Case Study of Reader/Writer System

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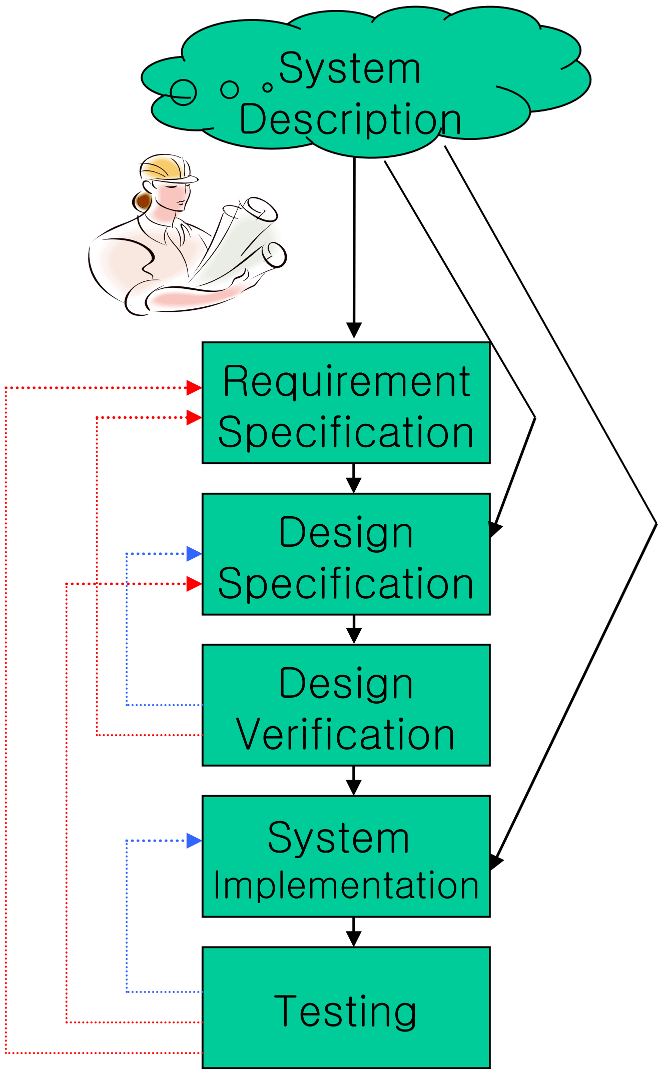
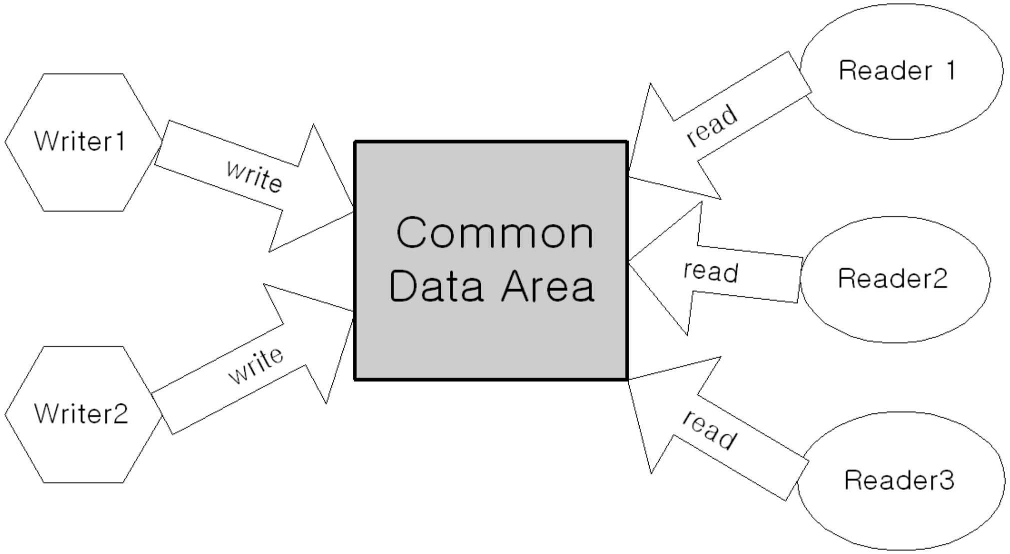
Outlines

- System Description
- Formal Requirement Specification
- Formal Design Specification
- Formal Verification
- Testing



Multiple Reader/Writer System

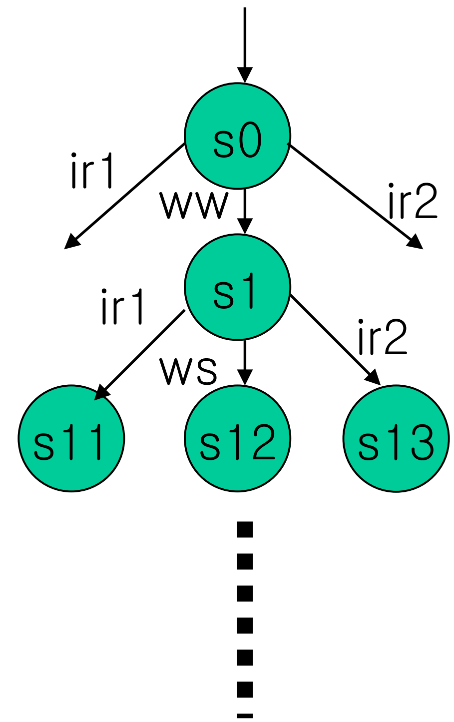


- System requirement
  - Concurrency (CON)
  - Exclusive writing (EW)
  - High priority of writer (HPW)

