

Cryptography - Part 1

Feb. 20, 2025

Recap question:

Feb. 20, 2025

A disease in the SIR model is estimated to have parameter values $\beta = 2$ and $\gamma = 1$. What is the herd immunity threshold for the disease?

The herd immunity threshold is

$$p_c = 1 - \frac{1}{R_0}.$$

To compute this, we need to find the value of R_0 . We use the definition of R_0 :

$$R_0 = \frac{\beta}{\gamma} = \frac{2}{1} = 2$$

so

$$p_c = 1 - \frac{1}{R_0} = 1 - \frac{1}{2} = \frac{1}{2}.$$

Cryptography - Part 1

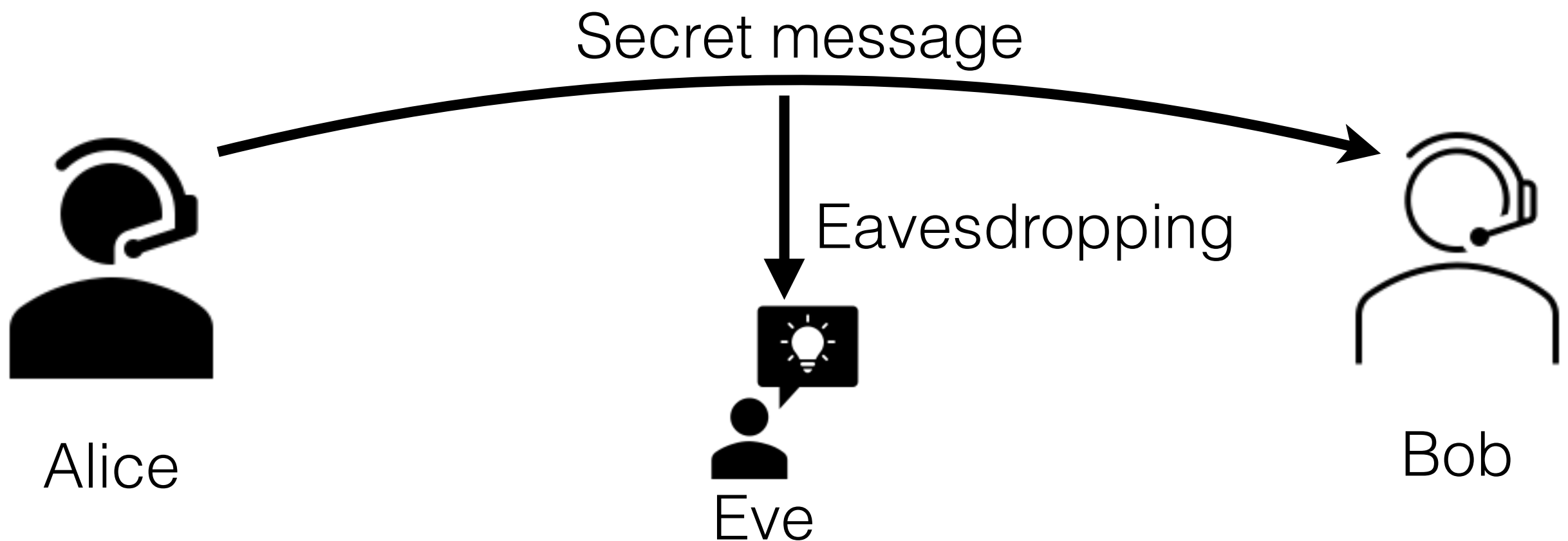
Feb. 20, 2025

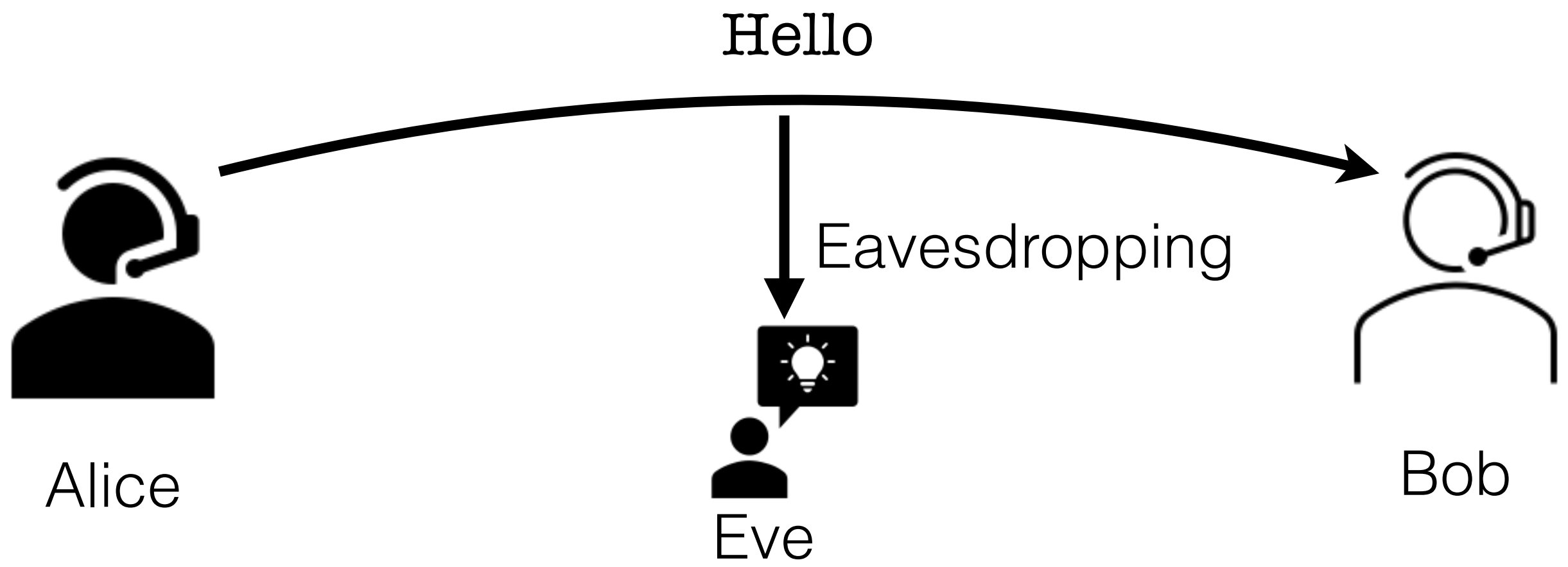
By the end of this lecture, you will be able to:

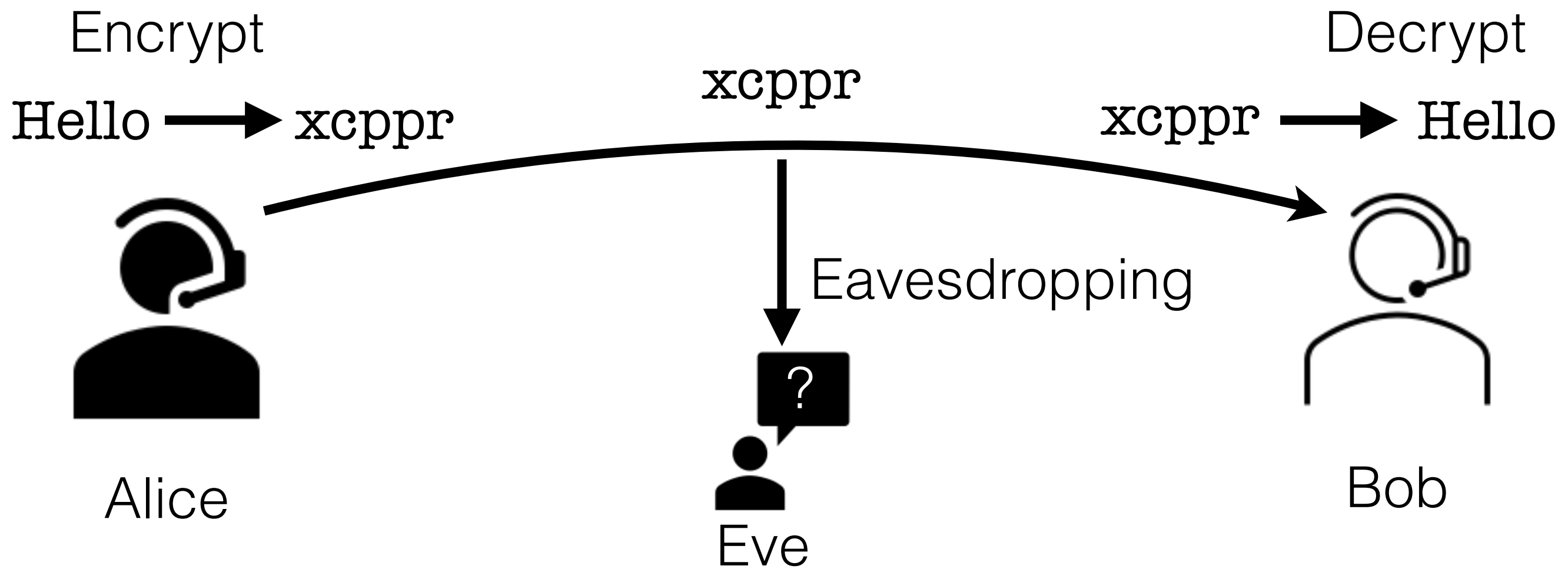
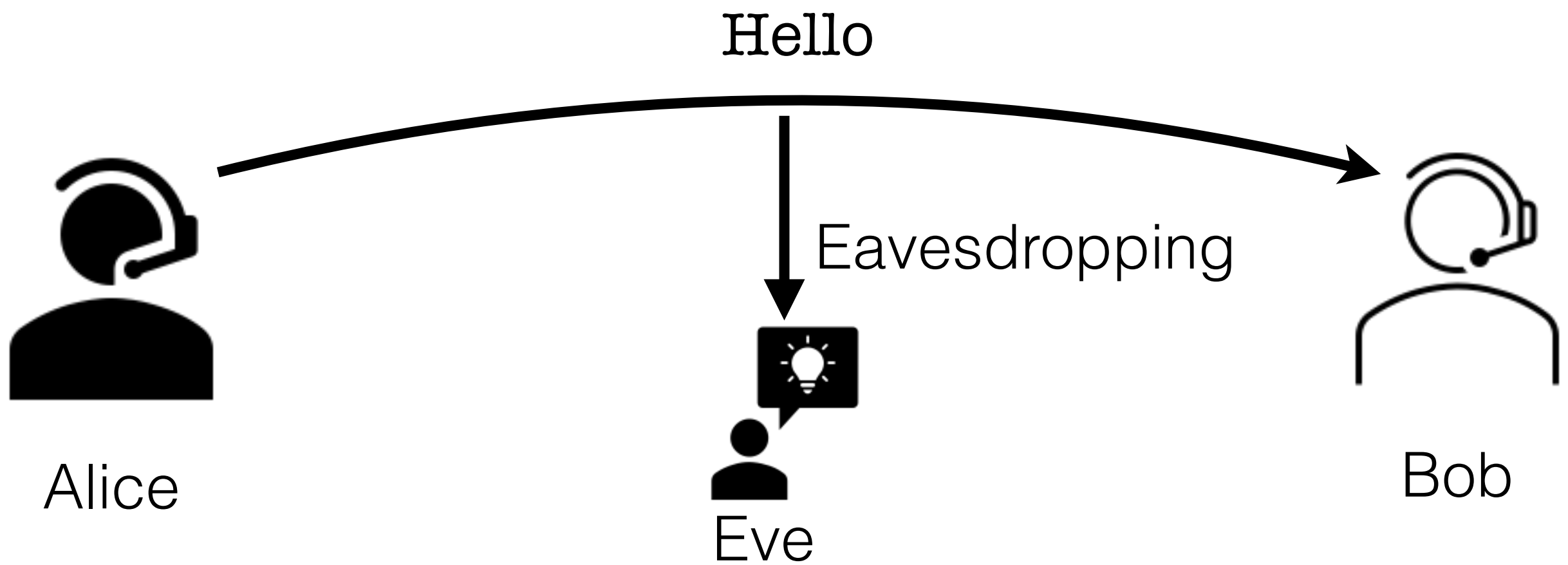
1. Define substitution ciphers and Caesar ciphers
2. Decrypt Caesar ciphers using the brute-force method
3. Decrypt substitution ciphers using frequency analysis

Cryptography is “the art of writing in secret characters”. A cryptographer encodes messages before they are transmitted so that even if the encrypted message is intercepted by a hostile party, its meaning will still remain secret.

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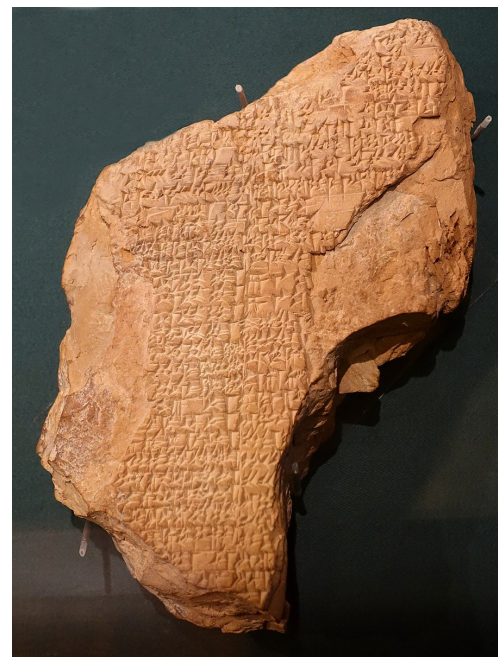
In principle, only “friends” of the original cryptographer, who knows the secret recipe for decoding or decrypting, can decode the encrypted message to the original plain text.

A “code breaker” seeks to detect patterns in the encrypted messages that will lead to sufficient understanding of the encryption scheme to enable the discovery of a decryption method.

Where is cryptography used?

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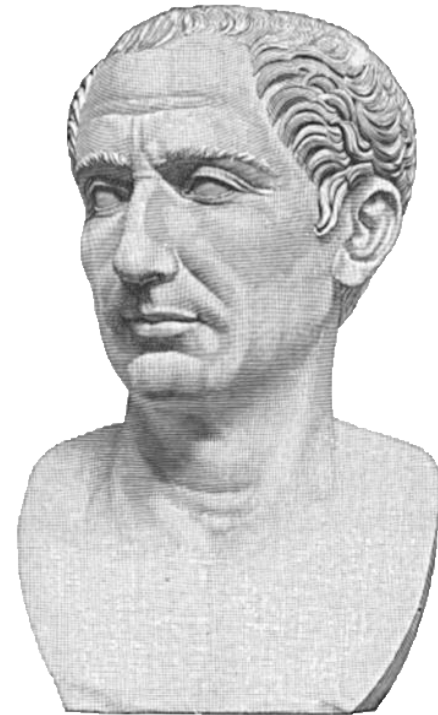
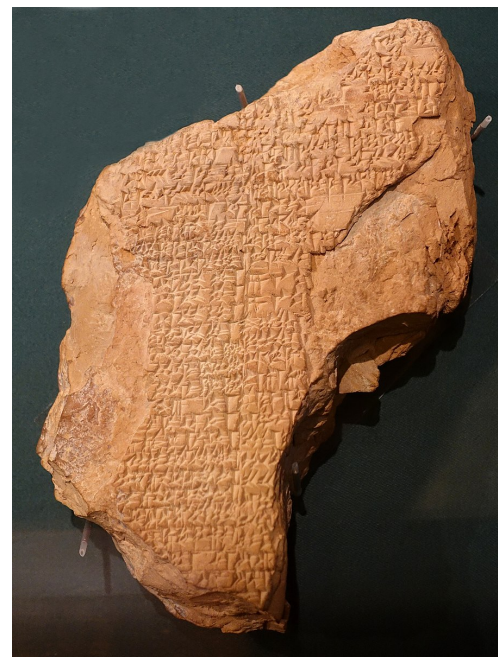
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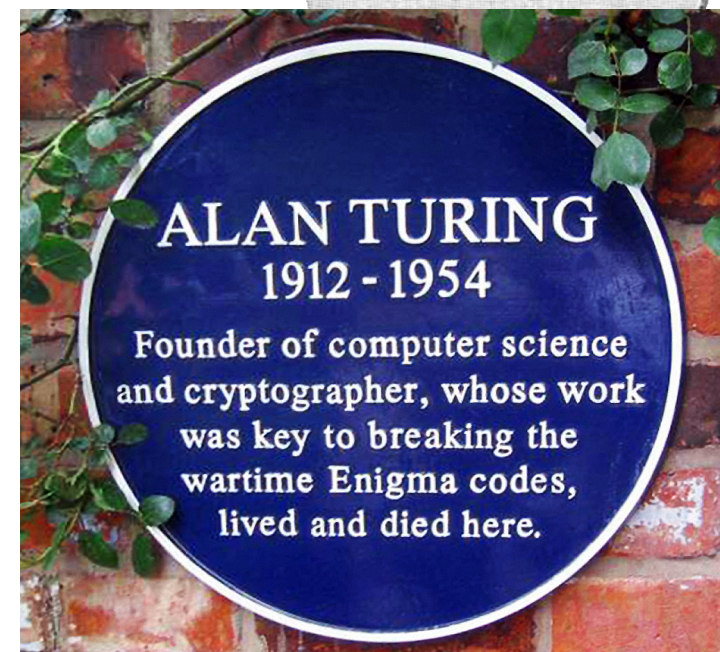
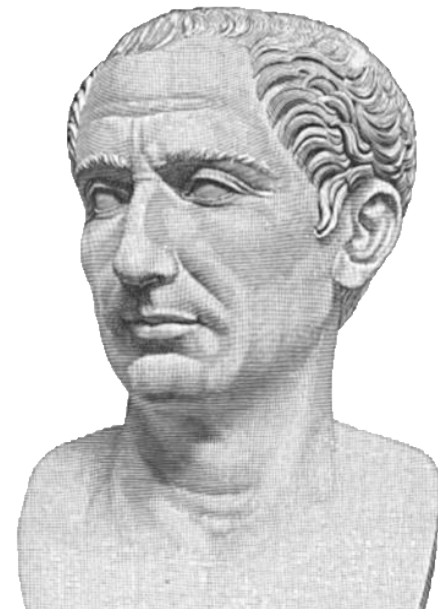
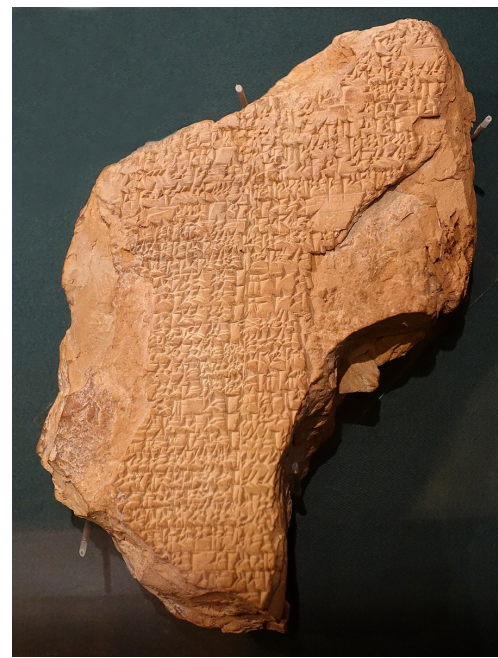
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⋮

