ELEN30011 EDM Task

- Xiufu SUN 1372750
- Wenyang SUN 1354302

1.1

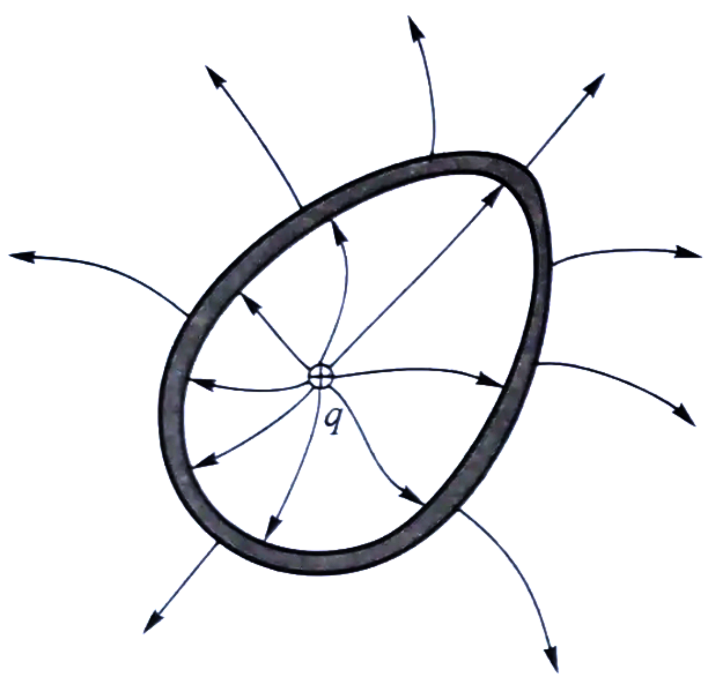
1.1.1

The electric field is given as:

$$\mathbf{E} = \mathbf{E}(r, \phi, \theta) = E(r)\hat{\mathbf{r}}$$

Gauss's law states that the electric flux through a closed surface is proportional to the amount of charge. Because of the symmetry of the sphere, the charge is evenly distributed on the surface of the shell. Hence, the electric field must be radial. To apply Gauss's law effectively in this configuration, the flux is equal everywhere on the sphere.
If the electric field had components in directions other than the radial direction, these components would cancel out when integrated over a symmetric spherical surface, leaving only the radial component. Hence, $\mathbf{E} = E(r)\hat{\mathbf{r}}$.

Moreover, if the shell is egg-shaped, its elctric field will be like this:



1.1.2

$$\rho_{in}(r) = (\frac{Q}{\pi a^4})r \quad Cm^{-3}$$

$$\begin{aligned}
Q_{in}(r) &= \int_0^r \int_0^{2\pi} \int_0^{\pi} \rho_{in}(\alpha)\alpha^2 sin\theta d\theta d\phi d\alpha \\
&= 2\pi \int_0^r \int_0^{\pi} (\frac{Q}{\pi a^4})\alpha^3 sin\theta d\theta d\alpha \\
&= 4\pi(\frac{Q}{\pi a^4}) \int_0^r \alpha^3 d\alpha \\
&= Q(\frac{r}{a})^4 & C
\end{aligned}$$

1.1.3

In 1.1.2, we get when $r \in [0, a]$, $Q_{in}(r) = Q(\frac{r}{a})^4$ C.

If $r \in (a, b)$, $\rho = 0$, Q will be:

$$\begin{aligned}
Q_{in}(r) &= \int_0^a \int_0^{2\pi} \int_0^{\pi} \rho_{in}(\alpha)\alpha^2 sin\theta d\theta d\phi d\alpha \\
&= 2\pi \int_0^a \int_0^{\pi} (\frac{Q}{\pi a^4})\alpha^3 sin\theta d\theta d\alpha \\
&= 4\pi(\frac{Q}{\pi a^4}) \int_0^a \alpha^3 d\alpha \\
&= Q(\frac{a}{a})^4 \\
&= Q & C
\end{aligned}$$

1.1.4

- Based on Gauss's law in its integral form states:

$$\oint_{\partial V} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{in}}{\epsilon},$$

1. **For $r \in [0, a]$:**

$$Q_{in}(r) = Q(\frac{r}{a})^4 \quad C$$

   Gauss's law:

$$\oint_{\partial V} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{in}}{\epsilon},$$

