THE DYNAMICS OF A FISH STOCK EXPLOITED IN TWO FISHING ZONES

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ABSTRACT

This work presents a specific stock-effort dynamical model. The stocks correspond to two populations of fish moving and growing between two fishery zones. They are harvested by two different fleets. The effort represents the number of fishing boats of the two fleets that operate in the two fishing zones. The bioeconomical model is a set of four ODE's governing the fishing efforts and the stocks in the two fishing areas. Furthermore, the migration of the fish between the two patches is assumed to be faster than the growth of the harvested stock. The displacement of the fleets is also faster than the variation in the number of fishing boats resulting from the investment of the fishing income. So, there are two time scales: a fast one corresponding to the migration between the two patches, and a slow time scale corresponding to growth. We use aggregation methods that allow us to reduce the dimension of the model and to obtain an aggregated model for the total fishing effort and fish stock of the two fishing zones. The mathematical analysis of the model is shown. Under some conditions, we obtain a stable equilibrium, which is a desired situation, as it leads to a sustainable harvesting equilibrium, keeping the stock at exploitable densities.

KEYWORDS: Dynamical population, fishing efforts, metapopulation, time scales, aggregation method, equilibrium, stability.

1. INTRODUCTION

The basic subdivision of fishing zones of a coastal state consists of the artisanal fishery which operates up to 3 miles from the coast, the coastal fishery that is between 3 and 12 miles from the coast and the high sea fishery which is beyond 12 miles from the coast. The adjacent coastal state is the owner of the resource evolving in this Exclusive Economical Zone (E.E.Z.). They are responsible for the management of the fishery in its E.E.Z. which is shared between different kinds of fisheries. So, in order to bring the situation under control, it is important to have a good knowledge of the global evolution of the resource and of the activity related to its exploitation. The fishery management authorities must deal with the possible fishery conflicts resulting for instance from the common exploitation of two fishing zones. Theoretically, each kind of fleet operates in its zone according to its own fishery characteristics. But in practice, the fish stock does not remain in a given area and frequently moves between two adjacent zones. Also, the fleets do not hesitate in crossing the invisible boundary between two adjacent zones in order to increase their catch. Therefore, it is interesting

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to build a bio-economical model capable of describing the stock-effort dynamic between two adjacent zones.

The research of a stable equilibrium point of a bio-economical model leads to the identification of a sustainable yield level which guarantees a continuity in the activity. At the same time, keeping the fish population at an equilibrium level, allows a permanent and balanced regeneration of the stock. These are ecological and biological interests in the analysis of a bio-economical model of a resource distributed over two fishing areas for the safeguard of the total resource.

The metapopulation concept is widely and firmly established in population biology. There are many works which explore the scope of the metapopulation concept and its applications, and we refer for example to Gilpin and Hanski (1991); Hastings and Harrison (1994), Hanski and Gilpin, (1997).

Thus, in the metapopulation approach, the populations are spatially structured into assemblages of local breeding populations and the migration among the local populations has some effects on local dynamics, including the possibility of population reestablishment following extinction. The book by Hanski and Gilpin (1997) presents a review of metapopulation biology and mathematical models of metapopulations and offers a more complete explanation of metapopulation approach and its conceptual domain.

In our approach, the population (the stock of fishes) is distributed on two spatial patches but with frequent migrations between them. However, the growth conditions and the fishing efforts are different in the two patches. The aim of this work is to study the effect of fast boat migrations between two fishing areas. It is also the aim to study the distribution of the fishing efforts on the stability of the system, that is the existence in the long term of a stable equilibrium for the fishing fleet exploiting the fish stock in the total zone.

In the next section, we construct a dynamical system describing the evolution of both fish stocks (in two fishing zones) and fishing efforts. The two fishing zones, which we call patches, are adjacent and have a small width in the sea (see Figure 1). Classical models and methods for the study of dynamical systems describing the evolution of biological populations can be found in (Edelstein-Keshet, 1988; Murray, 1989; Arrowsmith and Place, 1992).