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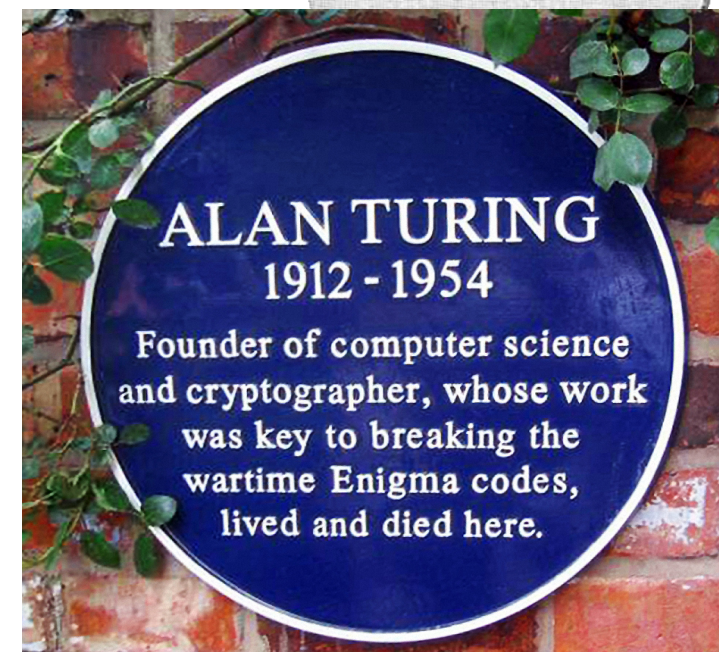
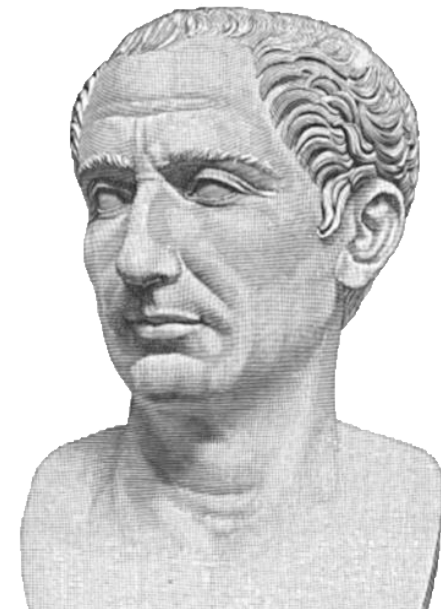
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Today: logging into websites, sending emails, WhatsApp, verifying credit card information, cloud storage of pictures, ...



Unsolved historical encryption techniques

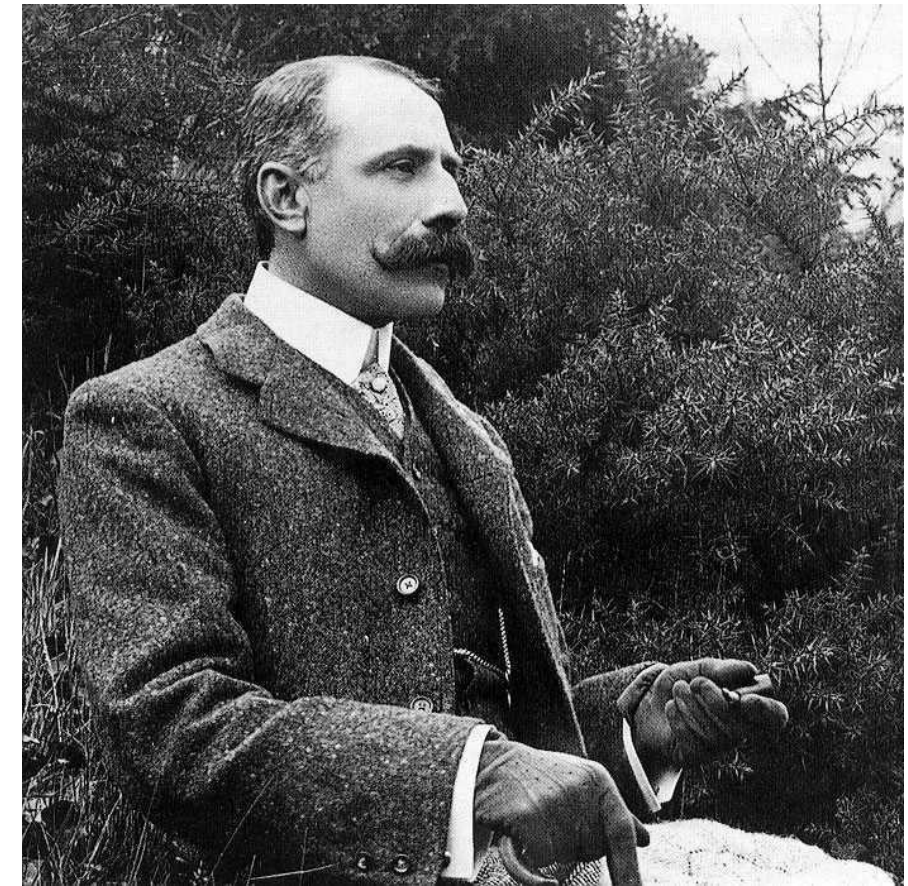
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202, 35, 10, 2, 41, 17, 84, 221, 736, 820, 214, 11, 60, 760.

THE
BEALE PAPERS,
CONTAINING
AUTHENTIC STATEMENTS
REGARDING THE
TREASURE BURIED
IN
1819 AND 1821,
NEAR
BUFORDS, IN BEDFORD COUNTY, VIRGINIA,
AND
WHICH HAS NEVER BEEN RECOVERED.
PRICE FIFTY CENTS.
LYNCHBURG:
VIRGINIAN BOOK AND JOB PRINT,
1881.

https://commons.wikimedia.org/wiki/File:Beale_1.svg

Unsolved historical encryption techniques

Letter written by composer
Edward Elgar to Dora Penny



https://commons.wikimedia.org/wiki/File:Edward_Elgar.jpg

Handwritten text in a cursive script, likely the Dorabella cipher, consisting of three lines of illegible characters.

<https://en.wikipedia.org/wiki/File:Dorabella-cipher-image.png>

Handwritten signature or date, possibly "The 14.97".