Solving $\oint_{\partial V} \mathbf{E} \cdot d\mathbf{A}$:

$$\begin{aligned}\oint_{\partial V} \mathbf{E} \cdot d\mathbf{A} &= \iint \mathbf{E} \cdot r^2 sin\phi d\phi d\theta \hat{\mathbf{r}} \\ &= 4\pi r^2 \mathbf{E}(r)\end{aligned}$$

Hence, when $r \in [0, a]$:

$$E(r) = \frac{Q(\frac{r}{a})^4}{4\pi\epsilon r^2} = \frac{Q}{4\pi\epsilon a^2}\left(\frac{r}{a}\right)^2.$$

2. **For** $r \in (a, b]$:

$$Q_{in}(r) = Q \quad C$$

Gauss's law:

$$\oint_{\partial V} \mathbf{E} \cdot d\mathbf{A} = 4\pi r^2 \mathbf{E}(r) = \frac{Q}{\epsilon}$$

Hence, when $r \in (a, b)$:

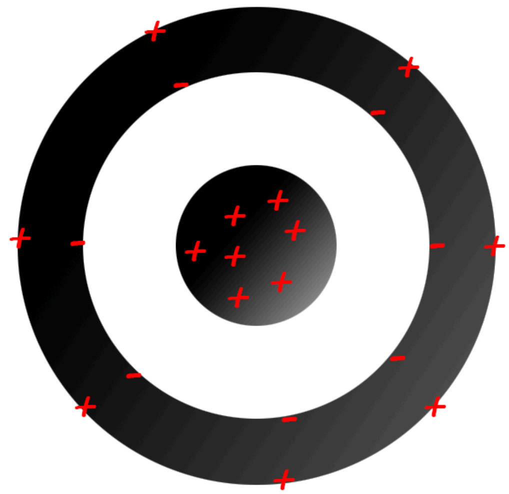
$$E(r) = \frac{Q}{4\pi\epsilon r^2} = \frac{Q}{4\pi\epsilon a^2}\left(\frac{a}{r}\right)^2.$$

The electric field $\mathbf{E}(r)$ in the interior and immediately surrounding the solid sphere is derived using Gauss's law and is given by:

$$\mathbf{E}(r) = \frac{Q}{4\pi\epsilon a^2}\begin{cases}\left(\frac{r}{a}\right)^2 \hat{r}, & r \in [0, a], \\ \left(\frac{a}{r}\right)^2 \hat{r}, & r \in (a, b].\end{cases}$$

