

# A Robust Mixed $H_2/H_\infty$ Tracking Control for 3-DOF Permanent Magnet Spherical Actuator

Shaoyong Cai, Jingmeng Liu\*, Dong Xu, Weihai Chen

**Abstract**—This paper addresses a robust mixed  $H_2/H_\infty$  control for 3-DOF Permanent Magnet (PM) Spherical Actuator in order to improve its trajectory tracking performance. The dynamic model of the PM spherical actuator is a multi-variable nonlinear system contains uncertainties such as model errors and disturbances inevitably, which will gravely influence the performance of the control system. Therefore, a mixed  $H_2/H_\infty$  control algorithm which consists of  $H_2$  and  $H_\infty$  control theory is design to compensate for these uncertainties. The robust mixed  $H_2/H_\infty$  control algorithm aim at attenuate the effects of the model uncertainties and external perturbations. To illustrate the effectiveness of the proposed control algorithm, both simulations and experiments are conducted. The results have shown that the proposed control algorithm has better trajectory tracking performance and strong robustness to uncertainties.

## I. INTRODUCTION

With the development of modern industrial technology, the demand for a multiple-degree-of-freedom motion is increasing need in many occasions. Spherical actuators can produce three degree-of-freedom (DOF) rotational motions in one joint. Compared to the traditional three-DOF actuators which are realized by connecting several single-axis motors in parallel or in series, it has overcomes disadvantages such as slow dynamic response, lack of dexterity and low inertia moment. Therefore, it has a wide potential applications in robot manipulators, vision systems, machining tools, automation technology, omnidirectional wheels, precision assembling, etc.

Research on multi-DOF actuators have been actively carried out that can be dated back to mid-1950s. Williams et al. [1] realized the first multi-DOF actuator which firstly can produce 2-DOF motions in one joint. So far, spherical actuators with various structures, operating principles and position detection sensors have been proposed[2], [3]. In this paper, we focus on the permanent magnet (PM) spherical actuator because of its simple structure, rapid response and high flux density. Thought our previous work, we have designed a spherical actuator which can realize spinning and tilting motion, and the orientation can be measured through a novel spherical joint [4], [5].

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Since applications of the spherical actuator need high trajectory tracking accuracy, many control methods have been developed to achieve high performance. The proportional derivative (PD) control scheme is a typical feedback control approach, which has simplicity in design and implementation and can obtain relatively acceptable tracking performances with appropriate control gains [6]. However, the PM spherical actuator includes nonlinearities and uncertainties which will serious influence the trajectory tracking performances of the PD control scheme in high accuracy required occasion.

To solve these problems, the computed torque method (CTM) is proposed to linearize and decouple the dynamics of a PM spherical motor, but it is a model-based control scheme that can not get acceptable trajectory tracking performance, because of the dynamic modeling errors cannot be avoided due to both structural and nonstructural uncertainties[7], [8]. In order to make suitable compensation for these uncertainties, a lot of intelligent control schemes have been employed in recent years. In [9], The neural networks (NNs) are also proposed for the control of PM spherical actuator which aim at handling complicated dynamic systems with learning good control parameters. However, large networks require extensive training data and is difficult to achieve acceptable convergence time, so its have a restrict to application for real time control of PM spherical actuator systems.

These issues motivated us to develop a control scheme which can cope with complex dynamic systems with disturbances of the PM spherical actuator in real time. In recent years, the mixed  $H_2/H_\infty$  optimal control has become more and more popular in the control of linear and nonlinear systems and has been successfully response in the field of quadrotors[10], [11], which is intuitively an good choice for PM spherical actuator to improve its trajectory tracking performance. In typical  $H_2$  and  $H_\infty$  optimal control designs, which can not total satisfy to attenuate the effects due to the totally external disturbance and model uncertainties[12], [13]. But mixed  $H_2/H_\infty$  tracking controller can estimating the effects of both the internal plant uncertainties and periodic-disturbances[14], [15], [16]. In this paper, a robust mixed  $H_2/H_\infty$  framework is introduced into the dynamic tracking control of PM spherical actuator, which can cope with the model uncertainty and external disturbances, and also can have a fast convergence.

The reminder of this paper is organized as follows. Section II presents the structure and working principle of the PM spherical actuator, the dynamic model and the torque model.

