For office use only	Team Control Number	For office use only
T1 ________________	**27505**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**A**	F4 ________________

Summary

Direction in traffic rules has long been debated and adapted worldwide.

In the first part, in order to judge the keep-right-except-to-pass rule, we build and analyze four mathematical and mechanical models qualitatively and quantitatively. Firstly, we build the overtaking model, quantifying safety as the safe following distance, so as to examine the tradeoff between traffic flow and safety. We find that the roles of under- and over-posted speed limit are to secure safety and improve traffic flow respectively. Secondly, we establish the rule evaluation function to analyze the right-most rule in light and heavy traffic. Evaluation result shows that there are two different densities for a given performance value between 0.21~0.73.

Then, we build the Cellular Automata & Markov model to evaluate the new rule we come up with, in three-lane freeway. CA simulation results show that the new rule can promote traffic flow and safety more efficiently than the former one. Besides, we set up the Fluid-Mechanics model revealing that viscous resistance has a huge retardation effect on traffic density and speed in both low and high density areas, with the consideration of the impact of overtaking. Furthermore, the Dynamic Disturbance model is established to illustrate that more lanes contribute to relieving traffic jam. It also reveals the importance and practicability of chaotic parameter control.

In the second part, considering left norm countries, we put ourselves in their position and put forward several useful recommendations. Firstly, the new rule can be carried over with series changes of orientation and custom, without considering other factors. Secondly, the vision distributions of left- and right-mounted steering wheel vehicle are different. Thirdly, Coriolis effect suggests that in the Northern Hemisphere, left-mounted steering wheel with vehicle running on right side is optimal and vice versa.

In the last part, we take a deep insight into the intelligent transportation system. Firstly, the intelligent vehicle can generate and track dynamic path during overtaking by means of cubic spline curve fitting. It is the core of intelligent system and direction of future vehicle development, not to mention its military value potentially. Secondly, the intelligent system can get optimal rate of vehicle in different lanes using Genetic Algorithm, which overcomes the uncertainty of diversion relying upon human judgment. Thirdly, intelligent vehicle and system are practical in either left or right norm countries, making traffic rules more flexible.

In conclusion, we sincerely hope that our profound research and insightful paper can give contribution to the improvement of local traffic rule design and global transportation system construction.

1 Introduction

The Geneva Convention on Road Traffic (1949) has been ratified by 95 countries and requires each ratifying country to have a uniform direction of traffic in the country. Article 9(1) provides that:All vehicular traffic proceeding in the same direction on any road shall keep to the same side of the road, which shall be uniform in each country for all roads. Domestic regulations concerning one-way traffic shall not be affected. **Figure1-1.** is the world map showing the driving directions for all countries and any changes that have occurred in the past starting with Finland's change in 1858.

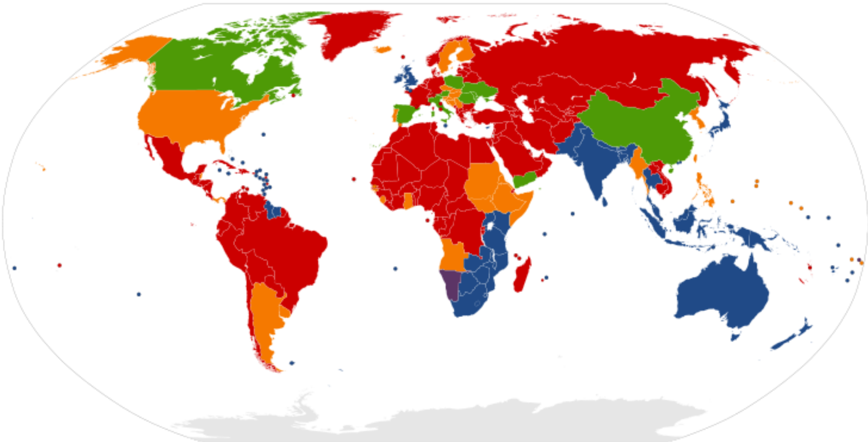


Figure1-1. World map

Has always driven on the right (RHT).
Originally drove on the left, but now drives on the right.
Has always driven on the left (LHT).
Originally drove on the right, but now drives on the left.
Once had different rules of the road (depending on one's location), but now drives on the right.

The right-hand traffic rule is listed in detail in below.

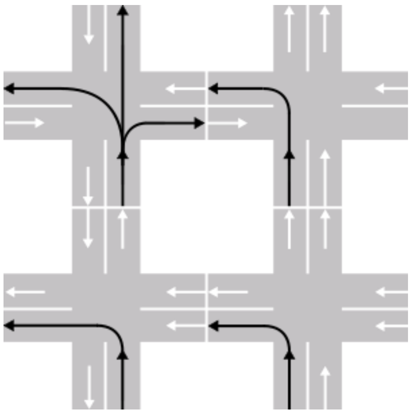


Figure 1-2. Right-hand traffic

- All traffic is generally required to keep right unless overtaking.
- Oncoming traffic is seen coming from the left.
- Left-turning traffic must cross oncoming traffic.
- Most traffic signs facing motorists are on the right side of the road.

- Traffic on roundabouts (traffic circles or rotaries) goes counterclockwise.
- Pedestrians crossing a two-way road look first for traffic from their left.
- The lane designated for normal driving and turning right is on the right.
- Most dual carriageway (divided highway) exits are on the right
- Other vehicles are generally overtaken (passed) on the left, though in some circumstances overtaking on the right is permitted.
- Most vehicles have the driving seat on the left.
- A right turn at a red light may be allowed after stopping.
- On roads without a footpath pedestrians may be advised to walk on the left.

2 Analysis

In countries where driving automobiles on the right is the rule (that is, USA, China and most other countries except for Great Britain, Australia, and some former British colonies), multi-lane freeways often employ **a rule that requires drivers to drive in the right-most lane unless they are passing another vehicle, in which case they move one lane to the left, pass, and return to their former travel lane.** The rule is called **the Keep-Right-Except-To-Pass Rule**.

Most people are right-handed, which leads to a natural tendency to favor one side of the road or another depending on the means of transportation being used. The commonplace driving mode worldwide is left-mounted steering wheel with vehicle running on right side.

Firstly, we establish the performance evaluation function to analyze the performance of this rule in light and heavy traffic. We build and analyze four mathematical models to examine tradeoffs between traffic flow and safety, the role of under-posted and over-posted speed limits. The result shows that this rule is not effective in promoting better traffic flow. Furthermore, we suggest a new rule that can promote greater traffic flow and safety.

Secondly, in countries where driving automobiles on the left is the norm, our solution can be carried over with changes of orientation and custom, without considering other factors. Considering vision distribution, vision distributions of left- and right-mounted steering wheel vehicle are different. If taking consideration of the Coriolis acceleration, the rule would need additional requirements. In the Northern Hemisphere, left-mounted steering wheel with vehicle running on right side is optimal. Nevertheless, in the Southern Hemisphere, right-mounted steering wheel with vehicle running on left side is optimal.

Lastly, the rule as stated above relies upon human judgment for compliance. If vehicle transportation on the same roadway was fully under the control of an intelligent system, which is either part of the road network or imbedded in the design of all vehicles using the roadway, this would change the results of our earlier analysis to a great extent.

The logic of the simulation process is detailed in **Figure 2-1.**

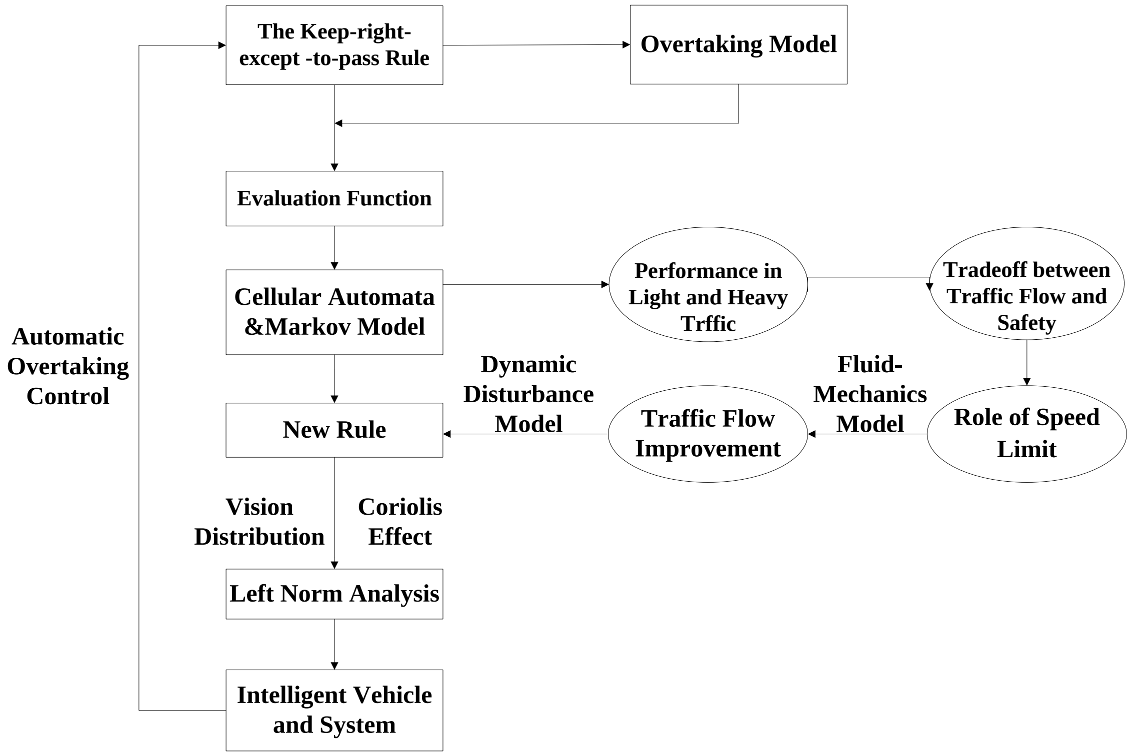


Figure2-1. Simulation of the process of modeling

3 Overtaking Model

3.1 Analysis

Based on former research, taking full consideration of the safe following distance of driving and overtaking, we divide the overtaking process into three stages: changing lane, passing and merging, and add different speed limits to different lanes as shown in **Figure 3-1.** Aimed at overtaking as many vehicles as possible, combined with the safe distance during the process of following driving and the relationship between the passing vehicle and the passed vehicle, the new overtaking model is built. The model provides judgments to decrease the frequency of traffic accident due to inappropriate overtaking action.

