

LibIAPWS

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Abstract

LibIAPWS is a C++ library to implement IAPWS [1] (International Association for the Properties of Water and Steam) formulation. It contains parametrization for thermodynamic properties, including enthalpy, entropy, heat capacities and latent heat, as functions of temperature, pressure and mass density of H₂O. Coexisting phases among water vapor, liquid and ice are also computed and tabulated.

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1 Library functions

1.1 R1-76 (2014) - IAPWS::Lib76 class

IAPWS R1-76 [2] release provides parametrization for the surface tension σ of the interface between water vapor and liquid. There is one function `get_tension_surf`, which takes temperature T as input and returns the surface tension.

function	input	output
<code>get_tension_surf</code>	temperature T (K)	surface tension σ (J/m ²)

Table 1: Library functions in IAPWS::Lib76 class

1.2 R6-95 (2018) - IAPWS::Lib95 class

IAPWS R6-95 [3] release provides parametrization for the specific Helmholtz free energy f and thermodynamic quantities of water vapor and liquid. The mass density ρ and temperature T are considered as independent variables. The parametrized functions for single-phase are listed in Table 2.

function	input	output
<code>get_param_f</code>	mass density ρ (kg/m ³) temperature T (K)	specific Helmholtz free energy f (J/kg)
<code>get_param_g</code>		specific Gibbs free energy g (J/kg)
<code>get_param_pressure</code>		pressure p (Pa = N/m ²)
<code>get_param_dpess_drho</code>		derivative of pressure $\left. \frac{\partial p}{\partial \rho} \right _T$ (m ² /sec ²)
<code>get_param_erg_int</code>		specific internal energy u (J/kg)
<code>get_param_entropy</code>		specific entropy s (J/kg K)
<code>get_param_enthalpy</code>		specific enthalpy h (J/kg)
<code>get_param_heat_c_v</code>		specific isochoric heat capacity c_v (J/kg K)
<code>get_param_heat_c_p</code>		specific isobaric heat capacity c_p (J/kg K)
<code>get_param_speed_sound</code>		speed of sound w (m/sec)

Table 2: Library functions for single-phase thermodynamic quantities in IAPWS::Lib95 class

In addition, one can utilize the parametrized Helmholtz free energy to obtain coexisting phases (saturation curve) between water vapor and liquid. In IAPWS::Lib95 class, iterative root-finding algorithms are implemented to specify the saturation curve. (More details about the Maxwell construction can be found in the following sections.) Table 3 presents functions to handle the tabulated saturation curve. One can call `make_tab_coex` to produce a table for saturation curve by specifying number of temperature bins and maximum temperature. The minimum temperature for tabulation is set to be that of the triple point $T_{\text{trip}} = 273.16$ K.

It is possible to expedite calculations in `make_tab_coex` function, by providing a function pointer to better initial guess for the saturation pressure. One option is to use `get_param4_sat_pressure` function in IAPWS::Lib97 class and it is demonstrated in `make_tab_IAPWS95.cpp` source file. `export_tab_coex` and `import_tab_coex` functions can be used to write and read the tabulated saturation curve, respectively.

function	input	output
<code>make_tab_coex</code>	number of temperature bins (<code>int</code>) maximum temperature (K) function pointer to initial guess for the saturation pressure (<code>double (*ptr)(double)</code>), default : NULL)	<code>void</code>
<code>export_tab_coex</code>	name of output file (<code>char *</code>)	<code>void</code>
<code>import_tab_coex</code>	name of input file (<code>char *</code>)	<code>void</code>

Table 3: Tabulation of coexisting phases (saturation curve between vapor and liquid) in IAPWS::Lib95 class

Once the tabulated saturation curve is produced or imported, one can call functions listed in Table 4 to obtain thermodynamic quantities on the saturation curve. The tabulated saturation curve is interpolated by means of natural cubic spline. All functions take temperature T (in K) as the input parameter.

function	input	output
<code>get_coex_pressure</code>	temperature T (K)	saturation pressure p_{coex} (Pa)
<code>get_coex_mden_vap</code>		mass density of vapor ρ_{vap} (kg/m ³)
<code>get_coex_mden_liq</code>		mass density of liquid ρ_{liq} (kg/m ³)
<code>get_coex_enthalpy_vap</code>		specific enthalpy of vapor h_{vap} (J/kg)
<code>get_coex_enthalpy_liq</code>		specific enthalpy of liquid h_{liq} (J/kg)
<code>get_coex_entropy_vap</code>		specific entropy of vapor s_{vap} (J/kg K)
<code>get_coex_entropy_liq</code>		specific entropy of liquid s_{liq} (J/kg K)
<code>get_coex_heat_latent</code>		specific latent heat $h_{\text{latent;vap-liq}}$ (J/kg)

Table 4: Thermodynamic quantities for coexisting phases (saturation curve between vapor and liquid) in `IAPWS::Lib95` and `IAPWS::Lib97` classes

1.3 R7-97 (2012) - `IAPWS::Lib97` class

IAPWS R7-97 [4] release provides parametrization for the specific Gibbs free energy g and thermodynamic quantities of water vapor and liquid. In this release, the T - p plane ($273.15 \text{ K} \leq T \leq 1073.15 \text{ K}$, $p \leq 100 \text{ MPa}$ and $1073.15 \text{ K} < T \leq 2273.15 \text{ K}$, $p \leq 50 \text{ MPa}$) is split into a few regions and each region has its own parametrization for thermodynamic quantities.

One can use functions listed in Table 5 to obtain thermodynamic quantities for single-phase states for all regions. In `IAPWS::Lib97` class, temperature T and pressure p are considered as independent variables. In addition, it is possible to specify whether one tries to obtain the metastable-vapor state. If a boolean value is not provided as input, this flag is set to be `false` by default. If it is set to be `true`, functions listed in Table 5 return thermodynamic quantities in the metastable-vapor region (bounding region 2).