Section III develops the robust mixed  $H_2/H_\infty$  tracking control scheme. IV and V presents the result of simulation and experiments. Finally, the concluding remarks are summarized in Section VI.

## II. DYNAMIC MODELING AND TORQUE MODELING

### A. PM Spherical Actuator

The CAD model of the PM spherical actuator is shown in Fig. 1. It consists of a ball-shaped rotor with 8 PM poles arranged along the equatorial plane of the rotor and 30 coils distributed in three layers symmetrically about the equatorial plane of the stator. The rotor, incorporated with a passive spherical joint including a 2-axis tilt sensor and a rotary encoder for measuring the orientation of the rotor to have a basis for the closed-loop system.

The rotor motion is generated by the attraction or repulsion forces between the coils and permanent magnets. As shown in Fig. 2, When the currents of the stator coils are activated by a specific approach, the rotor can be driven by the electromagnetic force between stator coils and PM poles to realize corresponding motion. Therefore, the rotor can produce corresponding motion within the workspace by activating the input current of stator coils in a specific approach[17].

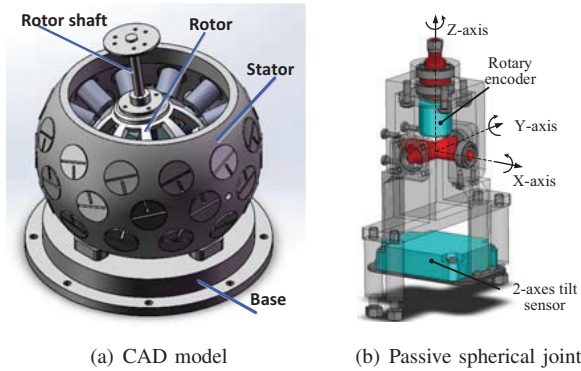


Fig. 1. The mechanical structure of the PMSA

### B. Dynamic Modeling

Consider the rotor of the spherical motor moving in free space. Fig. 3 shows the coordinates transformation from rotor frame to stator frame that the rotation matrix can be get through rotations around the axes of the stator at three times. With these rotations, the rotation matrix  $R$  is given by:

$$R = \begin{bmatrix} c\beta c\gamma & -c\beta s\gamma & s\beta \\ c\gamma s\alpha s\beta + c\alpha s\gamma & c\alpha c\gamma - s\alpha s\beta s\gamma & -c\beta s\alpha \\ -c\gamma s\alpha s\beta + s\alpha s\gamma & c\gamma s\alpha + c\alpha s\beta s\gamma & c\alpha c\beta \end{bmatrix}$$

with  $c \cdot = \cos(\cdot)$ ,  $s \cdot = \sin(\cdot)$  and  $\alpha, \beta, \gamma$  are Euler angles.

According to Lagrange's equation, the dynamic model of the spherical actuator can be derived as follow:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q, \dot{q}) = \tau_c \quad (1)$$

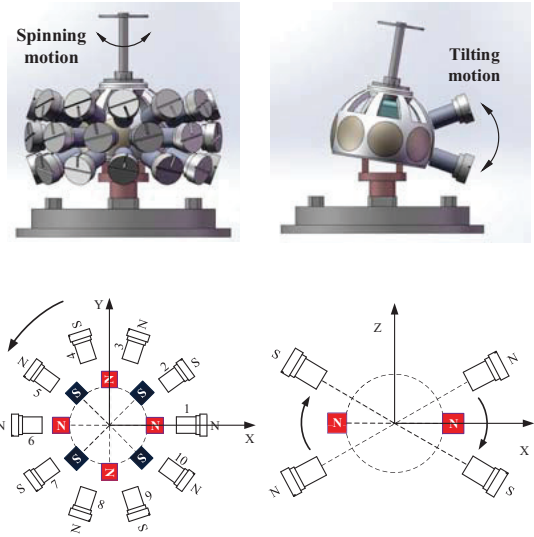


Fig. 2. The 3-DOF motion of spherical actuator

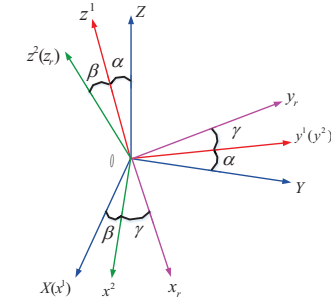


Fig. 3. Coordinate definition

where  $M(q)$  and  $C(\dot{q}, q)$  are the inertia matrix and centrifugal and Coriolis force matrix respectively while  $N(q, \dot{q})$  is gravity terms.  $q = [\alpha \ \beta \ \gamma]^T$  is the Euler angle vector, and  $\tau = [\tau_\alpha \ \tau_\beta \ \tau_\gamma]^T$  is the control torque.

