Supporting Information Appendix: 'Early warning signals of regime shifts in coupled human-environment systems'

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1 Uncoupled Environment Model: Equilibria and Stability

The differential equation for forest cover F is:

$$\dot{F} = RF(1-F) - \frac{hF}{F+s}, \tag{1}$$

where parameters are defined in the main text. We assume $h > 0$, $R > 0$ and $s > 0$.

From equation(1), there is one equilibrium F^* of no forest:

$$B_1 = 0 \tag{2}$$

and two equilibria where F^* may be nonzero:

$$\begin{aligned} B_2 &= \frac{r(1-S) + \sqrt{r^2(1-s)^2 - 4r(h-rs)}}{2r} \\ &= \frac{r(1-S) + \sqrt{r^2(1+s)^2 - 4rh}}{2r}, \end{aligned} \tag{3}$$

$$\begin{aligned} B_3 &= \frac{r(1-S) - \sqrt{r^2(1-s)^2 - 4r(h-rs)}}{2r} \\ &= \frac{r(1-S) - \sqrt{r^2(1+s)^2 - 4rh}}{2r}. \end{aligned} \tag{4}$$

Here B_2 is biologically meaningful if either of the following conditions are satisfied:
(1) $h < rs$ and $s < 1$,
(2) $h > rs$, $s < 1$ and $r^2(1-s)^2 > 4r(h-rs)$ or $r^2(1+s)^2 > 4rh$.
and B_3 is biologically meaningful when condition (2) is satisfied. B_2 and B_3 are identical when $r^2(1-s)^2 = 4r(h-rs)$ or $r^2(1+s)^2 = 4rh$, yielding the bifurcation point $h_{crit} = r(1+s)^2/4$.

From local stability analysis, it can be shown that B_1 is stable when $h > rs$. B_2 is locally asymptotically stable when it exists ($0 \leq B_2 \leq 1$) and B_3 is always unstable.

2 Coupled Human Environment System Model: Equilibria and Stability

The differential equations for forest cover F and the proportion of individuals with conservationist opinions x are given by:

$$\dot{x} = kx(1-x)\left[\delta(2x-1) + \frac{1}{F+c} - w\right] \tag{5}$$

$$\dot{F} = rF(1-F) - \frac{h(1-x)F}{F+S}, \tag{6}$$

where model parameters are defined in the main text.

From equations (5) and (6), there are three equilibria (F^*, x^*) where there are no conservationists in the population. One has no forest cover,

$$A_1 = (0,0), \tag{7}$$

the second has nonzero forest cover,

$$A_2 = (F_1, 0) \tag{8}$$

and the third may also have nonzero forest cover,

$$A_3 = (F_2, 0) \tag{9}$$

where F_1 and F_2 are identical to B_2 and B_3 in the uncoupled case, since the model reduces to the uncoupled case when $x = 0$.

Here A_2 is biologically meaningful if either of the following are satisfied:
(1) $h < rs$ and $s < 1$,
(2) $h > rs$, $s < 1$ and $r^2(1-s)^2 > 4r(h-rs)$ or $r^2(1+s)^2 > 4rh$.
A_3 is biologically meaningful when when condition (2) is satisfied and A_2, A_3 are identical when $r^2(1-s)^2 = 4r(h-rs)$ or $r^2(1+s)^2 = 4rh$, as before.

Also, there are two equilibria where the entire population practices forest conservation. One corresponds to no forest cover,

$$A_4 = (0, 1), \tag{10}$$

and the other corresponds to full forest cover,

$$A_5 = (1, 1). \tag{11}$$

Finally, there is one equilibrium where forest cover is zero, but the proportion of conservationists is nonzero:

$$A_6 = \left(0, \frac{w + \delta - \frac{1}{c}}{2\delta}\right). \tag{12}$$

A_6 is biologically meaningful when $w + \delta > \frac{1}{c}$ and $\frac{w+\delta-\frac{1}{c}}{2\delta} < 1$ are satisfied.