Note that, due to the fact that the specific Helmholtz free energy is parametrized in region 3 as a function of mass density ρ and temperature T , iterative root-finding algorithms are employed in this region. It ensures the integrity with other regions such that one can obtain thermodynamic quantities as functions of T and p .

function	input	output
<code>get_region</code>	temperature T (K) pressure p (Pa) flag for metastable state (bool, default : <code>false</code>)	integer assigned to region in the T - p plane (<code>int</code>) (returns 0 if it is out of the validity range)
<code>get_param_g</code>		specific Gibbs free energy g (J/kg)
<code>get_param_vol_spec</code>		specific volume v (m ³ /kg)
<code>get_param_mdensity</code>		mass density ρ (kg/m ³)
<code>get_param_erg_int</code>		specific internal energy u (J/kg)
<code>get_param_entropy</code>		specific entropy s (J/kg K)
<code>get_param_enthalpy</code>		specific enthalpy h (J/kg)
<code>get_param_heat_c_p</code>		specific isobaric heat capacity c_p (J/kg K)
<code>get_param_heat_c_v</code>		specific isochoric heat capacity c_v (J/kg K)
<code>get_param_speed_sound</code>		speed of sound w (m/sec)

Table 5: Library functions for single-phase thermodynamic quantities in `IAPWS::Lib97` class

The saturation curve between $T = 623.15 \text{ K}$ and $T_{\text{crit}} = 647.096 \text{ K}$ belongs to region 3 and thermodynamic quantities on this portion of the saturation curve must be found by implementing the Maxwell construction. Functions listed in Table 6 can be used to handle the tabulated saturation curve. The minimum and maximum temperature of tabulation are set to be 623.15 and 647.096 K, respectively. Once one write the table by calling `export_tab_coex`, the same file can be read by `import_tab_coex` function without generating the table from the scratch.

Once the tabulated saturation curve is produced or imported, one can call functions listed in Table 4 to obtain thermodynamic quantities on the saturation curve. For temperature $T \leq 623.15 \text{ K}$, parametrization for region 1 and 2, along with `get_param4_sat_pressure` function, are used to calculate thermodynamic quantities. For higher temperature $623.15 \text{ K} < T \leq 647.096 \text{ K}$, the tabulated saturation curve is interpolated by means of natural cubic spline. All functions take temperature T (in K) as the input parameter.

function	input	output
<code>make_tab_coex</code>	number of temperature bins (<code>int</code>)	<code>void</code>
<code>export_tab_coex</code>	name of output file (<code>char *</code>)	<code>void</code>
<code>import_tab_coex</code>	name of input file (<code>char *</code>)	<code>void</code>

Table 6: Tabulation of coexisting phases (saturation curve between vapor and liquid) in `IAPWS::Lib97` class

1.4 R10-06 (2009) - `IAPWS::Lib06` class

IAPWS R10-06 [5] release provides parametrization for the specific Gibbs free energy and thermodynamic quantities of H₂O Ice Ih. Temperature T and pressure p are considered as independent variables. The parametrized functions for single-phase are listed in Table 7.

function	input	output
<code>get_param_g</code>	temperature T (K) pressure p (Pa)	specific Gibbs free energy g (J/kg)
<code>get_param_f</code>		specific Helmholtz free energy f (J/kg)
<code>get_param_mdensity</code>		mass density ρ (kg/m ³)
<code>get_param_entropy</code>		specific entropy s (J/kg K)
<code>get_param_heat_c_p</code>		specific isobaric heat capacity c_p (J/kg K)
<code>get_param_enthalpy</code>		specific enthalpy h (J/kg)
<code>get_param_erg_int</code>		specific internal energy u (J/kg)
<code>get_param_coeff_alpha</code>		cubic expansion coefficient $\alpha = \frac{1}{v} \left. \frac{\partial v}{\partial T} \right _p$ (K ⁻¹)
<code>get_param_coeff_beta</code>		pressure coefficient $\beta = \left. \frac{\partial p}{\partial T} \right _v$ (Pa/K)
<code>get_param_comp_kappa_T</code>		isothermal compressibility $\kappa_T = -\frac{1}{v} \left. \frac{\partial v}{\partial p} \right _T$ (Pa ⁻¹)
<code>get_param_comp_kappa_s</code>		isentropic compressibility $\kappa_s = -\frac{1}{v} \left. \frac{\partial v}{\partial p} \right _s$ (Pa ⁻¹)

Table 7: Library functions for single-phase thermodynamic quantities in `IAPWS::Lib06` class

In addition, `IAPWS::Lib06` class comes up with calculation functionalities regarding the sublimation curve (coexisting phases) between water vapor and ice. Table 8 presents functions to handle the tabulated sublimation curve. Specifying and tabulating the sublimation curve requires information on the specific Gibbs free energy of vapor, which is provided by `IAPWS::Lib95` class. When one calls `make_tab_coex` function to generate a table for the sublimation curve, it is necessary to provide a pointer to `IAPWS::Lib95` object as an input. The maximum temperature for tabulation is set to be that of the triple point $T_{\text{trip}} = 273.16$ K. `export_tab_coex` and `import_tab_coex` functions can be used to write and read the tabulated sublimation curve, respectively.

function	input	output
<code>make_tab_coex</code>	pointer to <code>IAPWS::Lib95</code> object (<code>IAPWS::Lib95 *ptr</code>)	<code>void</code>
	number of temperature bins (<code>int</code>)	
	minimum temperature (K)	
<code>export_tab_coex</code>	name of output file (<code>char *</code>)	<code>void</code>
<code>import_tab_coex</code>	name of input file (<code>char *</code>)	<code>void</code>

Table 8: Tabulation of coexisting phases (sublimation curve between vapor and ice) in `IAPWS::Lib06` class

Once the tabulated sublimation curve is produced or imported, one can call functions listed in Table 9 to obtain thermodynamic quantities on the sublimation curve. The tabulated sublimation curve is interpolated by means of natural cubic spline. All functions take temperature T (in K) as the input parameter.

