MATH 141: Midterm 1

Name: Key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

(a) Suppose you write

$$(x+y)^2z^2 = x^2 + y^2z^2$$

What are the two errors you made?

① Violated LoE 4; x and y are terms. You cannot distribute exponents to terms.

② Violated distributive law. $(x+y)^2$ is a factor that generates 3 terms when expanded. Multiplying ≥ 2 terms requires parentheses.

(b) True or false: We can simplify

$$\frac{\overbrace{(x+1)(x-2)}^{\text{term}} + \overbrace{(x-2)(x+3)}^{\text{term}}}{(x+1)^2}$$

by crossing out the $x+1$.

False. $(x+1)$ is only a factor in the context of the term $(x+1)(x-2)$ in the numerator, not the global numerator context.

(c) If $f(x) = x^2 - x$, evaluate $f(x-h)$ and fully expand + simplify.

$f(x-h) = (x-h)^2 - (x-h)$

do not forget.

$= \boxed{x^2 - 2xh + h^2 - x + h}$

(d) If $F(x) = \cos^2(x^3)$ find three functions f, g, h where $f \circ g \circ h = F$.

If $h(x) = x^3$, $g(x) = \cos(x)$, $f(x) = x^2$ then

$(f \circ g \circ h)(x) = f(g(h(x))) = f(\underline{g(x^3)}) = f(\cos(x^3)) = (\cos(x^3))^2$

Start from innermost function

same notation

$= \cos^2(x^3)$

$= F(x)$ ✓

2. Suppose

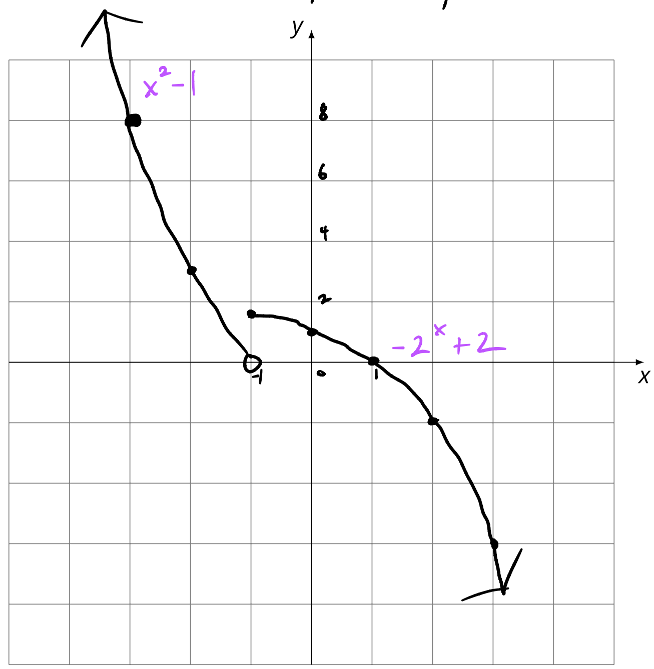
parabola parent shape

$$f(x) = \begin{cases} x^2 - 1 & x < -1 \\ -2^x + 2 & x \geq -1 \end{cases}$$

exponential parent shape

(a) Sketch a graph of $f(x)$.

x	$f(x)$
-3	$(-3)^2 - 1 = 8$
-2	3
-1	$-2^{-1} + 2 = -\frac{1}{2} + 2 = \frac{3}{2}$
0	$-2^0 + 2 = 1$
1	$-2^1 + 2 = 0$
2	-2



(b) What is $f(-1)$?

$$f(-1) = -2^{-1} + 2 = -\frac{1}{2} + 2 = \boxed{\frac{3}{2}}$$

(c) Does $\lim_{x \to -1} f(x)$ exist? Why or why not?

No. From the graph $\lim_{x \to -1^-} f(x) = 0$ but

$$\lim_{x \to -1^+} f(x) = \frac{3}{2}$$

Since $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$, we conclude

$\lim_{x \to -1} f(x)$ does not exist.

3. Perform the given instruction. Remember to use the relevant laws/properties and fully simplify. deal w/ exponents first before the fractions.

(a) Simplify: $\left(\frac{x+1}{(x-1)}\right)^2 \cdot \left(\frac{(x-1)(x+1)}{x+2}\right)^{-2}$

LoE 6
$$= \frac{(x+1)^2}{(x-1)^2} \cdot \left(\frac{x+2}{(x-1)(x+1)}\right)^2$$

LoE 5
$$= \frac{(x+1)^2}{(x-1)^2} \cdot \frac{(x+2)^2}{(\underbrace{(x-1)(x+1)}_{\text{factors}})^2}$$

LoE 4
$$= \frac{(x+1)^2}{(x-1)^2} \cdot \frac{(x+2)^2}{(x-1)^2 \cdot (x+1)^2}$$

frac law #1
$$= \frac{\cancel{(x+1)^2}(x+2)^2}{\cancel{(x+1)^2} \cdot (x-1)^4}$$

frac law #5
$$= \boxed{\frac{(x+2)^2}{(x-1)^4}}$$

(b) Expand: $\overset{(A-B)^2}{(x-2)^2}(x+3) + (x-3)(x+2)$

dist, $A^2-2AB+B^2$
$$= (x^2 - 4x + 4)(x+3) + (x-3)\cdot x + (x-3)\cdot 2$$

do not forget.