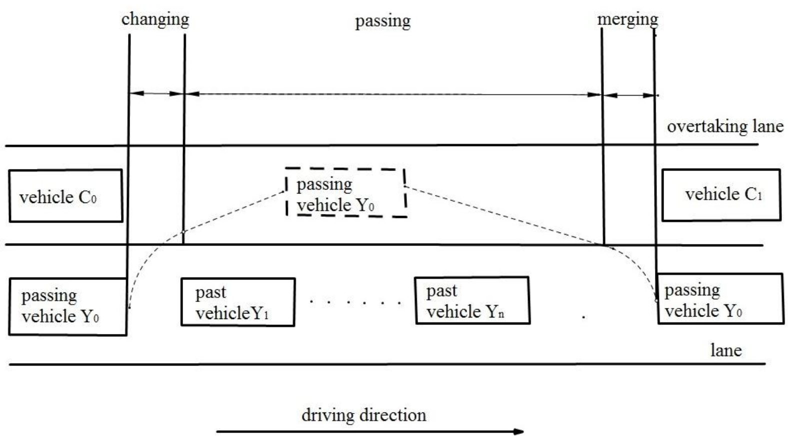


Figure 3-1. Three stages of overtaking process

3.2 Basic Assumptions

- The road is straight, the passing vehicle is in good condition and meets the passing requirements.
- During the overtaking process, no other vehicle is passing at the same time. Following vehicles behind the passing vehicle can all keep safe following distance with it.
- Other vehicles are running normally on the road. After finishing passing, the passing car returns to its former lane.
- The road is anisotropy, which means vehicles do not reverse.

3.3 Safe Following Distance

Two vehicles Y_0, Y_1 are running on the same lane with speed V_{Y_0}, V_{Y_1} and maximum braking acceleration a_{Y_0}, a_{Y_1} at a distance of L. The reaction time of driver and the coordination time of brake is t_r. The time of braking is t_d. The safe distance between two motionless vehicles is L_s.

Two vehicles on the same lane:

$$L_1 = V_{Y_0} t_r + \frac{V_{Y_0}^2}{2a_{Y_0}} + \frac{(V_{Y_0} - V_{Y_1}) t_d}{2} - \frac{V_{Y_0}^2}{2a_{Y_1}} + L_s \qquad \textbf{(3-1)}$$

$$L_2 = V_{Y_0} t_r + \frac{V_{Y_0}^2}{2}\left(\frac{1}{a_{Y_0}} - \frac{1}{a_{Y_1}}\right) + L_s \qquad \textbf{(3-2)}$$

$$L_3 = V_{Y_0}(t_d + t_r) + \frac{2V_{Y_0}V_{Y_1} - 2V_{Y_0}^2 - V_{Y_1}^2}{2a_{Y_1}} + \frac{V_{Y_0}^2}{2a_{Y_0}} - \frac{V_{Y_1} t_d}{2} + L_s \qquad \textbf{(3-3)}$$

The safe following distance: $L = \begin{cases} L_1, V_{Y_1} < V_{Y_0} \\ L_2, V_{Y_1} = V_{Y_0} \\ L_1, V_{Y_1} > V_{Y_0} \end{cases}$ **(3-4)** as shown in **Figure 3-2.**

Two vehicles on adjacent lanes:

The safe following distance on the former lane: $d_s = L + {L_{c_i}}/{2\cos\theta} - {L_{c_i}}/{2}$ **(3-5)**

θ (degree): vehicle’s angle with horizontal.

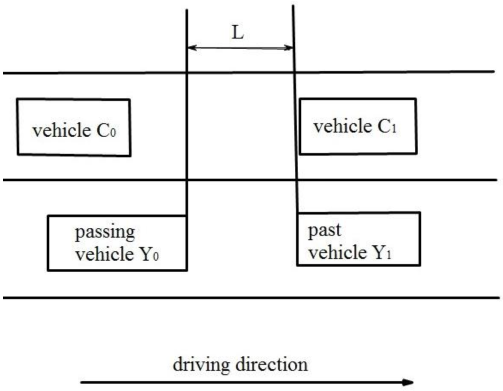
It is also called preventing horn touch distance, as shown in **Figure 3-3.**

The safe following distance on the overtaking lane:

$$d_d = L + 2\left({L_{c_i}}/{2\cos\theta} - {L_{c_i}}/{2}\right) \qquad \textbf{(3-6)}$$

It is also called preventing scratches distance, as shown in **Figure 3-4.**

The running distance on the overtaking lane is d.



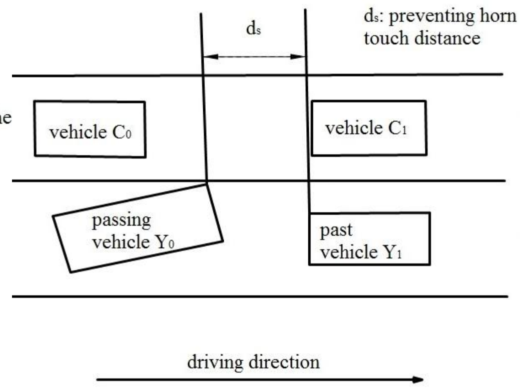


Figure 3-2.normal following distance Figure 3-3.preventing horn touch distance

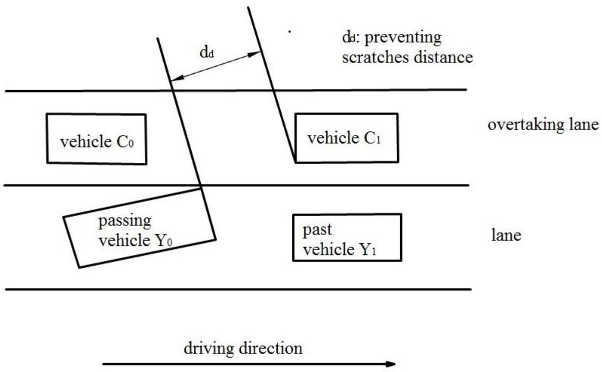


Figure 3-4. preventing scratches distance

3.4 Simulating Computation

Taking simulating calculation according to **(3-1)**~**(3-6)**, with t_r ranging from 0.8~1.0s, t_d ranging from 0.1~0.2s, a_{Y_0} , a_{Y_1} ranging from 6.25~7.95m/s.

The results are shown in **Table 3-1**.

Table 3-1. Safe distance when the vehicle overtaking

Surpassing speed v	60km/h		70km/h		80km/h	
	Following distance(m)	d (m)	Following distance(m)	d (m)	Following distance(m)	d (m)
passing vehicle	d_d =13.76	84.25	d_d =17.03	58.21	d_d =27.51	57.63
passed vehicle	d_s =12.30	/	d_s =12.30	/	d_s =12.30	/

The passing speed is less than the maximum speed limit. We conclude that a higher maximum speed limit requires a larger safe following distance d_d and a larger d **.**

This means that the lower the maximum speed limit is, the safer the vehicles are. The vehicles can run safely with a relatively small following distance.

The small safe following distance corresponds to a light traffic, while the large safe following distance corresponds to a heavy traffic. The result also examines tradeoffs between traffic flow and safety, based on the formula: $Q = vk$.

traffic flow: Q (pcu/h) , traffic speed: v (km/h), traffic flow density k : (pcu/km)
The safe following distance on the former lane can be transferred into traffic flow density through $k=\frac{1}{d_s}$. Therefore, the maximum traffic flow of the road is

$Q_{\max}=\frac{v_{\max}}{d_s}$ **(3-7)** Q_1 =4878.05pcu/km Q_2 =5691.06 pcu/km Q_3 =6504.07 pcu/km

The result $Q_1<Q_2<Q_3$ reveals that the safer the road is, the smaller the traffic flow is.

- The role of over-posted speed limit is to control the traffic flow and secure the road safety.
- The role of under-posted speed limit is to increase the traffic speed and promote the traffic flow.

4 Rule Evaluation Function

4.1 Traffic Speed-Flow Model

The speed-flow model structure under different traffic load conditions is reviewed by analyzing the traffic flow dissipation mechanism under peak hour over saturated traffic condition. A general speed-flow model under any traffic load conditions is established through curve fitting of large number of observed data of Beijing-Shanghai highway. The practical model parameters for each highway class under different design speed are also put forward. This model successfully solves the speed-forecasting problem of the traffic flow under peak hour over saturated condition.

$$\begin{cases} v=\dfrac{\alpha_1 v_s}{1+(Q/C)^{\beta}} \\ \beta=\alpha_2+\alpha_3(Q/C)^3 \end{cases} \quad \textbf{(4-1)}$$

where v_s is the design speed, C is the traffic capacity, α_1 , α_2 , α_3 are regression parameters.