In Section 3, we present the simplest case where fishing efforts are constants. In Section 4, we study the general case where fishing boats of each fleet are moving between the two fishing zones with constant migration rates, and we suppose that the variation of efforts in each fishing area depends on the fishery revenue, which can be considered as an investment.

We take advantage of the two time scales involved in the dynamics in order to reduce the dimension of the dynamical system. Aggregation methods are used (Auger and Roussarie, 1994; Auger and Poggiale, 1996a; Michalski *et al.*, 1997) in order to obtain a global dynamical model, which governs the total effort and stock of the two fishing zones. The study of the aggregated model leads to a stable stock-effort equilibrium.

Such a stable equilibrium state is a desirable situation, because it leads to a sustainable harvest, while keeping the stock at an exploitable density.

All parts of this work are provided with bio-economical interpretations.

2. PRESENTATION OF THE MODEL

We consider a biomathematical model which describes the dynamics of two fish populations x_1 and x_2 located in a border band between two fishing zones, and exploited by two fishing fleets which are represented by their fishing efforts E_1 and E_2. We assume that population x_1 situated in the first zone is harvested by fleet E_1, while population x_2 situated in the second zone is harvested by fleet E_2. The fish populations move and grow continually.

In this work, we present a new fishing model describing the time variation of the fishing efforts depending both on the fleet displacement and the fishing revenue.

Dynamics of the harvested resource stocks

The classical models of fisheries have been elaborated by Clark (1990). Clarke and Munro (1991), and Raïssi (1999) have studied variations of these models.

We assume in our model that in absence of migration between the two fishing zones, the evolution of the stocks is of a logistical type in each zone with a catch proportional to the stock and the fishing effort in this area. So it can be modeled as follows:

$$\begin{cases} \dot{x}_1 = r_1 x_1 (1 - \frac{x_1}{K_1}) - q_1 E_1 x_1 \\ \dot{x}_2 = r_2 x_2 (1 - \frac{x_2}{K_2}) - q_2 E_2 x_2 \end{cases} \tag{2.1}$$

where r_i $(i = 1,2)$ represents the intrinsic growth rate of the stock in zone i. Patches have distinct characteristics, so we suppose that parameters r_1 and r_2 are different. K_i $(i = 1,2)$ is the carrying capacity and parameter q_i $(i = 1,2)$ is the catchability coefficient of the fleet on patch i. The catchability is supposed to be constant, and in order to simplify the calculations, we choose $q_i = 1$ $(i = 1,2)$.

System (2.1) describes the fish stocks evolution in absence of migration of fish between the two patches. This evolution is related on the one hand to the natural growth (of logistic type) of the stocks that is described by the first term of the right hand side of system (2.1). On the other hand it is related to the catch rate described by the second term on the right hand side of system (2.1).

In the presence of migration of the fish populations between the two patches, a new term is added in system (2.1), which becomes as follows:

$$\begin{cases} \dot{x}_1 = R(kx_2 - k' x_1) + [r_1 x_1 (1 - \frac{x_1}{K_1}) - q_1 E_1 x_1] \\ \dot{x}_2 = R(k' x_1 - kx_2) + [r_2 x_2 (1 - \frac{x_2}{K_2}) - q_2 E_2 x_2] \end{cases} \tag{2.2}$$

where the coefficient k represents the migration rate from Zone 2 to Zone 1, while k' is the migration rate from Zone 1 to Zone 2. These coefficients are supposed to be constant. Auger and Poggiale (1996a) proposed quite a similar model for a single population distributed on two patches and connected by fast migrations but without exploitation of the stocks.

The term R is assumed to be a large positive constant ($R >> 1$), called the scaling factor between terms describing the migration and those describing the growth of the population (including the catch rate).

If we set:

$$\varepsilon := \frac{1}{R} \text{ and } \tau := \frac{t}{\varepsilon}$$

then system (2.2) can be rewritten as follows:

$$\begin{cases} \dfrac{dx_1}{d\tau} = (kx_2 - k' x_1) + \varepsilon[r_1 x_1 (1 - \dfrac{x_1}{K_1}) - q_1 E_1 x_1] \\ \dfrac{dx_2}{d\tau} = (k' x_1 - kx_2) + \varepsilon[r_2 x_2 (1 - \dfrac{x_2}{K_2}) - q_2 E_2 x_2] \end{cases} \tag{2.3}$$

where τ is a fast time scale with respect to t (so, t is the slow time scale).