Some (not very good) encryption techniques:

1. Ehay isay eryvay illysay

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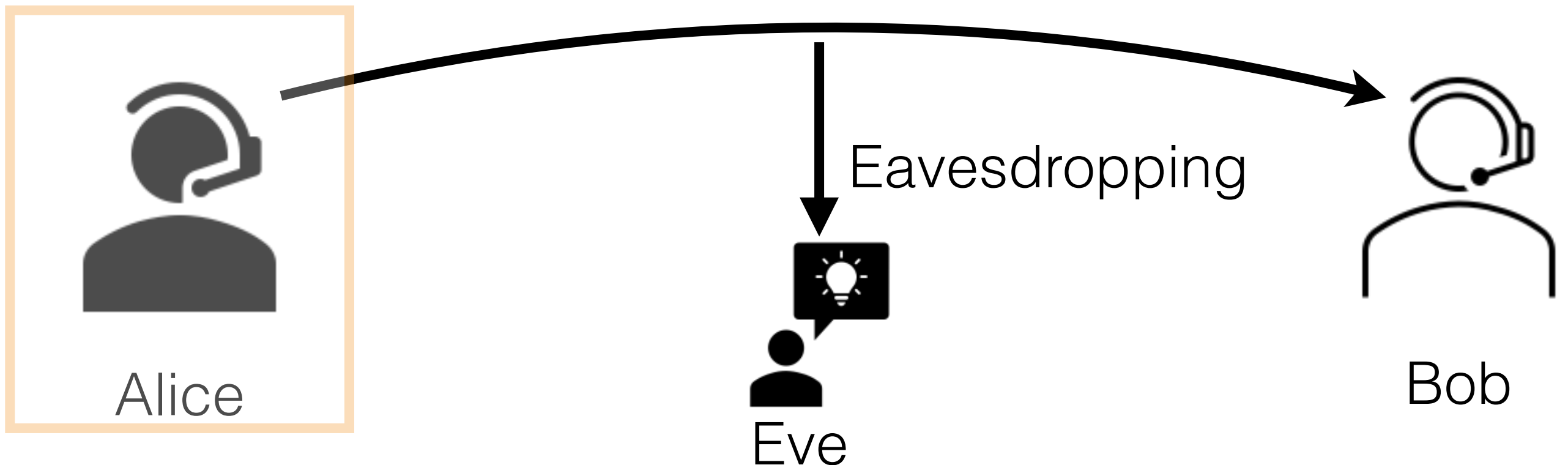
“Very secure encryption”

For the rest of today, we will discuss better techniques!

Old standards: substitution ciphers

The oldest schemes replace the letters in the message one by one, following a fixed recipe.

Encrypt



Old standards: substitution ciphers

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For example,

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

How do we encrypt “Math alive”?

Old standards: substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

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G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

Math alive

L

Old standards: substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

Math alive

Lg

Old standards: substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

Math alive

Lgx

Old standards: substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
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Math alive

Lgxu

Old standards: substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

Math alive

Lgxu g

Old standards: substitution ciphers

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Math alive

Lgxu gt

Old standards: substitution ciphers

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G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

Math alive

Lgxu gty

Old standards: substitution ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

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Lgxu gtyr

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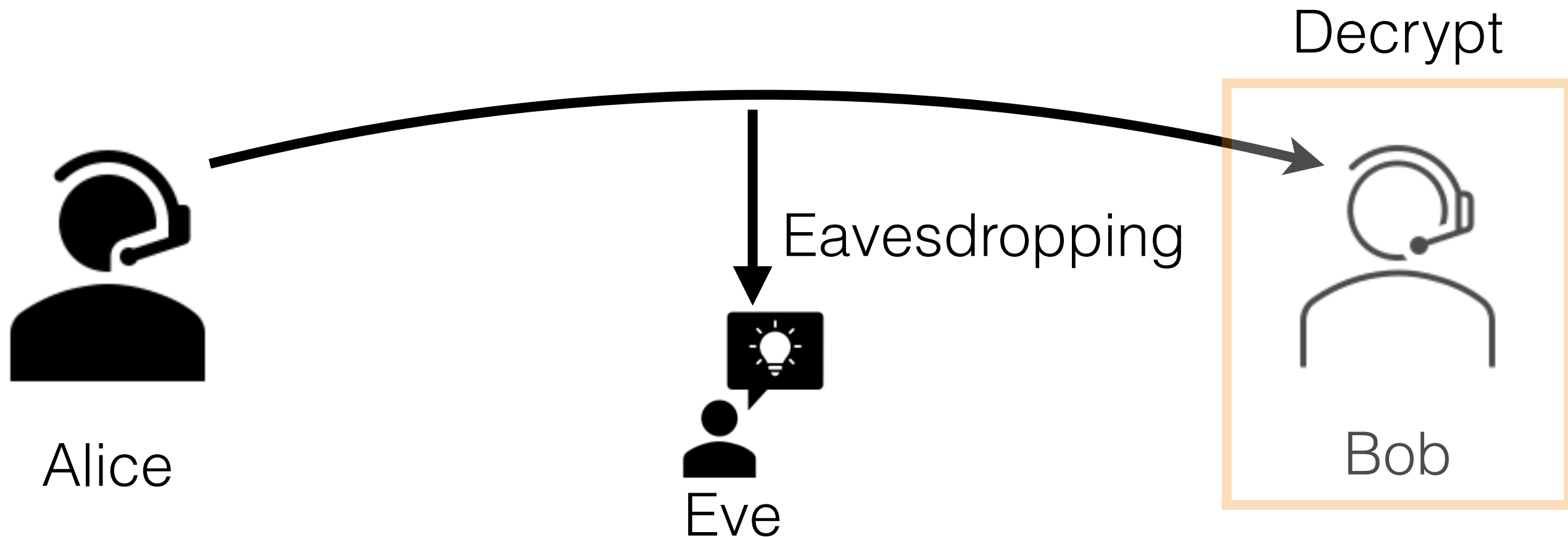
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

Math alive

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To decrypt, go the other way!



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G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

What does “QYMGNNAYVSATTHI” mean?

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
G	O	K	E	A	N	Q	U	Y	C	P	T	L	F	H	W	B	M	V	X	Z	R	I	D	S	J

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G

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GI

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GIR

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GIRA

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GIRAF

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

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GIRAFF

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GIRAFFE

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

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GIRAFFEI

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
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GIRAFFEIS

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

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GIRAFFEISY

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GIRAFFEISYE

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
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What does “QYMGNNAYVSATTHI” mean?

GIRAFFEISYEL

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
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A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

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GIRAFFEISYELLO

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
G O K E A N Q U Y C P T L F H W B M V X Z R I D S J

What does “QYMGNNAYVSATTHI” mean?

GIRAFFEISYELLOW

Special substitution cipher used by Julius Caesar

Caesar ciphers (circular shift of alphabets):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A

Shift by one step

Substitution cipher used by Julius Caesar

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A

Shift by one step

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B

Shift by two steps

Substitution cipher used by Julius Caesar

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A

Shift by one step

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B

Shift by two steps
⋮

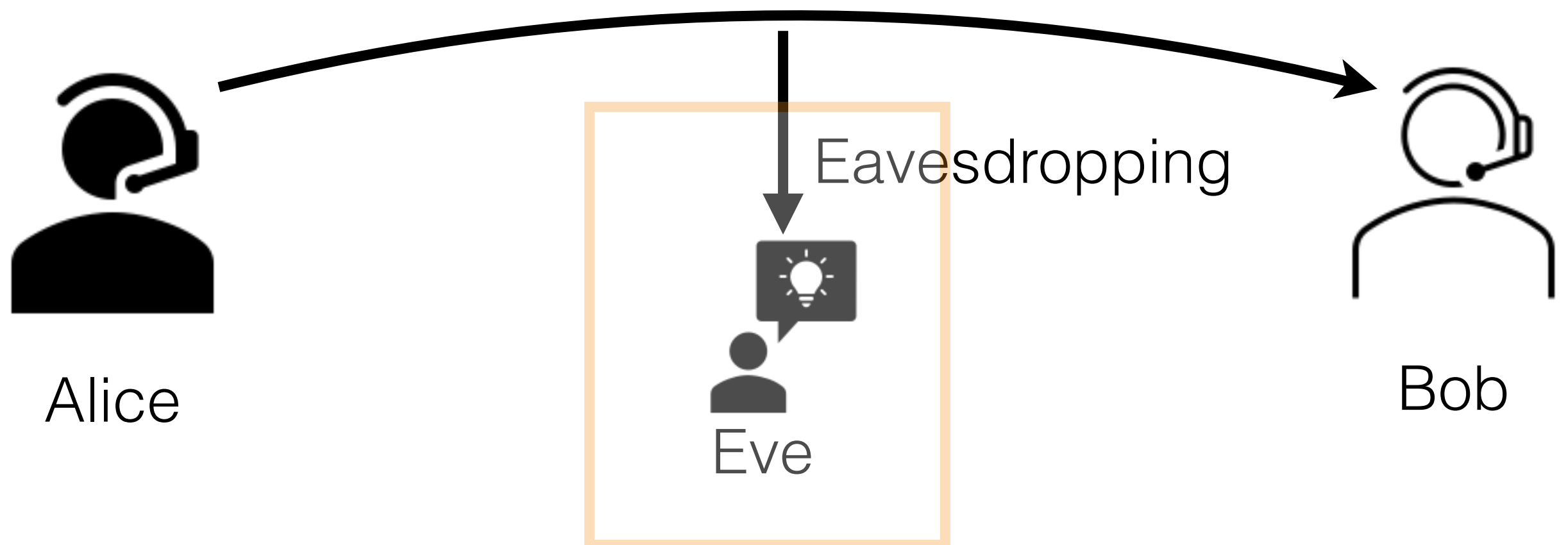
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Shift by 25 steps