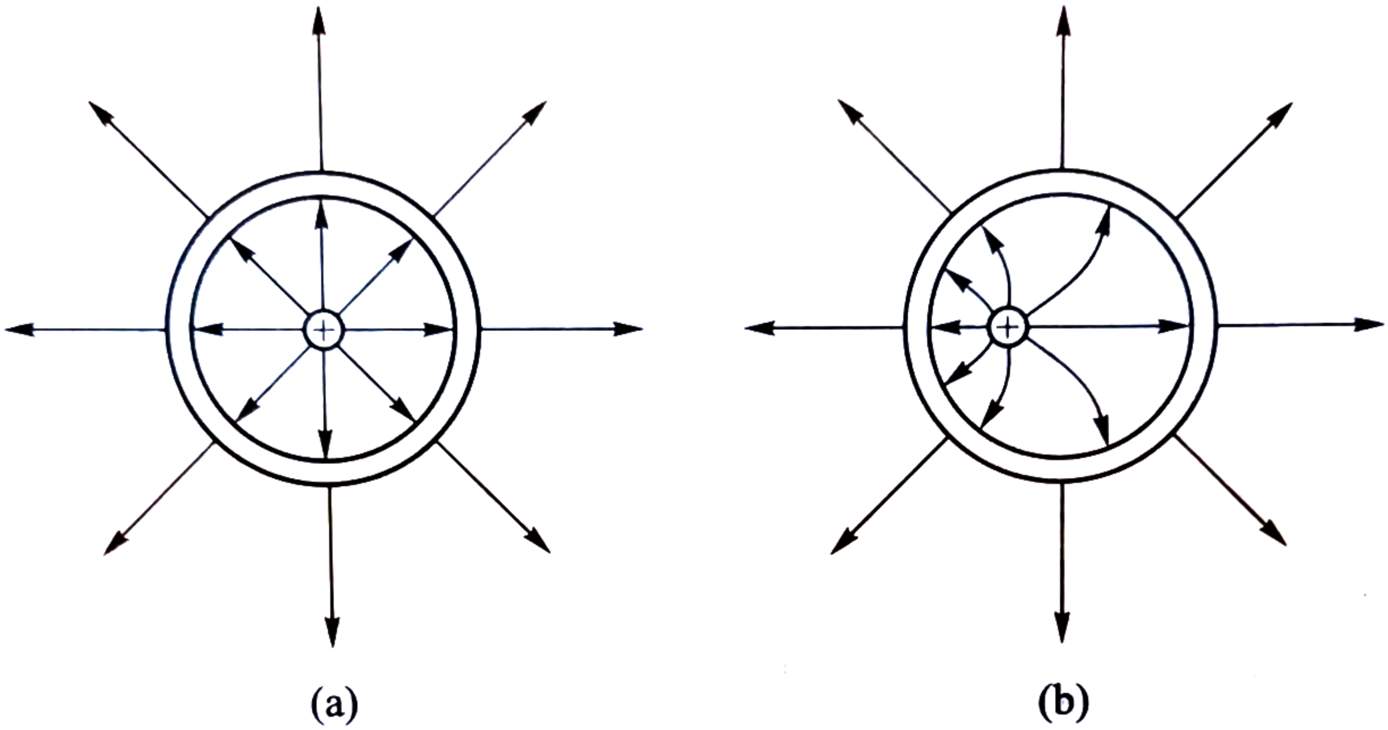
1.2

1.2.1



The charge enclosed by that surface is zero. From Gauss'law $\oint \mathbf{E} d\mathbf{A} = 0$ this implies that the electric field inside a sphere is zero. If we add an electron inside the shell, the electrons on the surface will experience a force. This will cause them to move on the surface such that the net forcefield becomes zero again. The charge will move freely to counteract any electric field inside. Therefore, when a conductive spherical shell is in electronic equilbrium, there should be no net electric field present inside. This is because the charge within the conductor will redistribute itself so as to create opposing electric fields within the candutert to cancel out any electric field present within it.

1.2.2



Based on the answer to 1.2.1,we knew that the value of electric field in the interval from $r \in (b, b+d)$ is zero. According to the formula of flux: $\oiint_{\mathbf{A}} \mathbf{E} d\mathbf{A}$, because of $\mathbf{E} = \mathbf{0}$ the flux $\oiint_{\mathbf{A}} \mathbf{E} d\mathbf{A} = 0$. $Q = \mathbf{E} \cdot \phi$ because of $\mathbf{E} = 0, Q = 0$, thus the total charge contained with this test surface must be zero.

1.2.3

If a conductor is placed in an electric field, the free electrons inside the conductor will move (positive charge moves through the electric line, negative charge moves against), which is called '**electrostatic induction**'.

In this question, negative charge moves to the inner surface ($r = b$), positive charge moves to the outer surface ($r = b + d$), until the opposite electric field strengths cancel each other out, the total electric field strength in the conductor becomes zero (a state of **equilibrium**). When $r \in (b, b+d)$, the electric field is zero. Based on Gauss' law, we know that the charge encased in the closed surface is also zero. Hence, the charge on the inner surface is $-Q$. Accordingly, $+Q$ on the outer surface.

Since the spherical shell is uniform and symmetrical, charge is uniformly distributed on the surface (Or, the tip of the conductor accumulates more charge). Hence, $-Q$ must be uniformly distributed on the inner surface, $+Q$ must be uniformly distributed on the outer surface.