Similarly, the melting curve (coexisting phases) between water liquid and ice is found and tabulated in `IAPWS::Lib06` class. The relevant functions are listed in Tables 10 and 11. Note that the melting curve is tabulated and interpolated as function of pressure p , and the minimum pressure is that of the triple point $p_{\text{trip}} = 611.654771007894$ Pa.

function	input	output
<code>get_coex_pressure</code>	temperature T (K)	sublimation pressure p_{coex} (Pa)
<code>get_coex_mden_vap</code>		mass density of vapor ρ_{vap} (kg/m ³)
<code>get_coex_mden_ice</code>		mass density of ice ρ_{ice} (kg/m ³)
<code>get_coex_enthalpy_vap</code>		specific enthalpy of vapor h_{vap} (J/kg)
<code>get_coex_enthalpy_ice</code>		specific enthalpy of ice h_{ice} (J/kg)
<code>get_coex_entropy_vap</code>		specific entropy of vapor s_{vap} (J/kg K)
<code>get_coex_entropy_ice</code>		specific entropy of ice s_{ice} (J/kg K)
<code>get_coex_heat_latent</code>		specific latent heat $h_{\text{latent;vap-ice}}$ (J/kg)

Table 9: Thermodynamic quantities for coexisting phases (sublimation curve between vapor and ice) in `IAPWS::Lib06` class

function	input	output
<code>make_tab_melt</code>	pointer to <code>IAPWS::Lib95</code> object (<code>IAPWS::Lib95 *ptr</code>) number of pressure bins (<code>int</code>) maximum pressure (Pa)	<code>void</code>
<code>export_tab_melt</code>	name of output file (<code>char *</code>)	<code>void</code>
<code>import_tab_melt</code>	name of input file (<code>char *</code>)	<code>void</code>

Table 10: Tabulation of coexisting phases (melting curve between liquid and ice) in `IAPWS::Lib06` class

1.5 R12-08 (2008) - `IAPWS::Lib08V` class

IAPWS R12-08 [6] release provides parametrization for the shear viscosity μ as a function of the mass density ρ and temperature T . Functions to be used are listed in Table 12. Note that one also needs to have an instance of `IAPWS::Lib95` class to implement the critical enhancement. There is a class (pointer) variable `ptr_lib95_` to connect a `IAPWS::Lib95` object and one can call `set_ptr_lib95` function to set it up.

2 Derivation for thermodynamic quantities

2.1 From the Helmholtz free energy $f(\rho, T)$

If one has mass density ρ (equivalently specific volume $v = 1/\rho$) and temperature T as independent variables, thermodynamic quantities can be derived from the specific Helmholtz free energy f

$$\frac{f(\rho, T)}{RT} = \phi \left(\delta \equiv \frac{\rho}{\rho_*}, \tau \equiv \frac{T_*}{T} \right) \quad (1)$$

where R is the specific gas constant. In addition, dimensionless quantities δ and τ are introduced in terms of reference mass density ρ_* and temperature T_* , respectively. For instance, specific entropy s and pressure p can be given by first derivatives of the Helmholtz free energy.

$$s = - \left. \frac{\partial f}{\partial T} \right|_{\rho} = R(\tau \phi_{\tau} - \phi) \quad \text{where} \quad \phi_{\tau} \equiv \frac{\partial \phi}{\partial \tau} \quad (2)$$

$$p = - \left. \frac{\partial f}{\partial v} \right|_T = \rho^2 \left. \frac{\partial f}{\partial \rho} \right|_T = \rho RT \delta \phi_{\delta} \quad \text{where} \quad \phi_{\delta} \equiv \frac{\partial \phi}{\partial \delta} \quad (3)$$

The specific internal energy u and enthalpy h can be subsequently obtained as

$$u = f + Ts = RT \tau \phi_{\tau} \quad (4)$$

$$h = u + pv = RT(\tau \phi_{\tau} + \delta \phi_{\delta}) . \quad (5)$$

The isochoric heat capacity c_v is given by the partial derivative of u with respect to T .

$$c_v = \left. \frac{\partial u}{\partial T} \right|_{\rho} = -R \tau^2 \phi_{\tau\tau} \quad \text{where} \quad \phi_{\tau\tau} = \frac{\partial^2 \phi}{\partial \tau^2} \quad (6)$$

function	input	output
<code>get_melt_temperature</code>	pressure p (Pa)	melting temperature T_{melt} (K)
<code>get_melt_mden_liq</code>		mass density of liquid ρ_{liq} (kg/m ³)
<code>get_melt_mden_ice</code>		mass density of ice ρ_{ice} (kg/m ³)
<code>get_melt_enthalpy_liq</code>		specific enthalpy of liquid h_{liq} (J/kg)
<code>get_melt_enthalpy_ice</code>		specific enthalpy of ice h_{ice} (J/kg)
<code>get_melt_entropy_liq</code>		specific entropy of liquid s_{liq} (J/kg K)
<code>get_melt_entropy_ice</code>		specific entropy of ice s_{ice} (J/kg K)
<code>get_melt_heat_latent</code>		specific latent heat $h_{\text{latent,liq-ice}}$ (J/kg)

