MATH 118: Final Prep Suggestions Key

Guaranteed Problems

The following concepts will definitely be tested on the final:

* Evaluate and simplify the average rate of change for $f(x)$ on $(x, x+h)$
* Solving all types of equations (linear, rational, quadratic, root, exponential, logarithmic)
* Evaluate (plug in) and simplify functions
* Graphing functions, including transformed ones and piecewise
* Expanding (distributive law) and factoring (4 methods)
* Complete factorization over $\mathbb{R}$ vs. $\mathbb{C}$
* Increasing, decreasing, local minima and maxima
* Exponential functions and logarithms
* Simplifying fractional expressions (#4 in Practice Midterm 1)
* Simplifying expressions which require GCF factoring (#2b Practice Midterm 1)
* Polynomial long division, factor/remainder theorem
* Being able to use properties, definitions, and laws correctly, such as Laws of Exponents.
  All of these are located in the Compendium! (click me)

This is **not** an exhaustive list. More concepts will be tested!

To prepare, consider the practice final to be:

* Practice Midterm 1
* Practice Midterm 2
* Practice problems from the last two weeks of class (the rest of this document)

See next page for problems. Good luck studying!

1. Find the vertical and horizontal asymptotes for the following rational functions.

   (a) $\dfrac{3x-2}{x-3}$

   Horizontal:

   Here $n = 1 = m$. So $\boxed{y = \dfrac{a_n}{b_m} = \dfrac{3}{1} = 3}$

   Vertical:

   Set denominator = 0 and solve.

   $x - 3 = 0$ so $\boxed{x = 3}$.

   (b) $\dfrac{4x^4 - 3x}{2x^3 + 5}$

   Horizontal Asymptote:

   Here $n = 4, m = 3$. $\boxed{\text{No horizontal asymptote}}$

   Vertical Asymptote:

   Set denominator = 0 and solve.

$$\begin{aligned} 2x^3 + 5 &= 0 \\ 2x^3 &= -5 \\ x^3 &= \frac{-5}{2} \\ \sqrt[3]{x^3} &= \sqrt[3]{-\frac{5}{2}} \end{aligned}$$

$$\boxed{x = \sqrt[3]{-\frac{5}{2}}}$$

   (c) $\dfrac{x+4}{(x-2)(x-3)} = \dfrac{x+4}{x^2-5x+6}$

   Horizontal Asymptote:

   Here $n = 1, m = 2$. $\boxed{\text{y = 0}}$

   Vertical Asymptote:

   Set denominator = 0 and solve.

$$(x-2)(x-3) = 0$$

$$x - 2 = 0 \qquad x - 3 = 0$$

$$\boxed{x = 2} \qquad \boxed{x = 3}$$

2. Describe a possible formula of a rational function with:

   * vertical asymptotes $x = -1$ and $x = 1$
   * horizontal asymptotes $y = 0$ and
   * a hole at $x = 2$.

$$\boxed{r(x) = \frac{x-2}{(x-2)(x+1)(x-1)}}$$

3. Evaluate the following:

(a) $\log_6 36 = \boxed{2}$ because $6^2 = 36$

(b) $e^{\ln 53} = \boxed{53}$

(c) $\log_2 \frac{1}{4}$

$$\begin{aligned}\log_2 \frac{1}{4} &= \log_2 \frac{1}{2^2}\\ &= \log_2 2^{-2}\\ &= \boxed{-2}\end{aligned}$$

(d) $\log_3 \sqrt{3}$

$$\begin{aligned}\log_3 \sqrt{3} &= \log_3 3^{\frac{1}{2}}\\ &= \boxed{\frac{1}{2}}\end{aligned}$$

(e) $\ln\left(\ln e^{e^{32}}\right)$

Start with the inside. Using Laws of Logarithms:

$$\begin{aligned}\ln\left(\ln e^{e^{32}}\right) &= \ln\left(e^{32} \cdot \ln e\right)\\ &= \ln\left(e^{32} \cdot 1\right)\\ &= \ln\left(e^{32}\right)\\ &= \boxed{32}\end{aligned}$$

(f) $\log_3 27^4$

Rewrite inside exponential with base 3 using $27 = 3^3$:

$$\begin{aligned}\log_3 27^4 &= \log_3 \left(3^3\right)^4\\ &= \log_3 3^{3\cdot 4}\\ &= \log_3 3^{12}\\ &= \boxed{12}\end{aligned}$$