Multiple Reader/Writer System

- 2 versions of HPW
  - HPW#1
    - While no reader is reading the common data area (CDA), if a writer has tried to write to CDA at the time instance T, no reader should read CDA after T until the writer completes writing.
  - HPW#2
    - While no reader is reading CDA, if a writer has tried to write to CDA at the time instance T <u>and no reader is waiting to read CDA before T,</u> no reader should read CDA after T until the writer completes writing. (i.e., respecting first-come-first-serve)

- 1 writer and 2 readers system
  - Execution tree
  - RW system has 9 events
    - {ir1,rs1,re1,ir2,rs2,re2,ww,ws,we}
  - A state s = $(n_{ir1}, n_{rs1}, n_{ir2}, n_{rs2}, n_{ww}, n_{ws})$
    - s0 = (0,0,0,0,0,0)
    - s1 = (0,0,0,0,1,0)
    - s11=(1,0,0,0,1,0)
    - s12= (0,0,0,0,0,1)



Valid execution paths

Defn 1 (An execution path) *An execution tree is a labeled transition system* (S, T_Σ) *where* S *is a set of states and* $T_\Sigma : S \times \Sigma \times S$ *is a set of transition over* S *with a set of label* Σ. *A state* s *consists of the following 6 integer variables*

$$s \stackrel{def}{=} (n_{ir1}, n_{rs1}, n_{ir2}, n_{rs2}, n_{ww}, n_{ws})$$

An execution path $\sigma = s_0 s_1 \ldots s_n$ *is a sequence of states in an execution tree.* σ_{s_i} *denotes the* i *th state of* σ.

Defn 2 (Definition of a state) $\#ir1(\sigma_{s_0}) \stackrel{def}{=} 0$. $\#ir1(\sigma_{s_i}) \stackrel{def}{=}$ *a number of event* $ir1$ *in an event trace* $\rho = l_0 \ldots l_{i-1}$ *such that* $\sigma_{s_i} \xrightarrow{l_i} \sigma_{s_{i+1}}$ *where* $i > 0$. *Similarly defined are* $\#rs1, \#re1, \#ir2, \#rs2, \#re2, \#ww, \#ws,$ *and* $\#we$.

Defn 2 (Definition of a state) $\#ir1(\sigma_{s_0}) \stackrel{def}{=} 0$. $\#ir1(\sigma_{s_i}) \stackrel{def}{=}$ *a number of event* $ir1$ *in an event trace* $\rho = l_0 \ldots l_{i-1}$ *such that* $\sigma_{s_i} \xrightarrow{l_i} \sigma_{s_{i+1}}$ *where* $i > 0$. *Similarly defined are* $\#rs1, \#re1, \#ir2, \#rs2, \#re2, \#ww, \#ws,$ *and* $\#we$.

State σ_s *of an execution path* σ *consists of the following 6 variables*

$$
\begin{aligned}
n_{ir1}(\sigma_s) &\stackrel{def}{=} \#ir1(\sigma_s) - \#rs1(\sigma_s) \\
n_{rs1}(\sigma_s) &\stackrel{def}{=} \#rs1(\sigma_s) - \#re1(\sigma_s) \\
n_{ir2}(\sigma_s) &\stackrel{def}{=} \#ir2(\sigma_s) - \#rs2(\sigma_s) \\
n_{rs2}(\sigma_s) &\stackrel{def}{=} \#rs2(\sigma_s) - \#re2(\sigma_s) \\
n_{ww}(\sigma_s) &\stackrel{def}{=} \#ww(\sigma_s) - \#ws(\sigma_s) \\
n_{ws}(\sigma_s) &\stackrel{def}{=} \#ws(\sigma_s) - \#we(\sigma_s)
\end{aligned}
$$

Initial state $\sigma_{s_0} \stackrel{def}{=} (0, 0, 0, 0, 0, 0)$

$n_{ir1}(\sigma_s)$ indicates whether there is "active" $ir1$ in an execution path $s_0 \ldots s$. We can think that ith occurence of $rs1$ "cancels" the ith occurence of $ir1$. $n_{ir1}(\sigma_s) = 1$ means that $ir1$ occurs i times and $rs1$ occurs $(i-1)$ times upto state σ_s, which means that $ir1$ is "active".