dist
$$= (x^2 - 4x + 4)\cdot x + (x^2 - 4x + 4)\cdot 3 + x^2 - 3x + 2x - 6$$

dist
$$= x^3 - 4x^2 + 4x + 3x^2 - 12x + 12 + x^2 - x - 6$$

$$= \boxed{x^3 - 9x + 6}$$

(c) Completely factor (you should have four factors): $x^4 - 5x^2 + 4$

3 term. Let $y = x^2$. Then

$$x^4 - 5x^2 + 4 = y^2 - 5y + 4 \leftarrow \begin{matrix} 1 & -4 \\ 1 & -1 \end{matrix}$$

$$\overset{\text{new } X}{=} (y-4)(y-1)$$

2 term, $A^2 - B^2$

$$= (x^2-4)(x^2-1)$$

$$\overset{A^2-B^2}{=} \boxed{(x-2)(x+2)(x-1)(x+1)}$$

means convert terms $\rightarrow$ factor and cancel if you can.

(d) $\boxed{\text{Simplify:}}$ $\dfrac{x^2h + 2xh + h}{h} \overset{\text{GCF}}{=} \dfrac{\cancel{h}(x^2 + 2x + 1)}{\cancel{h}}$

$$\underset{\#5}{\overset{\text{frac law}}{=}} x^2 + 2x + 1$$

$$\underset{\text{w/ new } X}{\overset{(A+B)^2}{=}} \boxed{(x+1)^2}$$

4. Draw the graph of a function which satisfies the following:

   (a) $f(0) = -1$

   (b) $f(3) = 1$

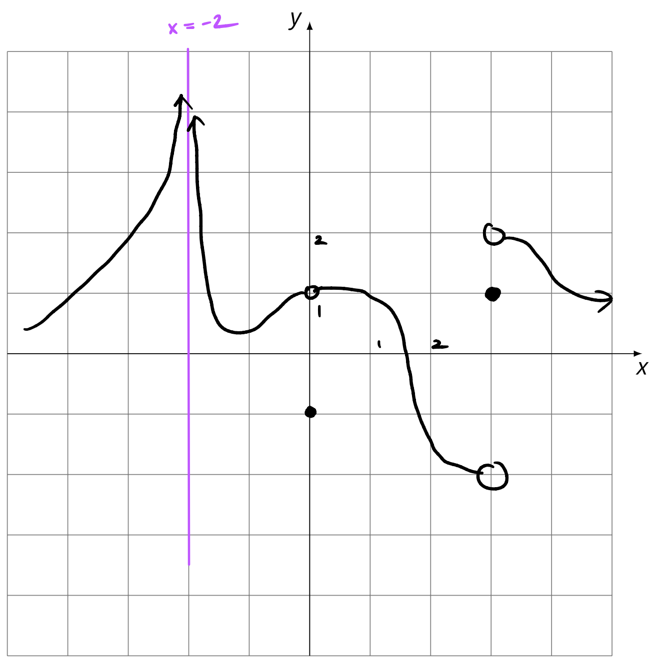
   (c) $\lim_{x \to 0} f(x) = 1$

   (d) $\lim_{x \to 3^-} f(x) = -2$

   (e) $\lim_{x \to 3^+} f(x) = 2$

   (f) $\lim_{x \to -2} f(x) = \infty$

Answers may vary.

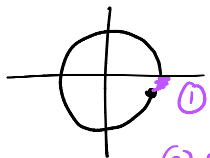


5. If

$$f(x) = \frac{1}{x} \qquad g(x) = \cos(x) \qquad h(x) = \sin(x) \qquad j(x) = e^x$$

Evaluate and fully simplify the following:

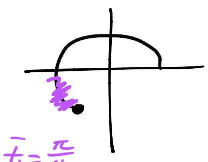
(a) $h\left(\frac{11\pi}{6}\right) = \sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = \boxed{-\frac{1}{2}}$



① $\bar{t} = \frac{\pi}{6}$

② sin − in IV

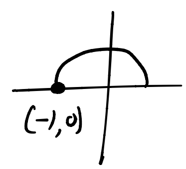
(b) $g\left(\frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$



① $\bar{t} = \frac{\pi}{4}$

② cos − in III

(c) $g(\pi \cdot j(0)) = g\left(\pi \cdot e^0\right) = g(\pi) = \cos(\pi) = \boxed{-1}$



(d) $\frac{f(x+h) - f(x)}{h} = \frac{\overbrace{\frac{1}{(x+h)} - \frac{1}{x}}^{\text{compound fraction}}}{h}$

compound fraction, LCD of nested fractions is $x \cdot (x+h)$

multiply by 1 $= \frac{x \cdot (x+h)}{x \cdot (x+h)} \cdot \frac{\frac{1}{(x+h)} - \frac{1}{x}}{h}$

frac law #1 $= \frac{x(x+h)\left(\frac{1}{(x+h)} - \frac{1}{x}\right)}{x \cdot h \cdot (x+h)}$ do not forget.

dist $= \frac{x \cdot \cancel{(x+h)} \cdot \frac{1}{\cancel{(x+h)}} - \cancel{x}(x+h) \cdot \frac{1}{\cancel{x}}}{x \cdot h\,(x+h)}$

frac law #5 and dist $= \frac{x - x - h}{x \cdot h\,(x+h)}$

$= \frac{-\cancel{h}}{x \cdot \cancel{h} \cdot (x+h)}$

frac law #5 $= \frac{-1}{x\,(x+h)}$

$= \boxed{-\frac{1}{x\,(x+h)}}$