Also the interior steady states (F^*, x^*) are given by

$$\dot{x} = \frac{[w + \delta - \frac{1}{F^*+c}]}{2\delta} \tag{13}$$

and

$$a_1 F^{*3} + a_2 F^{*2} + a_3 F^* + a_4 = 0 \tag{14}$$

where

$$
\begin{aligned}
a_1 &= \frac{r}{h} && (15)\\
a_2 &= -\frac{r}{h}(1-s-c) && (16)\\
a_3 &= -\left[\frac{w+\delta}{2\delta}+\frac{rs}{h}+\frac{rc}{h(1-s)}-1\right] && (17)\\
a_4 &= \frac{1}{2\delta}+c-\frac{c(w+\delta)}{2\delta}-\frac{rcs}{h} && (18)\\
&= \frac{1}{2\delta}[1-c(w+\delta)]+c\left(1-\frac{rs}{h}\right). && (19)
\end{aligned}
$$

If $rs > h$ and $w + \delta > \frac{1}{c}$ then $a_1 > 0$, $a_2 >$ or < 0, $a_3 < 0$, $a_4 < 0$. In either of the cases, using Descartes rule, F has one positive root, two or zero negative roots. Positive F and corresponding x value results a unique interior equilibrium.

From local stability analysis A_1 is locally asymptotically stable (LAS) if $h > rs$ and $\delta+w > \frac{1}{c}$ are satisfied. A_2 is LAS if it exists $(0 \le F_1 \le 1)$ and satisfies $w+\delta > \frac{1}{F_1+c}$. A_5 is locally asymptotically stable if it exists and satisfies $\delta + \frac{1}{1+c} > w$. A_3, A_4, A_6 are unstable.

3 Additional Methods

3.1 Model Fitting

Longitudinal data on old growth forest cover from 1933 to 2004 in the Pacific Northwest (below) were obtained from Ref. [1]

Year	1933	1945	1992	2004
Percentage Forest Cover	0.58	0.406	0.182	0.225

These data were interpolated to the yearly level using PCHIP in Matlab to generate a dataset for use in model fitting. Similarly, longitudinal data on population attitudes toward forest conservation in Oregon from 1933 to 2004 (below) were obtained from Ref. [3]

Year	1933	1986	1999	2004
Percentage Supporting Conservation	0	0.24	0.75	0.76

These data were interpolated to the yearly level using PCHIP in Matlab. The coupled HES model (section S2) was fitted to the two interpolated datasets via least squares minimization using a grid sweep with a resolution of 10 equally spaced parameter values across each parameter range tested, yielding a minimal absolute error of 2.2918 over 144 data points. Forest cover and human opinion data were weighted equally in the fitting. The differential equations were solved using the fourth-fifth order adaptive Runge-Kutta method in Matlab (ode45). Best fitting values appear in the main text, Table 1, and the best fitting model time series and the interpolated data appear in Figure S1.

3.2 Model Simulation and Numerical Analysis

Bifurcation diagrams of the deterministic equations for the uncoupled model (Equation (1)) and coupled model (Equations (5),(6)) were generated using XPP-AUT [4]. For generating time series of stochastic dynamics, Equations (1), (5) and (6) were solved using the Euler-Maruyama method with a timestep of 0.25 days and additive Gaussian white noise of amplitude $\sigma = 0.001$ [7]. Stochastic simulations were coded in the C programming language with reference to Numerical Recipes in C [6, 8].

3.3 Early Warning Signals

We used standard approaches to detect early warning signals [2]. The original time series were first smoothed by applying a Gaussian smoothing window (bandwidth=10% of length of dataset from the initial time to the bifurcation point). The smoothed time series was then subtracted from the raw time series to generate a residual time series. The lag-1 autocorrelation of the residual time series was then computed using a rolling window of width=50% of length of dataset from from the initial time to the bifurcation point. The Kendall tau rank correlation coefficient [5] was also computed to provide a statistic concerning temporal trends in lag-1 autocorrelation. All analyses were coded in the C programming language with reference to Numerical Recipes in C [6, 8].