Table 11: Thermodynamic quantities for coexisting phases (melting curve between liquid and ice) in `IAPWS::Lib06` class

function	input	output
<code>set_ptr_lib95</code>	pointer to an instance of <code>Lib95</code> class (<code>IAPWS::Lib95 *ptr</code>)	<code>void</code>
<code>get_param_viscosity</code>	mass density ρ (kg/m ³) temperature T (K)	shear viscosity μ (Pa · sec)

Table 12: Library functions for the shear viscosity in `IAPWS::Lib08V` class

The isobaric heat capacity c_p is defined as the partial derivative of h with respect to T , while the pressure is kept constant.

$$\begin{aligned}
c_p &= \left. \frac{\partial h}{\partial T} \right|_p \\
&= \lim_{\Delta T \rightarrow 0} \frac{h(\rho + \Delta\rho, T + \Delta T) - h(\rho, T)}{\Delta T} \quad \text{where} \quad \left. \frac{\partial p}{\partial \rho} \right|_T \Delta\rho + \left. \frac{\partial p}{\partial T} \right|_\rho \Delta T = 0.
\end{aligned} \tag{7}$$

$$= \left. \frac{\partial h}{\partial T} \right|_\rho - \left. \frac{\partial h}{\partial \rho} \right|_T \frac{\partial p / \partial T|_\rho}{\partial p / \partial \rho|_T} \tag{8}$$

$$= c_v + \frac{1}{\rho} \left. \frac{\partial p}{\partial T} \right|_\rho - \left(\left. \frac{\partial u}{\partial \rho} \right|_T + \frac{1}{\rho} \left. \frac{\partial p}{\partial \rho} \right|_T - \frac{p}{\rho^2} \right) \frac{\partial p / \partial T|_\rho}{\partial p / \partial \rho|_T} = c_v + \frac{1}{\rho} \left(\frac{p}{\rho} - \rho \left. \frac{\partial u}{\partial \rho} \right|_T \right) \frac{\partial p / \partial T|_\rho}{\partial p / \partial \rho|_T} \tag{9}$$

Derivatives of p and u can be written in terms of ϕ as following.

$$\rho \left. \frac{\partial u}{\partial \rho} \right|_T = RT \delta\tau \phi_{\delta\tau} \quad \text{where} \quad \phi_{\delta\tau} = \frac{\partial^2 \phi}{\partial \delta \partial \tau} \tag{10}$$

$$\frac{1}{\rho} \left. \frac{\partial p}{\partial T} \right|_\rho = R(\delta\phi_\delta - \delta\tau \phi_{\delta\tau}) \tag{11}$$

$$\left. \frac{\partial p}{\partial \rho} \right|_T = RT(2\delta\phi_\delta + \delta^2\phi_{\delta\delta}) \quad \text{where} \quad \phi_{\delta\delta} = \frac{\partial^2 \phi}{\partial \delta^2} \tag{12}$$

which lead one to obtain

$$c_p = c_v + R \frac{(\delta\phi_\delta - \delta\tau \phi_{\delta\tau})^2}{2\delta\phi_\delta + \delta^2\phi_{\delta\delta}}. \tag{13}$$

The speed of sound w can be computed by taking derivative of p with respect to ρ , while the specific entropy is kept constant.

$$\begin{aligned}
w^2 &= \left. \frac{\partial p}{\partial \rho} \right|_s \\
&= \lim_{\Delta\rho \rightarrow 0} \frac{p(\rho + \Delta\rho, T + \Delta T) - p(\rho, T)}{\Delta\rho} \quad \text{where} \quad \left. \frac{\partial s}{\partial \rho} \right|_T \Delta\rho + \left. \frac{\partial s}{\partial T} \right|_\rho \Delta T = 0
\end{aligned} \tag{14}$$

$$= \left. \frac{\partial p}{\partial \rho} \right|_T - \frac{1}{\rho} \left. \frac{\partial p}{\partial T} \right|_\rho \frac{\rho \partial s / \partial \rho|_T}{\partial s / \partial T|_\rho} \tag{15}$$

Derivatives of s can be written in terms of ϕ as following.

$$\rho \left. \frac{\partial s}{\partial \rho} \right|_T = -R (\delta \phi_\delta - \delta \tau \phi_{\delta\tau}) \quad (16)$$

$$\left. \frac{\partial s}{\partial T} \right|_\rho = -\frac{R}{T} \tau^2 \phi_{\tau\tau} \quad (17)$$

which result in the following expression for w .

$$w^2 = RT \left[2 \delta \phi_\delta + \delta^2 \phi_{\delta\delta} - \frac{(\delta \phi_\delta - \delta \tau \phi_{\delta\tau})^2}{\tau^2 \phi_{\tau\tau}} \right] \quad (18)$$

2.2 From the Gibbs free energy $g(T, p)$

If one has pressure p and temperature T as independent variables, thermodynamic quantities can be derived from the specific Gibbs free energy g

$$\frac{g(T, p)}{RT} = \gamma \left(\tau \equiv \frac{T_*}{T}, \Pi \equiv \frac{p}{p_*} \right) \quad (19)$$

where we have a dimensionless quantity Π defined in terms of the reference pressure p_* . One can obtain the specific entropy s and specific volume v from first derivatives of the Gibbs free energy.

$$s = - \left. \frac{\partial g}{\partial T} \right|_p = R (\tau \gamma_\tau - \gamma) \quad \text{where} \quad \gamma_\tau \equiv \frac{\partial \gamma}{\partial \tau} \quad (20)$$

$$v = \left. \frac{\partial g}{\partial p} \right|_T = \frac{RT}{p} \Pi \gamma_\Pi \quad \text{where} \quad \gamma_\Pi \equiv \frac{\partial \gamma}{\partial \Pi} \quad (21)$$

which lead to the following expressions for the specific enthalpy and internal energy.

