MATH 141: Quiz 7

Name: key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Find all local minima and maxima of the function

$$f(x) = \frac{x^2+1}{x}$$

First Derivative Test.

① find potential locations of local extrema.

$$f'(x) = \frac{x \cdot \frac{d}{dx}[x^2+1] - (x^2+1)\frac{d}{dx}[x]}{x^2}$$

$$= \frac{x \cdot 2x - (x^2+1)\cdot 1}{x^2}$$

$$= \frac{2x^2 - x^2 - 1}{x^2}$$

$$= \frac{x^2-1}{x^2} = \frac{(x-1)(x+1)}{x^2}$$

ⓐ solve $f'(x) = 0$

$$x^2 \cdot \frac{(x-1)(x+1)}{x^2} = 0 \cdot x^2$$

$$(x-1)(x+1) = 0$$

$$\boxed{x = -1, 1}$$

ⓑ find where $f'(x)$ DNE.

Denominator can't be 0 so

$$x^2 = 0$$

$$\boxed{x = 0}$$

② Discriminate the potential locations of local extrema with sign diagram

f': $+$ | -1 | $-$ | 0 | $-$ | 1 | $+$

$$f'(-2) = \frac{(-2-1)(-2+1)}{+} = +$$

$$f'(-\tfrac{1}{2}) = \frac{-\cdot +}{+} = -$$

$$f'(\tfrac{1}{2}) = \frac{-\cdot +}{+} = -$$

$$f'(2) = \frac{+\cdot +}{+} = +$$

∴, local maximum of

$$f(-1) = \frac{(-1)^2+1}{-1} = \frac{1+1}{-1} = -2$$

local minimum of $f'(1) = \dfrac{1^2+1}{1} = 2$

2. Determine the intervals of concavity of

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

① Find inflection points

$f'(x) = 6x^2 - 18x + 12$

$f''(x) = 12x - 18 = 6(2x-3)$

ⓐ Solve $f''(x) = 0$

$6(2x-3) = 0$

$2x - 3 = 0$

$x = \frac{3}{2}$

ⓑ find when $f''(x)$ DNE

not applicable, $f''(x)$ has domain $\mathbb{R}$.

② Sign diagram of f''

f'': $-$ on the left of $\frac{3}{2}$, $+$ on the right of $\frac{3}{2}$

$f''(0) = 6\cdot(2\cdot 0 - 3) = -$

$f''(2) = 6\cdot(2\cdot 2 - 3) = +$

Concave up on $(\frac{3}{2}, \infty)$

Concave down on $(-\infty, \frac{3}{2})$