4 Probabilistic Sensitivity Analysis for Eigenvalues

Variance in stochastic dynamics can also be influenced by the presence of basins of attraction of alternative stable states, i.e., non-local effects. To partially rule out non-local effects on the variance statistic, we compared the eigenvalues of the coupled and uncoupled systems close to the bifurcation points in Figure 5c,d,e,f. We hypothesized that if the reduced variance of the coupled system close to the bifurcation point is due to muting, then we should observe that the eigenvalues of the coupled system are more negative than the eigenvalue of the uncoupled system, giving rise to tighter stochastic dynamics around the equilibrium in the coupled case.

We tested this probabilistically. For each parameter value used to generate Figure 5c,d,e,f (and for harvesting efficiency h set at 95% of its value at the bifurcation point h_{crit}) we defined a uniform interval centred on the parameter value, with a width of $\pm 50\%$ of the parameter value. We sampled from the uniform distribution for each parameter value of the model and computed the eigenvalues of the equilibria.

The eigenvalue of equilibrium B_2 of the uncoupled model (see above) is

$$\lambda_{B_2} = \frac{-hmB_2}{r(B_2+s)^2(1-B_2)}$$

where $m = \sqrt{r^2(1-s)^2 - 4r(h-rs)}$ or $\sqrt{r^2(1+s)^2 - 4rh}$.

The eigenvalues of equilibrium $A_2 = (F_1, 0)$ of the coupled model (where $F_1 = B_2$ as noted above) are

$$\lambda_{A_{2,1}} = \lambda_{B_2}$$

and

$$\lambda_{A_{2,2}} = -k\left(\delta - 1/(F+c) + w\right)$$

We repeated this procedure 100 times, throwing out results where one of the eigenvalues was positive (thus yielding an unstable equilibrium). We computed

$$\Delta E_1 \equiv \lambda_{A_{2,1}} - \lambda_{B_2} \equiv 0$$

and

$$\Delta E_2 \equiv \lambda_{A_{2,2}} - \lambda_{B_2}$$

for each iteration. We found that it was indeed the case that the eigenvalues were more negative in the coupled model than the uncoupled model. In particular, the average and standard deviation of ΔE_2 were -0.082 ± 0.065, and ΔE_2 was negative in 92% of the parameter sets tested.

Similar results were found comparing B_2 to $A_5 = (1,1)$ at the baseline parameter values in Table 1. The eigenvalues for A_5 are

$$\lambda_{A_{5,1}} = -r$$

and

$$\lambda_{A_{5,2}} = -k\left(\delta + \frac{1}{1+c} - w\right)$$

We found the average and standard deviation of ΔE_1 were -0.020 (± 0.005) with ΔE_1 negative in 100% of the parameter sets tested; the average and standard deviation of ΔE_2 were -0.091 (± 0.022) with ΔE_2 negative in 98% of the parameter sets tested.

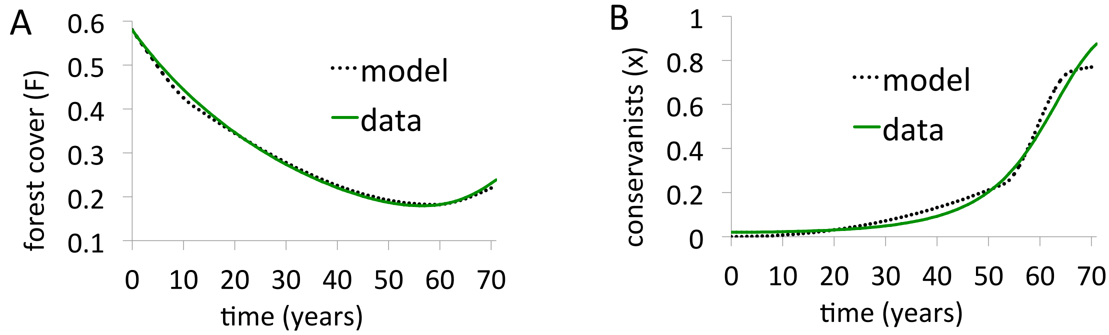

Figure S1: Comparison of data time series and best-fitting model time series for (a) forest cover F and (b) conservationist opinion x . See section S3 for fitting methodology.