$$h = g + Ts = RT \tau \gamma_\tau \quad (22)$$

$$u = h - pv = RT (\tau \gamma_\tau - \Pi \gamma_\Pi) . \quad (23)$$

The isobaric heat capacity c_p is given by

$$c_p = \left. \frac{\partial h}{\partial T} \right|_p = -R \tau^2 \gamma_{\tau\tau} \quad \text{where} \quad \gamma_{\tau\tau} = \frac{\partial^2 \gamma}{\partial \tau^2} . \quad (24)$$

The isochoric heat capacity c_v is given by the partial derivative of u with respect to T , while the specific volume is kept constant.

$$\begin{aligned} c_v &= \left. \frac{\partial u}{\partial T} \right|_v \\ &= \lim_{\Delta T \rightarrow 0} \frac{u(T + \Delta T, p + \Delta p) - u(T, p)}{\Delta T} \quad \text{where} \quad \left. \frac{\partial v}{\partial T} \right|_p \Delta T + \left. \frac{\partial v}{\partial p} \right|_T \Delta p = 0 . \end{aligned} \quad (25)$$

$$= \left. \frac{\partial u}{\partial T} \right|_p - \left. \frac{\partial u}{\partial p} \right|_T \frac{\partial v / \partial T|_p}{\partial v / \partial p|_T} \quad (26)$$

$$= c_p - p \left. \frac{\partial v}{\partial T} \right|_p - \left(\left. \frac{\partial h}{\partial p} \right|_T - p \left. \frac{\partial v}{\partial p} \right|_T - v \right) \frac{\partial v / \partial T|_p}{\partial v / \partial p|_T} = c_p + \left(pv - p \left. \frac{\partial h}{\partial p} \right|_T \right) \frac{\partial v / \partial T|_p}{p \partial v / \partial p|_T} \quad (27)$$

Derivatives of h and v can be written in terms of γ as following.

$$p \left. \frac{\partial h}{\partial p} \right|_T = RT \Pi \tau \gamma_{\Pi\tau} \quad \text{where} \quad \gamma_{\Pi\tau} = \frac{\partial^2 \gamma}{\partial \Pi \partial \tau} \quad (28)$$

$$\left. \frac{\partial v}{\partial T} \right|_p = \frac{R}{p} \Pi (\gamma_\Pi - \tau \gamma_{\Pi\tau}) \quad (29)$$

$$p \left. \frac{\partial v}{\partial p} \right|_T = \frac{RT}{p} \Pi^2 \gamma_{\Pi\Pi} \quad \text{where} \quad \gamma_{\Pi\Pi} = \frac{\partial^2 \gamma}{\partial \Pi^2} \quad (30)$$

which result in the following expression for c_v .

$$c_v = c_p + R \frac{(\gamma_{\Pi} - \tau \gamma_{\Pi\tau})^2}{\gamma_{\Pi\Pi}} \quad (31)$$

The speed of sound can be computed in a similar manner as one does with the Helmholtz free energy.

$$\begin{aligned} w^2 &= \left. \frac{\partial p}{\partial \rho} \right|_s \\ &= -v^2 \left[\lim_{\Delta p \rightarrow 0} \frac{v(T + \Delta T, p + \Delta p) - v(T, p)}{\Delta p} \right]^{-1} \quad \text{where} \quad \left. \frac{\partial s}{\partial T} \right|_p \Delta T + \left. \frac{\partial s}{\partial p} \right|_T \Delta p = 0 \end{aligned} \quad (32)$$

$$= -v^2 \left(\left. \frac{\partial v}{\partial p} \right|_T - \left. \frac{\partial v}{\partial T} \right|_p \frac{\partial s / \partial p|_T}{\partial s / \partial T|_p} \right)^{-1} = (pv)^2 \left(pT \left. \frac{\partial v}{\partial T} \right|_p \frac{p \partial s / \partial p|_T}{T \partial s / \partial T|_p} - p^2 \left. \frac{\partial v}{\partial p} \right|_T \right)^{-1} \quad (33)$$

Derivatives of s can be written in terms of γ as following.

$$p \left. \frac{\partial s}{\partial p} \right|_T = -R \Pi (\gamma_{\Pi} - \tau \gamma_{\Pi\tau}) \quad (34)$$

$$T \left. \frac{\partial s}{\partial T} \right|_p = -R \tau^2 \gamma_{\tau\tau} \quad (35)$$

which lead to the following expression for the speed of sound.

$$w^2 = RT \left[\frac{\gamma_{\Pi}^2}{(\gamma_{\Pi} - \tau \gamma_{\Pi\tau})^2 / (\tau^2 \gamma_{\tau\tau}) - \gamma_{\Pi\Pi}} \right] \quad (36)$$

3 Phase diagram from R6-95 and R10-06

IAPWS R6-95 [3] release provides parametrization for the specific Helmholtz free energy f of water vapor and liquid for wide range of temperature T and mass density ρ . Thermodynamic quantities, such as pressure, internal energy, enthalpy, entropy and heat capacity, can be obtained from derivatives of the Helmholtz free energy. When a system undergoes first-order phase transition between water vapor and liquid, the saturation pressure p_{coex} remains constant while the mass density changes between ρ_{vap} and ρ_{liq} . Such coexisting phase can be determined by applying the Maxwell construction.

