

$$f(x) = x^5 + x^3 + x + \frac{1}{x}$$

is even, odd, or neither.

$$f(-x) = (-x)^5 + (-x)^3 + (-x) + \frac{1}{-x}$$

$$= -x^5 - x^3 - x - \frac{1}{x}$$

$$= -\underbrace{\left(x^5 + x^3 + x + \frac{1}{x}\right)}$$

$$= -\overbrace{f(x)}$$

$\boxed{\text{Since } f(-x) = -f(x) \text{ the function is odd.}}$

(c) Find the domain for each of the following functions:

i. $f(x) = x^2$

$\boxed{\mathbb{R}}$

ii. $g(x) = \frac{1}{x}$

Problems

division by 0 : $x = 0$

Remove from $\mathbb{R}$

$\boxed{(-\infty, 0) \cup (0, \infty)}$

iii. $h(x) = \frac{1}{\sqrt{x}}$

Problems

(a) division by 0 :

$\sqrt{x} = 0$

$(\sqrt{x})^2 = 0^2$

$x = 0$

(b) Square root of negative

$x < 0$

Remove from $\mathbb{R}$

$\boxed{(0, \infty)}$

iv. $f(x) = \frac{1}{x^2 - 3x + 2}$

1 −2

1 −1

$= \frac{1}{(x-2)(x-1)}$

Problems

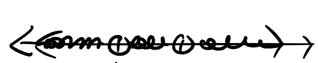
Division by 0:

$(x-2)(x-1) = 0$

$x - 2 = 0 \quad , \quad x - 1 = 0$

$x = 2, 1$

Remove from $\mathbb{R}$



$\boxed{(-\infty, 1) \cup (1, 2) \cup (2, \infty)}$

5. If $f(x) = 1 - x$ and $g(x) = x^2 - x$, find the following and **fully expand**:

(a) $f(x) - 3 \cdot g(x)$

common mistake : -3 is a factor of $g(x)$.
$(f(x) - 3)$ is NOT a factor of $g(x)$.
Why: by negative law #1:

$$f(x) - 3 \cdot g(x) = \underbrace{f(x)}_{\text{term}} + \underbrace{(-1) \cdot 3 \cdot g(x)}_{\text{term}}$$

$$f(x) - 3g(x) = 1 - x - 3(x^2 - x)$$

$$= 1 - x - 3x^2 + 3x$$

$$= \boxed{-3x^2 + 2x + 1}$$

(b) $f(x)g(x)$

common mistake : forget parentheses.

$$f(x) \cdot g(x) = (1-x)(x^2 - x)$$

$$\overset{\text{dist}}{=} (1-x) \cdot x^2 + (1-x) \cdot (-x)$$

$$\overset{\text{dist}}{=} x^2 - x^3 - x + x^2 = \boxed{-x^3 + 2x^2 - x}$$

(c) $f \circ g$

$$(f \circ g)(x) = f(g(x)) = f(x^2 - x) = 1 - (x^2 - x) = \boxed{-x^2 + x + 1}$$

(d) $g(x+h) - g(x) = \overset{(A+B)^2}{(x+h)^2} - (x+h) - (x^2 - x)$

$$\overset{\text{dist}}{=} x^2 + 2xh + h^2 - x - h - x^2 + x$$

$$= \boxed{2xh + h^2 - h}$$