

6.

$$\text{i) } m(a+bx) = \frac{1}{N} \sum_{i=1}^N (a+bx_i)$$

$$= \frac{1}{N} \left(aN + b \sum_{i=1}^N x_i \right)$$

$$= a + b \left(\frac{1}{N} \sum_{i=1}^N x_i \right)$$

$$= a + b \underline{\underline{m(x)}}$$

$$\text{ii) } \text{cov}(x, x) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(x_i - m(x))$$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2$$

$$= s^2$$

$$\text{iii) } \text{cov}(x, a+bx) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(a+bx_i - m(a+bx_i))$$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(a+bx_i - (a+bm(x)))$$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(b(y_i - m(y)))$$

$$= \frac{b}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y))$$

$$= \underline{\underline{b \text{cov}(x, y)}}$$

6.

$$\text{iv) } \text{cov}(a+bx, a+by) = \frac{1}{N} \sum_{i=1}^N (a+bx_i - m(a+bx_i))(a+by_i - m(a+by_i))$$

$$= \frac{1}{N} \sum_{i=1}^N (a+bx_i - a-bm(x))(a+by_i - b-m(y))$$

$$= \frac{1}{N} \sum_{i=1}^N (b(x_i - m(x))(y_i - m(y)))$$

$$= b^2 \text{cov}(x, y)$$

v) $X = \sum_{i=1}^N x_i$, $m(X) = \frac{1+3}{2} = 2$

$$m^2(X) = 2^2 = 4$$

$$\sqrt{m(X)} = \sqrt{\frac{1+3}{2}} = \sqrt{10}$$

$$= 3 \neq \sqrt{10}$$

$$X = \sum_{i=1}^N x_i, \text{ so } \sqrt{m(X)} = \sqrt{\frac{1+3}{2}} = \sqrt{10}$$

v) Yes, $\text{med}(x)$ is a measurement of center of points. If all points are shifted ($+a$) and scaled (bx) then median follows because its an odd/even point so it shifts as well

$$IQR = Q_3 - Q_1$$

$$\text{so } Q_3(a+bx) - Q_1(a+bx)$$

$$= a + bQ_3 - a - bQ_1$$

$$= b(Q_3 - Q_1)$$

$$= bIQR$$

$$\text{Not } a+bIQR$$