4.2 Evaluation Function

Take safe following distance, traffic flow and traffic speed as three indexes to evaluate the performance of the right-most rule. Set weight1/2,1/3,1/6 to three indexes to show their different importance. The evaluation function after normalization is

$$Y=\frac{1}{2}\frac{d}{d_s}+\frac{1}{3}\frac{Q}{C}+\frac{1}{6}\frac{v}{v_s} \quad \textbf{(4-2)}$$

We choose v_s =80km/h, then we get C =2000pcu/h, α_1 =1, α_2 ==1.88, α_3 =4.90 looking up to the parameter table and d_s =12.30 from **Table 3-1**. From above, $d=\frac{1}{k}=\frac{v}{Q}$, so we get Y as a function of Q . The curve is shown in **Figure 4-1**.(the value of traffic flow in the figure has to be transferred to get its real value)

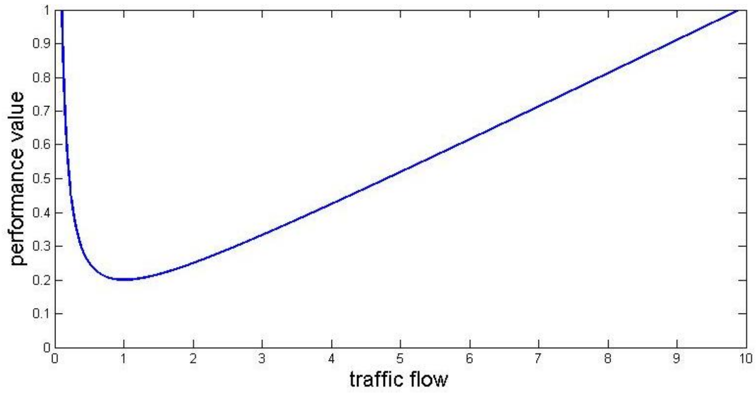


Figure 4-1.

4.3 Evaluation Result

Proceed from reality, we set the speed range 20~140km/h, and get traffic flow range 400~2600pcu/h. Therefore, the minimum performance value Y =0.23 when Q=1200pcu/h and the maximum performance value Y =0.78 when Q=400pcu/h.
The performance value varies with traffic density k as shown in **Table 4-1.**

Table 4-1.

k	2.86	3.28	4.33	5.46	7.52	9.67	11.03	12.11	13
Y	0.78	0.69	0.21	0.26	0.33	0.42	0.51	0.69	0.73

Therefore, the performances of this rule in light and heavy traffic are different. The minimum Y = 0.21 when k =4.33. There will be two different densities for a given performance value between 0.21~0.73.

4.4 Disadvantage of the Right-most Rule

If the freeway consists of only two lanes, the right-most rule is effective enough in promoting better traffic flow. Nevertheless, in reality, the freeway always consists of multi-lanes. Therefore, the right-most rule would be ineffective, taking no use of the rest lanes. Based on the actual situation of highways in China, we suggest a new rule which could promote greater traffic flow.
The new rule is embedded in the three-lane freeway. Lane 1 is the overtaking lane, lane 2 and lane 3 are the normal driving lanes identically. (numbered from left to right)

5 Cellular Automata &Markov Model

5.1 Cellular Automata Model

To evaluate the rule mentioned in the question, we established the Cellular Automata model to make the comparison between the former rule and the new rule which we have come up with. We suppose that the traffic road consists of three lanes, marked by first, second and third from left to right. Every lane is regarded as the discrete lattice chain in one dimension and every lattice grid point could only be taken up by one vehicle. According to the former rule, the speed limit in each lane is same. In order to analyze the rule numerically, the overtaking part should be divided into two steps: normal driving and lane changing.
In the first step, the speed and location can be renewed by the following algorithm:

1) Speed: regulated by the current speed and available space ahead.

$$if\,(V_j^{(i)}(t) \geq gap_j^{(i)}(t)),\, V_j^{(i)}(t+1) = \begin{cases} \mathrm{gap}_j^{(i)}(t)-1, & p \\ gap_j^{(i)}(t), & 1-p \end{cases}$$

$$else\ if\,(V_j^{(i)}(t) < V_{\max}),\, V_j^{(i)}(t+1) = \begin{cases} V_j^{(i)}(t), & p \\ V_j^{(i)}(t)+1, & 1-p \end{cases}$$

$$else\ \ (V_j^{(i)}(t) = V_{\max}),\, V_j^{(i)}(t+1) = \begin{cases} V\max-1, & p \\ V\max, & 1-p \end{cases}$$

2) Location: $X_j^{(i)}(t+1) = X_j^{(i)}(t) + V_j^{(i)}(t+1)$,where $V_j^{(i)}(t)$ is the speed of the ith vehicle on the jth lane, $X_j^{(i)}(t)$ is the location of the ith vehicle on the jth lane and $gap_j^{(i)}(t) = X_j^{(i+1)}(t) - X_j^{(i)}(t)$, p is the deceleration probability of vehicle.

In the second step, drivers are supposed to change their lanes following the safety and overtaking principles:

For the rule mentioned in the question:

1) The rule of turning right in the first lane: with all the requirements and inequalities in the **Figure5-1.** met, the vehicle should immediately alter its current lane to the second one in order to achieve the CA simulation of a normal overtaking action.

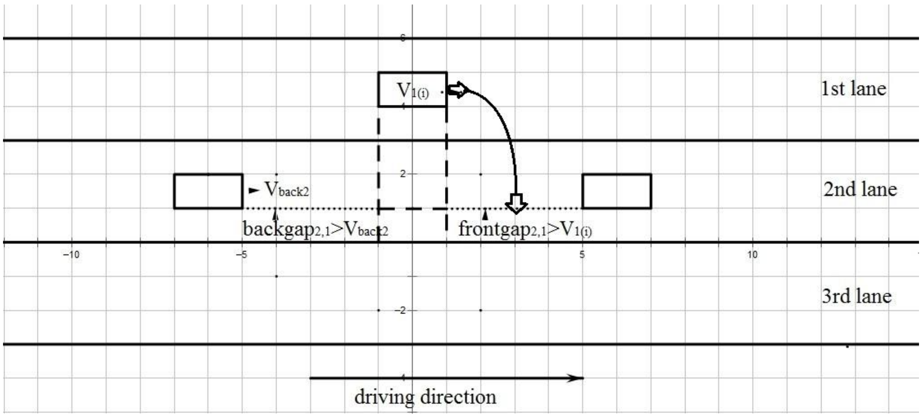


Figure5-1.

2) The rule of turning left in the second lane: All the requirements and inequalities in the **Figure5-2.** should be met in order to achieve the CA simulation of a normal overtaking action.

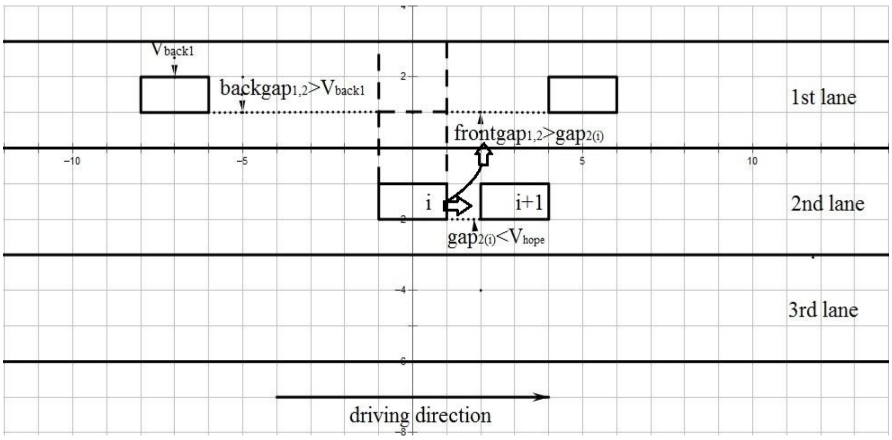


Figure5-2.

3) The rule of turning right in the second lane: with all the requirements and inequalities in the **Figure5-3**. met, the vehicle should immediately alter its current lane to the third one to achieve the CA simulation of a normal overtaking action.

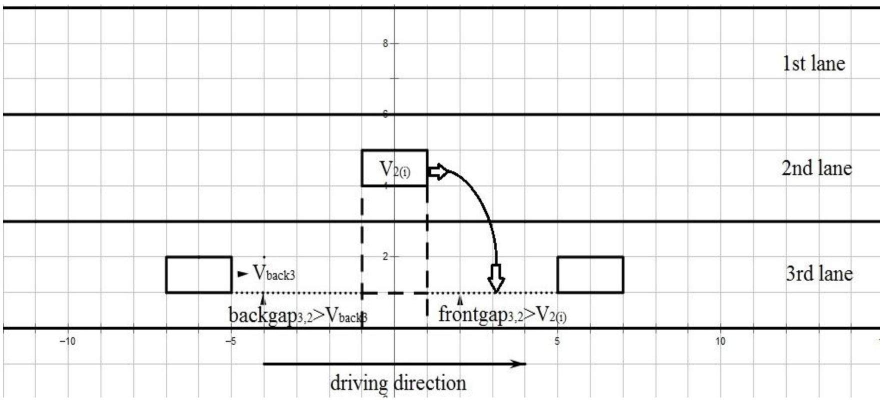


Figure5-3.

4) The rule of turning left in the third lane(right-most): All the requirements and inequalities in the **Figure5-4.** should be met in order to achieve the CA simulation of a normal overtaking action.

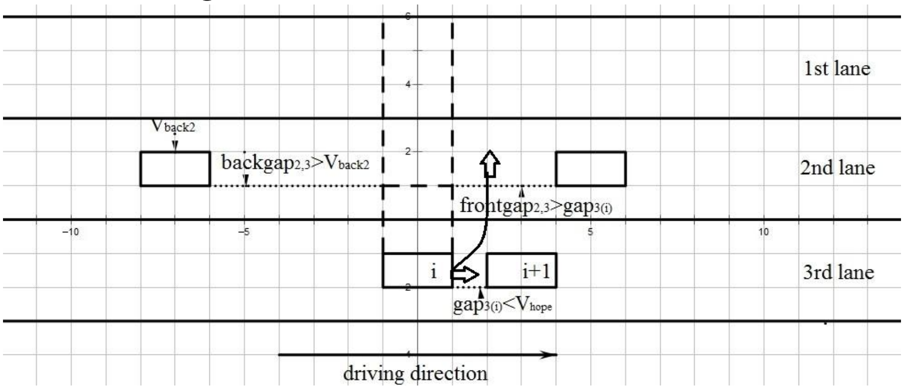


Figure5-4.