Thus, each equation of system (2.3) is composed of two parts: the first one is called the fast part or fast migration and the second one is called the slow part or slow growth term. This is a consequence of the fact that we have assumed that the displacement of the two populations is faster than their natural growth. This hypothesis is realistic as the border band between the two patches has a small width.

Dynamics of the fishing effort

We suppose that fishing efforts represent the number of the fishing boats in each fleet and that the fishing revenue is completely invested for strengthening the fleets at each time. In the absence of fishing boats migration between the two fishing zones, the fishing efforts evolution can be modeled as follows:

$$\begin{cases} \dot{E}_1(t) = (p_1 q_1 x_1 - c_1) E_1(t) \\ \dot{E}_2(t) = (p_2 q_2 x_2 - c_2) E_2(t) \end{cases} \tag{2.4}$$

where p_i (i = 1,2) represents the unit price of the catch in zone i, while c_i (i = 1,2) is the unit cost of the fishing effort in zone i. These parameters are determined by the market, and for simplicity, they are supposed to be constant.

The fishing stocks as well as the fishing fleets can be different in the two areas, so the unit prices and the unit costs can be different too (i.e., $p_1 \neq p_2$ and $c_1 \neq c_2$). Since the fishing fleets are located in a border band of a small width, the boats move quickly between the two patches and operate on the two fishing zones, in order to increase their revenue. Therefore, system (2.4) becomes:

$$\begin{cases} \dot{E}_1(t) = R[mE_2 - m' E_1] + E_1(p_1 q_1 x_1 - c_1) \\ \dot{E}_2(t) = R[m' E_1 - mE_2] + E_2(p_2 q_2 x_2 - c_2) \end{cases} \tag{2.5}$$

where the parameter m' represents the migration rate of the fishing boats operating from Zone 1 to Zone 2, while parameter m represents the migration rate of the fishing boats operating in the reverse direction.

Using the fast time τ leads to the following system:

$$\begin{cases} \dfrac{dE_1}{d\tau} = [mE_2 - m'\,E_1] + \varepsilon E_1(p_1q_1x_1 - c_1) \\ \dfrac{dE_2}{d\tau} = [m'\,E_1 - mE_2] + \varepsilon E_2(p_2q_2x_2 - c_2) \end{cases} \tag{2.6}$$

System (2.6) is still composed of two parts: a fast part describing the rapid displacement of boats between the two fishing patches and a slow part describing the growth of the revenue generated by the exploitation on each area.

We also assume that:

$$\text{for } i = 1,2,: \quad E_i \in [0, E_i^{\max}] \tag{2.7}$$

This last hypothesis means that the fishing effort E_i cannot exceed the quantity E_i^{max} (for $i = 1,2$).

Dynamics of the detailed system stock-effort

Combining system (2.3) with system (2.6), we obtain the complete dynamical system studied in this work:

$$\begin{cases} \dfrac{dx_1}{d\tau} = (kx_2 - k'\,x_1) + \varepsilon[rx_1(1 - \dfrac{x_1}{K_1}) - E_1x_1] \\ \dfrac{dx_2}{d\tau} = (k'\,x_1 - kx_2) + \varepsilon[rx_2(1 - \dfrac{x_2}{K_2}) - E_2x_2] \\ \dfrac{dE_1}{d\tau} = [mE_2 - m'\,E_1] + \varepsilon E_1(p_1x_1 - c_1) \\ \dfrac{dE_2}{d\tau} = [m'\,E_1 - mE_2] + \varepsilon E_2(p_2x_2 - c_2) \end{cases} \tag{2.8}$$

System (2.8) describes the evolution of both fish resources and fishing efforts: x_1, x_2, E_1 and E_2. Let $x(t) = x_1(t)+x_2(t)$ be the total stock while the full effort is given by: $E(t) = E_1(t)+E_2(t)$.