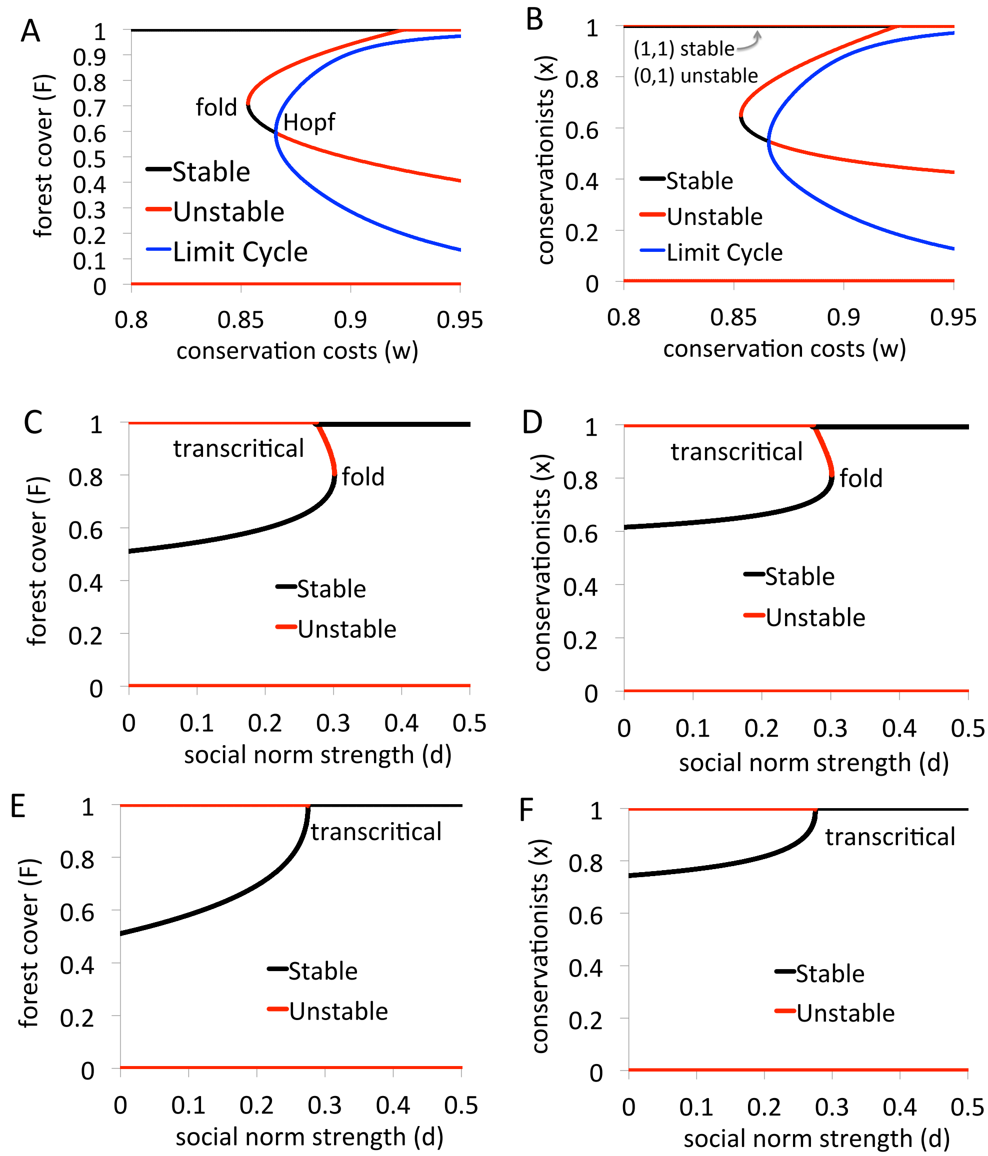

Figure S2: Bifurcation diagrams of forest cover F (a,c,e) and prevalence of conservationists x (b,d,f) in the coupled HES model, versus (a,b) conservation cost w for $d = 0.3$ (blue lines correspond to minimum and maximum of stable limit cycle); (c,d) social norms strength d for $w = 0.9$, $h = 0.10$/yr; (e,f) social norms strength d for $w = 0.9$, $h = 0.15$/yr. All other parameter values are as in Table 1.