$$p_{\text{coex}}(T) = p(\rho_{\text{vap}}, T) = p(\rho_{\text{liq}}, T) \quad (37)$$

$$p(\rho, T) = - \left. \frac{\partial f}{\partial v} \right|_T = \rho^2 \left. \frac{\partial f}{\partial \rho} \right|_T \quad (38)$$

$$\begin{aligned} f(\rho_{\text{liq}}, T) - f(\rho_{\text{vap}}, T) &= p_{\text{coex}} (v_{\text{vap}} - v_{\text{liq}}) \\ &= p_{\text{coex}} \left(\frac{1}{\rho_{\text{vap}}} - \frac{1}{\rho_{\text{liq}}} \right) \quad \text{where} \quad \text{specific volume } v = \frac{1}{\rho}. \end{aligned} \quad (39)$$

The specific latent heat h_{latent} , which is amount of heat required to transform unit mass of liquid water into vapor in this case, is given by difference in specific enthalpy or entropy.

$$h_{\text{latent;vap-liq}}(T) = h(\rho_{\text{vap}}, T) - h(\rho_{\text{liq}}, T) = T [s(\rho_{\text{vap}}, T) - s(\rho_{\text{liq}}, T)] \quad (40)$$

One can demonstrate that equation (40) is equivalent to what is given by the Clapeyron equation. Let us consider infinitesimally small deviations $\Delta \rho_{\text{vap}}$, $\Delta \rho_{\text{liq}}$ and ΔT in mass densities and temperature, respectively. Then left- and right-hand sides (LHS and RHS) of equation (39) become

$$\begin{aligned} \text{LHS} &= f(\rho_{\text{liq}} + \Delta \rho_{\text{liq}}, T + \Delta T) - f(\rho_{\text{vap}} + \Delta \rho_{\text{vap}}, T + \Delta T) \\ &= f(\rho_{\text{liq}}, T) - f(\rho_{\text{vap}}, T) \\ &\quad + \Delta T \left[\left. \frac{\partial f}{\partial T} \right|_{\rho} (\rho_{\text{liq}}, T) - \left. \frac{\partial f}{\partial T} \right|_{\rho} (\rho_{\text{vap}}, T) \right] + \Delta \rho_{\text{liq}} \left. \frac{\partial f}{\partial \rho} \right|_T (\rho_{\text{liq}}, T) - \Delta \rho_{\text{vap}} \left. \frac{\partial f}{\partial \rho} \right|_T (\rho_{\text{vap}}, T) \\ &= f(\rho_{\text{liq}}, T) - f(\rho_{\text{vap}}, T) + \Delta T [s(\rho_{\text{vap}}, T) - s(\rho_{\text{liq}}, T)] - p_{\text{coex}} \left(\frac{\Delta \rho_{\text{vap}}}{\rho_{\text{vap}}^2} - \frac{\Delta \rho_{\text{liq}}}{\rho_{\text{liq}}^2} \right) \end{aligned} \quad (41)$$

$$\text{RHS} = p_{\text{coex}} \left(\frac{1}{\rho_{\text{vap}}} - \frac{1}{\rho_{\text{liq}}} \right) + \Delta T \frac{dp_{\text{coex}}}{dT} \left(\frac{1}{\rho_{\text{vap}}} - \frac{1}{\rho_{\text{liq}}} \right) - p_{\text{coex}} \left(\frac{\Delta \rho_{\text{vap}}}{\rho_{\text{vap}}^2} - \frac{\Delta \rho_{\text{liq}}}{\rho_{\text{liq}}^2} \right) \quad (42)$$

By equating LHS and RHS, one obtains the Clapeyron equation for the latent heat.

$$h_{\text{latent};\text{vap-liq}}(T) = \left(\frac{1}{\rho_{\text{vap}}} - \frac{1}{\rho_{\text{liq}}} \right) T \frac{dp_{\text{coex}}}{dT} \quad (43)$$

Once `LibIAPWS` library is built, one can run `make_tab_IAPWS95.exec` executable to obtain the coexistence curve (saturation curve) between water vapor and liquid. It produces a text file `tab_coex_IAPWS95.txt` which contains the tabulated coexistence curve as functions of temperature.

The phase diagram can be extended to incorporate the ice phase, by implementing IAPWS R10-06 [5] release. The specific Gibbs free energy g is parametrized as function of temperature T and pressure p , and coexistence curve can be obtained from continuity condition of the Gibbs free energy. The specific Gibbs free energy g_{fluid} for fluid (vapor or liquid) can be obtained from the parametrization in R6-95 release and it must match to g_{ice} provided in R10-06 release.

$$\begin{aligned} g_{\text{ice}}(T, p_{\text{coex}}) &= g_{\text{fluid}}(\rho_{\text{fluid}}, T) \\ &= f_{\text{fluid}}(\rho_{\text{fluid}}, T) + \frac{p_{\text{coex}}}{\rho_{\text{fluid}}} \end{aligned} \quad (44)$$

For given temperature and pressure, mass density ρ_{fluid} of fluid is found by equation (38) and used to compute g_{fluid} . The specific latent heat between fluid and ice can be found in a similar manner as equation (40).

$$h_{\text{latent};\text{fluid-ice}}(T) = h_{\text{fluid}}(\rho_{\text{fluid}}, T) - h_{\text{ice}}(T, p_{\text{coex}}) = T [s_{\text{fluid}}(\rho_{\text{fluid}}, T) - s_{\text{ice}}(T, p_{\text{coex}})] \quad (45)$$

After running `make_tab_IAPWS95.exec` to tabulate the saturation curve, one can run `make_tab_IAPWS06.exec` executable to obtain the coexistence curve between water fluid and ice. It produces two text file `tab_coex_IAPWS06.txt` and `tab_melt_IAPWS06.txt`. `tab_coex_IAPWS06.txt` provides the coexistence curve (sublimation curve) between water vapor and ice, while `tab_melt_IAPWS06.txt` contains the coexistence curve (melting curve) between water liquid and ice. The resulting phase diagram on the T - p plane is presented in Figure 1.