In practical spherical actuator systems, the external disturbances, the perturbations of system parameters are inevitable. Hence, the dynamic model Eq. (1) is modified as:

$$\hat{M}(q)\ddot{q} + \hat{C}(q, \dot{q})\dot{q} + \hat{N}(q, \dot{q}) = \tau_c - \tau_d \quad (2)$$

where  $\hat{M}(q) = M(q) + \Delta M(q)$  is the actual inertial matrix;  $\hat{C}(q, \dot{q}) = C(q, \dot{q}) + \Delta C(q, \dot{q})$  is the actual coriolis matrix;  $\hat{N}(q, \dot{q}) = N(q, \dot{q}) + \Delta N(q, \dot{q})$  is the actual gravity matrix. Herein,  $\Delta M(q)$ ,  $\Delta C(q, \dot{q})$ ,  $\Delta N(q, \dot{q})$  denote the uncertainty and perturbation of system. By letting  $n((q, \dot{q}, \ddot{q}) = \tau_d - \Delta M\ddot{q} - \Delta C\dot{q} - \Delta N$  denote the combined disturbance due to the parameter uncertainties and external disturbances, then Eq. (2) can be rewritten as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q, \dot{q}) - n(q, \dot{q}, \ddot{q}) = \tau_c \quad (3)$$

### C. Torque Modeling

The torque model of the spherical motor is aim at establishing the relationship between the current input of the stator coils and the torque output at a specified rotor orientation. The torque has a linear property with respect to the current input and the total torque can be calculated by summing up every coils which is given as follow:

$$T = [T_x \quad T_y \quad T_z]^T = GI \quad (4)$$

where the torque matrix  $G = [G_1 \dots G_j \dots G_N]$  and the current inputs is  $I = [I_1 \dots I_j \dots I_N]^T$ .  $N$  is the number of the coils that be 15 because the coils are divided into 15 groups.

Herein,  $I_j$  is the current input of the  $j^{th}$  coil while  $G_j$  describes the torque contribution matrix of the  $j^{th}$  coil, which is given as:

$$G_j = \begin{cases} \sum_{i=1}^8 (-1)^{i-1} f(\varphi_{ij})(r_i \times s_j / |r_i \times s_j|), & r_i \times s_j \neq 0 \\ 0, & r_i \times s_j = 0 \end{cases} \quad (5)$$

where  $f(\varphi_{ij})$  is a torque function obtained by using a finite element(FE) method base on curve fitting of the computed data between one PM pole and one coil,  $\varphi_{ij} = \cos^{-1}(r_i, s_j)$  is the separation angle between the PM pole and the coil.

Therefore, according to Eq. (4), given desired control torque  $T$  at a specific rotor orientation, the current of the coils  $I$  can be calculated by[7]:

$$I = G^T (GG^T)^{-1} T \quad (6)$$

### III. DESIGN OF ROBUST MIXED $H_2/H_\infty$ TRACKING CONTROLLER

With dynamics and torque model, the solution of control for spherical actuator can be executing better. To control the nonlinear PM spherical actuator system with uncertainties such as modeling errors and disturbances, a robust mixed  $H_2/H_\infty$  tracking control law is developed as follows[18].