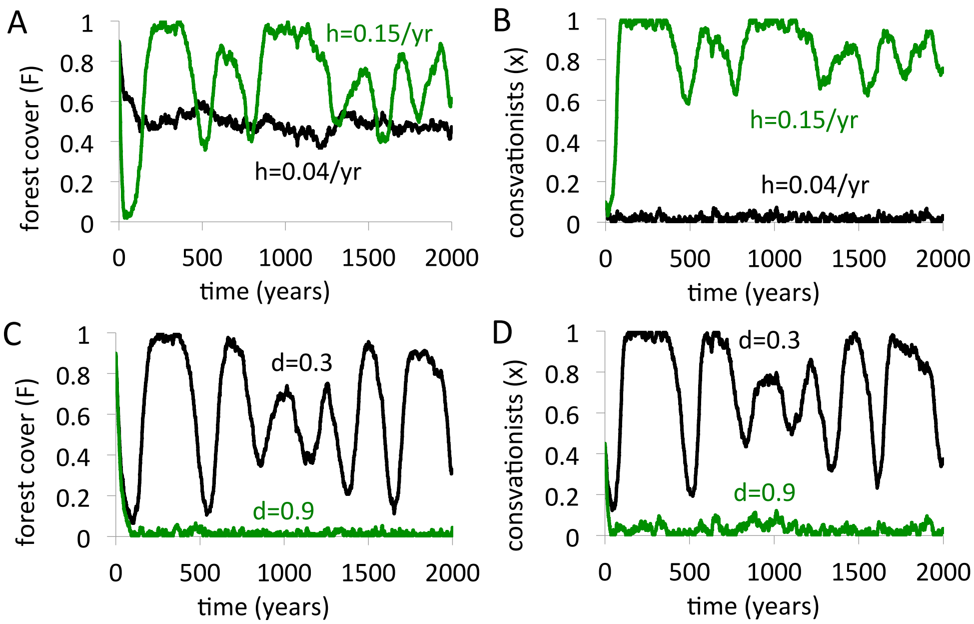

Figure S3: Long-term dynamics for different social and harvesting parameters in the coupled HES model. Subpanels show time series of (a,c) forest cover F and (b,d) conservationist opinion x versus time t, for (a,b) high and low harvesting efficiency h and for (c,d) high and low social norm strength d . Parameter values are $h = 0.1$/year, $d = 0.3$, $w = 1$ unless otherwise stated in subpanels, with remaining parameters listed in Table 1.