(g) $\log_3 90 - \log_3 10$

$$\begin{aligned}\log_3 90 - \log_3 10 &= \log_3 \frac{90}{10}\\ &= \log_3 9\\ &= \log_3 3^2\\ &= \boxed{2}\end{aligned}$$

(h) $\log_{32} 16 + \log_{32} 2$

$$\begin{aligned}\log_{32} 16 + \log_{32} 2 &= \log_{32}\left(16 \cot 2\right)\\ &= \log_{32} 32\\ &= \boxed{1}\end{aligned}$$

4. Expand
$$\ln \frac{3x^2}{(x-1)^4}$$
using Laws of Logarithms.

$$\begin{aligned}\ln \frac{3x^2}{(x-1)^4} &= \ln 3x^2 - \ln(x-1)^4 \\ &= \ln 3 + \ln x^2 - 4\ln(x-1) \\ &= \boxed{\ln 3 + 2\ln x - 4\ln(x-1)}\end{aligned}$$

5. If $\$15,000$ is invested at a yearly interest rate of 4.2%, find the amount after 10 years if interest is compounded monthly. Leave expression as-is.

We have $P = 15000, r = 0.042, n = 12, t = 10$. Plugging in:

$$A(10) = P\left(1+\frac{r}{n}\right)^{nt} = \boxed{15000\left(1+\frac{0.042}{12}\right)^{12\cdot 10}}$$

6. Solve the following equations. Simplify until you reach an exponential or logarithm you cannot evaluate without a calculator.

(a) $7^{2x-3} = 7^{6+5x}$

We have

$$7^{2x-3} = 7^{6+5x}$$
$$\log_7 7^{2x-3} = \log_7 7^{6+5x}$$
$$(2x-3)\cdot\log_7 7 = (6+5x)\cdot\log_7 7$$
$$(2x-3)\cdot 1 = (6+5x)\cdot 1$$
$$2x-3 = 6+5x$$
$$-9 = 3x$$
$$\boxed{x=-3}$$

(b) $2 + 2^{3x} = 18$

We have

$$2 + 2^{3x} = 18$$
$$2^{3x} = 16$$
$$\log_2 2^{3x} = \log_2 16$$
$$3x\cdot\log_2 2 = \log_2 2^4$$
$$3x\cdot 1 = 4\cdot\log_2 2$$
$$3x = 4\cdot 1$$
$$\boxed{x=\frac{4}{3}}$$

(c) $\log_3(2-x) = 3$

Rewrite in exponential form and solve.

$$\log_3(2-x) = 3$$
$$3^3 = 2-x$$
$$9 = 2-x$$
$$\boxed{x=-7}$$

(d) $\log(x-4) = 1$

Rewrite in exponential form and solve.

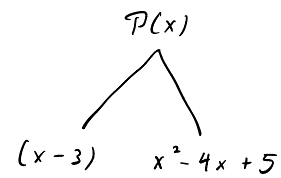
$$\log(x-4) = 1$$
$$10^1 = x-4$$
$$\boxed{16=x}$$

7. Consider $P(x) = x^3 - 7x^2 + 17x - 15$ where $P(3) = 0$.

(a) Find a complete factorization over $\mathbb{R}$.

$P(3) = 0$ so $(x-3)$ is a factor of $P(x)$.

$$
\begin{array}{r|rrrr}
 & x^2 & -4x & +5 & \\
\hline
x-3 & x^3 & -7x^2 & +17x & -15 \\
 & -(x^3 & -3x^2) & & \\
\hline
 & & -4x^2 & +17x & \\
 & & -(-4x^2 & +12x) & \\
\hline
 & & & 5x & -15 \\
 & & & -(5x & -15) \\
\hline
 & & & & 0
\end{array}
$$



So $P(x) = (x-3)(x^2-4x+5)$

$x^2 - 4x + 5$ is irreducible because $b^2 - 4ac = (-4)^2 - 4 \cdot 1 \cdot 5 = 16 - 20 < 0$.

answer : $\boxed{P(x) = (x-3)(x^2-4x+5)}$

(b) Find a complete factorization over $\mathbb{C}$.

Factor the irreducible $x^2 - 4x + 5$.

We solve $x^2 - 4x + 5 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm i\sqrt{4}}{2} = \frac{4 \pm 2i}{2} = \frac{\cancel{2}(2 \pm i)}{\cancel{2}} = 2 \pm i.$$

So a complete factorization over $\mathbb{C}$ is

$$P(x) = (x-3)(x-(2-i))(x-(2+i))$$

$$= \boxed{(x-3)(x-2+i)(x-2-i)}$$