ELEN30011 EDM Task

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1.1

(a)

$$(1, 0, 0)$$

(b)

$$(-1, 0, 0)$$

(c)

$$(0, -1, 3)$$

(d)

$$(0, 0, -2)$$

(e)

$$(-1, 0, 0)$$

1.2

(a)

$$(1, 0, \pi/2)$$

(b)

$$(1, \pi/2, \pi/2)$$

(c)

$$(1, 0, 0)$$

(d)

$$(\sqrt{2}, \pi/2, \pi/4)$$

(e)

$$(0, 0, 0)$$

2.1

(a) Let

$$x = rcos\phi, y = rsin\phi, z = z$$

Jacobin:

$$\begin{vmatrix} cos\phi & -rsin\phi & 0 \\ sin\phi & rcos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Let r = 1,

$$T(\phi) = \begin{pmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b)
We've got that

$$T(\phi) = \begin{pmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Hence,

$$(T^*(\phi))^T = \begin{pmatrix} cos\phi & -sin\phi & 0 \\ sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^*(\phi) = \begin{pmatrix} cos\phi & sin\phi & 0 \\ -sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$det[T(\phi)] = 1$$

Based on Cramer's rule,

$$T^{-1}(\phi) = \frac{1}{det} T^*(\phi)$$

Hence,

$$T^{-1}(\phi) = \begin{pmatrix} cos\phi & sin\phi & 0 \\ -sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c)

$$r = \sqrt{x^2 + y^2}$$
$$cos(\phi) = \frac{x}{\sqrt{x^2 + y^2}}$$
$$sin(\phi) = \frac{y}{\sqrt{x^2 + y^2}}$$

(d)

$$T^{-1}(\phi) = \begin{pmatrix} cos\phi & sin\phi & 0 \\ -sin\phi & cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S(P) = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 0 \\ -\frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.2

In Question 2.1(d), we've got

$$S(P) = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 0 \\ -\frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a) P = (0, -1, 0)

$$S(P) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) P = (1, 0, 0)

$$S(P) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) P = (-1, 0, 0)

$$S(P) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(d) P = (1, -1, 0)

$$S(P) = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(e) P = (0, 0, 0)

$$S(P) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3.1

(a)

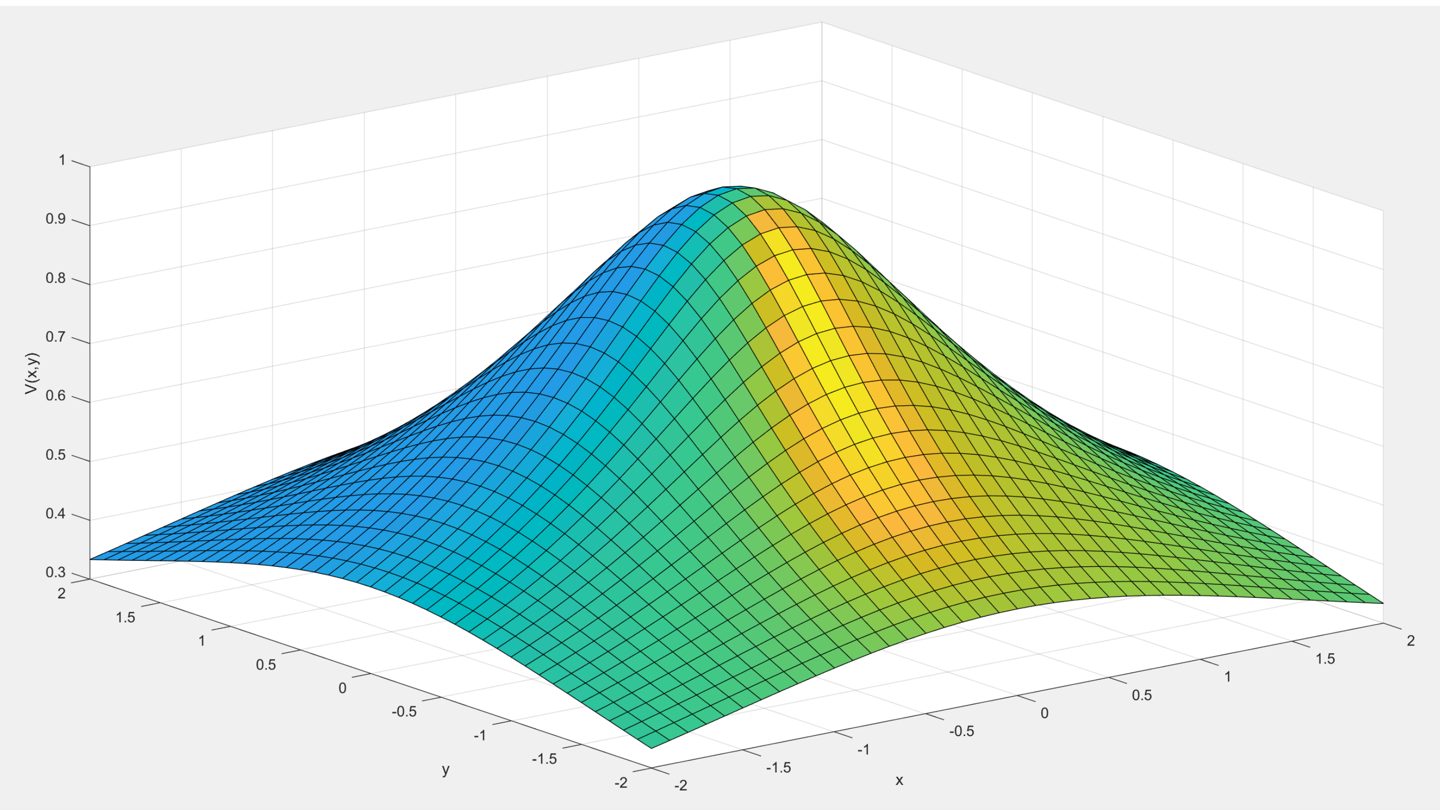
```
x = -2:.1:2;
y = -2:.1:2;
[xx, yy] = meshgrid(x, y);

size(xx)
size(yy)
```