We choose a value for the shift (the “key”) between 1 and 25 and get one cipher

Example: the following was encrypted with a Caesar cipher, but we do not know the key that was used. Decrypt it anyways!

nqpiecnewncvkqpu



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longcalculations



Brute force decryption: try every possible key.

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Brute force decryption: try every possible key.

Feasible for Caesar cipher (25 possible keys), but not for general substitution ciphers, since there are a total of

$$26 \cdot 25 \cdot \dots \cdot 2 \cdot 1 \approx 4 \cdot 10^{26}$$

keys.

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Feasible for Caesar cipher (25 possible keys), but not for general substitution ciphers, since there are a total of

$$26 \cdot 25 \cdot \dots \cdot 2 \cdot 1 \approx 4 \cdot 10^{26}$$

keys. Trying them all would take around 317 years on Earth's currently largest supercomputer.

zkbzkrphzkbihhwgrqwldlophqqrzwdnhphwrbrxuilkqlvkolqhrkpbkhduwlwe
uhdnvhyhubvwhswkdwlwdnhexwlpkrslqjwkdwkwhjdwhvwkhboowhoop
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fkrrvhbrxuodvwzrugvwklvlvwkhodvwwlphfdxvhbrxdqglzhzhuhheruqwrg
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If the language in which the plain text is written is known to the code-breaker, and if the messages contain a few sentences of text,

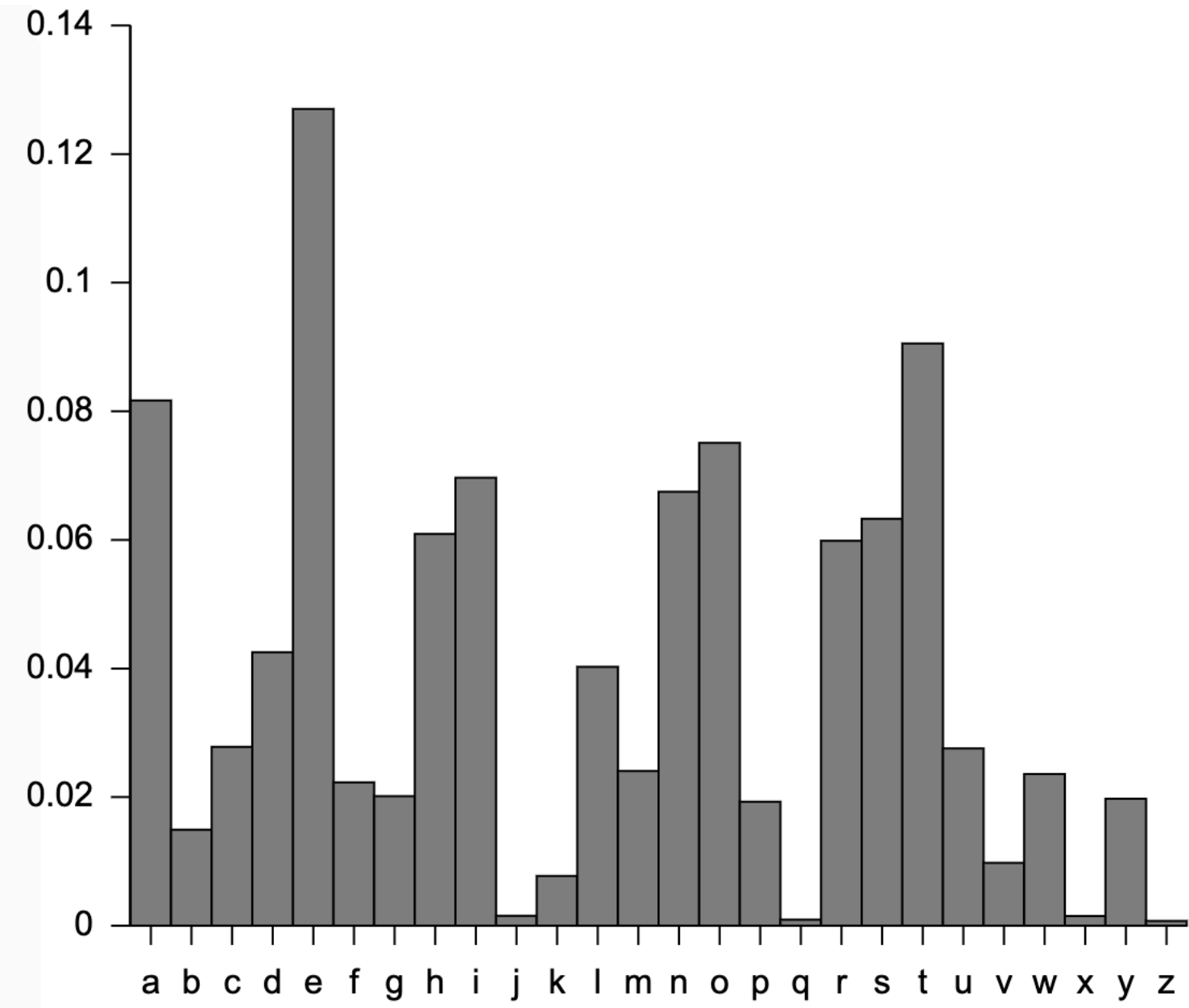
STANDARD					
LETTER	FREQUENCY				
a	.0761	i	.0734	r	.0615
b	.0154	j	.0015	s	.0650
c	.0311	k	.0065	t	.0933
d	.0395	l	.0411	u	.0272
e	.1262	m	.0254	v	.0099
f	.0234	n	.0711	w	.0189
g	.0195	o	.0765	x	.0019
h	.0551	p	.0203	y	.0172
		q	.0010	z	.0009

