MATH 141: Midterm 2

Name: _____

Directions:

* ⋆ Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* ⋆ **Remember to simplify each expression.**
* ⋆ If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* ⋆ Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

1. Suppose $f(x) = \sqrt{x}$.

    (a) What does the expression $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ represent?

    (b) Find
    $$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$$
    for the given function $f(x)$. You must use this limit definition to receive credit.

    (c) Find the equation of the tangent line of $f(x)$ at the point $(1, 1)$.

2. Short answer questions:

   (a) If a function $f(x)$ is continuous at $x = a$, must it be differentiable at $x = a$ as well? If not, draw a graph of a function that is continuous but not differentiable at $x = a$.

   (b) True or false:
   $$f(x) = \sin(x) + \frac{x}{x+1}$$
   is continuous on $\mathbb{R}$.

   (c) Given $f(x) = x$, find an equation of the normal line at $(3, 3)$.

3. Answer the following:

(a) Given a function $f(x)$, if
$$\lim_{x \to a} f(x) = \frac{0}{0}$$
what global factor do you need to manifest in the numerator and denominator and why?

(b) Find
$$\lim_{t \to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$$

(c) Find

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

4. Find the following derivatives. You are allowed to use the Differentiation Rules.

(a) $f(x) = \pi^2$

(b) $f(x) = x^2 \sin x$

(c) $f(x) = \dfrac{\sin(x^2)}{2 - \cos x}$

(d) $g(x) = \sqrt{\tan x^3}$

5. Given the implicit equation

$$\sqrt{xy} = x + y$$

Find $\dfrac{dy}{dx}$.