For the new rule:

The principles are almost the same as the points mentioned above but two: first of all, when the vehicle is in the second lane and there is no other vehicle in front of it, the driver would be inclined to go straight (with probability p_{22}) or turn left(with probability p_{23})$\left(p_{22}+p_{23}=1\right)$(**Figure5-5.**). In addition, the same principle is applied to the vehicle in the third lane with probability p_{32} and $p_{33}\left(p_{32}+p_{33}=1\right)$.

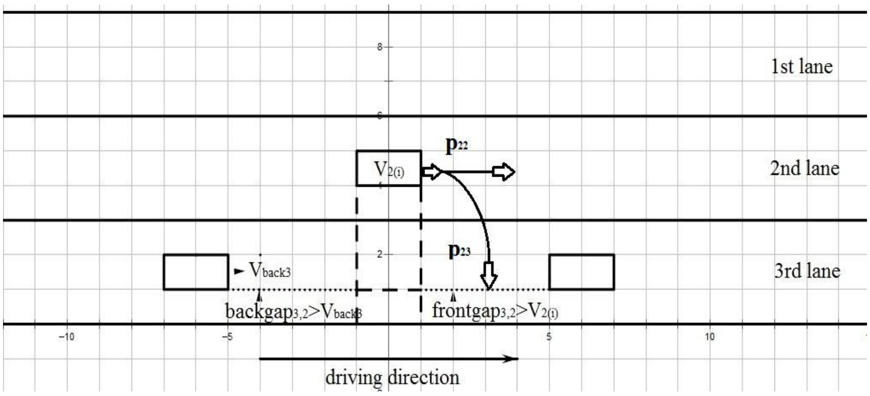


Figure5-5.

In order to get more precise results from the simulation, we define the next variables:

vehicle density: $\rho(t)=\dfrac{N(t)}{3L}$, total average speed: $\bar{V}=\dfrac{\sum_{J=1}^{3}\sum_{i=1}^{N_j(t)}V_j^{(i)}(t)}{N(t)}$

flow: $f(t)=\rho(t)\bar{V}$, total amount of vehicle $N(t)=\sum_{j=1}^{3}N_j(t)$, utilization rate: $\dfrac{N_j(t)}{N(t)}$

We assume that every lattice grid in this model has the length of 10 meter in reality and the gap of time ($\triangle t$) is nearly one second. The vehicle could pass almost three lattices at most for every time step ($V_{\max}\approx 3$). In this way, the maximum speed of the vehicle is 30 m/s i.e. 108 km/h. From the experience and statistic data, we could learn that 80 km/h meets the expectation of most of the drivers in freeway, which means $V_{ex}\approx 2.2$. In the simulation, we choose one thousand lattices to create a random part of freeway, whose length is 10km.

Based on the former rule, most of the drivers are required to choose the right-most lane(3rd lane in this model). Under this statement, we set up the initial densities and transition probabilities of each lane as follow: $\rho_{3init}:\rho_{2init}:\rho_{1init}=0.1:0.3:0.7$ and

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}=\begin{pmatrix} 0.1 & 0.1 & 0 \\ 0.9 & 0.2 & 0.2 \\ 0 & 0.7 & 0.8 \end{pmatrix}$$ from the statistic data.

On the other hand, as far as the new rule is concerned, owing to the two main lanes, the initial densities and transition probability matrix should be regulated as a reasonable assumption: $\rho_{3init}:\rho_{2init}:\rho_{1init}=0.2:0.4:0.4$ and

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}=\begin{pmatrix} 0.1 & 0.2 & 0 \\ 0.9 & 0.4 & 0.5 \\ 0 & 0.4 & 0.5 \end{pmatrix}.$$

Notice: every sample has been simulated for ten thousand periods and we choose the one thousand periods at random. Then we process the data by averaging them.

The results of CA simulation by Matlab are represented:

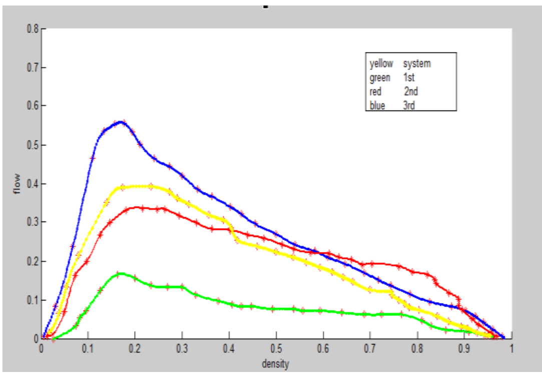


Figure 5-6.1 former rule(density-flow)

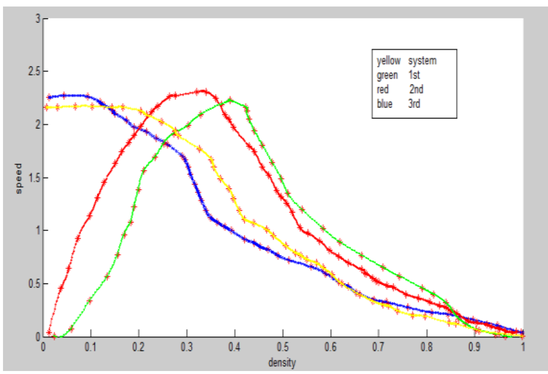
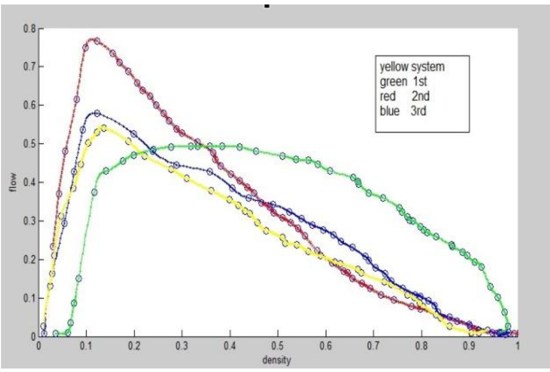


Figure5-6.2 former rule(density-speed)



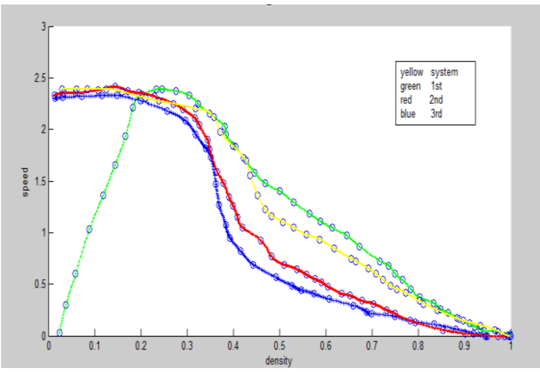


Figure 5-7.1 new rule (density-flow) **Figure5-7.2 new rule (density-speed)**

From the pictures, we could draw the conclusion that under the light traffic circumstance, the new rule presents a more even traffic flow for three lanes leading to a greater total amount of the freeway flow. As for the heavy traffic, the flow in every lane has been enhanced especially for the first lane. In this way, the utilization of this lane is also improved, which could prevent some traffic accidents from happening, and the security index of the freeway is enhanced correspondently.

5.4 Markov Model

With regard to the probability of lane changing, an appropriate Markov model could be established to serve as a judgment for the evaluation of diverse rules. Firstly, these three lanes can be viewed as three states, which constitute the following set (the set is aperiodic and irreducible obviously):

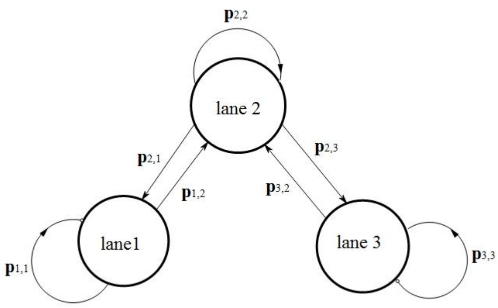


Figure 5-8.

Secondly, we find that diverse combination of $p_{i,j}$ stands for the corresponding traffic rules. Having analyzed the data collected from the freeway under this rule, we conclude the transition matrix.

The former rule:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.2 & 0 \\ 0.8 & 0.2 & 0.2 \\ 0 & 0.6 & 0.8 \end{pmatrix}$$

$$\rho_1 = 0.2\rho_1 + 0.8\rho_2 + 0\rho_3$$

$$\rho_2 = 0.2\rho_1 + 0.2\rho_2 + 0.6\rho_3$$

$$\rho_3 = 0\rho_1 + 0.2\rho_2 + 0.8\rho_3$$

$$\rho_1 + \rho_2 + \rho_3 = 1$$

The new rule:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.4 & 0 \\ 0.9 & 0.3 & 0.5 \\ 0 & 0.3 & 0.5 \end{pmatrix}$$

$$\rho_1 = 0.1\rho_1 + 0.9\rho_2 + 0\rho_3$$

$$\rho_2 = 0.4\rho_1 + 0.3\rho_2 + 0.3\rho_3$$

$$\rho_3 = 0\rho_1 + 0.5\rho_2 + 0.5\rho_3$$

$$\rho_1 + \rho_2 + \rho_3 = 1$$

After simplifying and caltulating by Matlab, we get the results as follow:

```
>>  a=[-0.8,0.2,0;0.8,-0.8,0.2;1,1,1];
b=[0;0;1];
x=linsolve(a,b)

x =

    0.0588
    0.2353
    0.7059
```

```
>> a=[-0.9,0.4,0;0.9,-0.7,0.5;1,1,1];
b=[0;0;1];
x=linsolve(a,b)

x =

    0.2174
    0.4891
    0.2935
```

From the results we could learn that, comparing relative density of each lane, the new rule allow the first and second lane to play a more significant role in alleviating the traffic pressure, enhancing the overall traffic flow consequently. What's more, in the light of the advantages of Markov chain model, which indicates the various rules could be turned into diverse combinations of transition probabilities, we could even figure out the most optimal rule for every distinct region.
In a nutshell, new rule could promote the traffic flow more efficiently than the former rule.