In the next sections, we take advantage of the two time scales to obtain a model that governs the total stock and fishing efforts at a slow time scale. This aggregated model is an approximation of the complete model and we refer to (Poggiale, 1994; Michalski *et al.*, 1997) for aggregation methods. We recall that the link between the aggregated model and the detailed system is based on the application of an adequate version (Poggiale, 1994) of the Center Manifold Theorem (Fenichel, 1971; Auger and Roussarie, 1994; Auger and Poggiale, 1996a, 1996b).

3. CONSTANT FISHING EFFORT

This simple case occurs when fishing efforts E_1 and E_2 are constants (Figure 1). The system reduces to the two previous equations (2.3) in which E_1 and E_2 are supposed to remain constant.

In order to study the evolution of the full level stock x, it is necessary to look for the equilibrium of the fast part, called: the fast equilibrium. We obtain this equilibrium by neglecting the slow growth in system (2.2) which is equivalent to set $\varepsilon = 0$ in system (2.3).

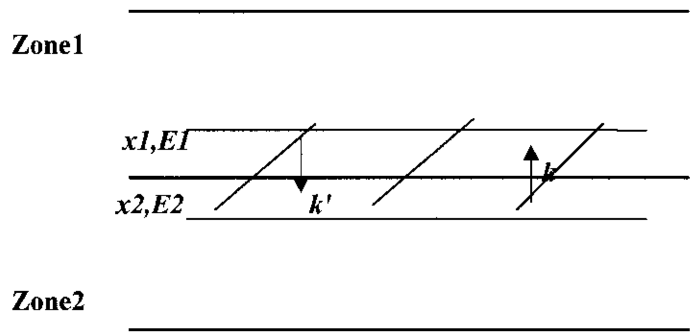

Figure 1. Illustration of two adjacent fishing zones with a small width in the sea. E_1 and E_2 are supposed constant in this case, so there is no migration of fish boats.

The fast equilibrium

A simple calculation leads to the following fast equilibrium:

$$x_1{}^* = v_1 x \text{ and } x_2{}^* = v_2 x$$

where

$$v_1 = \frac{k}{k+k'} \text{ and } v_2 = \frac{k'}{k+k'}$$

represent the proportions of stock on each patch at the fast equilibrium. It can be simply shown that this fast equilibrium is hyperbolically stable.

The aggregated system

By substituting this fast equilibrium x_1^* and x_2^* into the equation of the full fish resource, we obtain the aggregated model:

$$\dot{x}(t) = rx(1-\frac{x}{K}) - Ex + o(\varepsilon) \tag{3.1}$$

where v_1 and v_2 are as above and:

$$\begin{cases} r = r_1 v_1 + r_2 v_2 \\ K = \dfrac{r}{\dfrac{r_1 v_1^2}{K_1} + \dfrac{r_2 v_2^2}{K_2}} \\ E = v_1 E_1 + v_2 E_2 \end{cases} \tag{3.2}$$

Therefore, we have two equilibrium points: $x^* = 0$ which is an unstable equilibrium and $x^* = K(1\text{-}E/r)$. Two cases can occur:

- $E < r$, so the equilibrium $x^* > 0$ and is stable.
- $E > r$, so the equilibrium $x^* < 0$ and the origin is stable.

As a consequence, we obtain a condition for the fishing efforts in the two areas in order to have a sustainable fishing activity:

$$E_1k+E_2k' < r_1k+r_2k' \tag{3.3}$$

This condition is important and it shows that the fishing efforts must be shared in the two areas in a certain way in order to maintain the stock at a positive equilibrium. Otherwise, the stock is doomed to extinction.

4. NON CONSTANT FISHING EFFORTS

In this section, we consider whether the fishing efforts depend on time and whether the migration rates of these fishing efforts are constants (Figure 2).

In this case, we study the complete previous system (2.8).