Considering dynamics model, define the states variables  $x_1(t) = q(t)$ ,  $x_2(t) = \dot{q}(t)$ , then the tracking error define as follows:

$$\tilde{x} = \begin{bmatrix} \dot{\tilde{q}} \\ \tilde{q} \end{bmatrix} = \begin{bmatrix} x_2 - \dot{q}_d \\ x_1 - q_d \end{bmatrix} \quad (7)$$

By using Eq. (8), the error dynamic equation is computed as:

$$\dot{\tilde{x}} = A(q, \dot{q})\tilde{x} + B(\ddot{q}_d, \dot{q}_d, \dot{q}, q) + CM^{-1}(q)(\tau_c + n(q, \dot{q}, \ddot{q})) \quad (8)$$

where

$$A(\cdot) = \begin{bmatrix} -M^{-1}(q)C(q, \dot{q}) & 0_{n \times n} \\ I_{n \times n} & 0_{n \times n} \end{bmatrix} \quad (9)$$

$$B(\cdot) = \begin{bmatrix} -\ddot{q}_d - M^{-1}(q)(C(q, \dot{q})\dot{q}_d + N(q, \dot{q})) \\ 0_{n \times n} \end{bmatrix} \quad (10)$$

$$C = \begin{bmatrix} I_{n \times n} \\ 0_{n \times n} \end{bmatrix} \quad (11)$$

Then, we further define  $r(t) = t_{11}(\dot{q}(t) - \dot{q}_d(t)) + t_{12}(q(t) - q_d(t))$  and introduce a state space transformation matrix  $T$ :

$$T = \begin{bmatrix} T_{11} & T_{12} \\ 0_{n \times n} & I_{n \times n} \end{bmatrix} = \begin{bmatrix} t_{11}I_{n \times n} & t_{12}I_{n \times n} \\ 0_{n \times n} & I_{n \times n} \end{bmatrix} \quad (12)$$

where  $t_{11}$  and  $t_{12}$  are two positive constants.

Then, Eq. (8) can be further expressed as:

$$\dot{\tilde{x}} = A_T(\tilde{x}, t) + B_T(\tilde{x}, t)T_{11}(-F(x_e) + \tau_c + n) \quad (13)$$

where  $x_e = [x_1^T, x_2^T, q_r^T, \dot{q}_r^T, \ddot{q}_r^T]^T$ ,

$$A_T(\cdot) = T^{-1} \begin{bmatrix} M^{-1}C & 0_{n \times n} \\ T_{11}^{-1} & -T_{11}^{-1}T_{12} \end{bmatrix} T \quad (14)$$

$$B_T(\cdot) = T^{-1} \begin{bmatrix} I_{n \times n} \\ 0_{n \times n} \end{bmatrix} M^{-1} \quad (15)$$

$$F(\cdot) = M(\ddot{q}_d - T_{11}^{-1}T_{12}\dot{\tilde{q}}) + C(\dot{q}_d - T_{11}^{-1}T_{12}\tilde{q}) + N \quad (16)$$

#### A. Problem Formulation

For the system with parameter perturbations and exterior disturbances, the mixed  $H_2/H_\infty$  tracking control design is produced that the robust controller can achieve the  $H_2$  optimal tracking:

$$\min_{\mu(t)} \left\{ \tilde{x}^T(t_f)Q_{2f}\tilde{x}(t_f) + \int_{t_0}^{t_f} (\tilde{x}^T(t)Q_2\tilde{x}(t) + \mu^T(t)R_2\mu(t))dt \right\} \quad (17)$$

with the  $H_\infty$  disturbance elimination mechanism:

$$\max_{\mu(t)} \left\{ \tilde{x}^T(t_f)Q_{1f}\tilde{x}(t_f) + \int_{t_0}^{t_f} (\tilde{x}^T(t)Q_1\tilde{x}(t) + \mu^T(t)R_1\mu(t) - v^2n^T(t)n(t))dt \right\} \leq \tilde{x}^T(t_0)P\tilde{x}(t_0) \quad (18)$$

where  $t_0$  and  $t_f$  mean the initial and final time of the robust control operation.

Then, we establish two performance indexes  $J_1$  and  $J_2$ , which  $J_1$  is defined as:

$$J_1(\mu, n) \cong \tilde{x}^T(t_f)Q_{1f}\tilde{x}(t_f) + \int_{t_0}^{t_f} [\tilde{x}^T(t)Q_1\tilde{x}(t) + \mu^T(t)R_1\mu(t) - v^2n^T(t)n(t)]dt \quad (19)$$

and  $J_2$  is formulated as:

$$J_2(\mu, n) \cong \tilde{x}^T(t_f)Q_{2f}\tilde{x}(t_f) + \int_{t_0}^{t_f} [\tilde{x}^T(t)Q_2\tilde{x}(t) + \mu^T(t)R_2\mu(t)]dt \quad (20)$$

The mixed  $H_2/H_\infty$  tracking control objective can be expressed as:

$$J_2(\mu^*, n^*) \leq J_2(\mu, n^*), \forall \mu(t) \in L_2[t_0, t_f] \quad (21)$$

$$J_1(\mu^*, n^*) \geq J_1(\mu^*, n), \forall n(t) \in L_2[t_0, t_f] \quad (22)$$

where  $\mu^*$ ,  $n^*$  represent the optimal input and the worst possible disturbance, severally.