- Valid execution path σ
  - An execution path $\sigma = s_0 . s_1 \ldots s_n$
    - σ_{si} denotes the i th state of σ
  - Definition of a state s_i.
    - $\#ir1(\sigma_{s0}) = 0$
    - $\#ir1(\sigma_{si}) = \#$ of ir1 in a trace $l_0 \ldots l_{i-1}$ s.t. $\sigma_{si} -l_i\!-\!> \sigma_{si+1}$
    - $n_{ir1}(\sigma_s) = \#ir1(\sigma_s) - \#rs1(\sigma_s)$
    - $n_{rs1}(\sigma_s) = \#rs1(\sigma_s) - \#re1(\sigma_s)$
    - $n_{ir2}(\sigma_s)$, $n_{rs2}(\sigma_s)$, $n_{ww}(\sigma_s)$, $n_{ws}(\sigma_s)$ are defined similarly

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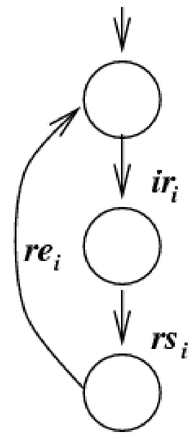
Valid execution path σ

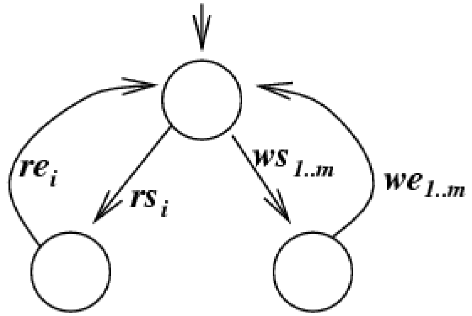
Correct Event Ordering

- For all state s_i in σ
  - $n_{ir1}(s_i)\geq 0 \wedge n_{rs1}(s_i)\geq 0 \wedge n_{ir1}(s_i)+n_{rs1}(s_i) \leq 1$
  - $n_{ir2}(s_i)\geq 0 \wedge n_{rs2}(s_i)\geq 0 \wedge n_{ir2}(s_i)+n_{rs2}(s_i) \leq 1$
  - $n_{ww}(s_i)\geq 0 \wedge n_{ws}(s_i)\geq 0 \wedge n_{ww}(s_i)+n_{ws}(s_i) \leq 1$

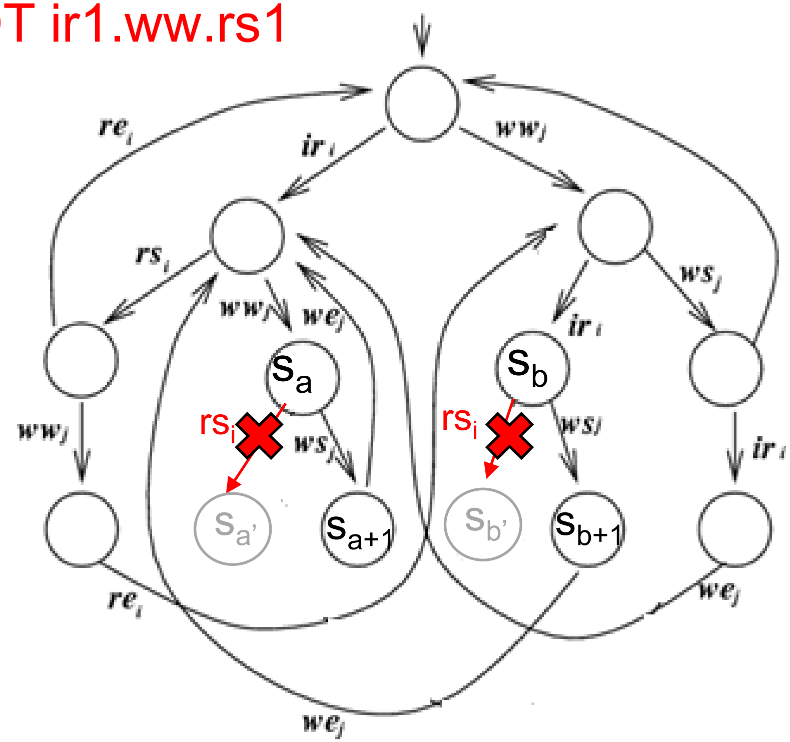


Exclusive Writing

- For all state s_i in σ
  - $n_{ws}(s_i)=1 \rightarrow (n_{rs1}(s_i)=0 \wedge n_{rs2}(s_i)=0)$