5.5 New Rule Expansion

The above three-lane freeway can be expanded to four-lane freeway used in China widely. Lane 1 is the overtaking lane, lane 2 and lane 3 are the normal driving lanes, lane 4 is the slow lane. Normally, small cars always run on lane 2, and buses and trucks always run on lane 3. Regularly, there is no vehicle on lane 4. If a vehicle breaks down, it moves to lane 4 for repair. Or, if a traffic accident happens, the accident vehicles move to lane 4 to deal with the accident.

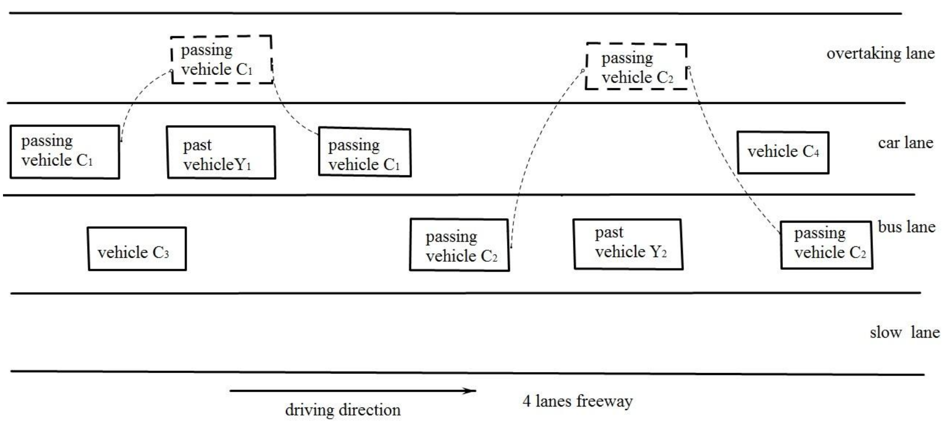


Figure 5-9. New rule on four-lane freeway

5.6 Advantage of New Rule

- New rule makes use of more lanes than the right-most rule on four-lane freeway (3>2), so it is more effective in promoting better traffic flow.
- New rule stipulates an overtaking lane, where the traffic density is relatively low. Therefore, the probability of accident during overtaking is largely reduced.
- New rule allows a slow lane for vehicle repairing and accident handling, thus improves the traffic safety to a great extent.

6 Fluid-Mechanics Model

6.1 Hydrodynamics Theories

Fluid mechanics is the branch of physics that studies fluids and the forces on them. Fluid mechanics can be divided into fluid statics, the study of fluids at rest; fluid kinematics, the study of fluids in motion; and fluid dynamics, the study of the effect of forces on fluid motion. It is a branch of continuum mechanics, a subject which models matter without using the information that it is made out of atoms, that is, it models matter from a macroscopic viewpoint rather than from a microscopic viewpoint. Fluid mechanics, especially fluid dynamics, is an active field of research with many unsolved or partly solved problems. Fluid mechanics can be mathematically complex, and can be best solved by numerical methods, typically using computers. A modern discipline, called computational fluid dynamics (CFD), is devoted to this approach to solving fluid mechanics problems.

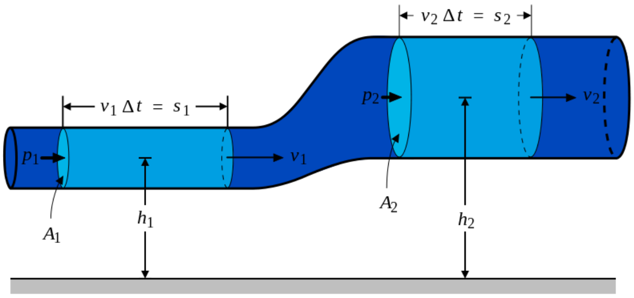


Figure 6-1. Fluid mechanics

6.2 Viscous Resistance

Viscous resistance comes from the mixture of vehicles and overtaking. The change along the direction of traffic flow is defined as on-way friction. The change due to overtaking partly is defined as local friction. So viscous resistance is defined as

$$\tau_w = \left[\mu(u-u_f)+n\right]\frac{\partial k}{\partial x} \quad \textbf{(6-1)}$$

Substitute into the acceleration formula.

$$a=\frac{du}{dt}=\frac{du(x,t)}{dt}=\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}\bullet\frac{dx}{dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x} \quad \textbf{(6-2)}$$

$$\frac{du}{dt}=\frac{1}{T}\left[u_e(k)-u(x,t)\right]+\tau_w \quad \textbf{(6-3)}$$

We get the final mechanical model, including a continuous function.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\frac{1}{T}\left[u_e(k)-u\right]+\left[\mu(u-u_f)+n\right]\frac{\partial k}{\partial x} \quad \textbf{(6-4)}$$

$$\frac{\partial k}{\partial t}+\frac{\partial q}{\partial x}=\frac{dr}{dx} \quad \textbf{(6-5)}$$

6.3 Simulation Result and Discussion

Simulate the traffic flow of a 6km highway in 25minutes, the results are shown in **Table 6-1.**

Table 6-1. Simulation result

	Density(pcu/km)	Speed(km/h)	Traffic flow(pcu/h)
Initial value	52.10	45.10	2339.00
Stable value	25.40	60.10	1551.00
Ratio	0.50	1.33	0.66

The result shows that in low density area, overtaking has a small influence on traffic speed. While in high density area, overtaking has a huge influence on traffic speed. Considering the impact of overtaking, viscous resistance has a huge retardation effect on traffic density and speed in both low and high density areas. But its effect is not obvious in high speed area (with free traffic flow). Taking the impact of mixed traffic flow and overtaking into consideration, the variation trend of three elements of traffic flow flattens out.

7 Traffic Flow Dynamic Disturbance Model

7.1 Analysis

Selecting data from traffic videos, we build new equations of continuity, together with momentum equations to form an integrated macroscopic traffic flow model.

Considering a one-direction four-lane road with total length L, with unimpeded velocity $u_f = 88.5$ km/h and jam density $\rho_j = 143$ pcu/km. Assuming that the original smooth traffic flow is disturbed, the traffic flow density varies partly. We will discuss the regularity of the development and evolution of the disturbance wave.

7.2 Discrete Method

Considering the merging term, when the traffic density of the lane is smaller than adjacent lane, there will be vehicles merging in. Thus, we extend the model from single lane road to four-lane road. According to the principle of the uniformity of dimension, we deduce the emerging function.

$$S_{1,j}^{n} \bullet \Delta t = \begin{Bmatrix} -k_1 \rho_{2,j}^{n}, & \rho_{1,j}^{n} > 0.55(\rho_{1,j}^{n} + \rho_{2,j}^{n}) \\ k_2 \rho_{1,j}^{n}, & \rho_{1,j}^{n} < 0.45(\rho_{1,j}^{n} + \rho_{2,j}^{n}) \\ 0, & otherwise \end{Bmatrix} \quad \text{(7-1)}$$

$$S_{3,j}^{n}\bullet\Delta t=\begin{cases}-k_3\rho_{2,j}^{n}, & \rho_{3,j}^{n}>0.55(\rho_{3,j}^{n}+\rho_{2,j}^{n})\\ k_4\rho_{3,j}^{n}, & \rho_{3,j}^{n}<0.45(\rho_{3,j}^{n}+\rho_{2,j}^{n})\\ 0, & otherwise\end{cases} \quad \text{(7-2)}$$

$$S_{2,j}^{n}\bullet\Delta t=-(S_{1,j}^{n}\bullet\Delta t+S_{3,j}^{n}\bullet\Delta t) \quad \text{(7-3)}$$

$k_1 \sim k_4$ are related with the traffic flow density. From traffic videos taken from several typical roads of Beijing and Shanghai, we analyze a large amount of data and divide the traffic flow into four types.

- Extremely low density ($0\le\rho<0.1$, ρ is the ratio of real density and jam density) There are less than 14 vehicles per km. Vehicles seldom change lanes, so $k_1 \sim k_4$ are all zero.
- Low density ($0.1\le\rho<0.3$) There are 14~43 vehicles per km. The probability of lane changing is 6% on average. Referring to references, we get $k_i=\lambda_i\frac{\Delta x}{240}(i=1,\cdots,4)$, $\lambda_1 \sim \lambda_4$ values 0.012,0.048,0.048,0.012 respectively.
- High density ($0.3\le\rho\le0.5$) There are 43~72 vehicles per km. The probability of lane changing is 3.6% on average. Referring to references, we get $k_i=\lambda_i\frac{\Delta x}{240}(i=1,\cdots,4)$, $\lambda_1 \sim \lambda_4$ values 0.008,0.028,0.028,0.008 respectively.
- Extremely high density ($0.5<\rho\le1$) There are more than 72 vehicles per km. Although drivers want to change lanes eagerly, they can hardly do so due to the extremely small distance between vehicles. So vehicles seldom change lanes, $k_1 \sim k_4$ are all valued zero approximately.

7.3 Equilibrium Function

Different equilibrium functions have different influence to the disturbance. So we consider the following three equilibrium functions.