### B. Solution

In order to solve the problem of Eq. (17), we let  $R_1 = R_2 = R$  and assign  $Q_2 = \beta Q_1$  with  $0 < \beta < 1$ . By Cholesky decomposition,  $Q_1$  can be written as:

$$Q_1 = \begin{bmatrix} L_{11}^T L_{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & L_{22}^T L_{22} \end{bmatrix} \quad (23)$$

Hence, the transformation matrix  $T$  and input weighting matrix  $R$  can be resolved through the calculation and presented as:

$$T = \begin{bmatrix} \nu\sqrt{1-\beta}L_{11} & \nu\sqrt{1-\beta}L_{22} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix} \quad (24)$$

$$R = \nu^2 \begin{pmatrix} \beta - 1 \\ \beta - 2 \end{pmatrix} I_{3 \times 3} \quad (25)$$

For conforming to Eq. (12), we make  $L_{11} = cI_{3 \times 3}$  and  $L_{22} = dI_{3 \times 3}$ , Therefore, we now procure  $t_{11}$  and  $t_{12}$  in Eq. (12) where:

$$t_{11} = c\nu\sqrt{1-\beta} \quad (26)$$

$$t_{12} = d\nu\sqrt{1-\beta} \quad (27)$$

Eventually, it can be proved that[11], [14], [18]:

$$\mu^* - R^{-1}B^T T e(t) \quad (28)$$

$$n^* = \frac{1}{\nu^2} B^T T e(t) \quad (29)$$

are the solutions of the mixed  $H_2/H_\infty$  control tracking problem.

## IV. SIMULATION

To better evaluate effectiveness and robustness of the proposed control algorithm, simulation is carried out in this section. Because the modeling error and the disturbance are main factors in affect the trajectory tracking performance, simulations are mian focus on these two aspects.

The desired trajectory is:

$$q = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0.2 * t * \sin(\pi t) \\ 0.2 * \cos(\pi t) \\ 2\pi t \end{bmatrix}, t \in [0, 5]$$

The modeling errors are set as follows:

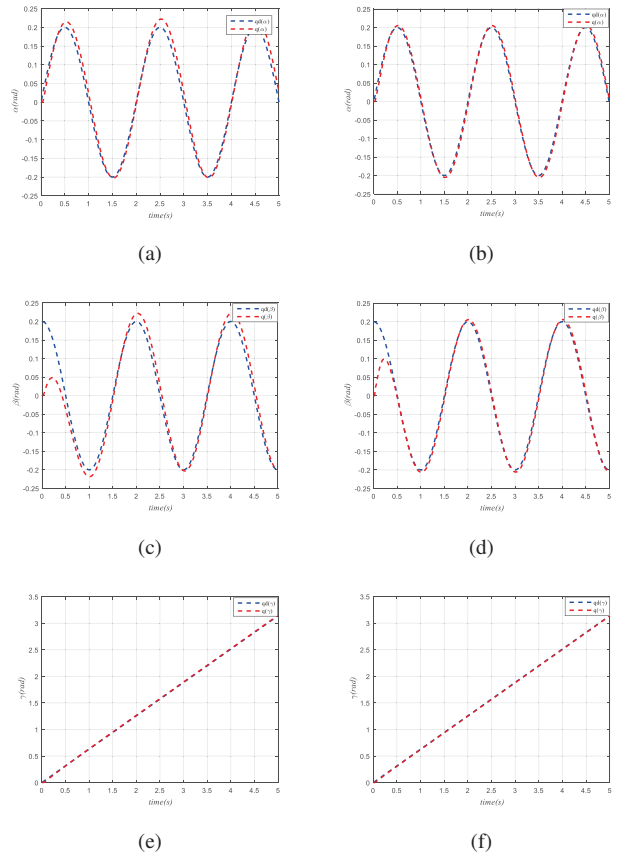


Fig. 4. Tracking performance with the PD control (a) $\alpha$  angle, (c) $\beta$  angle, (e) $\gamma$  angle and Tracking performance with the proposed control (b) $\alpha$  angle, (d) $\beta$  angle, (f) $\gamma$  angle