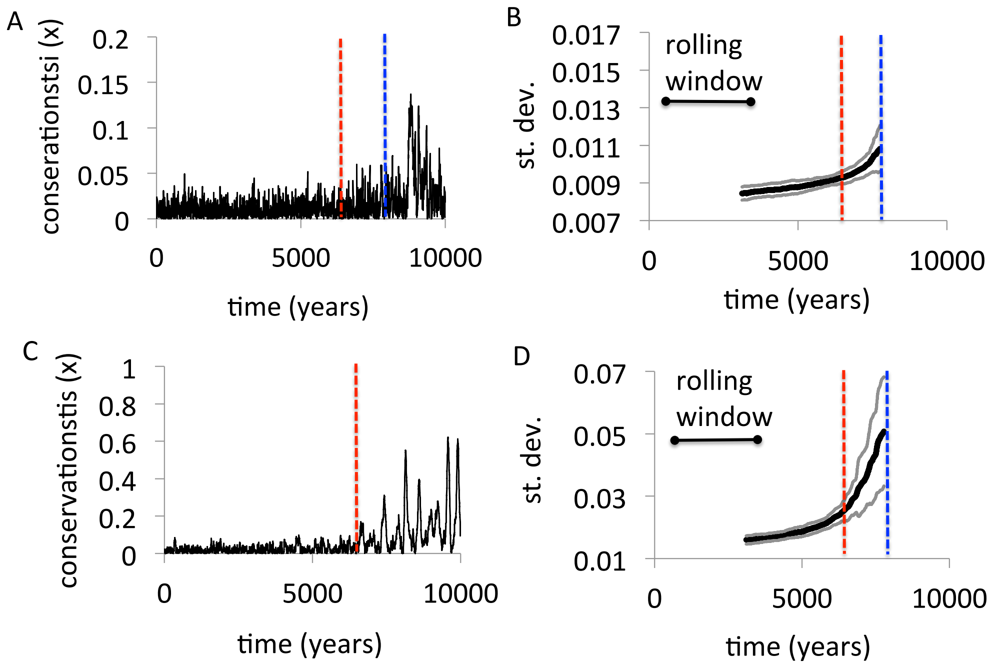

Figure S4: Early warning signals in the prevalence of conservationists, x. Subpanels show (a) collapse of forest in the coupled HES model for $w = 1.5$, $d = 0.2$ and (b) the early warning signal; (c) conservation of forest in the coupled HES model for $w = 1.0$, $d = 0.1$ and (d) the early warning signal. Blue dashed line is bifurcation point for system in (a,b), red dashed line is bifurcation point for system in (c,d). Other parameter values are as in Table 1, except h increases linearly from 0.03/yr to 0.055/yr over the simulated time span. See SI Appendix: Methods for details on generation of stochastic dynamics and computation of residual standard deviation.