Which output is:

```
ans =

    41    41

ans =

    41    41
```

(b)

```
x = -2:.1:2;
y = -2:.1:2;
[xx, yy] = meshgrid(x, y);

size(xx)
size(yy)

zz = 1./sqrt(1 + xx.^2 + yy.^2);
figure(1);
surfl(xx, yy, zz);
xlabel('x');
ylabel('y');
zlabel('V(x,y)');
grid on;
```



Based on the picture above, it has been shown that the surface exhibit a maximum.

After checked the value "zz" in workspace, we get the maximum point is $(0, 0, 1)$.

For certain plane, origin can always be the point with the highest electrostatic potential. If a charge moves in any direction on its x-y plane, the electric field does positive work on it.

(c)
A circle.

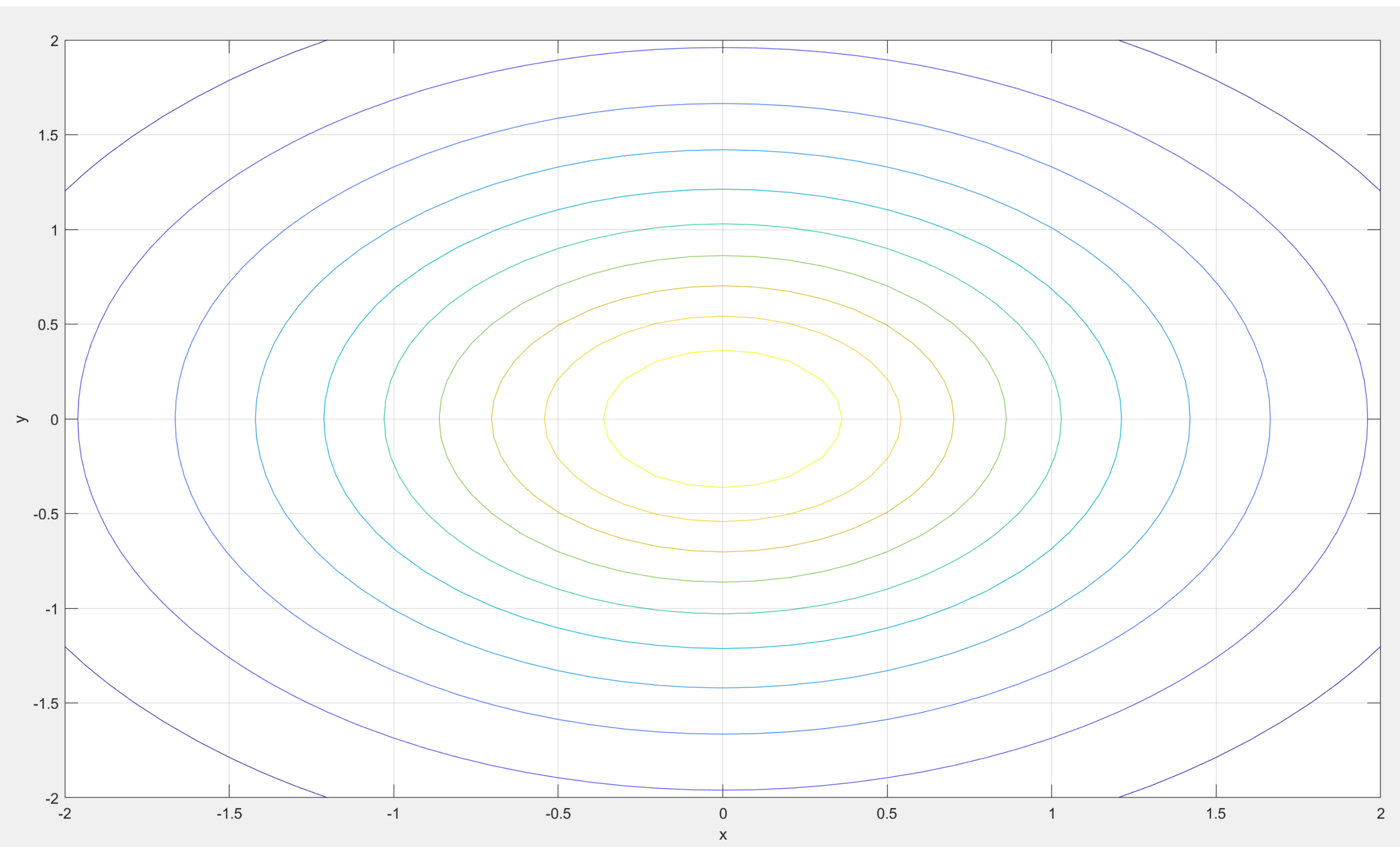
$$V = \frac{1}{\sqrt{1+x^2+y^2}}$$
$$\sqrt{1+x^2+y^2} = \frac{1}{V}$$
$$x^2+y^2 = \frac{1}{V^2} - 1$$