$$\begin{aligned} \Delta M(q) &= 0.25 * M(q), \Delta C(q, \dot{q}) = 0.25 * C(q, \dot{q}), \\ \Delta N(q, \dot{q}) &= [0.001; 0.001; 0.001]. \end{aligned}$$

The random external disturbance torque is set as:

$\tau_d = r * [\cos(\pi t), \exp(-\pi t), \sin(2\pi t)]$  where  $r = 0.01$  is randomly distributed in  $(-0.01, 0.01)$  (Nm).

The gains of PD controller are  $K_p = \text{diag}[100, 100, 100]$  and  $K_d = \text{diag}[40, 40, 40]$ , whereas the tuned parameters for robust mixed  $H_2/H_\infty$  control are  $\nu = 2$ ,  $\beta = 0.5$ ,  $c = 1$ ,  $d = 1$ . The principle inertial moments of the rotor are  $J_1 = 2.219e-003$  (kg·m<sup>2</sup>),  $J_2 = 2.176e-003$  (kg·m<sup>2</sup>),  $J_3 = 2.256e-003$  (kg·m<sup>2</sup>).

Fig. 4(a)(c)(e) show the tracking performance under the PD control method in  $\alpha$ ,  $\beta$  and  $\gamma$  angle while Fig. 4(b)(d)(f) denotes the result of proposed control.  $q$  denotes the actual output, and  $q_d$  is the desired trajectory. It can be seen that the position tracking errors are much large. It can be observed that the actual trajectory fits the desired trajectory well under the proposed control method, and the maximum position tracking error during the motion is smaller than  $0.1(\text{rad})$ . The result show that proposed the proposed algorithm can give much better performances in the presence of model uncertainty and random non-repetitive external disturbances.



## V. EXPERIMENTS

Fig. 5(a) shows the experimental prototype and control system of the spherical actuator. The block diagram of the control system consists of a personal computer(PC), a digital signal processor TMS320F28335(DSP), a field programmable gate array EP4CE10F17C8N(FPGA), V/I converting circuit, orientation and current measurement module(see Fig. 5(b)). A graphical user interface (GUI) program is developed on the PC in the VS2016 environment programmed by C++. Hence, the control parameter setting, control mode selection, command sending and status displaying can be easily done through GUI.

The DSP and FPGA constitute the core control module of current controller(see Fig. 5(c)). More specifically, DSP is responsible for communicating with computer, algorithm computing while FPGA is in charge of driving the AD5370 chip and receiving and processing sensors information which consists of orientation module and current sampling. The D/A chip AD5370 is applied to offer 40 channels with 16-bit resolution digital-to-analog converting for generating multi-channel and bipolar current simultaneously. Then, the V-I converting circuit can transform the voltage obtained from AD5370 into a proportional current. The ADS8364 is an A/D chip that can offer 6-channel output with a 16-bit A/D converting resolution.

To observe the control performance preliminarily, a circular trajectory in the X-Y plane is conducted. In this experiment, a load weighting 0.1 kg is fixed on the output shaft. The desired trajectory is set as:

$$q = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 5 * \cos(\frac{2*\pi t}{30} - \frac{\pi}{2}) \\ 5 * \sin(\frac{2*\pi t}{30} - \frac{\pi}{2}) + 5 \end{bmatrix}, t \in [0, 30]$$

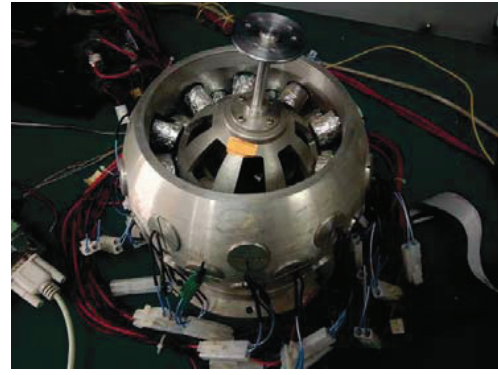
Fig. 6(a) shows the trajectory tracking performance of the PD control scheme. It is obvious that the maximum tracking errors of both  $\alpha$  and  $\beta$  are more than  $0.5^\circ$  due to the fact dynamic model is uncertain and disturbances.