Payne equilibrium function:

$$U_{e,i}=\min\left\{u_f,u_f\left[1.94-6\frac{\rho_i}{\rho_j}+8\frac{\rho_i^2}{\rho_j^2}-3.93\frac{\rho_i^3}{\rho_j^3}\right]\right\}, i=1,2,3,4 \quad \text{(7-4)}$$

Lee equilibrium function: $$U_{e,i}=u_f\frac{1-\rho_i/\rho_j}{1+100\,\rho_i^4/\rho_j^4}, i=1,2,3,4 \quad \text{(7-5)}$$

KK equilibrium function: $$U_{e,i}=u_f\left(\frac{1}{1+e^{\frac{\rho_i/\rho_j-0.25}{0.06}}}-0.00000372\right), i=1,2,3,4 \quad \text{(7-6)}$$

7.4 Initial and Boundary Conditions

According to the disturbance form, we set the initial condition as the following disturbance function.

$$\rho_i\left(x,0\right)=\begin{cases}\rho_{0,i}\left[1-\beta\sin\frac{\pi(x-x_0)}{l_0}\right], x_0-l_0\le x\le x_0 \\ \rho_{0,i}\left[1+\frac{1}{2}\beta\sin\frac{\pi(x-x_0)}{l_0}\right], \begin{matrix} x_0<x\le x_0+2l_0\rho_{0,i}, \\ 0<x<x_0-l_0 or\ x_0+2l_0<x<L\end{matrix}\end{cases} \quad \textbf{(7-7)}$$

$\rho_{0,i}$ is the original uniform traffic flow density, β is the disturbance strength. We suppose that disturbance only appears in lane 2 and 3, so in lane 1and 4 $\rho_i\left(x,0\right)=\rho_{0,i}$. The initial speed and traffic flow density satisfy Greenshields Model.

$$u_i(x,0)=u_f\left[1-\frac{\rho_i\left(x,0\right)}{\rho_j}\right], i=1,2,3,4 \quad \textbf{(7-8)}$$

In order to stimulate longer roads than reality, we adopt cyclic boundary condition on the one side, and homogeneous Neumann condition on the other. When the disturbance wave spreads down, we get

$$\begin{cases}\frac{\partial\rho_i}{\partial x}(1,t)=\frac{\partial u_i}{\partial x}(1,t)=0 \\ \rho_i\left(0,t\right)=\rho_i\left(1,t\right) \quad ,i=1,2,3,4\left(t>0\right) \\ u_i(0,t)=u_i(1,t)\end{cases} \quad \textbf{(7-9)}$$

When the disturbance wave spreads up we get

$$\begin{cases}\frac{\partial\rho_i}{\partial x}(1,t)=\frac{\partial u_i}{\partial x}(1,t)=0 \\ \rho_i\left(0,t\right)=\rho_i\left(1,t\right) \quad ,i=1,2,3,4\left(t>0\right) \\ u_i(0,t)=u_i(1,t)\end{cases} \quad \textbf{(7-10)}$$

Because the solution conditions are either Dirichlet condition or Neumann condition, which are easy to discrete, we omit the discrete forms.

7.5 Result Discussion

Under low and high traffic density, we calculate using different equilibrium functions. The dimensionless time step is $\Delta t=2\times10^{-4}$ and space step is $\Delta x=0.00233$, which equals to actual time step $\Delta t=4.3s$ and space step $\Delta x=35m$. The vehicle changes lane at x =2, and the changes of disturbance factor β with x are shown in **Figure 7-1.**

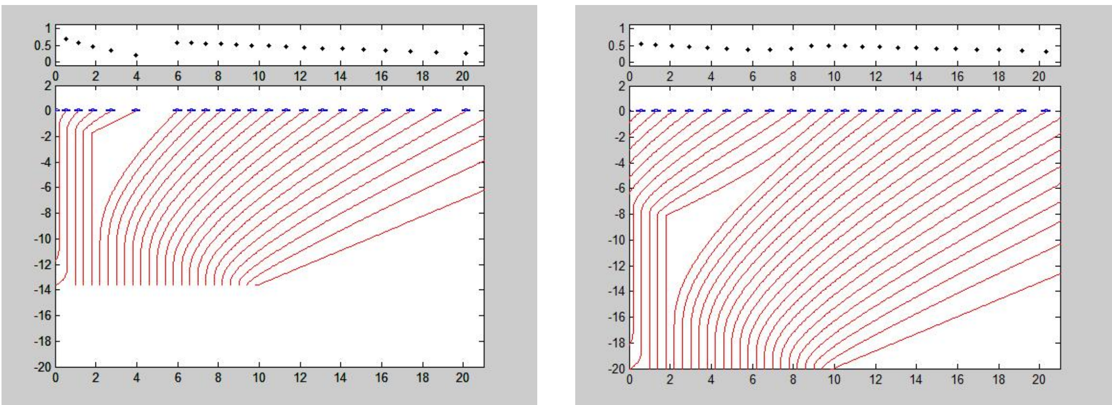


Figure 7-1.

Therefore, we get the following conclusions.

- Under low and high traffic density, the speeds of disturbance wave are nearly the same in four-lane and single-lane roads. But the amplitude during propagation is more easily decreased in four-lane roads, which illustrates that more lanes contribute to relieving traffic jam.
- In simulating traffic flow with uneven left-passing and right-passing probability, the new emerging function model has superior advantage.
- The bigger the traffic flow is, the larger the disturbance is. The larger the disturbance is, the easier traffic congestion appears.
- With the increase of traffic density, the chaos appears. The model reveals the importance and practicability of chaotic parameter control.

8 Left Norm Analysis

8.1 Change of Orientation and Custom

In countries where driving automobiles on the left is the norm, our solution can be carried out with series changes of orientation and custom, without considering other factors.

- Vehicle running on left side should be set with right-mounted steering wheel so as to make drivers see more clearly when overtaking.
- Of the four-lane freeway, lane 4 is the surpassing lane, lane 2 and lane 3 are the normal driving lanes, lane 1 is the slow lane. Normally, small cars always run on lane 3, and buses and trucks always run on lane 2. (numbered from left to right)
- The transportation infrastructure should be built on the right side of drivers so as to be seen more clearly.
- In England, drivers normally use left hand to shift gears and right hand to handle the steering wheel, with left-hand left-foot braking and right-foot accelerating. However, they have to change their habit just to the opposite under right-hand rule.
- In England, pedestrians crossing a two-way road look first for traffic from their right. However, they have to change their habit looking left first.

The change of orientation is for the sake of safety, thus reduces the probability of traffic accidents. Nevertheless, the change of custom may be quite difficult for people, especially seniors.

8.2 Vision Distribution

The vision distributions of left-mounted steering wheel vehicle and right-mounted steering wheel vehicle with a speed of 70km/h are shown in Figure 8-1.
(1:direct vision area with both eyes, 2:direct vision area with single eye, 3:blind area, 4:glancing area with both eyes, 5:indirect back vision area, 6:indirect side vision with both eyes)

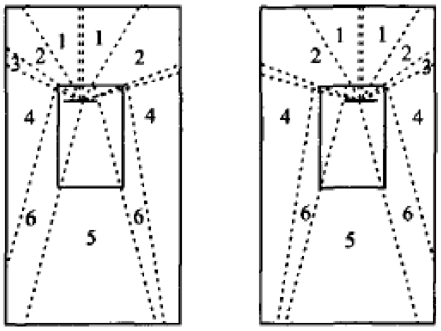


Figure 8-1.

After changing to right running rule, left-mounted steering wheel vehicle has a better left-side vision, which is favorable to overtaking. Nevertheless right-mounted steering wheel vehicle has a better right-side vision, which is favorable to watching the non-motor vehicles and pedestrians and avoiding traffic accidents.

8.3 Coriolis Effect

The earth rotates from west to east, so for objects moving along the surface of the earth, the motion of the earth is carrier motion. When carrier motion is rotation with a fixed axis, the instantaneous absolute acceleration is $a_a = a_e + a_r + a_c$, where a_e, a_r, a_c are carrier acceleration, relative acceleration and Coriolis acceleration.
The moving object in the Northern Hemisphere has a tendency of turning right. To sustain vehicle's normal running, the ground must provide a friction to balance the effect of Coriolis acceleration as shown in **Figure 8-2**. The action points of friction and Coriolis acceleration have a distance, which cause a torque making the vehicle has a tendency of overturning to the right.

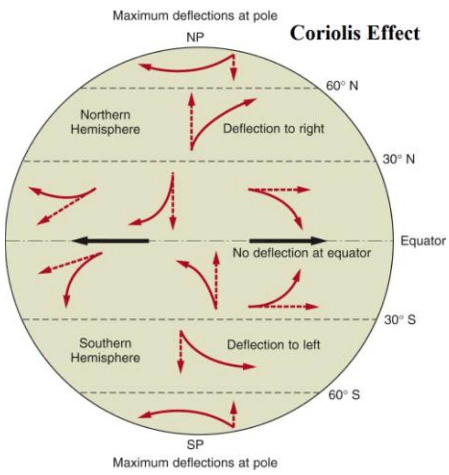


Figure 8-2.

Take the Beijing-Guangzhou highway as an example, the vehicle running from north to south. If the ground friction is too small to sustain the Coriolis acceleration of normal running, the vehicle will turning right instead of turning left and affecting the normal running of the left side vehicles, thus avoids traffic accidents as shown in **Figure 8-3**.

