MATH 118: Midterm 1 Key

Name: ______________________________

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Short answer questions:

   (a) True or False: We are allowed to use exponent laws in the following way:

$$\left(\frac{x-2}{x^2y}\right)^2 = \frac{x^2-2^2}{x^4y^2}$$

   $\boxed{\text{False.}}$ The x and 2 in the numerator are terms. Exponents = multiplication = **factors**. Properties for factors do not interact with terms.

   There is **only one property** where terms and factors interact. It is the distributive property.

   (b) True or false: We can simplify

$$\frac{\overbrace{(x-1)^2x}^{\text{Term}} - \overbrace{1}^{\text{Term}}}{(x-1)}$$

   by crossing out the $(x-1)$ (the one in parentheses).

   $\boxed{\text{False.}}$ The numerator has global terms. Therefore you cannot have global factors. You can only cancel global factors.

   (c) Suppose

$$f = 2x(x-1) \qquad g = 2x^2 - 2x$$

   Expand and simplify $f - g$.

   Don't forget to distribute the negative to **both terms.**

$$\begin{aligned} f - g &= 2x(x-1) - (2x^2 - 2x) \\ &= 2x^2 - 2x - 2x^2 + 2x \\ &= \boxed{0} \end{aligned}$$

2. Factor and simplify:

(a) $x^2 - 4$

Two term factoring problem. Can't use GCF.

Using $A^2 - B^2$ where $A = x$ and $B = 2$, we have

$$\overset{A^2}{x^2} - \overset{B^2}{4} = \boxed{(x-2)(x+2)}$$

(b) $x^2 - 7x^2 - x + 7$

First, combine like terms x^2 and $-7x^2$:

$$x^2 - 7x^2 - x + 7 = -6x^2 - x + 7$$

Three term factoring problem. Can't use GCF.

Use new X method: $a = -6, b = -1, c = 7$

One diagonal product near $b - 1$ is $-1 \cdot 7$, so try it:

$$\begin{array}{ccc} 6 & & 7 \\ | & \times & | \\ \text{-1} & & 1 \end{array}$$

which works, cross-product and sum and $6 \cdot 1 + (-1) \cdot 7 = -1 = b$

$\boxed{(6x+7)(-x+1)}$ or factor out the negative, $\boxed{-(6x+7)(x-1)}$

(c) $\underbrace{2(x-2)(2x+1)^2}_{\text{Term}} + \underbrace{4(2x+1)(x-2)^2}_{\text{Term}}$

Two term factoring problem. Can use GCF, common factor is $2(x-2)(2x+1)$. Undo distributive law:

$$\begin{aligned} 2(x-2)(2x+1)^2 + 4(2x+1)(x-2)^2 &= 2(x-2)(2x+1)\big[(2x+1) + 2(x-2)\big] \\ &= 2(x-2)(2x+1)(2x+1+2x-4) \\ &= \boxed{2(x-2)(2x+1)(4x-3)} \end{aligned}$$

3. Expand and simplify:

(a) $(x+h)-1-(x-1)$

Note that $(x+h)-1=(x+h)+(-1)$. Do not distribute negatives backwards; the -1 is a different term.

$$\begin{aligned}(x+h)-1-(x-1) &= x+h-1-x+1\\ &= \boxed{h}\end{aligned}$$

(b) $3(x+2)-2(2x+1)^2x$

Using $(A+B)^2$, I will expand $(2x+1)^2=4x^2+4x+1$ first. Remember the parentheses.

$$\begin{aligned}3(x+2)-2(2x+1)^2x &= 3x+6-2(4x^2+4x+1)x\\ &= 3x+6-8x^3-8x^2-4x\\ &= \boxed{-8x^3-8x^2-x+6}\end{aligned}$$

4. Simplify:

(a) $\dfrac{x}{2x-3} - \dfrac{x}{x+1}$

Left fraction missing $(x+1)$ as a factor, right missing $(2x+3)$ as a factor. Introduce and don't forget parentheses due to multiplying 2 terms:

$$\begin{aligned}
\frac{x}{2x-3} - \frac{x}{x+1} &= \frac{(x+1)}{(x+1)} \cdot \frac{x}{2x-3} - \frac{x}{x+1} \cdot \frac{(2x-3)}{(2x-3)} \\
&= \frac{(x+1)x}{(x+1)(2x-3)} - \frac{x(2x-3)}{(x+1)(2x-3)} \\
&= \frac{(x+1)x - x(2x-3)}{(x+1)(2x-3)} \\
&= \frac{x^2+x-2x^2+3x}{(x+1)(2x-3)} \\
&= \frac{-x^2+4x}{(x+1)(2x-3)} \\
&= \frac{-(x^2-4x)}{(x+1)(2x-3)} \\
&= \boxed{-\frac{x^2-4x}{(x+1)(2x-3)}}
\end{aligned}$$

