MATH 161: Quiz 7

Name: key

Directions:

- Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Differentiate the following:

1. $x^2 + y^3 = 1$

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[y^3] = \frac{d}{dx}[1]$$

$$2x + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -2x$$

$$\boxed{\frac{dy}{dx} = -\frac{2x}{3y^2}}$$

2. $2xy - y^3 = x$

product rule

$$2\frac{d}{dx}[xy] - \frac{d}{dx}[y^3] = \frac{d}{dx}[x]$$

$$2\left(x \cdot \frac{d}{dx}[y] + y \cdot \frac{d}{dx}[x]\right) - 3y^2 \cdot \frac{dy}{dx} = 1$$

$$2\left(x\frac{dy}{dx} + y\right) - 3y^2 \cdot \frac{dy}{dx} = 1$$

$$2x\frac{dy}{dx} + 2y - 3y^2\frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(2x - 3y^2) = 1 - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{1-2y}{2x-3y^2}}$$

3. $y = \cos xy$

chain rule

$$\frac{d}{dx}[y] = \frac{d}{dx}\left[\cos(xy)\right]$$

product rule

$$\frac{dy}{dx} = -\sin(xy)\cdot\frac{d}{dx}[xy]$$

$$\frac{dy}{dx} = -\sin(xy)\cdot\left(x\frac{d}{dx}[y] + y\cdot\frac{d}{dx}[x]\right)$$

$$\frac{dy}{dx} = -\sin(xy)\cdot\left(x\frac{dy}{dx} + y\right)$$

$$\frac{dy}{dx} = -\sin(xy)\times\frac{dy}{dx} - y\sin(xy)$$

$$\frac{dy}{dx} + \sin(xy)\times\frac{dy}{dx} = -y\sin(xy)$$

$$\frac{dy}{dx}\left(1 + x\sin(xy)\right) = -y\sin(xy)$$

$$\boxed{\frac{dy}{dx} = \frac{-y\sin(xy)}{1 + x\sin(xy)}}$$