Logic in Computer Science Coursework 1

Date of Handout: Saturday 09 October 2018
Date of Submission: Friday 15 October 2018

Instructions

I. **Submission should be made in the format of PDF by email to Dr. Bo Liu < liubocq@swu.edu.cn>**
II. **The PDF should be made by Microsoft Word or LaTex.**
III. **Late submission without giving an acceptable in advance will be recorded as a mark of 0 (zero).**
IV. **Discussions on the understanding of the questions are encouraged, but plagiarism is strictly not allowed.**

1. Instead of defining a subset of a set in terms of membership, we take the following definition

   A is a subset of B if $A \cap B = A$

   Prove this is equivalent to the original definition, i.e. A is a subset of B according to the above definition iff

   A is subset of B iff for any a,

2. Prove

3. Show that the set of all even numbers are countable (define a bijiection from this set to the set of natural numbers N).
4. Let U be the assumed universal set, for any subset V, define $\mathcal{A}^c = \mathcal{U}-\mathcal{A}$. Show the following laws, for any subsets $\mathcal{A}$ and B of $\mathcal{U}$

   $(\mathcal{A}\cup\mathcal{B})^c = \mathcal{A}^c\cap\mathcal{B}^c$ $(\mathcal{A}\cap\mathcal{B})^c = \mathcal{A}^c\cup\mathcal{B}^c$

5. Prove $\mathcal{A}\cap\mathcal{B}=\mathcal{B}-(\mathcal{B}-\mathcal{A})$

6. Given a finite universal set $\mathcal{U}$ of n elements. Represent a subset $\mathcal{A}$ of $\mathcal{U}$ with the bit string of length n, where the ith bit in the string is 1 if a_i belongs to $\mathcal{A}$ and is 0 if a_i does not belong to $\mathcal{A}$. Let $\mathcal{U} = \{1,2,3,4,5,6,7,8,9,10\}$, and the ordering of elements of $\mathcal{U}$ has the elements in increasing order; that is $a_i = i$.

   a. What bit string represents the subset of all odd integers in $\mathcal{U}$?
   b. What bit string represents the subset of all even integers in $\mathcal{U}$?
   c. What bit string represents the subset of all integers not exceeding 5 in $\mathcal{U}$
   d. What bit strings represent the union, intersection and difference of each pair of the above subsets?

7. Discuss how to represent a Class as relation (thus, a set) in a Java program. Use

examples to demonstrate your argument.