1.3

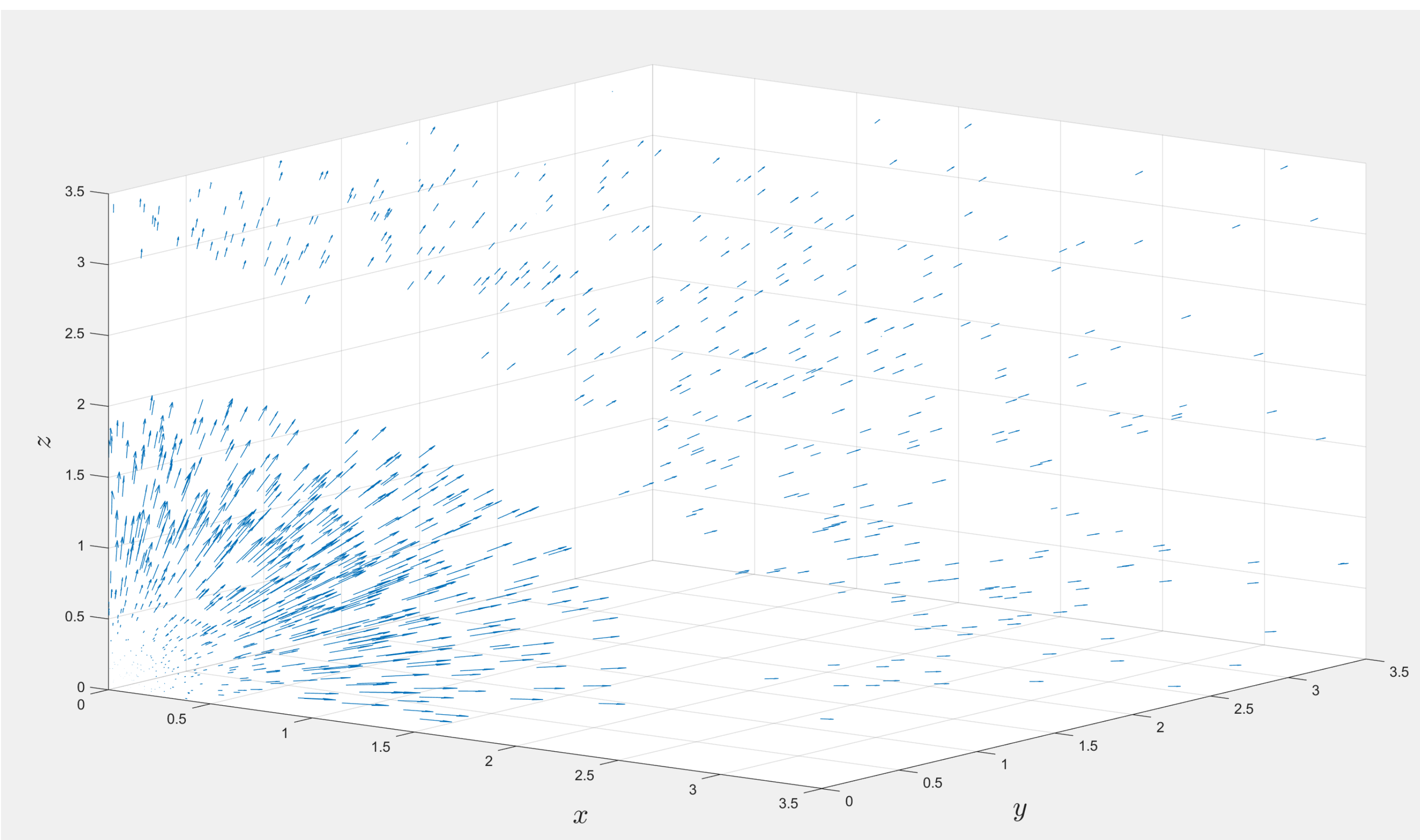
1.3.1

If $r \in (b, b+d)$, Q will be:

$$\begin{aligned}
Q(r) &= \int_0^a \int_0^{2\pi} \int_0^{\pi} \rho_{in}(\alpha)\alpha^2 sin\theta d\theta d\phi d\alpha \\
&= 2\pi \int_0^a \int_0^{\pi} (\frac{Q}{\pi a^4})\alpha^3 sin\theta d\theta d\alpha \\
&= 4\pi(\frac{Q}{\pi a^4}) \int_0^a \alpha^3 d\alpha \\
&= Q(\frac{a}{a})^4 \\
&= Q \qquad\qquad\qquad\qquad\qquad\qquad C
\end{aligned}$$

1.3.2

Yes.