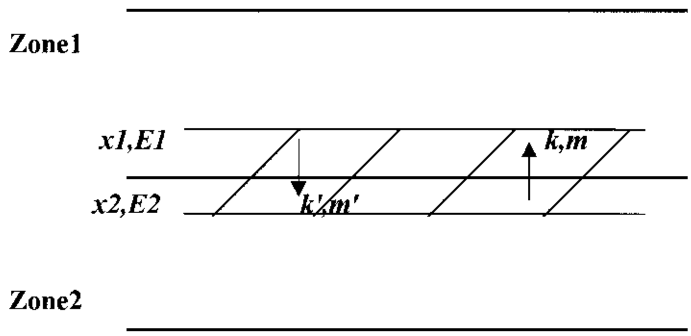

Figure 2. Illustration of two adjacent fishing zones with a small width in the sea. In this case, E_1 and E_2 are non constant, so migration rates of fishing efforts (m&m') appear.

The fast equilibrium

In this case, the fast equilibrium is given by the expressions:

$$\begin{cases} x_1{}^* = v_1x & x_2{}^* = v_2x \\ E_1{}^* = \eta_1 E & E_2{}^* = \eta_2 E \end{cases} \tag{4.1}$$

where we have stock and fishing efforts shared between the two zones with proportions v_1 and v_2 are as above and:

$$\begin{cases} \eta_1 = \dfrac{m}{m+m'} \\ \eta_2 = \dfrac{m'}{m+m'} \end{cases} \tag{4.2}$$

The aggregated system

The aggregated model is given by the following system:

$$\begin{cases} \dot{x}(t) = rx(1-\dfrac{x}{K}) - qEx + o(\varepsilon) \\ \dot{E}(t) = E(px-c) + o(\varepsilon) \end{cases} \tag{4.3}$$

where

$$\begin{cases} r = r_1 v_1 + r_2 v_2 \\ K = \dfrac{r}{\dfrac{r_1}{K_1} v_1^2 + \dfrac{r_2}{K_2} v_2^2} \\ q = \eta_1 c_1 + \eta_2 c_2 \end{cases} \tag{4.4}$$

and

$$\begin{cases} p = \eta_1 v_1 p_1 + \eta_2 v_2 p_2 \\ c = \eta_1 c_1 + \eta_2 c_2 \end{cases} \tag{4.5}$$

System (4.3) has nearly the same form as the slow parts of the complete system: however, a term of catchability q appears in the right hand side of the first equation of system (4.3).

Equilibrium points and asymptotical behavior

A similar prey-predator model can be found in Edelstein-Keshet (1988). Thus, we briefly present the main conclusions. The *x-nullclines* and *E-nullclines* are the following ones.:

- $x-nullclines$: $\dot{x}(t) = 0$ which means: $x = 0$ or $E = \dfrac{r}{q}(1-\dfrac{x}{K})$
- $E-nullclines$: $\dot{E}(t) = 0$ which means: $E = 0$ or $x = \dfrac{c}{p}$

There are three equilibrium points: (0,0),(K,0) and (c/p, r/q(1- c/pK).

The Jacobian matrix in the neighborhood of an equilibrium point (x^*,E^*) is:

$$Jac_{(x^*,E^*)} = \begin{pmatrix} r - 2\dfrac{r}{K}x^* - qE^* & -qx^* \\ pE^* & px^* - c \end{pmatrix} \tag{4.6}$$

- The origin (0,0) is always a saddle point.