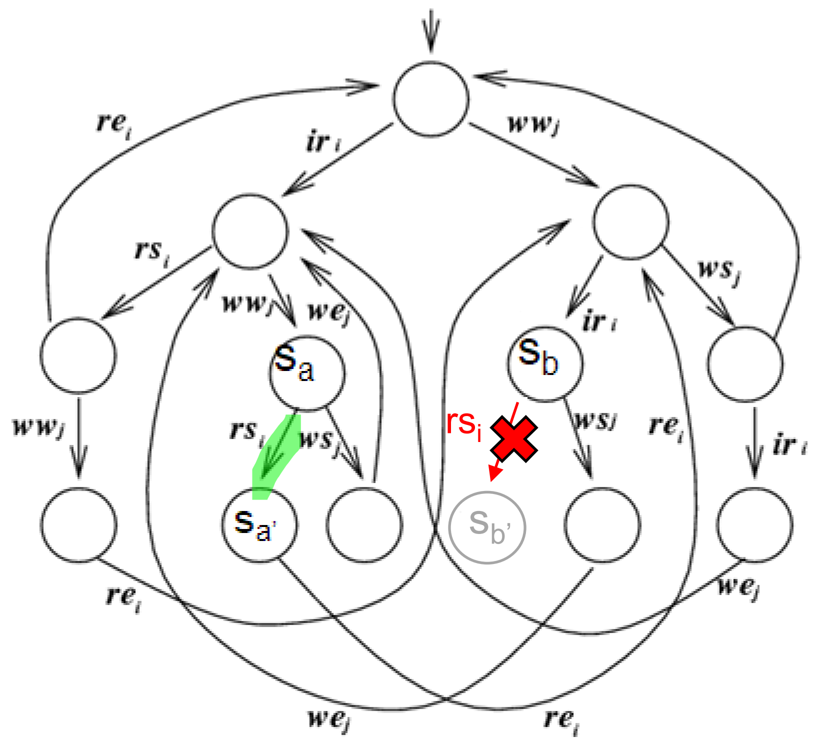
- Valid execution paths
  - High Priority of Writer #1 (HPW#1)
    - $(n_{ww}(s_i)=1 \wedge n_{rs1}(s_i)=0 \wedge n_{rs2}(s_i)=0)$ $\rightarrow (n_{rs1}(s_{i+1})=0 \wedge n_{rs2}(s_{i+1})=0)$
    - Ex. HPW#1 allow ir1.ww.ws, but NOT ir1.ww.rs1
    - Ex1. For a state s_a in the right LTS,
      - s_a -ws_j-> s_{a+1} is valid, since $n_{ww}(s_a)=1$, $n_{rs1}(s_a)=0$, $n_{rs2}(s_i)=0$, $n_{rs1}(s_{a+1})=0$, $n_{rs2}(s_{a+1})=0$
    - Ex2. For a state s_a in the right LTS,
      - s_a -rs_i-> $s_{a'}$ is **NOT** valid, since $n_{ww}(s_a)=1$, $n_{rs1}(s_a)=0$, $n_{rs2}(s_i)=0$, $n_{rs1}(s_{a'})=$**1**, $n_{rs2}(s_{a'})=0$