4 Comparison between R6-95 and R7-97

Provided that IAPWS R6-95 [3] and R7-97 [4] releases have significant overlap in the range of validity, it is worthwhile to have a closer look at difference between these two releases. One can compare thermodynamic quantities derived from R6-95 and R7-97 releases on the temperature-pressure (T - p) plane. Let us define relative deviation diff_{97-95} in a thermodynamic quantity \mathcal{Q} as

$$\text{diff}_{97-95} \mathcal{Q} \equiv \frac{\mathcal{Q}_{\text{R7-97}} - \mathcal{Q}_{\text{R6-95}}}{\mathcal{Q}_{\text{R7-97}} + \mathcal{Q}_{\text{R6-95}}} \quad (46)$$

Figure 2 shows relative deviations in the mass density ρ , specific enthalpy h and specific heat capacities. For the most of region, deviations in the mass density, specific energy and entropy are less than 0.05 %. The specific heat capacities c_p and c_v exhibit larger deviation, but they are still less than 0.5 %. Figure 3 presents relative deviations on the saturation curve (coexisting phases) between water vapor and liquid. For the most part of the curve, deviations in the saturation pressure, mass density and specific entropy are less than 0.05 %. Even though deviations increase as one gets closer to the critical temperature, they are still less than 1 %.

References

- [1] <https://iapws.org>
- [2] IAPWS, *Revised Release on Surface Tension of Ordinary Water Substance* (2014).
- [3] IAPWS, *Revised Release on the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use* (2018).
- [4] IAPWS, *Revised Release on the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam* (2012).
- [5] IAPWS, *Revised Release on the Equation of State 2006 for H₂O Ice Ih* (2009).
- [6] IAPWS, *Release on the IAPWS Formulation 2008 for the Viscosity of Ordinary Water Substance* (2008).

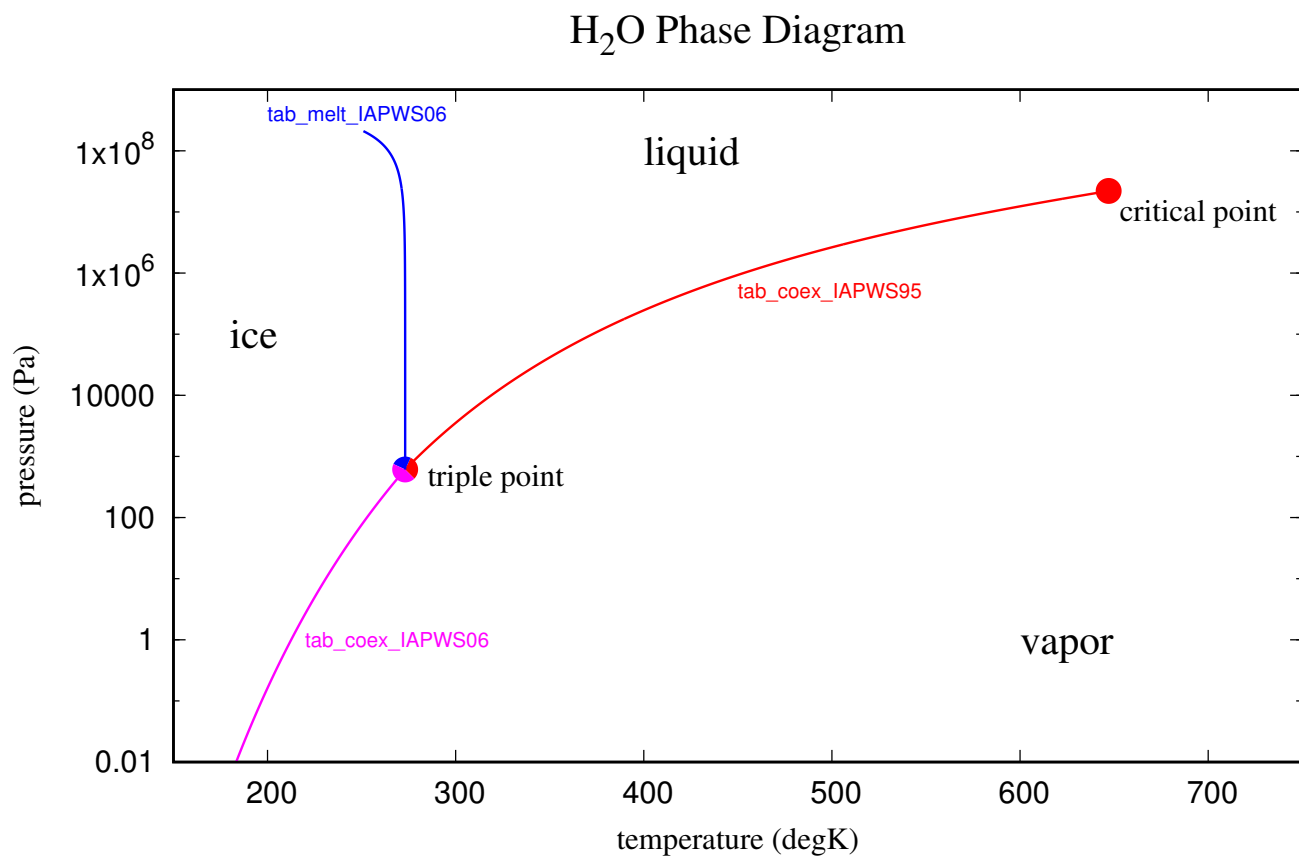


Figure 1: Phase diagram of H₂O derived from IAPWS R6-95 and R10-06 releases.