In contrast, Fig. 6(b) shows the trajectory tracking performance of the proposed control scheme. It can be observed that the actual trajectory fits the desired trajectory extremely well and the maximum tracking errors of both  $\alpha$  and  $\beta$  are smaller than  $0.2^\circ$ , which indicates that the proposed control have been effectively compensated for uncertainties and model errors.

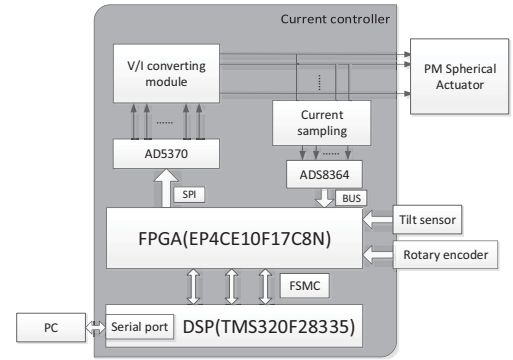
In summary, The experimental results show that compared to the PD control, the robust mixed  $H_2/H_\infty$  control algorithm can give much better performances, which demonstrate the effectiveness of the proposed algorithm..

## VI. CONCLUSION

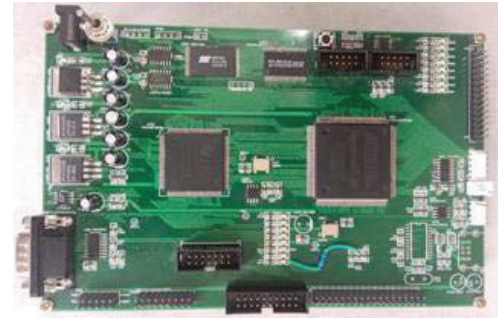
This paper mainly focuses on a robust mixed  $H_2/H_\infty$  control for 3-DOF (PM) Spherical Actuator in order to improve its trajectory tracking performance. Owing to complicated mechanical structure and electromagnetic field, there exist model error and disturbances during the control process. So we develop a robust mixed  $H_2/H_\infty$  control algorithm to



(a) Experimental prototype



(b) Block diagram of control system



(c) Core control module

Fig. 5. Prototype of PMSA and control system

compensate for these uncertainties. To illustrate the effectiveness of the proposed control algorithm, both simulations and experiments are conducted. The simulation results have shown that the proposed algorithm can effectively deal with the modeling uncertainties and external disturbances. In the end, an experimental platform, has been developed for demonstrating the practicability of the proposed algorithm.

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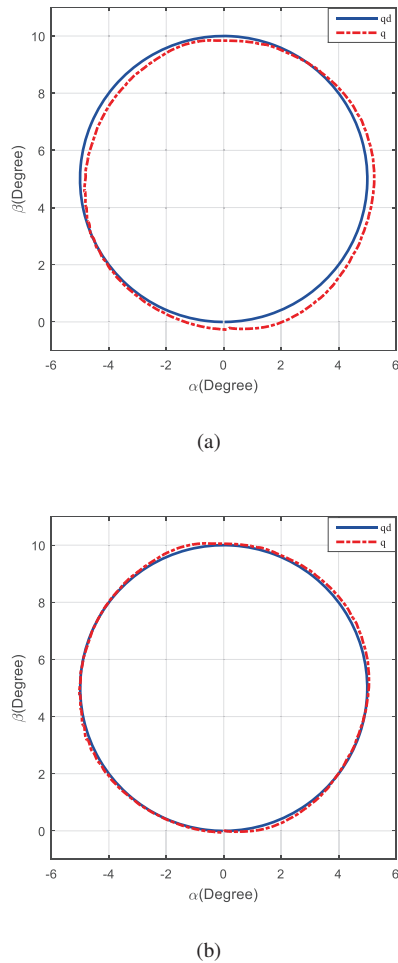


Fig. 6. Tracking performance with the PD control (a) and proposed control (b)

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