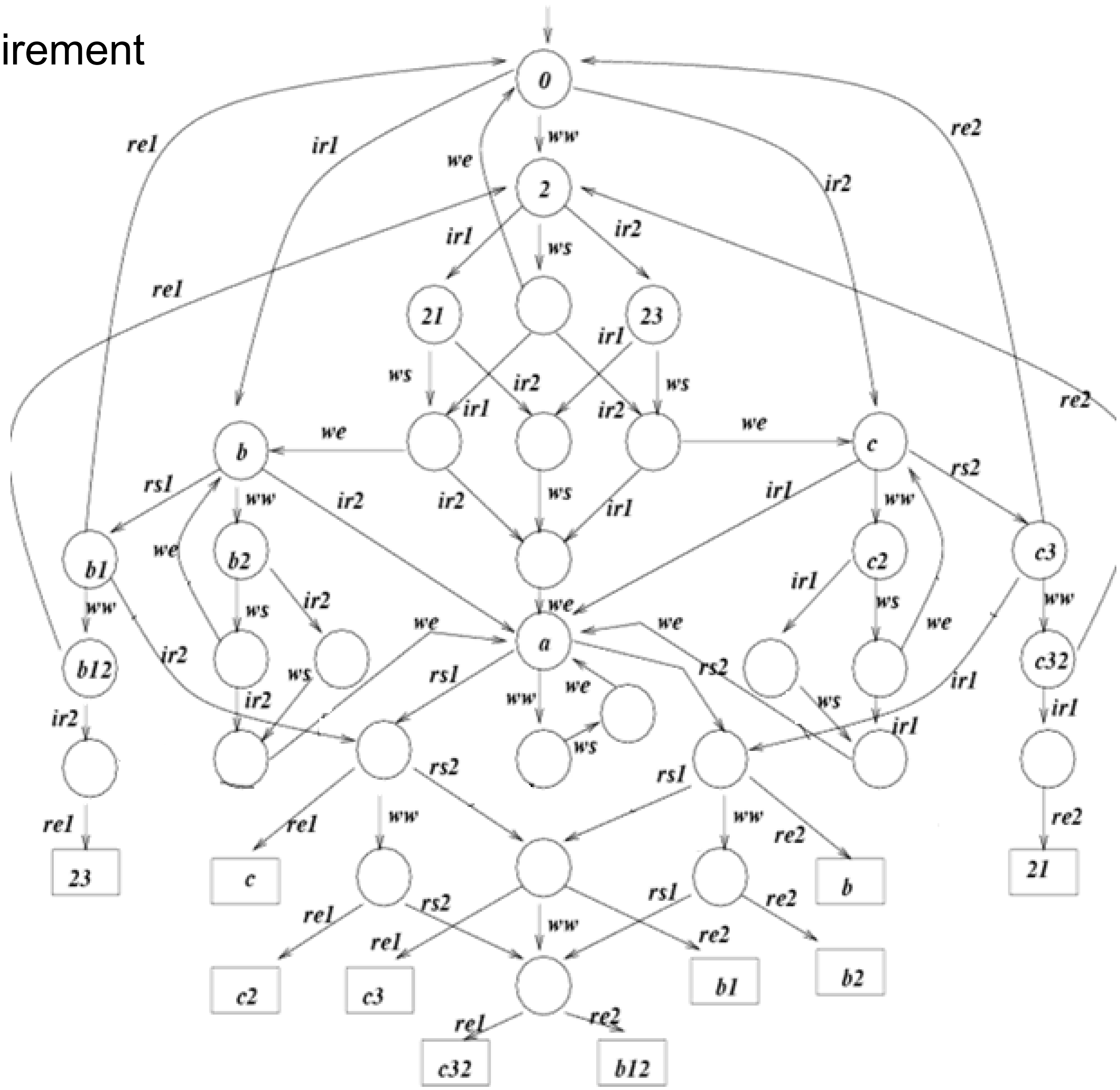
Valid execution paths

High Priority of Writer #2 (HPW#2)

- Difficult to formally specify HPW#2 in the given formal framework since we need to specify an order of events in a trace
  - Ex. we need to distinguish **ir1.ww.rs1** and **ww.ir1.rs1**

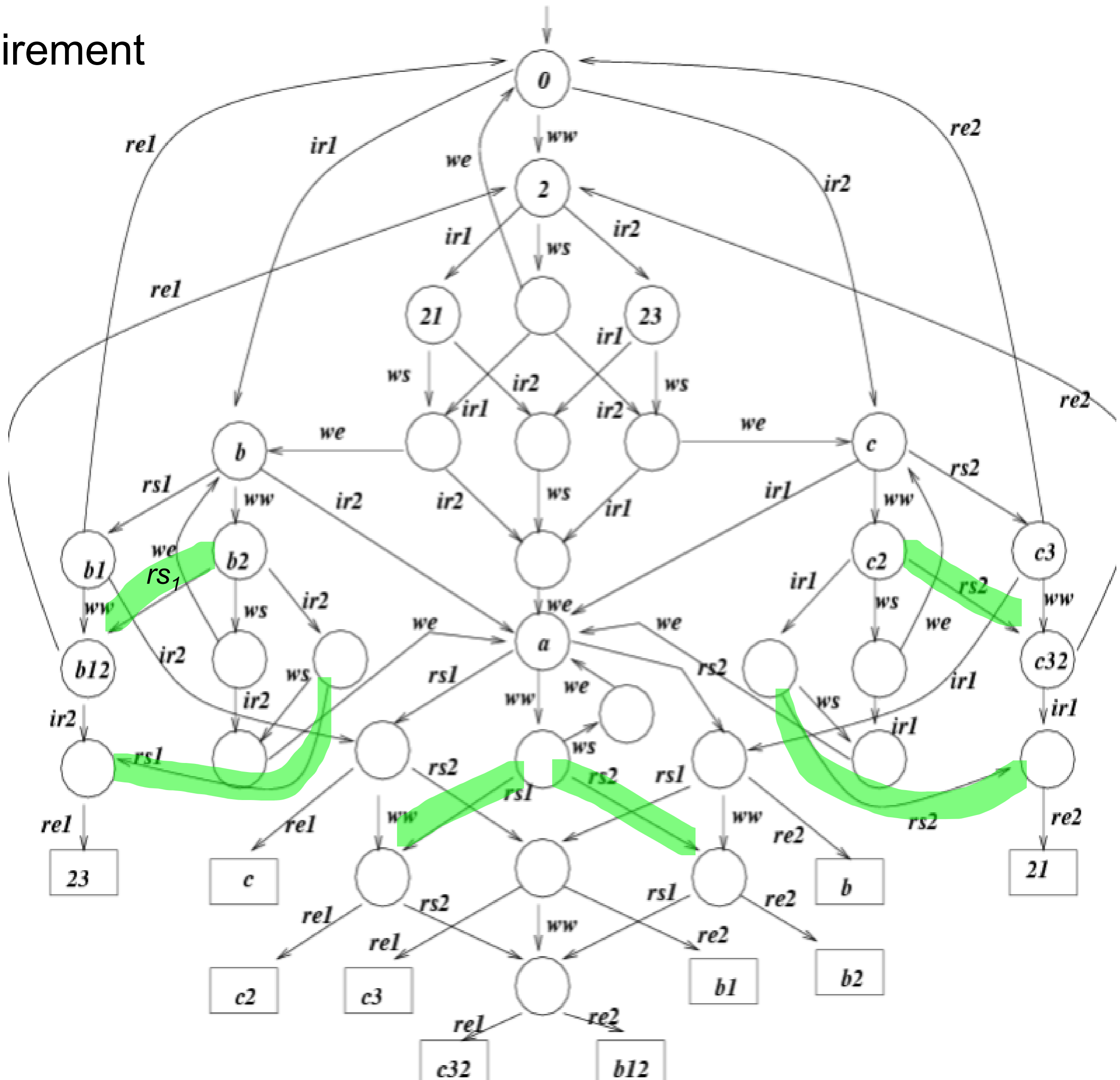


LTS for the Requirement
w/ **HPW#1**



Formal Requirement Specification (cont.)

