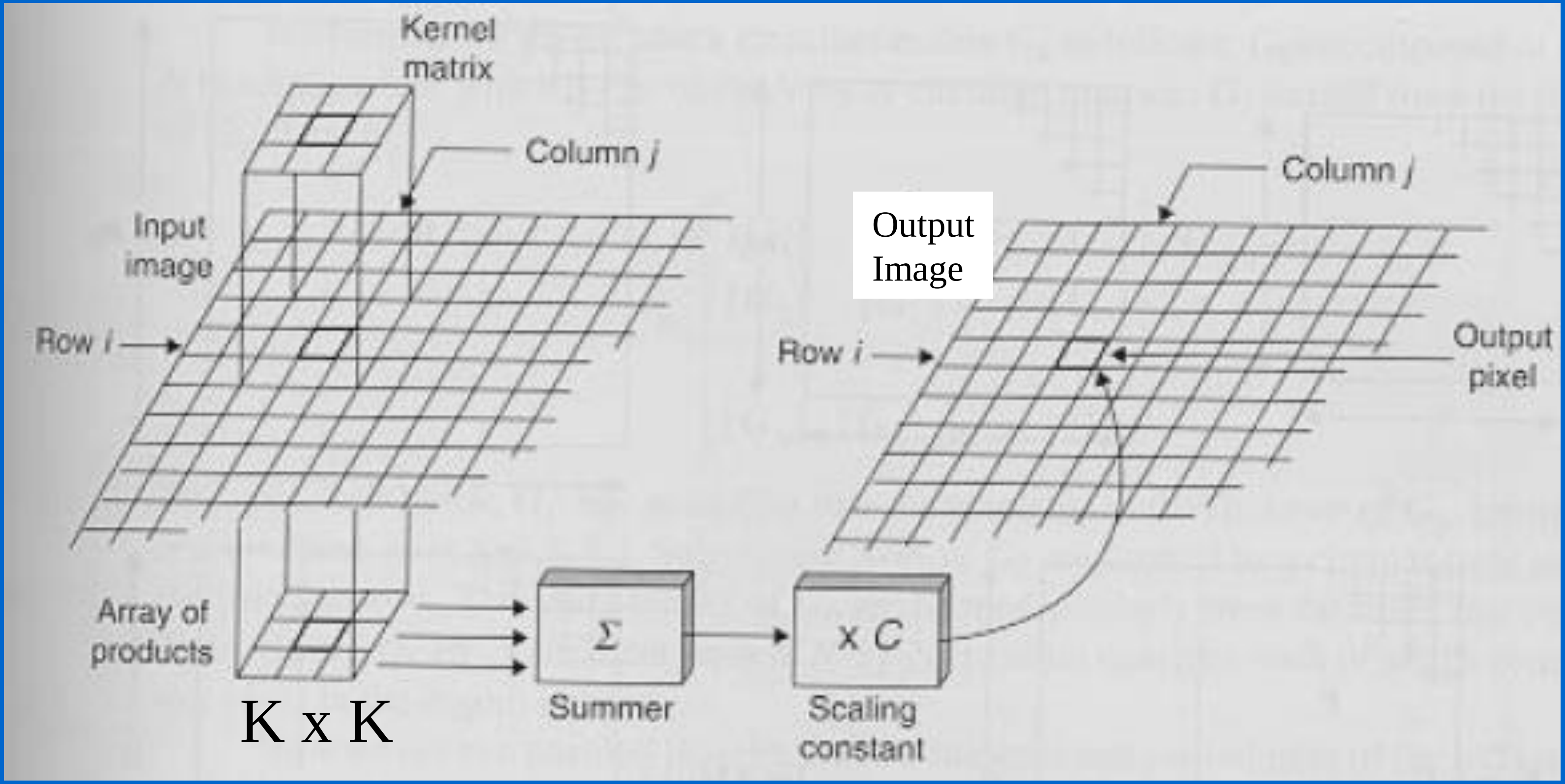
Correlation - Review



$$g(x,y) = w(x,y) \bullet f(x,y) = \sum_{s=-K/2}^{K/2} \sum_{t=-K/2}^{K/2} w(s,t) f(x+s, y+t)$$

Convolution - Review

- Same as correlation except that the mask is **flipped** both horizontally and vertically.

$$g(x,y) = w(x,y) * f(x,y) = \sum_{s=-K/2}^{K/2} \sum_{t=-K/2}^{K/2} w(s,t) f(x-s, y-t)$$

- Note that if w(x,y) is symmetric, that is w(x,y)=w(-x,-y), then convolution is equivalent to correlation!

1D Continuous Convolution - Definition

- Convolution is defined as follows:

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(a)g(x-a)da$$

- Convolution is commutative

$$f(x) * g(x) = g(x) * f(x)$$

Convolution Theorem

- Convolution in the spatial domain is equivalent to multiplication in the frequency domain.

$$f(x) * g(x) <\!\!-\!\!-\!\!> F(u)G(u)$$

$f(x) \longleftrightarrow F(u)$
$g(x) \longleftrightarrow G(u)$

- Multiplication in the spatial domain is equivalent to convolution in the frequency domain.

$$f(x)g(x) <\!\!-\!\!-\!\!> F(u) * G(u)$$

Efficient computation of (f * g)

- 1. Compute $\mathcal{F}(f(x))=F(u)$ and $\mathcal{F}(g(x))=G(u)$
- 2. Multiply them: $F(u)G(u)$ (i.e., complex multiplication)
- 3. Compute the inverse FT: $\mathcal{F}^{-1}(F(u)G(u))=f(x)*g(x)$