

MATH 141: Midterm 2

Name: _____

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * **Remember to simplify each expression.**
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
		80

1. Find the following derivatives. You are allowed to use the Differentiation Rules.

(a) $f(x) = 300$

(b) $f(x) = 5x^4 - x^2 + 3x$

(c) $f(x) = \frac{\sin^2(x)}{x^2}$

(d) $g(x) = x^2 \cos(x^2)$

(e) $f(x) = \left(\frac{x^2 - 1}{x^2 + 3} \right)^4$

2. The following three equations are in implicit form. Find $\frac{dy}{dx}$.

(a) $3x^2 + 2y = 2x^4 + 3y^2$

(b) $x^2 - 2xy + y^2 = 5$

(c) $\cos(xy) = 1 + \sin y$

3. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mph passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.

4. Short answer questions:

- (a) What are the two methods for finding local minimums and maximums called? Compare the strength of both methods.

- (b) Under which conditions are both the absolute minimum and maximum of $f(x)$ guaranteed to exist?

- (c) What is the method for finding absolute minimums and maximums called? How do you use it?

- (d) Suppose $f(x)$ is continuous on $[a, b]$. Sketch a graph of $f(x)$ which shows there does not necessarily need to be a c where

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

5. Suppose $f(x) = \frac{x}{x^2 + 1}$.

- (a) Find all intervals on which $f(x)$ is increasing and decreasing.
- (b) Find all local minimums and maximums.

6. Determine the intervals of concavity of

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

7. Sketch a possible graph of a function which satisfies the following:

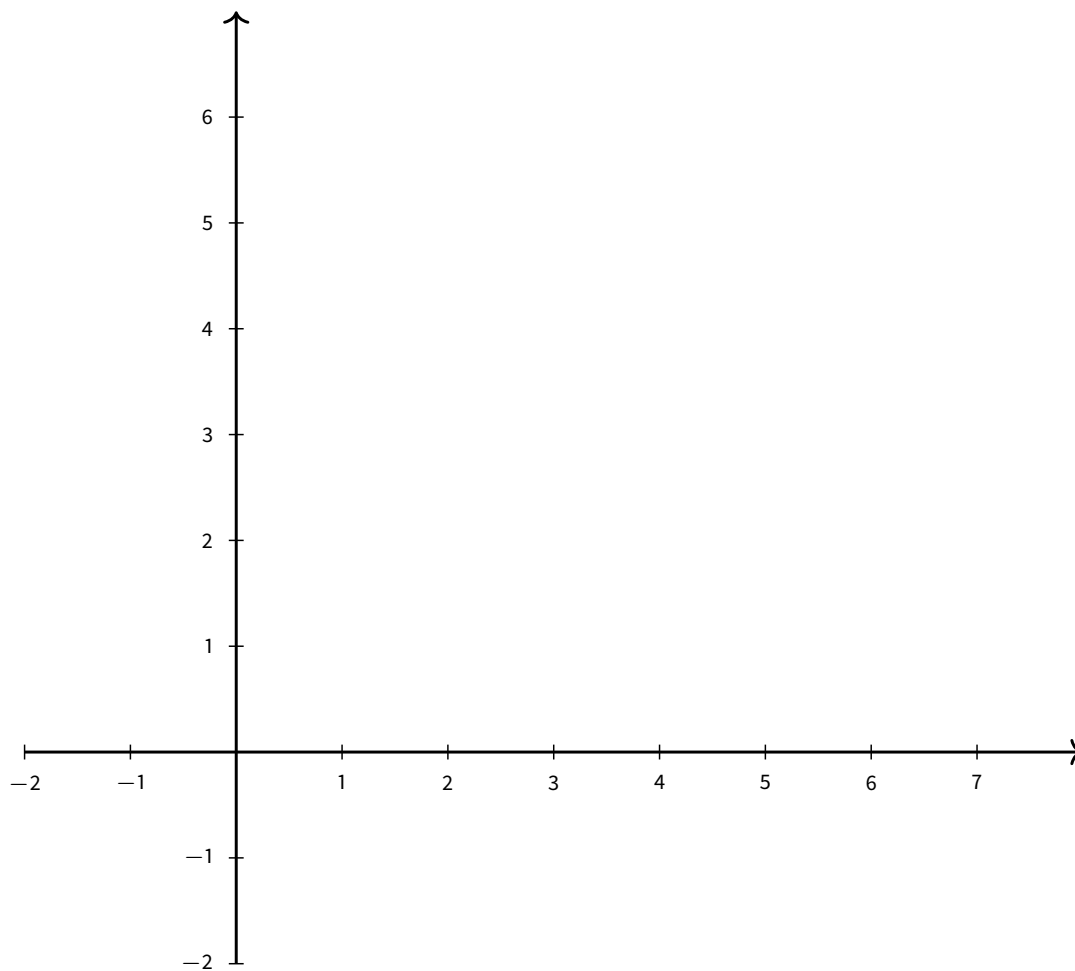
(a) $f(0) = 0, f(2) = 1, f'(2) = 0$

(b) $f'(x) < 0$ when $0 < x < 2$ and $f'(x) > 0$ when $x > 2$

(c) $f''(x) < 0$ when $0 < x < 1$ and $x > 4$

(d) $f''(x) > 0$ when $1 < x < 4$

(e) $\lim_{x \rightarrow \infty} f(x) = 5$



8. Suppose someone owns 4000 meters of fencing. They wish to create a rectangular piece of grazing land where one side is along a river. This means no fence is needed for that side. Moreover, they wish to subdivide the rectangle into three separate sections with two pieces of fence, both of which are parallel to the sides not along the river.

What are the dimensions of the largest area that can be enclosed?