LTS for the Requirement
w/ **HPW#2**



New transitions that are not allowed by HPW#1

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```
***********************************
* Requirement Specification w/ HPW#1  *
***********************************
proc ReqRW_HPW1 = ir1.B + ww.S2 + ir2.C
proc S2 = ir1.S21 + ws.S22 + ir2.S23
proc S21 = ws.S212 + ir2.S213
proc S22 = ir1.S212 + we.ReqRW_HPW1+
ir2.S232
proc S23 = ir1.S213 + ws.S232
proc S212 = we.B + ir2.S2123
proc S213 = ws.S2123
proc S232 = ir1.S2123 + we.C
proc S2123 = we.A

proc A = rs1.A1 + ww.A2 + rs2.A3
proc A1 = re1.C + ww.A12 + rs2.A13
proc A2 = ws.we.A
proc A3 = rs1.A13 + ww.A32 + re2.B
proc A12 = re1.C2 + rs2.A123
proc A13 = re1.C3 + ww.A123 + re2.B1
proc A32 = rs1.A123 + re2.B2
proc A123 = re1.C32 + re2.B12

proc B = rs1.B1 + ww.B2 + ir2.A
proc B1 = re1.ReqRW_HPW1 + ww.B12 +
ir2.A1
proc B2 = ws.B22 + ir2.B23
proc B12 = re1.S2 + ir2.B123
proc B22 = we.B + ir2.B223
proc B23 = ws.B223
proc B123 = re1.S23
proc B223 = we.A

proc C = ir1.A + ww.C2 + rs2.C3
proc C2 = ir1.C21 + ws.C22
proc C3 = ir1.A3 + ww.C32 +
re2.ReqRW_HPW1
proc C21 = ws.C221
proc C22 = ir1.C221 + we.C
proc C32 = ir1.C321 + re2.S2
proc C221 = we.A
proc C321 = re2.S21
```

```
*************************************************
* Requirement Specification w/ HPW#2  *
*************************************************
proc ReqRW_HPW2 = ir1.B + ww.S2 + ir2.C
proc S2 = ir1.S21 + ws.S22 + ir2.S23
proc S21 = ws.S212 + ir2.S213
proc S22 = ir1.S212 + we.ReqRW_HPW2+
ir2.S232
proc S23 = ir1.S213 + ws.S232
proc S212 = we.B + ir2.S2123
proc S213 = ws.S2123
proc S232 = ir1.S2123 + we.C
proc S2123 = we.A

proc A = rs1.A1 + ww.A2 + rs2.A3
proc A1 = re1.C + ww.A12 + rs2.A13
proc A2 = rs1.A12 + ws.we.A  + rs2.A32
proc A3 = rs1.A13 + ww.A32 + re2.B
proc A12 = re1.C2 + rs2.A123
proc A13 = re1.C3 + ww.A123 + re2.B1
proc A32 = rs1.A123 + re2.B2
proc A123 = re1.C32 + re2.B12

proc B = rs1.B1 + ww.B2 + ir2.A
proc B1 = re1.ReqRW_HPW2 + ww.B12 +
ir2.A1
proc B2 = rs1.B12 + ws.B22 + ir2.B23
proc B12 = re1.S2 + ir2.B123
proc B22 = we.B + ir2.B223
proc B23 = ws.B223 + rs1.B123
proc B123 = re1.S23
proc B223 = we.A

proc C = ir1.A + ww.C2 + rs2.C3
proc C2 = ir1.C21 + ws.C22 +rs2.C32
proc C3 = ir1.A3 + ww.C32 +
re2.ReqRW_HPW2
proc C21 = ws.C221 + rs2.C321
proc C22 = ir1.C221 + we.C
proc C32 = ir1.C321 + re2.S2
proc C221 = we.A
proc C321 = re2.S21
```

Formal Design Specification

- RW system designed in “Concurrent Programming in Java[Lea99]”
- proc S = (R1|R2|W|AR0|WW0|AW0| LOCK|SLEEP0)\ { dec_WW, inc_WW, dec_AW,inc_AW,…}

  proc R1 = …

  - Processes (R1, R2, W, Lock, etc) communicate each other through signals (dec_WW, inc_WW, etc)

  - variables in RW code are represented as processes (AR0, AW0, etc)

