1 Exponential Model

$$\mathbb{E}(y|x) = a_0 + a_1 a_2 \left(1 - \exp\{-x/a_2\} - a_3 \cdot \exp\left\{-\frac{1}{2}\left(\frac{x-a_4}{a_5}\right)^2\right\}\right) + a_6 a_8 x \log\left(1 + \exp\left\{\frac{x - x_7}{a_8}\right\}\right)$$

- a_0 = approximate value of $E(y|x)$ at $x = 0$
- a_1 = approximate slope of the curve near $x = 0$
- a_2 = scale (in dimensions of x) of the concavity of the curve near 0
- a_3 = minimum magnitude (in proportion of y) of the dip that occurs at the mid-high range of x
- a_4 = approximate center of the dip that occurs at the mid-high range of x
- a_5 = scale (in dimensions of x) of the dip that occurs at the mid-high range of x
- a_6 = slope of the curve in the limit of high values of x
- a_7 = position of the approximate 'knot' where the shape of the curve changes
- a_8 = scale (in dimensions of x) of how fast the slope changes

2 S-Curve Improvement

$$\mathbb{E}(y|x) = a_0 + logit^{-1}\left(\frac{x-a_1}{a_2}\right) \cdot a_3 \cdot a_4 \cdot \left(1 - \exp\{-x/a_4\} - a_5 \cdot \exp\left\{-\frac{1}{2}\left(\frac{x-a_6}{a_7}\right)^2\right\}\right)$$

- a_0 = approximate value of $E(y|x)$ at $x = 0$
- a_1 = approximate inflection point of S-curve
- a_2 = approximate scale of S-curve at inflection point
- a_3 = approximate slope of S-curve at inflection point
- a_4 = scale (in dimensions of x) of S-curve at inflection point
- a_5 = minimum magnitude (in proportion of y) of the dip that occurs at the mid-high range of x
- a_6 = approximate center of the dip that occurs at the mid-high range of x
- a_7 = scale (in dimensions of x) of the dip that occurs at the mid-high range of x