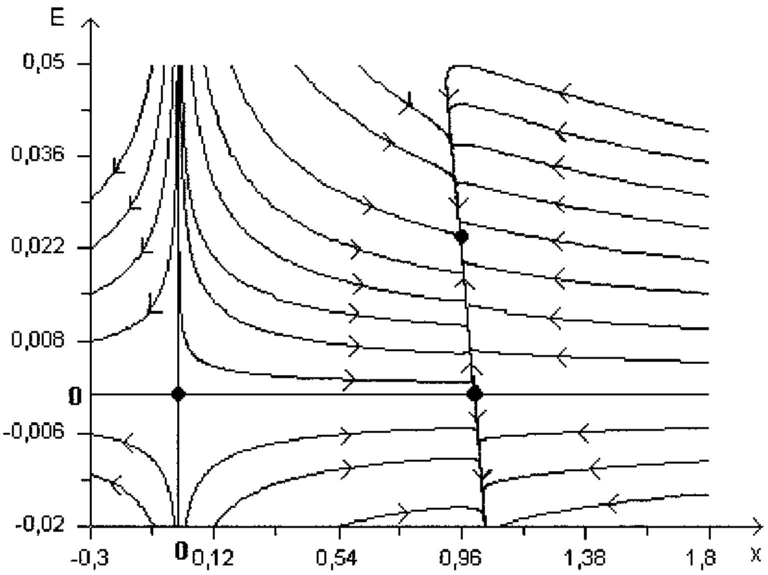

Figure 3. Results of simulations illustrating the case where $pK-c>0$ and $\Delta=(rc/pK)^2[1-4(pK/rc)(pK-c)]>0$. So, we obtain saddle points at (0,0) and $(K,0)$ and a stable node (which is a stable equilibrium) at $(c/p,(r/q)(1-c/pK))$. We have chosen values of the dynamical system as: $dx/dt = 0.5x(1\text{-}x)\text{-}xE$ and $dE/dt = E(0.21x\text{-}0.2)$; so $r = 0.5$, $K = 1$, $q = 1$, $p = 0.21$ and $c = 0.2$.

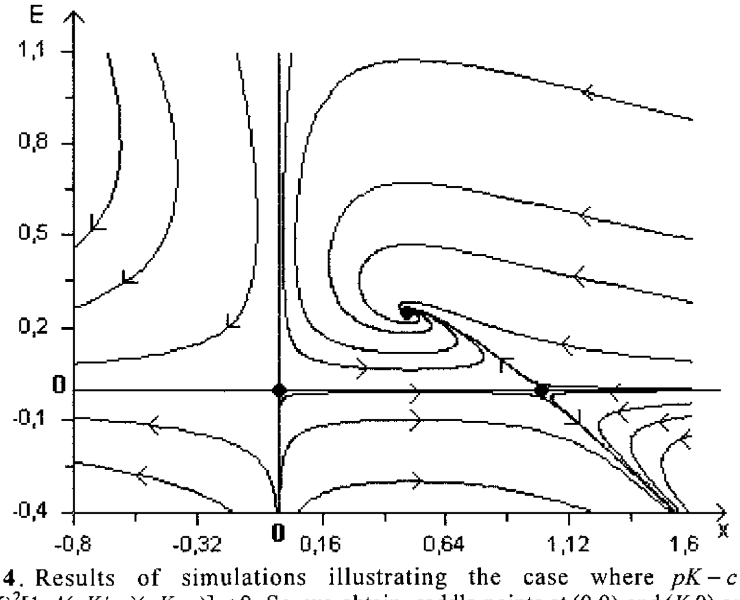

Figure 4. Results of simulations illustrating the case where $pK-c>0$ and $\Delta=(rc/pK)^2[1-4(pK/rc)(pK-c)]<0$. So, we obtain saddle points at (0,0) and $(K,0)$ and a stable focus (which is a stable equilibrium) at $(c/p,(r/q)(1-c/pK))$. We have chosen values of the dynamical system as: $dx/dt = 0.5x(1\text{-}x)\text{-}xE$ and $dE/dt = E(0.4x\text{-}0.2)$; so $r = 0.5$, $K = 1$, $q = 1$, $p = 0.4$ and $c = 0.2$.

First let us assume that pK-c > 0. This condition implies that the last equilibrium belongs to the positive quadrant. This hypothesis also means that the fleets will not fish if they are not assured of a minimal positive revenue. Then, we have the next results:

- The equilibrium point $(K,0)$ is a saddle point.
- The equilibrium point $(c/p, r/q(1-c/pK))$ is stable when it belongs to the positive quadrant (see Figure 3 or Figure 4).

If we now assume that pK-c < 0. Then, we have the next results (see also Figure 5):

- The equilibrium point $(K,0)$ is a stable node.
- The equilibrium point $(c/p, r/q(1-c/pK))$ does not belong to the positive quadrant.

