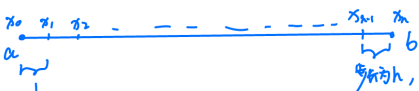


数值实验五——计算机作业

1. 计算圆周率

- a) 复合辛普森公式，当 $n=7$ 时，计算结果为 3.141592648320655
 b) 复合三点高斯-勒让德公式，当 $n=2401$ 时，计算结果为 3.141592663583750
 推导过程如下：

$$\int_a^b f(x) dx \stackrel{\eta = \frac{b-a}{2}t + \frac{a+b}{2}}{=} \int_{-1}^1 f\left(\frac{b-a}{2}t + \frac{a+b}{2}\right) \cdot \frac{b-a}{2} dt$$



$$\int_{x_0}^{x_1} f(x) dx \stackrel{\eta = \frac{x_1-x_0}{2}t + \frac{x_0+x_1}{2}}{=} \int_{-1}^1 f\left(\frac{x_1-x_0}{2}t + \frac{x_0+x_1}{2}\right) \cdot \frac{x_1-x_0}{2} dt = \frac{x_1-x_0}{2} \cdot \sum_{j=0}^2 w_j f\left(\frac{x_1-x_0}{2}t_j + \frac{x_0+x_1}{2}\right)$$

$$\Downarrow \text{当 } x_0=x_i, x_1=x_{i+1} \text{ 时.}$$

$$\int_{x_i}^{x_{i+1}} f(x) dx = \frac{x_{i+1}-x_i}{2} \cdot \sum_{j=0}^2 w_j f\left(\frac{x_{i+1}-x_i}{2}t_j + \frac{x_i+x_{i+1}}{2}\right) \stackrel{\eta = \frac{x_{i+1}-x_i}{2}t + \frac{x_i+x_{i+1}}{2}, x_i = x_i + h}{=} \frac{h}{2} \cdot \sum_{j=0}^2 w_j f\left(\frac{h}{2}t_j + x_i + \frac{h}{2}\right)$$

$$\Downarrow \text{当有 } n \text{ 段积分时.}$$

$$\int_a^b f(x) dx = \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} f(x) dx = \frac{b-a}{n} \sum_{i=0}^{n-1} \sum_{j=0}^2 w_j f\left(\frac{h}{2}t_j + x_i + \frac{h}{2}\right)$$

2. 计算年增长率

- a) 两点公式：

rate2 =

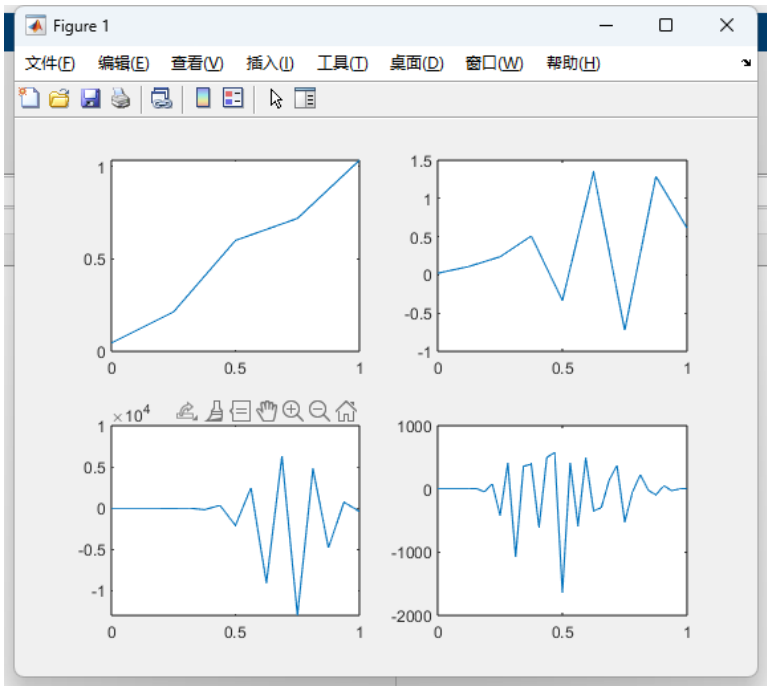
0.021052631578947	0.015760869565217	0.015680751173709
0.006899350649351	0.014426727410782	0.018978102189781
0.013775794757390	0.011029411764706	0.010993377483444

- b) 三点公式：

rate3 =

0.022039473684211	0.016576086956522	0.014647887323944
0.010227272727273	0.010440394836750	0.015792966157930
0.014863357501394	0.011568627450980	0.010463576158940
0.010381861575179		

3. 结算结果:



4	1.506568e+04	0.099311
8	4.521167e+08	1.475252
16	1.181506e+17	12974.164824
32	1.969206e+17	1639.845545