(b) $\dfrac{3(x+h)^2-2-(3x^2-2)}{h}$

Expand numerator because like terms are created.

$$\begin{aligned}
\frac{3(x+h)^2-2-(3x^2-2)}{h} &= \frac{3(x^2+2xh+h^2)-2-3x^2+2}{h} \\
&= \frac{3x^2+6xh+3h^2-3x^2}{h} \\
&= \frac{6xh+3h^2}{h} \\
&= \frac{\cancel{h}(6x+3h)}{\cancel{h}} \\
&= 6x+3h \\
&= \boxed{3(2x+h)}
\end{aligned}$$

(c) $\dfrac{\dfrac{1}{x+1}-\dfrac{1}{x-1}}{x-1}$

Compound fraction. Imagine the numerator is a separate problem.

Left missing $(x-1)$, right missing $(x+1)$.

$$
\begin{aligned}
\frac{\dfrac{1}{x+1}-\dfrac{1}{x-1}}{x-1} &= \frac{\dfrac{x-1}{x-1}\cdot\dfrac{1}{x+1}-\dfrac{1}{x-1}\cdot\dfrac{x+1}{x+1}}{x-1} \\
&= \frac{\dfrac{x-1}{(x-1)(x+1)}-\dfrac{x+1}{(x-1)(x+1)}}{x-1} \\
&= \frac{\dfrac{x-1-(x+1)}{(x-1)(x+1)}}{x-1} \\
&= \frac{\dfrac{x-1-x-1}{(x-1)(x+1)}}{x-1} \\
&= \frac{\dfrac{-2}{(x-1)(x+1)}}{x-1} \\
&= \frac{-2}{(x+1)(x-1)}\cdot\frac{1}{x-1} \\
&= \boxed{-\frac{2}{(x-1)^2(x+1)}}
\end{aligned}
$$

5. Perform the given instruction:

   (a) Simplify $-8^{\frac{2}{3}}$

   Memorize the Compendium. Follow the definitions of negative and fractional exponent:

$$
\begin{aligned}
-8^{\frac{2}{3}} &= (-1)\cdot 8^{\frac{2}{3}} \\
&= (-1)\cdot \sqrt[3]{8^2} \\
&= (-1)\cdot \sqrt[3]{(2^3)^2} \\
&= (-1)\cdot \sqrt{3}2^6 \\
&= (-1)\cdot 2^{\frac{6}{3}} \\
&= (-1)\cdot 2^2 \\
&= \boxed{-4}
\end{aligned}
$$

   (b) Rationalize the numerator and simplify:

$$\frac{\sqrt{x}-1}{x-1}$$

   Two term rationalization problem. Multiply by the conjugate radical and use $A^2 - B^2$:

$$
\begin{aligned}
\frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} &= \frac{(\sqrt{x})^2 - 1^2}{(x-1)(\sqrt{x}+1)} \\
&= \frac{1\cdot \cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)} \\
&= \boxed{\frac{1}{\sqrt{x}+1}}
\end{aligned}
$$

(c) Simplify:

$$\frac{x-1}{x^2+x}\cdot\frac{x^2}{x^2-x}\cdot\left(\frac{x^{99}-x^{32}+2\sqrt{x^{11}}}{x^{12345}-x+1}\right)^0$$

This is a fraction multiplication problem. According to 1.4: Simplifying, first factor each fraction, multiply, then cancel!

To factor, each problem is a two term factoring problem where you can use GCF.

Anything to the zeroth power is 1.

$$\begin{aligned}\frac{x-1}{x^2+x}\cdot\frac{x^2}{x^2-x}\cdot\left(\frac{x^{99}-x^{32}+2\sqrt{x^{11}}}{x^{12345}-x+1}\right)^0 &= \frac{x-1}{x(x+1)}\cdot\frac{x^2}{x(x-1)}\cdot 1\\ &= \frac{(x-1)x^2}{x(x+1)\cdot x(x-1)}\\ &= \frac{1\cdot\cancel{(x-1)}\cancel{x^2}}{\cancel{x}(x+1)\cdot\cancel{x}\cancel{(x-1)}}\\ &= \boxed{\frac{1}{x+1}}\end{aligned}$$