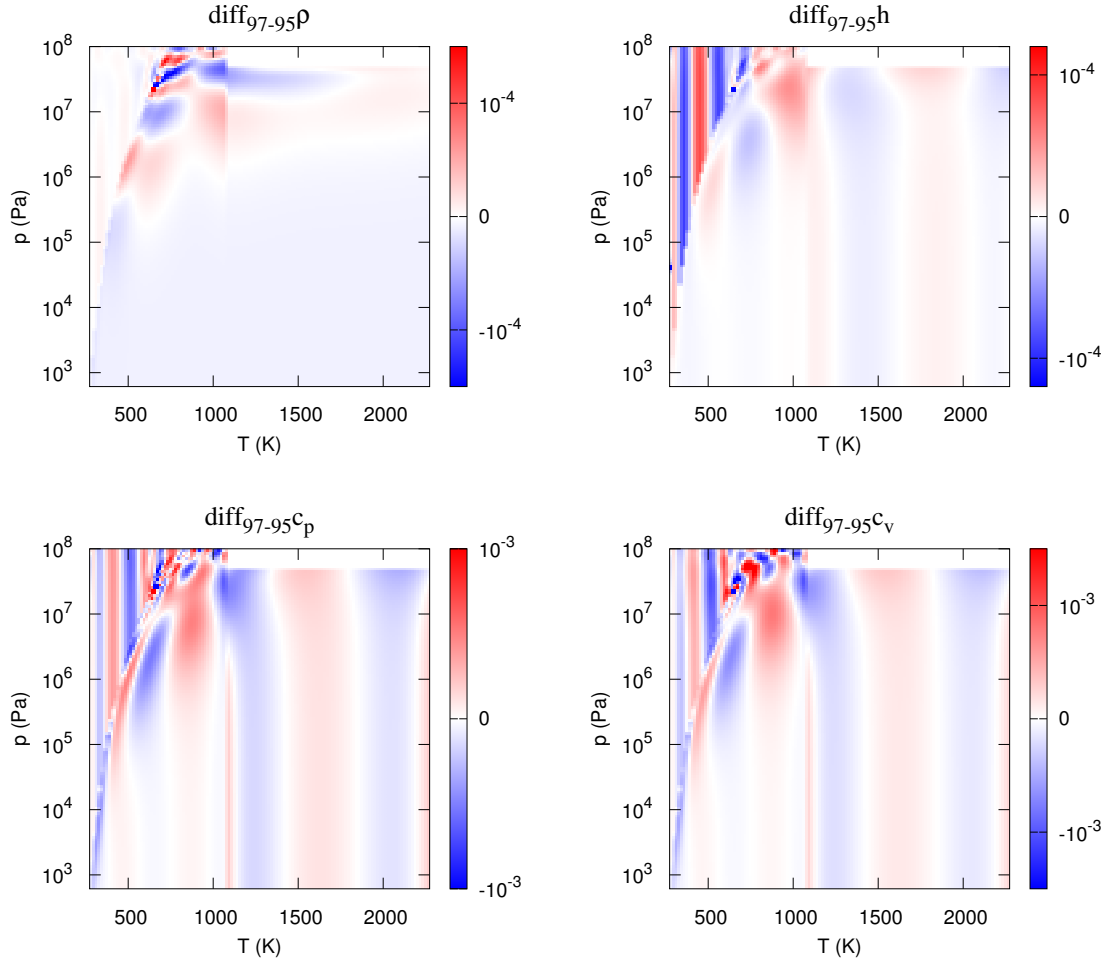


Figure 2: Relative deviations diff_{97-95} in thermodynamic quantities — mass density (top left), specific enthalpy (top right) and specific heat capacities (c_p and c_v at bottom left and right, respectively) — between R06-95 and R07-97.

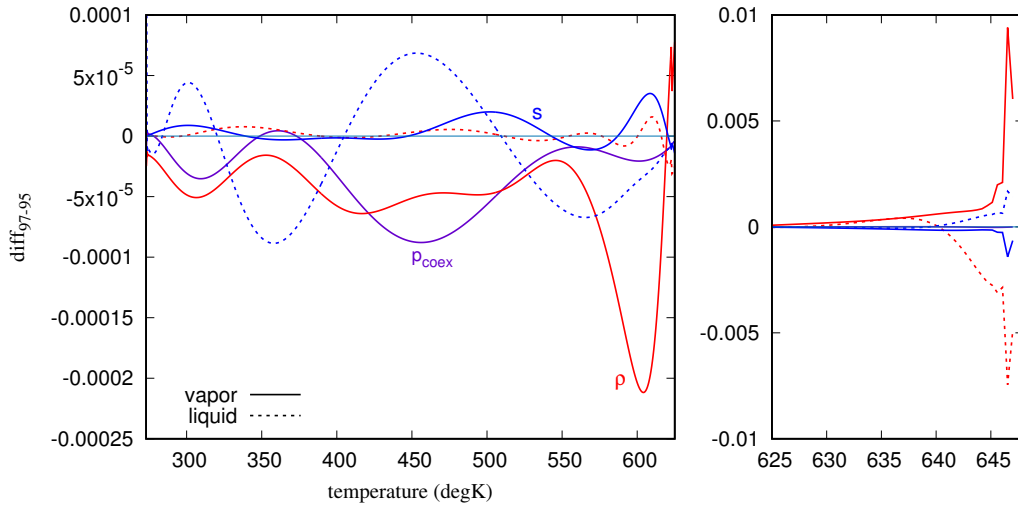


Figure 3: Relative deviations diff_{97-95} in thermodynamic quantities — pressure, mass density and specific entropy — on the saturation curve (coexisting phases) between water vapor and liquid. The solid and dashed curves correspond to vapor and liquid, respectively.