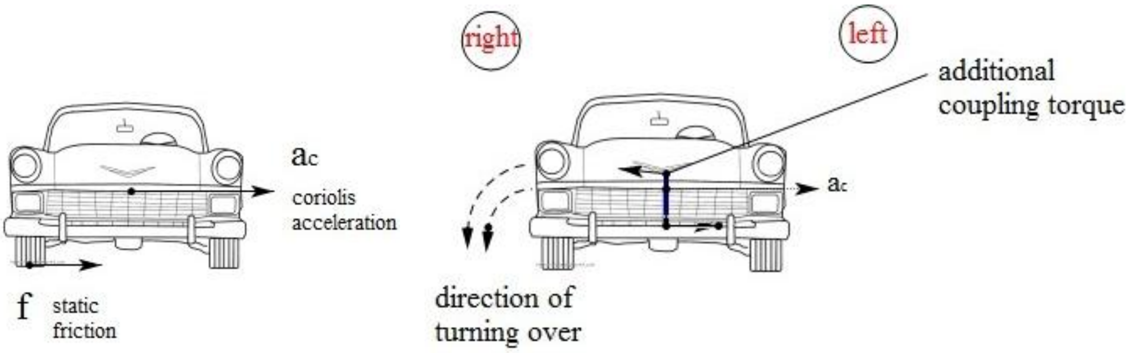


Figure 8-3.

According to the formula of Coriolis effect, $a_c = 2v_{veh} \cdot \omega_{ear} \cdot \sin\varphi$ where v_{veh} is the speed of vehicle, ω_{ear} is the angular speed of the earth and φ is the latitude of the freeway. Take England as an example, if the speed is $v_{veh} = 150km/h \approx 41.6667m/s$,then $\omega_{ear} = \frac{2\pi}{24\times3600} \approx 7.2722\times10^{-5} rad/s$ and the latitude of England is 60 degrees. In this way, $a_c = 5.2487\times10^{-3} m/s^2$. Under some extreme weather, like snowstorm, the friction on road would be reduced to large extent. For the region located in the Northern Hemisphere, overtaking from left is superior to right because the latter may lead to a larger acceleration to right and vice versa. Consequently, we could avoid those accidents caused by the inexperienced drivers or over-speed issues. We should consider Coriolis effect to potential technological improvement of speed in the future.

In conclusion, taking consideration of the Coriolis acceleration, the rule would need additional requirements for the sake of safety. In the Northern Hemisphere, left-mounted steering wheel with vehicle running on right side is optimal. Nevertheless, in the Southern Hemisphere, right-mounted steering wheel with vehicle running on left side is optimal.

9 Intelligent Vehicle

9.1 Autonomous Overtaking Control

In order to simulate driver's behavior truly, focusing on intelligent vehicle autonomous overtaking, a dynamic target position concept is proposed and cubic spline curves as path fitting curves of car is employed. An intelligent vehicle controller is designed by using the fuzzy logic as a control strategy, the T-S fuzzy model as its structure and an adaptive neural network as a means of adjusting membership function.

9.2 Dynamic Path Generation and Tracking

Simplified four-wheel kinetic model: $\begin{cases} x(k+1) = x(k) + v\Delta s\cos\left[\theta(k)\right] \\ y(k+1) = y(k) + v\Delta s\sin\left[\theta(k)\right] \\ \theta(k+1) = \theta(k) + v\Delta s\tan\alpha(k)/l \end{cases}$ **(9-1)**

Lane changing path is defined as the cubic polynomial:

$y = ax^3 + bx^2$ **(9-2)** $a = (x_t\tan\theta_t - 2y_t)/x_t^3, b = (3y_t - x_t\tan\theta_t)/x_t^2$

Calculate the steering angle when the controlled vehicle moving to the dynamic target location.

Differential equation: $\dot{x} = v\cos\theta, \dot{y} = v\sin\theta, \dot{\theta} = v\dfrac{\tan\alpha}{l}$ **(9-3)**

$\therefore \tan\theta = y' = 3ax^2 + 2bx$, $\sec^2\theta\dot{\theta} = 6ax\dot{x} + 2b\dot{x} = \dot{x}(6ax+2b)$ **(9-4)**,

$\therefore \sec^2\theta\dfrac{\tan\alpha}{\cos\theta} = (6ax+2b)l$ **(9-5)**

$\because x = 0, \theta = 0 \therefore \tan\alpha = 2bl, \alpha_t = \arctan(2bl)$ **(9-6)**

9.3 Simulation Result

Simulate the overtaking process of a vehicle, with all vehicles in uniform motion with speed of 15m/s. The changes of x (m), y (m),vehicle's angle with horizontal θ (degree) are shown in **Figure 9-1.**

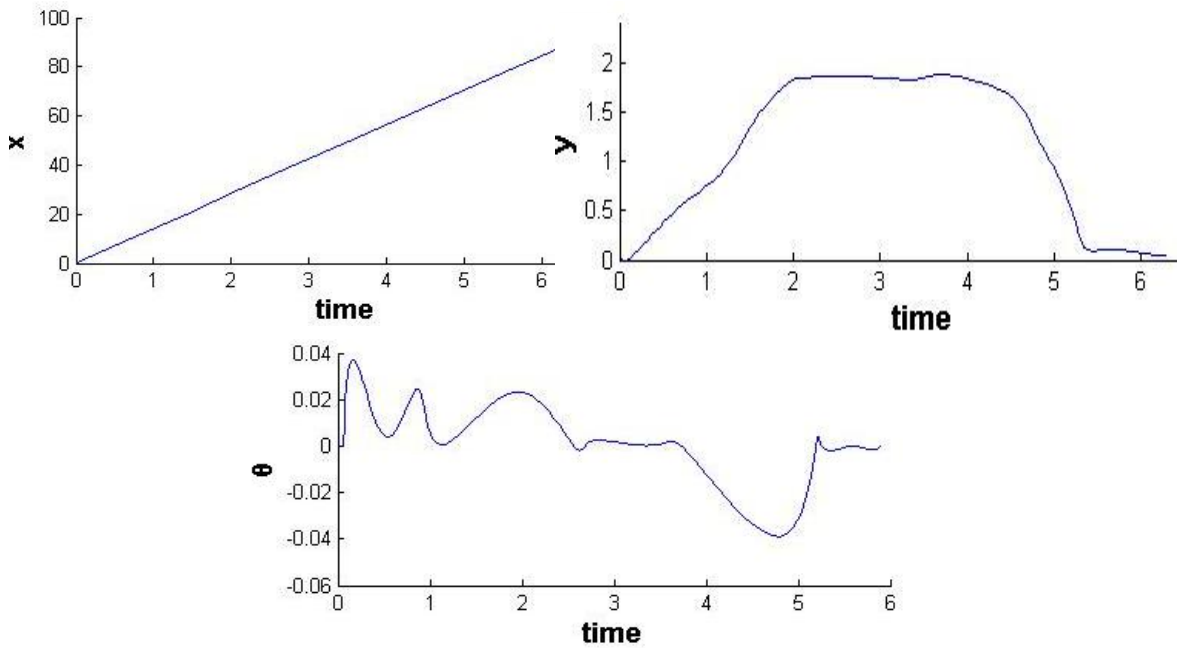


Figure 9-1.

Simulation result based on Simulink shows that the research is correct and feasible, and can simulate overtaking characteristic of vehicle in actual traffic situation well.

9.4 Strength of Intelligent Vehicle

- Autonomous overtaking control overcomes the inflexibility of magnetic marker guidance.
- Besides, it overcomes the drawback of inaccuracy of data collection from controller and the fixation of control rule.

- Simulation result shows that the controlled vehicle can change lane and overtake smoothly, thus the model can simulate the lateral motion and the driver's overtaking action ideally.
- The intelligent vehicle not only has special military value potentially, but also improves traffic flow and secures more safety. So it is the core of intelligent system and the direction of future vehicle development.

10 Intelligent Transportation System

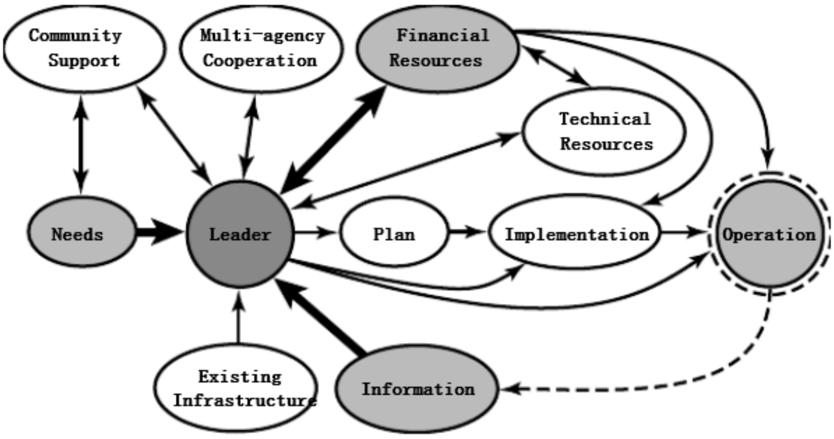


Figure 10-1.Requirements for successful ITS operation (Dahlgren 1996)

10.1 Segment Control Layer

The hierarchical control structure of multi-lane traffic flow is divided into the coordinated control layer, segment control layer and vehicle control layer. In our paper an optimal control model is proposed for the segment control layer. The objective of the model is to maximize the total traffic flow and the average speed in a segment and the decision variables are the rates of vehicle number in different lanes on the same segment. The number of decision variables in this model is greatly reduced compared with that of traditional linear model.

On segment i, the optimization objective is $\max J_2 = \sum_l (\varepsilon_1 q(t,i,l) + \varepsilon_2 v(t,i,l))$ **(10-1)**

Constraints are $\begin{cases} \rho(t,i,l) = k(t,i,l)\rho(t,i) \\ v(t,i,l) = V_{(i,l)}(\rho(t,i,l)) \\ q(t,i,l) = \rho(t,i,l)v(t,i,l) \\ \sum_l k(t,i,l) = 1 \end{cases}$

Typical relationship between v, ρ: $V_{(i,l)}(\rho) = v(i,l)b(i,l)\exp\left[-0.5(\frac{\rho}{\rho(i,l)})^2\right]$ **(10-2)**

$v(i,l)$ is the free traffic flow speed of lane (i,l), $\rho(i,l)$ is the critical density, $b(i,l)$ is speed-limit coefficient $(0 \le b(i,l) \le 1)$. They depend on the characteristic of lane (i,l), determined by traffic statistics.