代码块

文件名称: hw5_1_1.m	功能: 数值实验 5.1 (1) 的计算代码
<pre>clc,clear all format long; fun=@(x) 4./(1+x.^2); f1=integral(fun,0,1); n=4; f2=fsimpson(fun,0,1,n); delta=abs(f2-f1); while delta>1e-8 n=n+1; f2=fsimpson(fun,0,1,n); delta=abs(f2-f1); end n,f2 % 7 function I=fsimpson(fun,a,b,n) h=(b-a)/n; x=linspace(a,b,2*n+1); y=feval(fun,x); I=h/6*(y(1)+2*sum(y(3:2:2*n-1))+4*sum(y(2:2:2*n))+y(2*n+1)); end</pre>	

文件名称: hw5_1_3.m	功能: 数值实验 5.1 (3) 的计算代码
<pre>clc,clear all format long; w=[5 8 5]/9; t=[-sqrt(15) 0 sqrt(15)]/9; fun=@(x) 4./(1+x.^2); f1=integral(fun,0,1); n=3; f2=fglegendre(fun,w,t,0,1,n); delta=f2-f1; while delta>1e-8 n=n+1; f2=fglegendre(fun,w,t,0,1,n); delta=abs(f2-f1); end n,f2 %2401 function f=fglegendre(fun,w,t,a,b,n) h=(b-a)/n; f=0;</pre>	

```

    for i=0:n-1
        gp=h/2+(a+i*h)+h/2*t;
        f=f+fun(gp)*w*h/2;
    end
end

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文件名称: hw5_5.m	功能: 数值实验 5.5 的计算代码
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```

clc,clear all
format long;
y=1900:10:1990;
p=[76.0 92.0 106.5 123.2 131.7 150.7 179.3 204.0 226.5 251.4];
% 2points
L2=diff(p)./diff(y);
rate2=L2./p(1:end-1)
% 3points
Lleft=(-3*p(1)+4*p(2)-p(3))/20; % 左边界点采用向后 差分
Lright=(p(end-2)-4*p(end-1)+3*p(end))/20; %右边界点采用向前差分
Lmid=(-p(1:end-2)+p(3:end))/20;% 中点采用中心差分格式
L3=[Lleft Lmid Lright];
rate3=L3./p

```

文件名称: hw5_7.m	功能: 数值实验 5.7 的计算代码
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```

clc,clear all
warning off
n=[4 8 16 32];
hold on;
for i=1:4
    [x,U]=hw7(n(i));
    subplot(2,2,i)
    plot(x,U)
end
hold off;
function [x,U]=hw7(n)
    x=[linspace(0,1,n+1)]';
    y=((x.^2+1).^(3/2)-x.^3)/3;
    h=2/n;
    w=ones(n+1,1)*2/6;w(2:2:n)=4/6;w([1 n+1])=1/6;w=w*h;
    [X,S]=meshgrid(x);
    % This MATLAB function returns 2-D grid coordinates based on the
    % coordinates contained in vectors x and y.
    A=(X.^2+S.^2).^(1/2); % A*diag(d)*x==y
    U=(A\y)./w;
    fprintf('%3d %13.6e %13.6f\n',n,cond(A),norm(U-x,'inf'));
end

```

文件名称: check.m	功能: 验证部分手算作业的答案
<pre> %%% homework 5.5 clc,clear all format long; fun=@(x) exp(x); f1=fttrapz(fun,0,1,4); f2=fsimpson(fun,0,1,4); f3=exp(1)-1; deltaf1=f3-f1; deltaf2=f3-f2; %%% homework 5.8 clc,clear all format long; x1=0.5555556*(1/(0.7745967+2)+1/(2-0.7745967))+0.5*0.8888889 x2=0.2369269*(1/(0.9061798+2)+1/(2-0.9061798))+0.4786287*(1/(0.5384693+2)+1/(2-0.5384693))+0.5*0.5688889 %%% homework 5.9 clc,clear all fun=@(x) 1./sqrt(1+x.^2) f4=fqbx(f,fun,4) deltaf4=f4-2.62205755429213 %%% funtion function I=fttrapz(fun,a,b,n) h=(b-a)/n; x=linspace(a,b,n+1); y=feval(fun,x); I=h/2*(y(1)+2*sum(y(2:n))+y(n+1)); end function I=fsimpson(fun,a,b,n) h=(b-a)/n; x=linspace(a,b,2*n+1); y=feval(fun,x); I=h/6*(y(1)+2*sum(y(3:2:2*n-1))+4*sum(y(2:2:2*n))+y(2*n+1)); end function I=fqbx(f,fun,n) t=1:2:2*n+1; x=cos(t/(2*(n+1))*pi); y=feval(fun,x); I=pi/(n+1)*sum(y); end </pre>	