MATH 141: Quiz 7

Name: key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Given the function

$$f(x) = x^3 - x$$

(a) Find all local minima and maxima.

We use the first derivative test.

1 crit #'s

$f'(x) = 3x^2 - 1$

a Solve $f'(x) = 0$

$3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$

b $f'(x)$ DNE

N/A, $f'(x)$ is a polynomial.

2 sign diagram of $f'(x)$

f': $+$ | $-\frac{\sqrt{3}}{3}$ | $-$ | $\frac{\sqrt{3}}{3}$ | $+$

$f'(-10) = 3\cdot(-10)^2 - 1 = +$

$f'(0) = 3\cdot 0^2 - 1 = -$

$f'(10) = 3\cdot 10^2 - 1 = +$

$\therefore$ local maximum of

$f\left(-\frac{\sqrt{3}}{3}\right) = \left(-\frac{\sqrt{3}}{3}\right)^3 - \left(-\frac{\sqrt{3}}{3}\right)$

$= -\frac{3\sqrt{3}}{27} + \frac{\sqrt{3}}{3} \cdot \frac{3}{3}$

$= -\frac{\sqrt{3}}{9} + \frac{3\sqrt{3}}{9}$

$= \frac{3\sqrt{3} - \sqrt{3}}{9} = \boxed{\frac{2\sqrt{3}}{9}}$

local minimum of

$f\left(\frac{\sqrt{3}}{3}\right) = \left(\frac{\sqrt{3}}{3}\right)^3 - \frac{\sqrt{3}}{3}$

$= \frac{3\sqrt{3}}{27} - \frac{\sqrt{3}}{3} \cdot \frac{3}{3}$

$= \frac{\sqrt{3} - 3\sqrt{3}}{9} = \boxed{-\frac{2\sqrt{3}}{9}}$

continued on back...

(b) Find the intervals of concavity.

① inflection points

$f''(x) = 6x$

ⓐ solve $f''(x) = 0$

$6x = 0 \rightarrow x = 0$

ⓑ $f''(x)$ DNE

N/A

② sign diagram of f''

f'': $-$ on the left of 0, $+$ on the right of 0

$f''(-1) = 6 \cdot (-1) = -$

$f''(1) = 6 \cdot (1) = +$

$\therefore$ concave down on $(-\infty, 0)$

concave up on $(0, \infty)$

2. Use the second derivative test to show $g(x) = x^2$ has a local minimum at $x = 0$.

① Solve $g'(x) = 0$

$g'(x) = 2x$

$2x = 0$

$x = 0$

② test crit # $x = 0$

$f''(x) = 2$

$f''(0) = 2 > 0$

By the second derivative test

$f(0) = 0^2 = 0$ is a local minimum