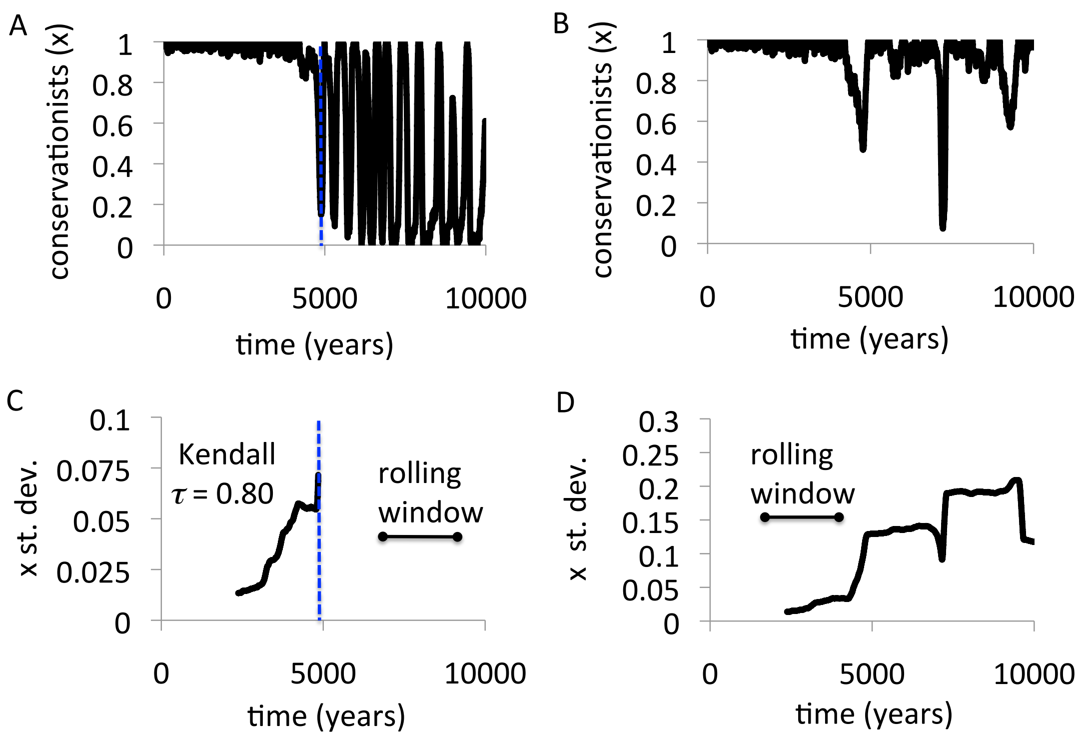

Figure S5: Subpanels show regime shift in conservationist dynamics, x versus t, in the baseline model (a), but persistent criticality in the extended (b), with corresponding plots of standard deviation over time in the extended model (c) and baseline model (d). $a = 0.9$, $b = 0.0013$. The residual standard deviation was calculated by applying a Gaussian smoothing window (bandwidth=10 % of length of dataset up to bifurcation point) to the x time series and subtracting it from the x time series, and computing the standard deviation of the resulting residual time series using a rolling window (width=50 % of length of dataset up to bifurcation point).

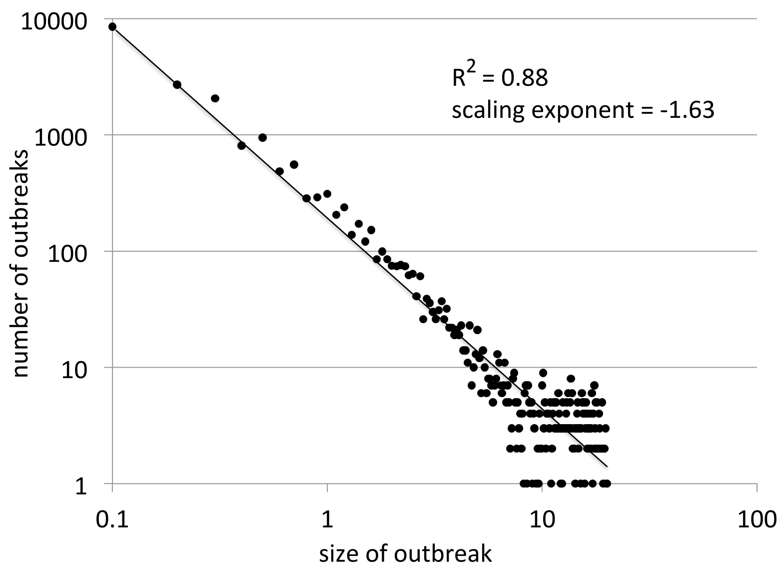

Figure S6: Power law behavior in extended model near critical point. Figure shows log-log plot of frequency distribution of deforestation outbreak size for case where time evolution of ω depends upon F. Figure was generated by simulating stochastic coupled HES model at Figure 6d parameter values 500 times using different random number seeds. The start of an outbreak was defined as a time when forest cover F fell below a threshold of $\overline{F}_{thr} = 0.98$, and the end of an outbreak was defined as when F moves back above $\overline{F}_{thr}$. Outbreak size was monitored between $t = 4000$ years and $t = 10000$ years. The outbreak size F_{out} was computed from the cumulative forest cover harvested during the outbreak, i.e., from the equation $dF_{out}/dt = \frac{h(1-x)F}{F+s}$ from the F equation of the coupled HES model. $p < 0.0001$.

References

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