2.1 Use the Taylor's series expansion method to a derive a second-order accurate central-difference approximation for $\frac{\partial u}{\partial x}$.

SOLUTION: To derive a second-order accurate central-difference approximation for $\frac{\partial u}{\partial x}$ using Taylor's series expansion, we can begin by expanding $u(x \pm \Delta x)$ in a Taylor series around x:

$$u(x + \Delta x) = u(x) + \Delta x \frac{\partial u}{\partial x}(x) + O(\Delta x^2),$$

$$u(x - \Delta x) = u(x) - \Delta x \frac{\partial u}{\partial x}(x) + O(\Delta x^2).$$

Next, we can subtract the second equation from the first to eliminate the even-order terms:

$$u(x + \Delta x) - u(x - \Delta x) = 2\Delta x \frac{\partial u}{\partial x}(x) + O(\Delta x^3).$$

Solving for $\frac{\partial u}{\partial x}(x)$, we get:

$$\frac{\partial u}{\partial x}(x) = \frac{u(x + \Delta x) - u(x - \Delta x)}{2\Delta x} + O(\Delta x^2).$$

This is the second-order accurate central-difference approximation for $\frac{\partial u}{\partial x}(x)$. The error term is proportional to Δx^2, which means that the approximation becomes more accurate as Δx gets smaller. However, this approximation is not as accurate as the third- or fourth-order approximations.