```
class RW {
    int activeReaders_ = 0;
    int activeWriters_= 0;
    int waitingReaders_= 0;
    int waitingWriters_ = 0;

    void read()    {
            beforeRead();
            read_();
            afterRead();
    }

    void beforeRead()        {…}
    void read_()             {…}
    void afterRead()         {…}
            …
}
```

Testing using Formal Specification

- Insert probe into the RW source code
  - Probe generates event signal
- Testing RW code utilizing formal requirement spec as a test oracle
  - Use CWB-NC based simulation feature
  - Inappropriate event signal means violation

```
public abstract class RW{
    protected int activeReaders_ = 0;
    protected int activeWriters_= 0;
    protected int waitingReaders_= 0;
    protected int waitingWriters_ = 0;

    public void read(String id)    {
        beforeRead();
        read_(id);
        afterRead();
    }
    protected synchronized void beforeRead(){
        Event("ir", pid);
        ...   }

    public void read_() {
        Event("rs", pid);
        ...   }

    protected synchronized void afterRead(){
        Event("re", pid);
        ...    }
... }
```

RW Java Code

```
public abstract class RW2  {
   protected int activeReaders_ = 0;  //threads executing read_
   protected int activeWriters_= 0;      //always 0 or 1
   protected int waitingReaders_= 0; //threads not yet in read_
   protected int waitingWriters_ = 0;  //same for write_

   protected abstract void read_(String id);
   protected abstract void write_(String id);

   void Event(String s){ }//System.out.println(s);}

   public void read(String id)    {
      beforeRead();
      read_(id); // Event("rs" , pid);
      afterRead();
   }

   public void write(String id)    {
      beforeWrite();
      write_(id); //  Event("ws", pid);
      afterWrite();
   }

   protected boolean allowReader()    {
      if (waitingWriters_ == 0 && activeWriters_ == 0)      {
         return true;
      }
      else
         return false;
   }

protected boolean allowWriter()    {
   if(activeReaders_ == 0 && activeWriters_ == 0) {
      return true;
   } else return false; }

protected synchronized void beforeRead()    {
   Event("ir" , pid);
  ++waitingReaders_;
   while(!allowReader())
       try{ wait();}
       catch(InterruptedException ex){}
   --waitingReaders_;
   ++activeReaders_;}

protected synchronized void afterRead()    {
   Event("re" , pid);
   --activeReaders_;
   notifyAll();}

protected synchronized void beforeWrite()    {
   Event("ww" , pid);
   ++waitingWriters_;
   while(!allowWriter())
      try{wait();}
      catch(InterruptedException ex){}
   --waitingWriters_;
   ++activeWriters_;}

protected synchronized void afterWrite()    {
   Event("we" , pid);
   --activeWriters_;
   notifyAll(); }
```

RW System Design

```
*********************************************
* RW system design of 2 Readers and 1 Writer *
* matching to the Java Implementation   *
*********************************************
proc DesignRW = (R1|R2|W|AR0|WW0|AW0|LOCK|SLEEP0)\
                {dec_WW, inc_WW, dec_AW,inc_AW, dec_AR, inc_AR,
                 zero_WW, zero_AW, zero_AR, non_zero_WW,non_zero_AW,
                 non_zero_AR, lock, unlock,
                 zero_sleep, one_sleep, two_sleep, dec_sleep, inc_sleep,
                 wake_up}

proc WW0 = zero_WW.WW0 + inc_WW.WW1
proc WW1 = dec_WW.WW0 + non_zero_WW.WW1

proc AW0 = zero_AW.AW0 + inc_AW.AW1
proc AW1 = dec_AW.AW0 + non_zero_AW.AW1

proc AR0 = zero_AR.AR0 + inc_AR.AR1
proc AR1 = dec_AR.AR0 + inc_AR.AR2
      + non_zero_AR.AR1
proc AR2 = dec_AR.AR1 + non_zero_AR.AR2

proc SLEEP0 = zero_sleep.SLEEP0 + inc_sleep.SLEEP1
proc SLEEP1 = one_sleep.SLEEP1 + inc_sleep.SLEEP2 + dec_sleep.SLEEP0
proc SLEEP2 = two_sleep.SLEEP2 + dec_sleep.SLEEP1

proc R1 =    'lock.ir1.
    (  'zero_WW.
                ('zero_AW.'inc_AR.'unlock.READ1
                   + 'non_zero_AW.'inc_sleep.'unlock.R1')
                + 'non_zero_WW.'inc_sleep.'unlock.R1')
proc R1' =    wake_up.'lock.
    (  'zero_WW.
                ('zero_AW.'inc_AR.'unlock.READ1
                   + 'non_zero_AW.'inc_sleep.'unlock.R1')
                + 'non_zero_WW.'inc_sleep.'unlock.R1')

proc R2 =    'lock.ir2.
    (  'zero_WW.
        ('zero_AW.'inc_AR.'unlock.READ2
           + 'non_zero_AW.'inc_sleep.'unlock.R2')
        + 'non_zero_WW.'inc_sleep.'unlock.R2')
proc R2' =    wake_up.'lock.
    (  'zero_WW.
        ('zero_AW.'inc_AR.'unlock.READ2
           + 'non_zero_AW.'inc_sleep.'unlock.R2')
        + 'non_zero_WW.'inc_sleep.'unlock.R2')

