MATH 141: Quiz 4

Name: key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. **Using the limit definition of a derivative, i.e.**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

find the derivative of the function $f(x) = 1 - x - x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

expand numerator to create global factor of h.

$$= \lim_{h \to 0} \frac{1 - (x+h) - (x+h)^2 - (1 - x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{1 - x - h - (x^2 + 2xh + h^2) - 1 + x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} \cancel{- x} - h \cancel{- x^2} - 2xh - h^2 \cancel{- 1} \cancel{+ x} \cancel{+ x^2}}{h}$$

$$= \lim_{h \to 0} \frac{-h^2 - h - 2xh}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-h - 1 - 2x)}{\cancel{h}}$$

$$\lim_{h \to 0} [-h - 1 - 2x]$$

$$= -2x - 1 - 0$$

$$= \boxed{-2x - 1}$$

2. Using the limit definition of a derivative, find an equation of the tangent line for the function $f(x) = (x-1)^2$ at the point $(2,1)$.

At point $(a, f(a))$, the tangent line has equation

$$y - f(a) = f'(a)\cdot(x-a)$$

Our point is $\underset{(a,\ f(a))}{(2, 1)}$ so we have

$$y - 1 = f'(2)\cdot(x-2)$$

The slope $f'(2)$ is the derivative at $a=2$ so

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{(2+h-1)^2-(2-1)^2}{h}$$

$$= \lim_{h\to 0}\frac{(1+h)^2-1^2}{h}$$

$$= \lim_{h\to 0}\frac{1+2h+h^2-1}{h}$$

$$= \lim_{h\to 0}\frac{2h+h^2}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$= \lim_{h\to 0}(2+h)$$

$$= 2+0$$

$$= \boxed{2}$$

So the tangent line has equation

$$y - 1 = 2\cdot(x-2)$$

$$y = 2x - 4 + 1$$

$$\boxed{y = 2x-3}$$