Using constraints to simplify the objective function and we get

$\max J_2 = \sum_l (\varepsilon_1 k(t,i,l)\rho(t,i) + \varepsilon_2) V_{(i,l)}(k(t,i,l)\rho(t,i))$ **(10-3)** $k(t,i,l) \ge 0, \sum_l k(t,i,l) = 1$

10.2 Genetic Algorithm

In the computer science field of artificial intelligence, Genetic Algorithm (GA) is a search heuristic that mimics the process of natural selection. This heuristic is routinely used to generate useful solutions to optimization and search problems. Genetic Algorithms belong to the larger class of evolutionary algorithms (EA), which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover.

The Genetic Algorithm, which can overcome the disadvantage of some optimal algorithm that may obtain local optimal solution, is employed to solve the nonlinear program model.

Identify fitness function $O_{new} = J_2 + J_{Penalty}$,where

$$J_{Penalty} = \begin{cases} 0, if\ \ k\ \ is\ \ available \\ r(\sum_l k(t,i,l) - 1), otherwise \end{cases}$$

r is penalty factor. So the nonlinear programming problem is transformed into a problem with region constraints only. $\max O_{new} = J_2 + J_{Penalty}$ **(10-4)** $k(t,i,l) \ge 0$

10.3 Simulation Computation

Assuming segment i consists of three lanes each with a length of 10km, parameters are as follow.

$$v(i,1) = 150, b(i,1) = 1, \rho(i,1) = 37.3$$
$$v(i,2) = 130, b(i,2) = 0.8, \rho(i,2) = 20$$
$$v(i,3) = 110, b(i,3) = 0.6, \rho(i,3) = 15$$

When $\rho(t,i)$ =20,60,120, the objective function figure in its domain of definition $\{(k_1,k_2) \mid k_1,k_2 \in [0,1], k_1 + k_2 \le 1\}$ is shown in **Figure 10-2.**

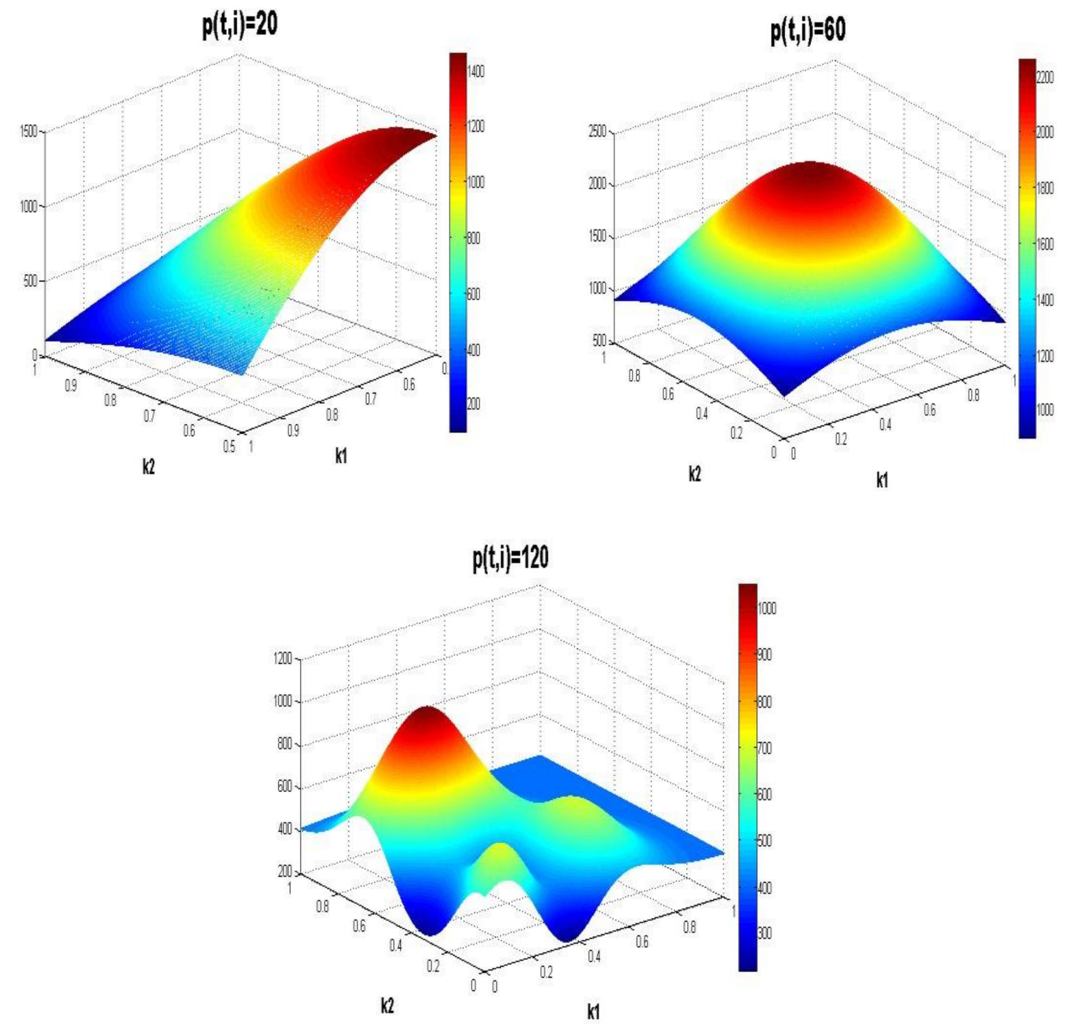


Figure 10-2. Objective function

Using genetic algorithm we get solutions when $\rho(t,i)$ takes different values. We find that the maximum value available for the objective function varies with the variation of average traffic density of the segment as shown in **Figure 10-3**.

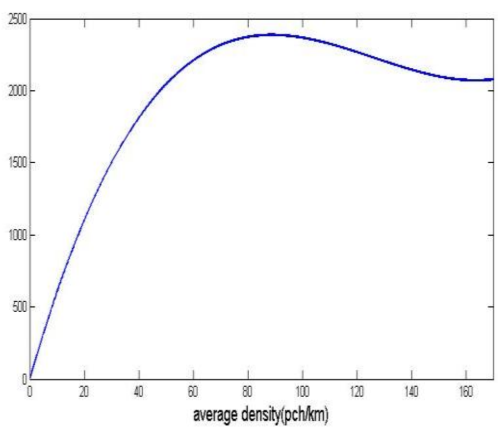


Figure 10-3.

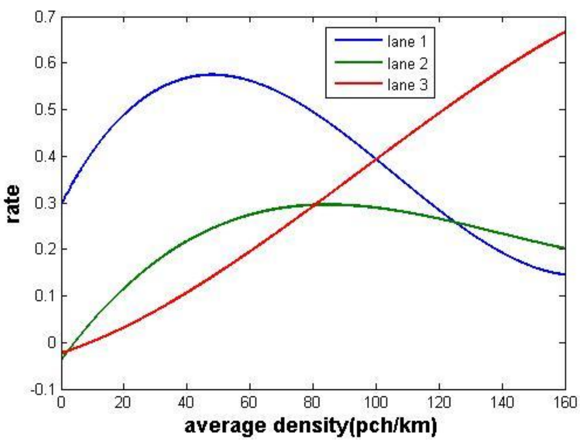


Figure 10-4.

When the objective function maximizes, the rates of the number of vehicles in different lanes change, as shown in **Figure 10-4.**

$$\rho(t,i) = 20 \Rightarrow k_1 = 0.91, k_2 = 0.09, k_3 = 0$$
$$\rho(t,i) = 60 \Rightarrow k_1 = 0.5447, k_2 = 0.2733, k_3 = 0.1820$$
$$\rho(t,i) = 120 \Rightarrow k_1 = 0.3069, k_2 = 0.1627, k_3 = 0.5303$$

The simulation examples demonstrate the effectiveness of the model and its solution algorithm.

10.4 Strengths of Intelligent System

- Our model aims to maximize the total traffic flow volume and the average speed in a segment, using rates of vehicles in different lanes as decision variables. The optimization model overcomes the uncertainty of vehicle diversion rate, thus it is suitable for the traffic flow real-time monitoring.
- The variables and constraints are largely reduced and the lane parameter value could be officially obtained, which make our model practical and easy to apply as part of the road network.
- The optimization model promotes traffic flow and average speed to the maximum, which overcomes the uncertainty of diversion relying upon human judgment.

11 Conclusion

Owing to the high traffic accident rate each year, the research focusing on traffic rules and traffic system is in urgent need and of significant importance.

Firstly, in order to analyze the performance of the keep-right-except-to-pass rule comprehensively, we build and analyze four mathematical models from different perspectives: overtaking model, Cellular Automata & Markov model, Fluid-Mechanics model and Dynamic Disturbance model. Then, we put forward our new rule to improve traffic flow and safety more effectively with verification.

Secondly, we analyze the left norm countries from three points of view: orientation and custom, vision distribution and Coriolis effect.

Lastly, based on the former modeling and analysis, we take a deep insight into the intelligent transportation system, hoping to make it feasible under different rule restrictions.

The intelligent vehicle and intelligent system are practical in either left or right norm countries, making traffic rules more adaptive and flexible. Therefore, they overcome the vision and geographical restricts and can be popularized extensively. Moreover, the intelligent vehicle and intelligent system could both promote traffic flow and safety to a great extent. Therefore, they should be implemented in the transportation system worldwide.

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