1. For $r \in [0, 1m]$: The electric field increases proportionally to $(r)^2$ and points radially outward. This is reflected by the denser field lines as r increases, indicating a stronger field.
2. For $r \in [1m, 2m]$: The field decreases as $(1/r)^2$ as r increases. The visualization shows this by the reduction in field line density, matching the expected behavior.
3. For $r \in (2m, 3m)$: The electric field is zero within the conducting shell, as shown by the absence of field lines in this region, consistent with the equation.
   4.For $r \geq 3m$: The electric field follows a $(1/r)^2$ relationship once again. The field lines spread out as the distance from the origin increases, showing a decrease in field strength that aligns with the equation (13).

```
figure(2);
elecField = sqrt(E_x.^2+E_y.^2+E_z.^2);
radius = sqrt(x.^2+y.^2+z.^2);
scatter(radius, elecField);
xlabel('$r = \sqrt{x^2 + y^2 + z^2}$','Interpreter','latex');
ylabel('Electric field');
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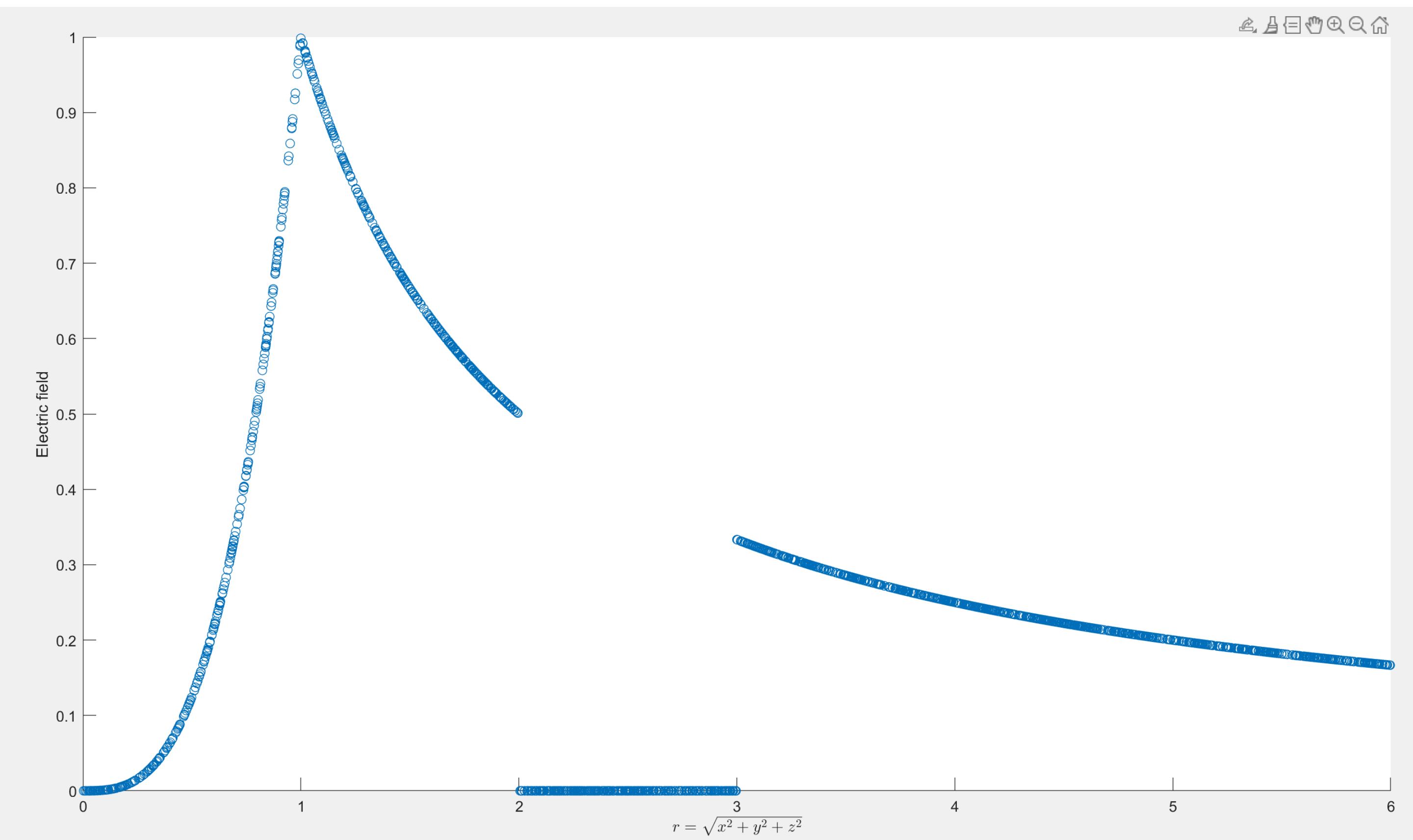

2.1

2.1.1

Since the direction and magnitude of the symmetric magnetic field of the cylinder, E depend solely on the radial distance r from the axis, independent of the Angle ϕ (as shown in Part 1). As H is perpendicular to E, H has no component in the r direction. J is uniform along the z direction, H has no component in the z direction either.

Using Ampere’s law:

$$\oint_{l(A)} \mathbf{H} \cdot d\mathbf{l} = \iint_{A(l)} \mathbf{J} \cdot d\mathbf{A} + \iint_{A(l)} \frac{d\mathbf{D}}{dt} \cdot d\mathbf{A}$$

The electric field does not change with time.

$$\oint_{l(A)} \mathbf{H} \cdot d\mathbf{l} = \iint_{A(l)} \mathbf{J} \cdot d\mathbf{A} = I$$

Using the right-hand rule, we can determine the direction and nature of the magnetic field generated by the current.

$$\mathbf{H}(r, \phi, z) = H_\phi \hat{\phi} \quad Am^{-1}$$

2.1.2

From Cartesian coordinates to cylindrical coordinates:

$$d\mathbf{A} = r d\theta dr \hat{\mathbf{z}}$$

Based on the definition of current:

$$\begin{aligned} I_{core}(r) &= \iint_{A(l)} \mathbf{J} \cdot d\mathbf{A} \\ &= \int_0^r \int_0^{2\pi} J_{core}(\alpha)\hat{\mathbf{z}} \cdot \alpha d\theta d\alpha \hat{\mathbf{z}} \\ &= \int_0^r \frac{3I\alpha^2}{a^3} d\alpha \\ &= I(\frac{r}{a})^3 \qquad\qquad\qquad\qquad A \end{aligned}$$

The total current is I:

$$I = I_{core}(a) = I(\frac{a}{a})^3 = I \quad A$$

2.1.3

We've already got $\oint_{l(A)} \mathbf{H} \cdot d\mathbf{l} = \iint_{A(l)} \mathbf{J} \cdot d\mathbf{A} = I$ in 2.1.1.

$r \in (0, a]$,

$$\iint_{A(l)} \mathbf{J} \cdot d\mathbf{A} = I(\frac{r}{a})^3 \quad A$$

When $r \in (a, b]$,

$$I = I_{core}(a) = I(\frac{a}{a})^3 = I \quad A$$

For $\oint_{l(A)} \mathbf{H} \cdot d\mathbf{l}$, we have known that $\mathbf{H}$ is in the ϕ direction.

$$\begin{aligned}\oint_{l(A)} \mathbf{H} \cdot d\mathbf{l} &= rH_\phi \int_0^{2\pi} d\phi \\ &= 2\pi r H_\phi \qquad\qquad A\end{aligned}$$

Hence, when $r \in (0, a]$

$$\begin{aligned}2\pi r H_\phi &= I(\frac{r}{a})^3 \\ \Rightarrow H_\phi &= I(\frac{r^2}{2\pi a^3})\end{aligned}$$

When $r \in (a, b]$

$$\begin{aligned}2\pi r H_\phi &= I \\ \Rightarrow H_\phi &= \frac{I}{2\pi r}\end{aligned}$$

Hence,

$$\mathbf{H}(r) = \frac{I}{2\pi a} \begin{cases} \left(\frac{r}{a}\right)^2 \hat{\phi}, & r \in [0, a], \\ \left(\frac{a}{r}\right) \hat{\phi}, & r \in (a, b]. \end{cases}$$

2.2

2.2.1

Based on the definition of current, when $r \in (b, (b+d)]$,

$$
\begin{aligned}
I_{shield} &= \iint_{A(l)} \mathbf{J} \cdot d\mathbf{A} \\
&= \int_b^{b+d} \int_0^{2\pi} J_{shield}(\alpha)\hat{\mathbf{z}} \cdot \alpha d\theta d\alpha \hat{\mathbf{z}} \\
&= \frac{-I}{\pi[(b+d)^2 - b^2]} \int_b^{b+d} 2\pi\alpha d\alpha \\
&= \frac{-I}{\pi[(b+d)^2 - b^2]} \times \pi[(b+d)^2 - b^2] \\
&= -I \qquad\qquad\qquad\qquad\qquad\qquad A
\end{aligned}
$$

2.2.2

Ampère's circuital law shows that,

$$\oint_{\mathbb{C}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I_{\mathrm{enc}}$$

When the loop contains both core and shield, $\sum I = I_{core} + I_{shield} = 0$.

Hence,

$$\oint_{\mathbb{C}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = 0$$

The magnetic field is zero outside the shield.