The condition $c < pK$ is thus necessary in order to be able to maintain fishing activity. This means that for a given spatial distribution of fishes in the two patches, i.e. for fixed stock migration rates k and k', we obtain a condition for the migration rates m and m' of the fleets which is the following one:

$$(mc_1 + m'c_2)\left(\frac{r_1k^2}{K_1} + \frac{r_2k'^2}{K_2}\right) < (r_1k + r_2k')(mkp_1 + m'k'p_2) \qquad (4.8)$$

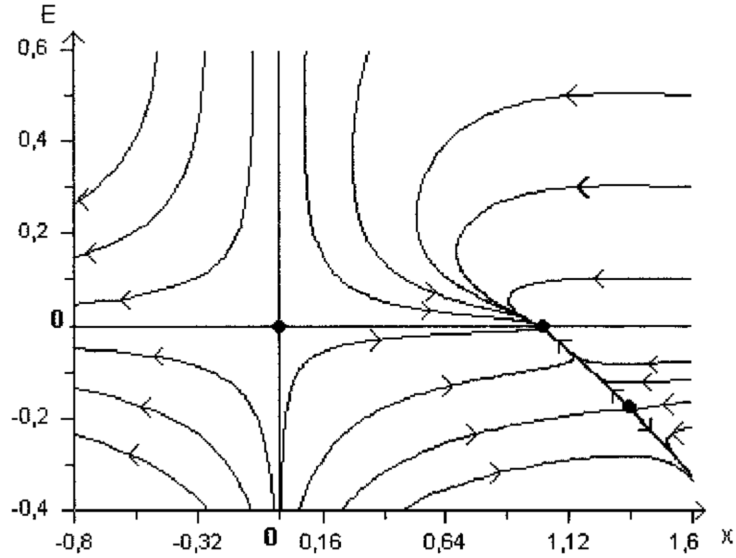

Figure 5. Results of simulations illustrating the case where $pK-c < 0$. So, we obtain saddle points at (0,0) and $(c/p,(r/q)(1-c/pK))$ and a stable node (which is a stable but not desired equilibrium, because the fishing effort is null) at $(K,0)$. We have chosen values of the dynamical system as: $dx/dt = 0.5x(1\text{-}x)\text{-}xE$ and $dE/dt = E(0.3x\text{-}0.4)$; so $r = 0.5$, $K = 1$, $q = 1$, $p = 0.3$ and $c = 0.4$.

This condition takes into account the biological characteristic of the species in the two zones, growth rates and carrying capacities, its spatial distribution in the two fishing zones and economical parameters such as the efforts and the cost of fishing in the two patches and how fleets must move so that a stable equilibrium can be reached with a positive stock and constant captures. This condition is important because it shows that to maintain a sustainable fishing activity for a given species, all control parameters must be chosen in order to satisfy the previous inequality (4.8).

5. CONCLUSION AND DISCUSSION

We built a dynamical model, describing at the same time the evolution and the migration of two fish populations and of two fleets harvesting in two connected patches.

The analysis of the aggregated method leads to a stable equilibrium. This is a good situation for fishing, because it allows a durability of the fishing activity and of the exploited species. However, our work shows that even in a simple system of two patches with constant migrations between the two areas, there are conditions on the fishing efforts and the migration rates of the fleets that must be verified in order to be able to maintain a sustainable fishing activity.

The generalization of this result to more than two exploited zones that could be represented as a set of N connected patches (N>2) is direct and should not present any difficulty.

The comprehension of this simple system is a first step for the study of more complex systems. So, our next work will consist of the study of a more complex situation where migration rates for fishing fleets are stocks-dependent. This situation will be more realistic than the case of constant rates, because the fleets move between different fishing zones with respect to the fishing density.

It appears to us interesting to introduce a control parameter in the model that will reflect the interference of different components of the dynamics of the exploited resource. This is due to the importance of the fishing activity throughout the world and the urgency of the elaboration of fisheries arrangement planning between different fishing zones of each coastal state. This will be the subject of our future research.

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