If the language in which the plain text is written is known

contains

ST
LETTER FR

a
b
c
d
e
f
g
h



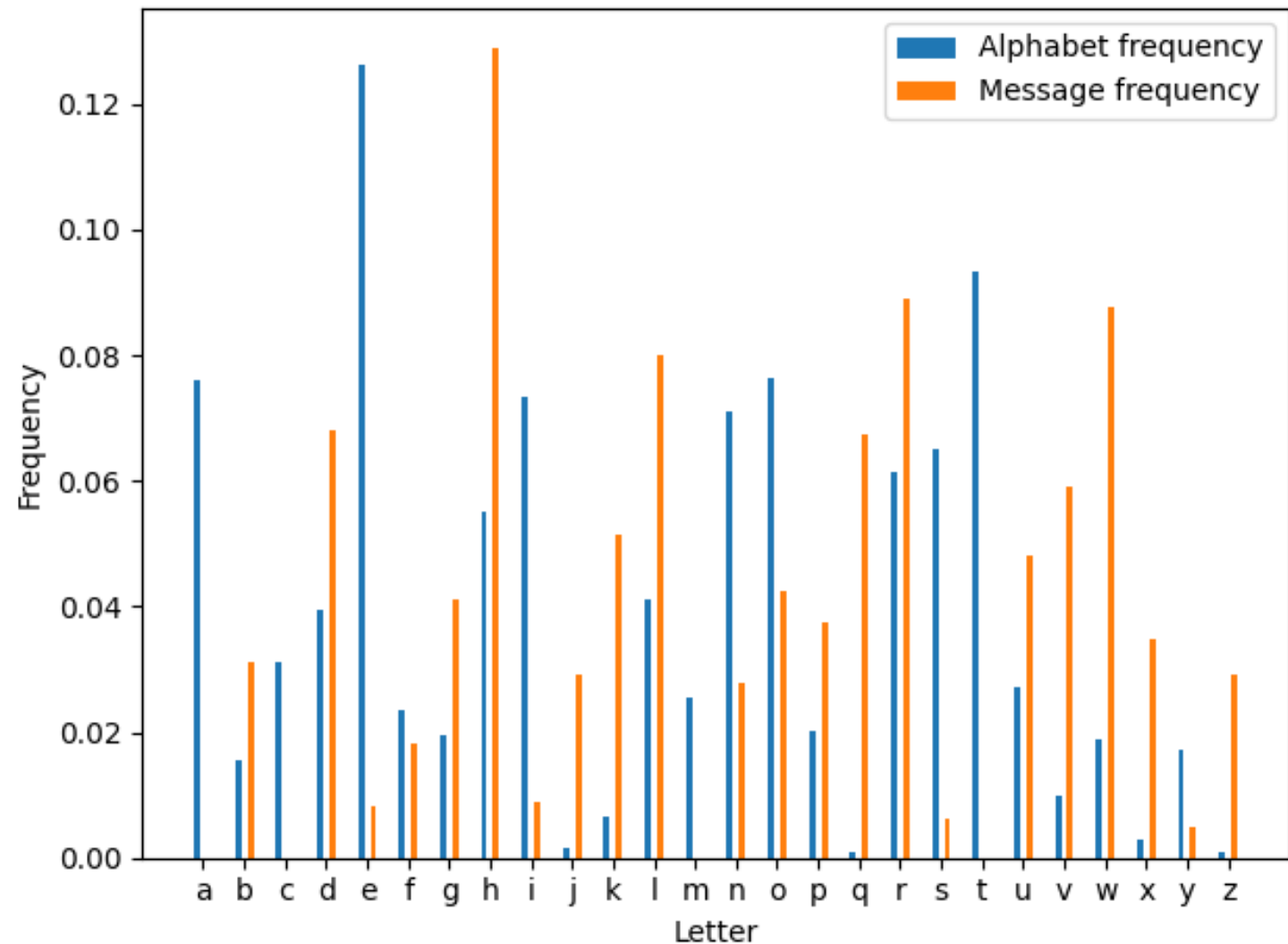
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.0650
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Decryption example using frequency analysis:

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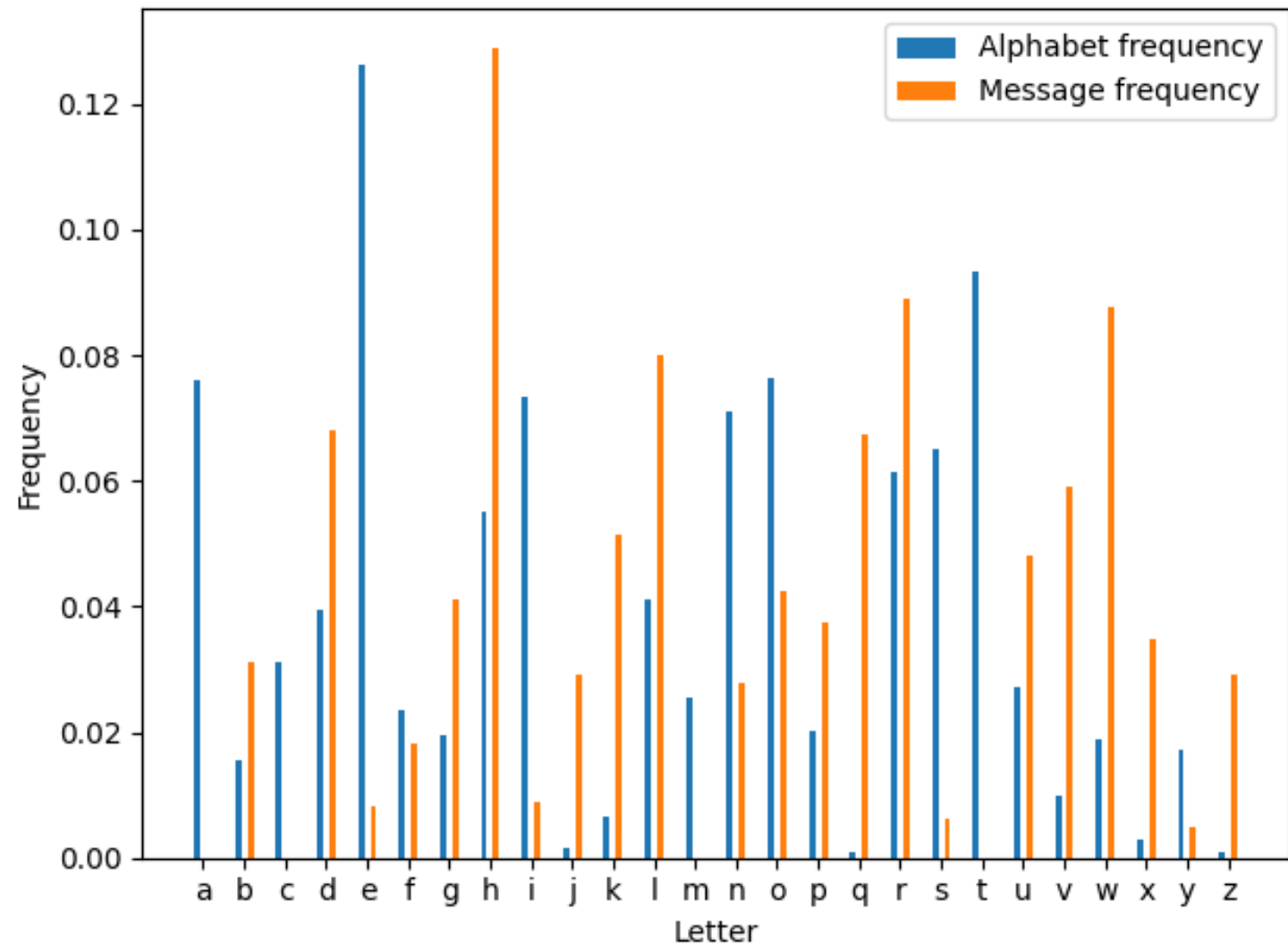
Decryption example using frequency analysis:

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lhzhzhuhheruqwrghh



Decryption example using frequency analysis:

zkbzkrphzkbihhwgrqwldlophqzwdnhphwrbrxuilqlvkolqhrkp
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whywhomewhyfeetdontfailmenowtakemetoyourfinishlineohmyheartitbreakseverystepthatitake...
— Lana Del Rey

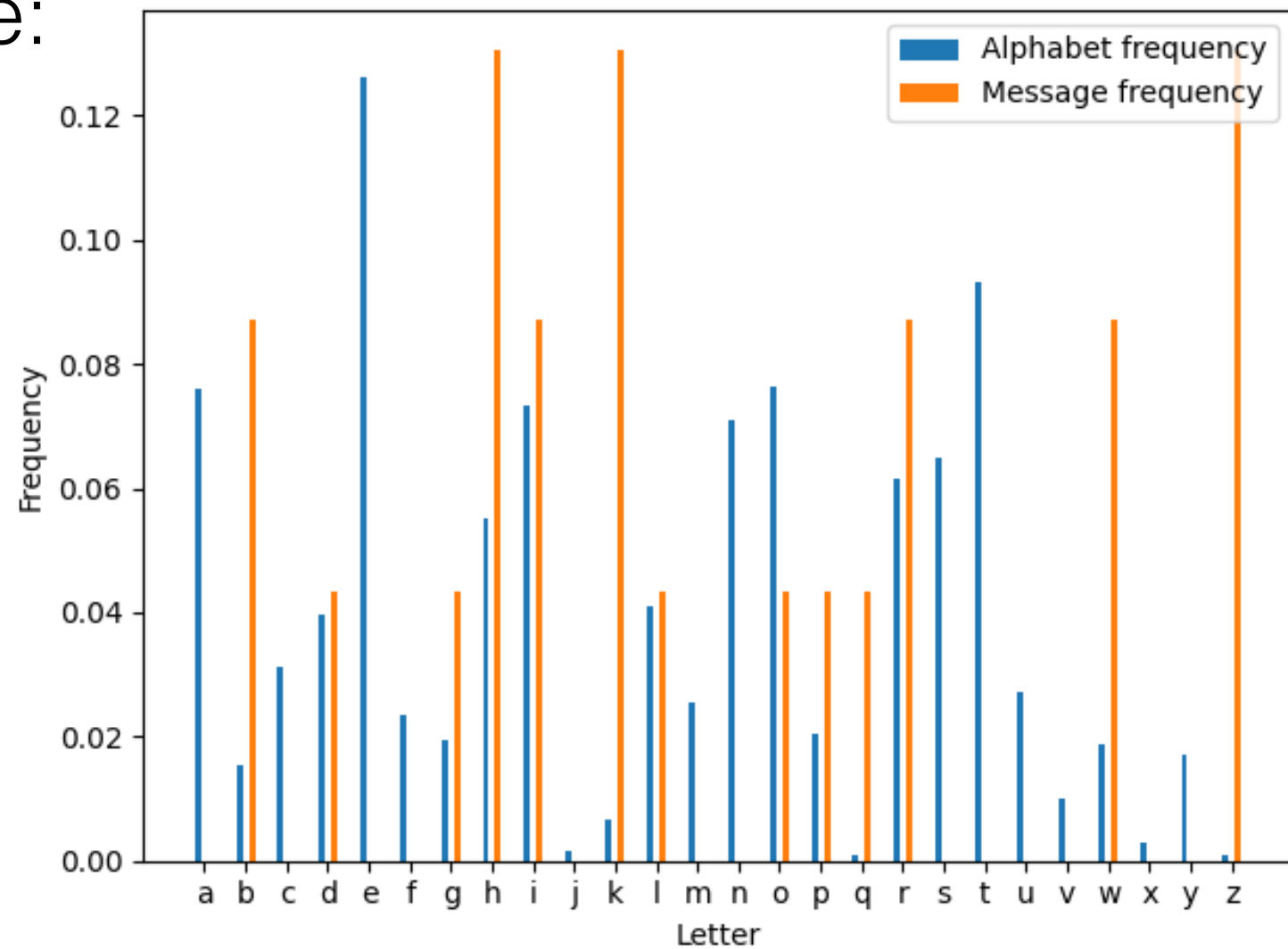
Decryption example using frequency analysis:

The first part of the message:

zkbzkrphzkbihhwgrqwidlo

“decrypts” to:

etoetihaetonaasridsnlcu



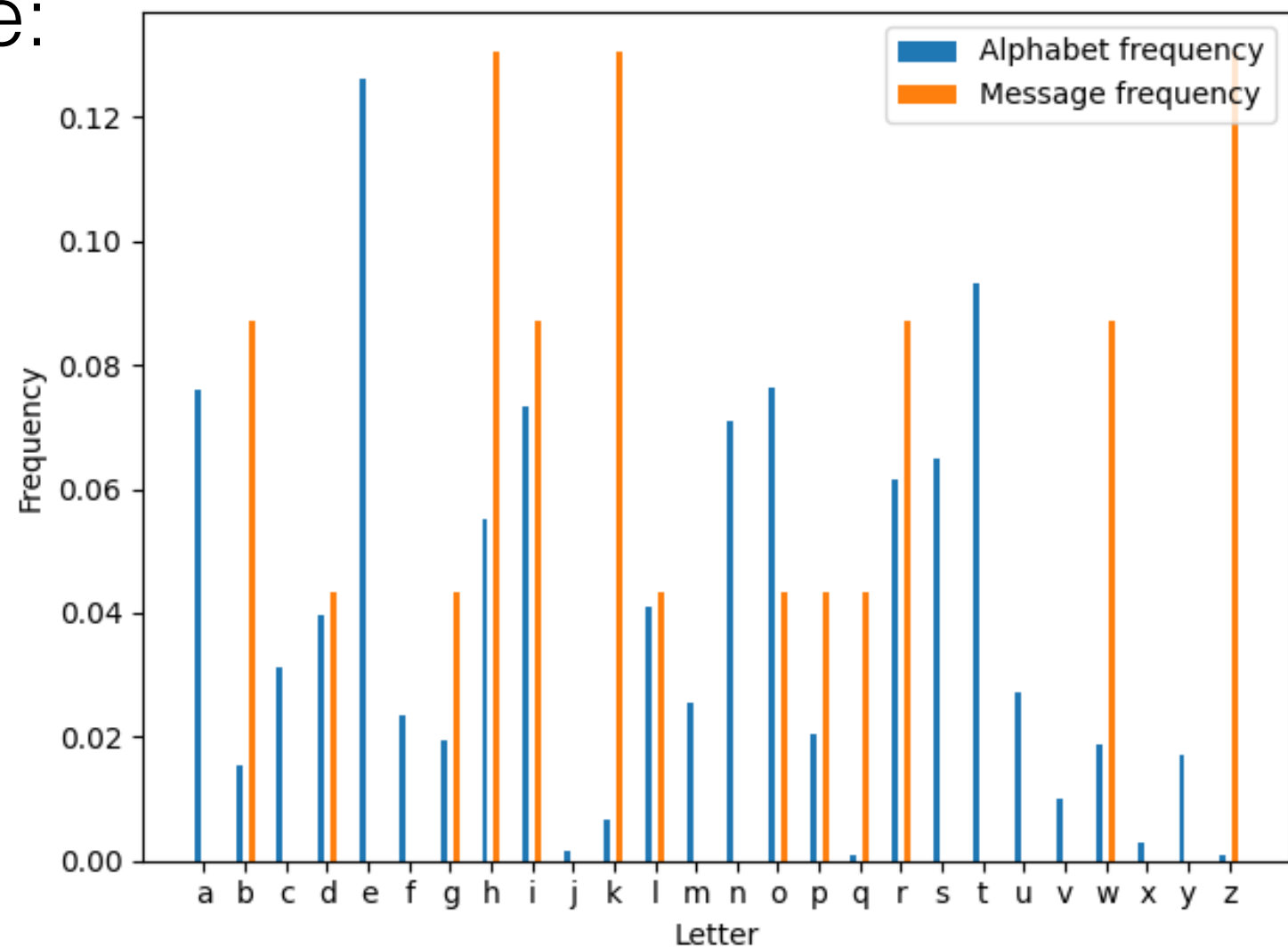
Decryption example using frequency analysis:

The first part of the message:

zkbzkrphzkbihhwgrqwidlo

“decrypts” to:

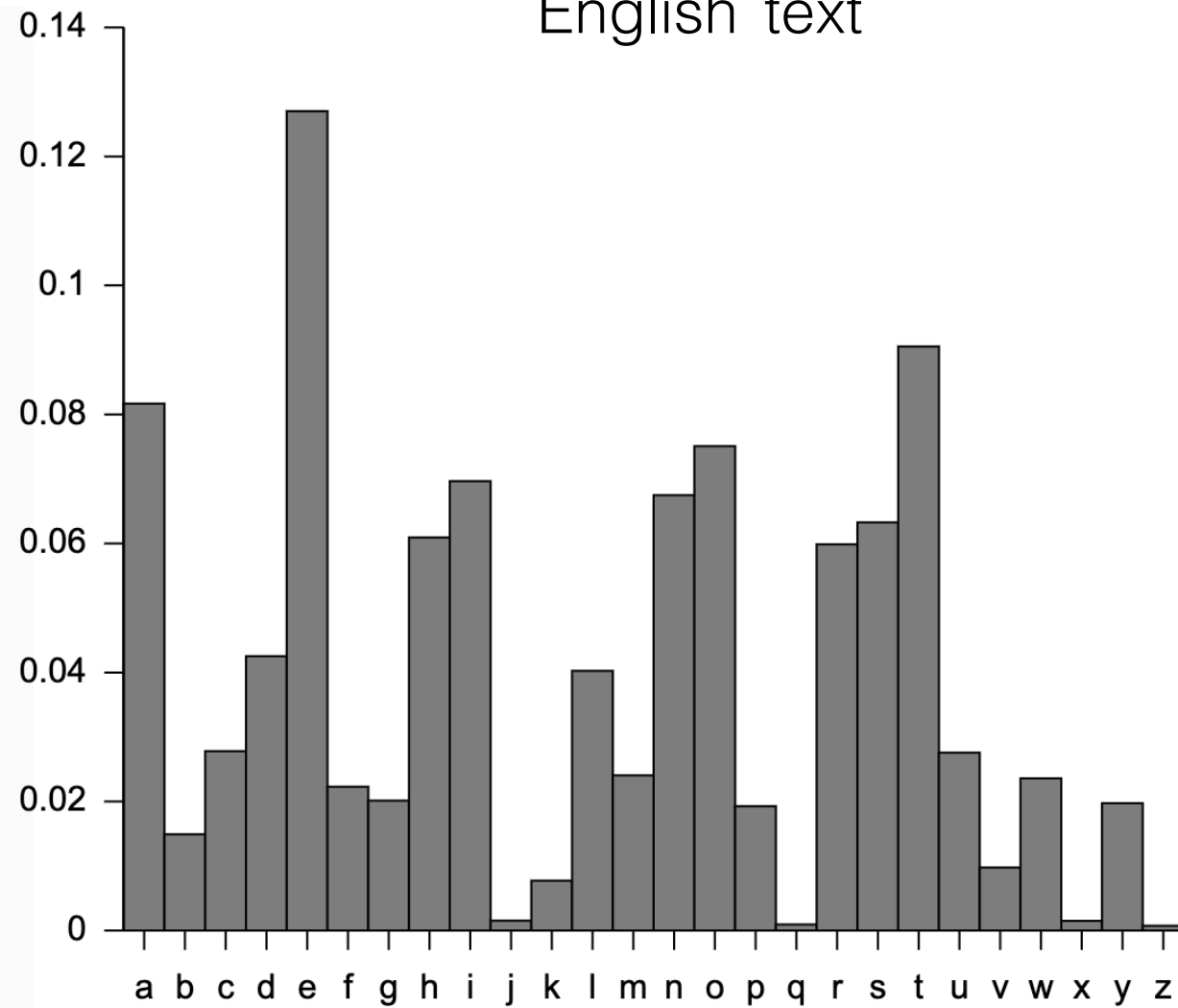
etoetihaetonaasridsnlcu



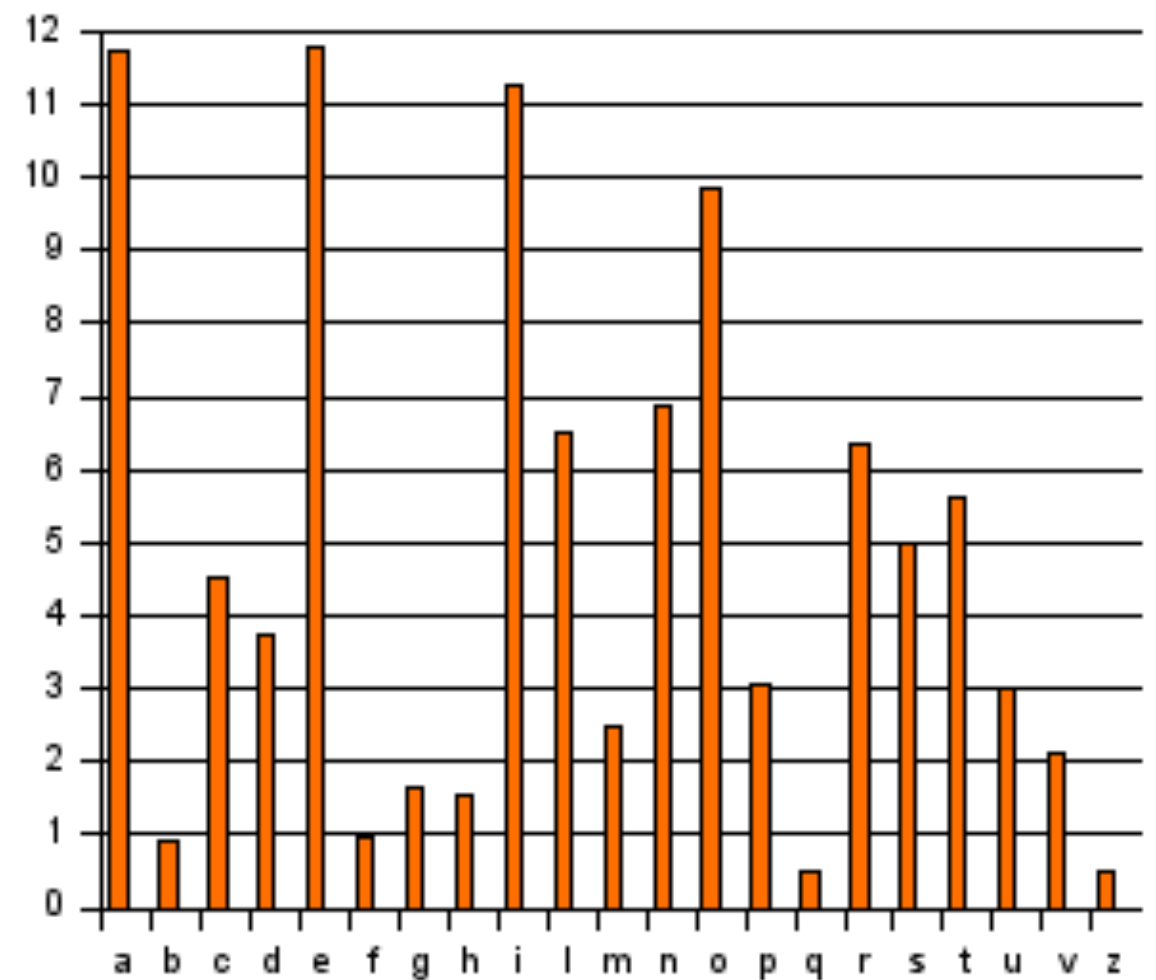
Does not work when message is too short!

Decryption example using frequency analysis:

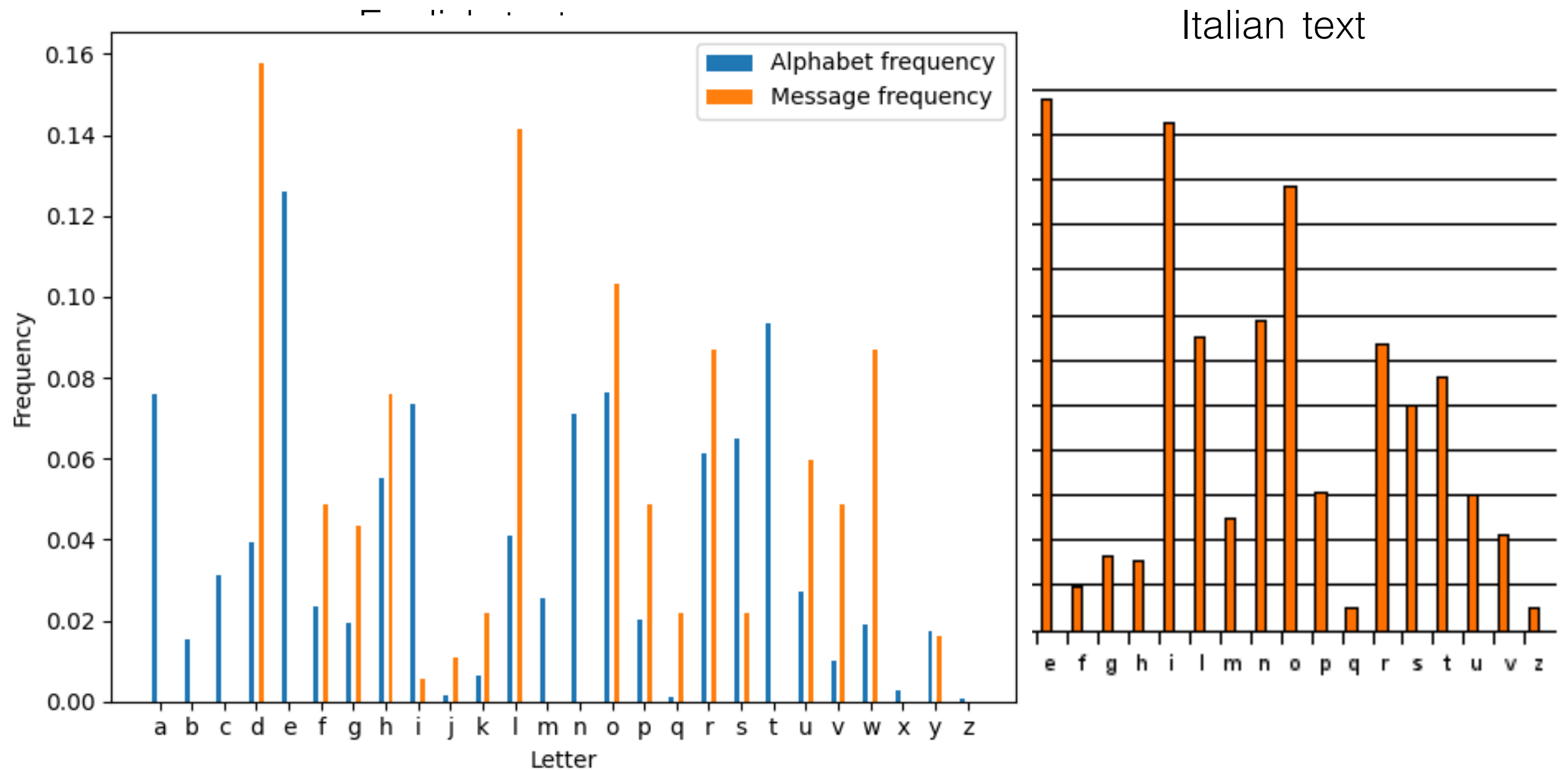
English text



Italian text



Decryption example using frequency analysis:



https://commons.wikimedia.org/wiki/File:Frequenze-alf_it.png

Does not work if message is in the wrong language!