MATH 141: Quiz 5

Name: key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

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1. Differentiate the following. You are allowed to use shortcuts.

(a) $g(x) = \dfrac{3x-1}{2x^2-x} - 4$

$$g'(x) = \frac{d}{dx}\left[\frac{3x-1}{2x^2-x} - 4\right]$$

$$= \frac{d}{dx}\left[\frac{3x-1}{2x^2-x}\right] - \frac{d}{dx}[4]$$

$$= \frac{(2x^2-x)\cdot\frac{d}{dx}[3x-1] - (3x-1)\cdot\frac{d}{dx}[2x^2-x]}{(2x^2-x)^2}$$

$$= \frac{3(2x^2-x) - (3x-1)\cdot(4x-1)}{(2x^2-x)^2}$$

$$= \frac{6x^2-3x - (12x^2-7x+1)}{(2x^2-x)^2}$$

$$= \frac{6x^2-3x-12x^2+7x-1}{(2x^2-x)^2}$$

$$= \boxed{\frac{-6x^2+4x-1}{(2x^2-x)^2}}$$

(b) $f(x) = 3(\sqrt[3]{x^2}-1)\sin(x)$

$$= 3\left(x^{\frac{2}{3}}-1\right)\sin(x)$$

$$f'(x) = \frac{d}{dx}\left[3\left(x^{\frac{2}{3}}-1\right)\sin(x)\right]$$

careful!

product rule generates two terms

$$= 3\cdot\frac{d}{dx}\Big[\underbrace{\left(x^{\frac{2}{3}}-1\right)}_{\text{left}}\underbrace{\sin(x)}_{\text{right}}\Big]$$

$$= 3\cdot\left(\left(x^{\frac{2}{3}}-1\right)\cdot\frac{d}{dx}[\sin(x)] + \sin(x)\cdot\frac{d}{dx}\left[\left(x^{\frac{2}{3}}-1\right)\right]\right)$$

$$= 3\left(\left(x^{\frac{2}{3}}-1\right)\cdot\cos(x) + \sin(x)\cdot\frac{2}{3}x^{-\frac{1}{3}}\right)$$

$$= 3\left(x^{\frac{2}{3}}\cos(x) - \cos(x) + \frac{2}{3\sqrt[3]{x}}\sin(x)\right)$$

$$= \boxed{3\sqrt[3]{x^2}\cos(x) - 3\cos(x) + \frac{2\sin(x)}{\sqrt[3]{x}}}$$

2. Given the function

$$f(x) = x^4 + \sin(x) - 237483$$

find the following:

(a) $f'(x) = 4x^3 + \cos(x)$

(b) $f''(x) = 12x^2 - \sin(x)$

(c) $f'''(x) = 24x - \cos(x)$

(d) $f^{(4)}(x) = 24 + \sin(x)$