ELEN30011 EDM Task

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1.1

$\boldsymbol{\nabla} \cdot \mathbf{F}$ (div $\mathbf{F}$) is a scalar, $\boldsymbol{\nabla} \times \mathbf{F}$ is a vector field.

Explaination: $\Downarrow$

1.2

Let $\mathbf{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a vector field with

$$\mathbf{F}(x, y, z) = F_x(x, y, z)\hat{\mathbf{x}} + F_y(x, y, z)\hat{\mathbf{y}} + F_z(x, y, z)\hat{\mathbf{z}}$$

The divergence of $\mathbf{F}$ (div $\mathbf{F}$) is

$$\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{F} &= (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \cdot (F_x, F_y, F_z) \\ &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \end{aligned}$$

$\boldsymbol{\nabla} \cdot \mathbf{F}$ is a scalar.

$$\begin{aligned} \boldsymbol{\nabla} \times \mathbf{F} &= (\frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}) \times (F_x(x, y, z)\hat{\mathbf{x}} + F_y(x, y, z)\hat{\mathbf{y}} + F_z(x, y, z)\hat{\mathbf{z}}) \\ &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \\ &= (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\hat{\mathbf{x}} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\hat{\mathbf{y}} + (\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x})\hat{\mathbf{z}} \end{aligned}$$

$\boldsymbol{\nabla} \times \mathbf{F}$ is a vector field.

1.3

(a)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$$

(b)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = 1\hat{\mathbf{x}} + z\hat{\mathbf{y}} + y\hat{\mathbf{z}}$$

(c)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = x\hat{\mathbf{x}} + (y + \frac{1}{2}z^2 siny)\hat{\mathbf{y}} - zcosy\hat{\mathbf{z}}$$

(d)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = \frac{2x}{x^2+y^2+z^2}\hat{\mathbf{x}} + \frac{2y}{x^2+y^2+z^2}\hat{\mathbf{y}} + \frac{2z}{x^2+y^2+z^2}\hat{\mathbf{z}}$$