proc W =    'lock.ww.'inc_WW.
    ( 'zero_AR.
        ('zero_AW.'dec_WW.'inc_AW.'unlock.WRITE
           +'non_zero_AW.'inc_sleep.'unlock.W')
        + 'non_zero_AR.'inc_sleep.'unlock.W')
proc W' =    wake_up.'lock.
    ( 'zero_AR.
        ('zero_AW.'dec_WW.'inc_AW.'unlock.WRITE
           +'non_zero_AW.'inc_sleep.'unlock.W')
        + 'non_zero_AR.'inc_sleep.'unlock.W')

proc READ1 = rs1.re1.'lock.'dec_AR.
        ('zero_sleep.'unlock.R1 +
         'one_sleep.'wake_up.'dec_sleep.'unlock.R1 +
         'two_sleep.'wake_up.'dec_sleep.'wake_up.'dec_sleep.'unlock.R1)
proc READ2 = rs2.re2.'lock.'dec_AR.
        ('zero_sleep.'unlock.R2+
         'one_sleep.'wake_up.'dec_sleep.'unlock.R2+
         'two_sleep.'wake_up.'dec_sleep.'wake_up.'dec_sleep.'unlock.R2)
proc WRITE = ws.we.'lock.'dec_AW.
        ('zero_sleep.'unlock.W +
         'one_sleep.'wake_up.'dec_sleep.'unlock.W +
         'two_sleep.'wake_up.'dec_sleep.'wake_up.'dec_sleep.'unlock.W)

proc LOCK = lock.unlock.LOCK
```

- May preorder (classical trace inclusion)
  - P $\cdot_{\text{may}}$ Q iff on T’(P)µ T’(Q)
    - Ex. le –S may “a.nil” “a.b.nil”
      - Since T’(a.nil) = {a}, T’(a.b.nil) = {a,a.b}
      - But not le –S may “a.b.nil” “a.nil”

Formal Verification Result

```
cwb-nc>
le -S may DesignRW ReqRW_HPW1
Building automaton...
......
States: 620
Transitions: 1016
Done building automaton.
Building automaton...
States: 34
Transitions: 69
Done building automaton.
Transforming automaton...
Done transforming automaton.
FALSE...
DesignRW has trace:
        ir1 ww rs1
ReqRW_HPW1 does not.
```

```
cwb-nc>
le -S may DesignRW ReqRW_HPW2

Building automaton...
......
States: 620
Transitions: 1016
Done building automaton.
Building automaton...
States: 34
Transitions: 75
Done building automaton.
Transforming automaton...
Done transforming automaton.
TRUE
```

DesignRW violates ReqRW_HPW1, because
(as a counter example ir1.ww.rs1 indicates)
R1 can read before W writes
if R1 performs ir1 before W performs ww.

DesignRW satisfies ReqRW_HPW2

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Formal Verification Result

```
cwb-nc>
le -S may ReqRW_HPW2 DesignRW
Building automaton...
States: 34
Transitions: 75
Done building automaton.
Building automaton...
......
States: 620
Transitions: 1016
Done building automaton.
Transforming automaton...
Done transforming automaton.
FALSE...
ReqRW_HPW2 has trace:
        ir1 ww ws
DesignRW does not.
```

See the following Java code to figure out why RW system cannot performs ir1.ww. ws

```
public void write(String id)   {
              beforeWrite();
              write_(id); //  Event("ws", pid);
              afterWrite();
   }
protected synchronized void beforeRead()   {
   Event("ir" , pid);
   ++waitingReaders_;
   while(!allowReader())
       try{ wait();}
       catch(InterruptedException ex){}
   --waitingReaders_;
   ++activeReaders_;}

protected synchronized void beforeWrite()   {
   Event("ww" , pid);
   ++waitingWriters_;
   while(!allowWriter())
      try{wait();}
      catch(InterruptedException ex){}
   --waitingWriters_;
   ++activeWriters_;}

protected boolean allowWriter()   {
   if(activeReaders_ == 0 && activeWriters_ == 0) {
      return true;
   } else return false; }
```

Formal Verification Result

- Regarding ir1.ww.rs1
  - Can DesignRW perform it?
  - Does ReqRW_HPW1 allow it?
  - Does ReqRW_HPW2 allow it?
- Regarding ir1.ww.ws
  - Can DesignRW perform it?
  - Does ReqRW_HPW1 allow it?
  - Does ReqRW_HPW2 allow it?
- Regarding ww.ir1.ws
  - Can DesignRW perform it?
  - Does ReqRW_HPW1 allow it?
  - Does ReqRW_HPW2 allow it?
- Regarding ww.ir1.rs1
  - Can DesignRW perform it?
  - Does ReqRW_HPW1 allow it?
  - Does ReqRW_HPW2 allow it?