Its radius is $\sqrt{\frac{1}{V^2}-1}$

If $V = c > 1$, radius will be an imaginary number, which is impossible here. Hence, c will never be greater than 1.

(d)

```
figure(2);
contour(xx, yy,zz, 10);
xlabel('x');
ylabel('y');
grid on;
```

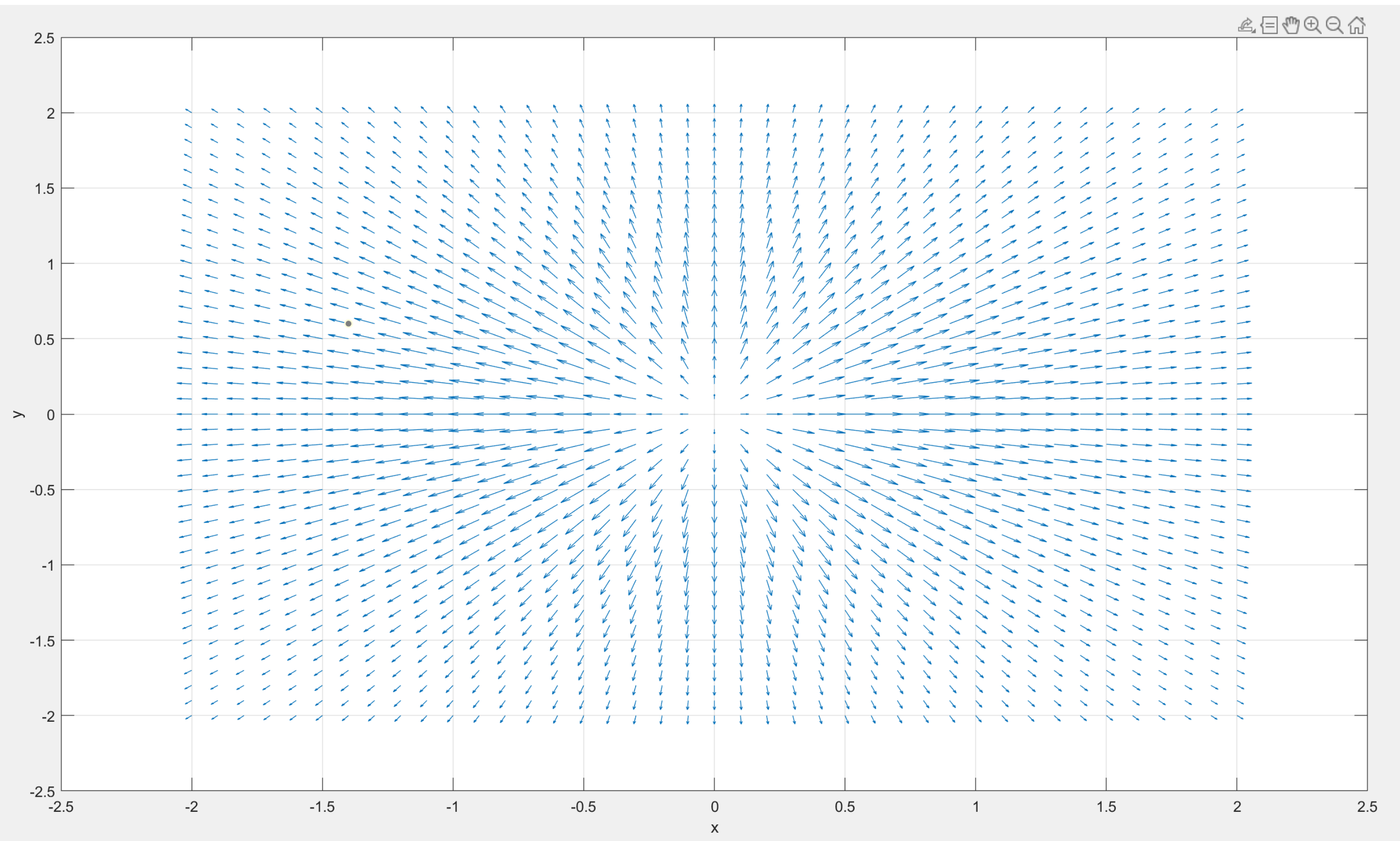


"10" means: Display 10 contour lines at automatically chosen levels (heights).

3.2

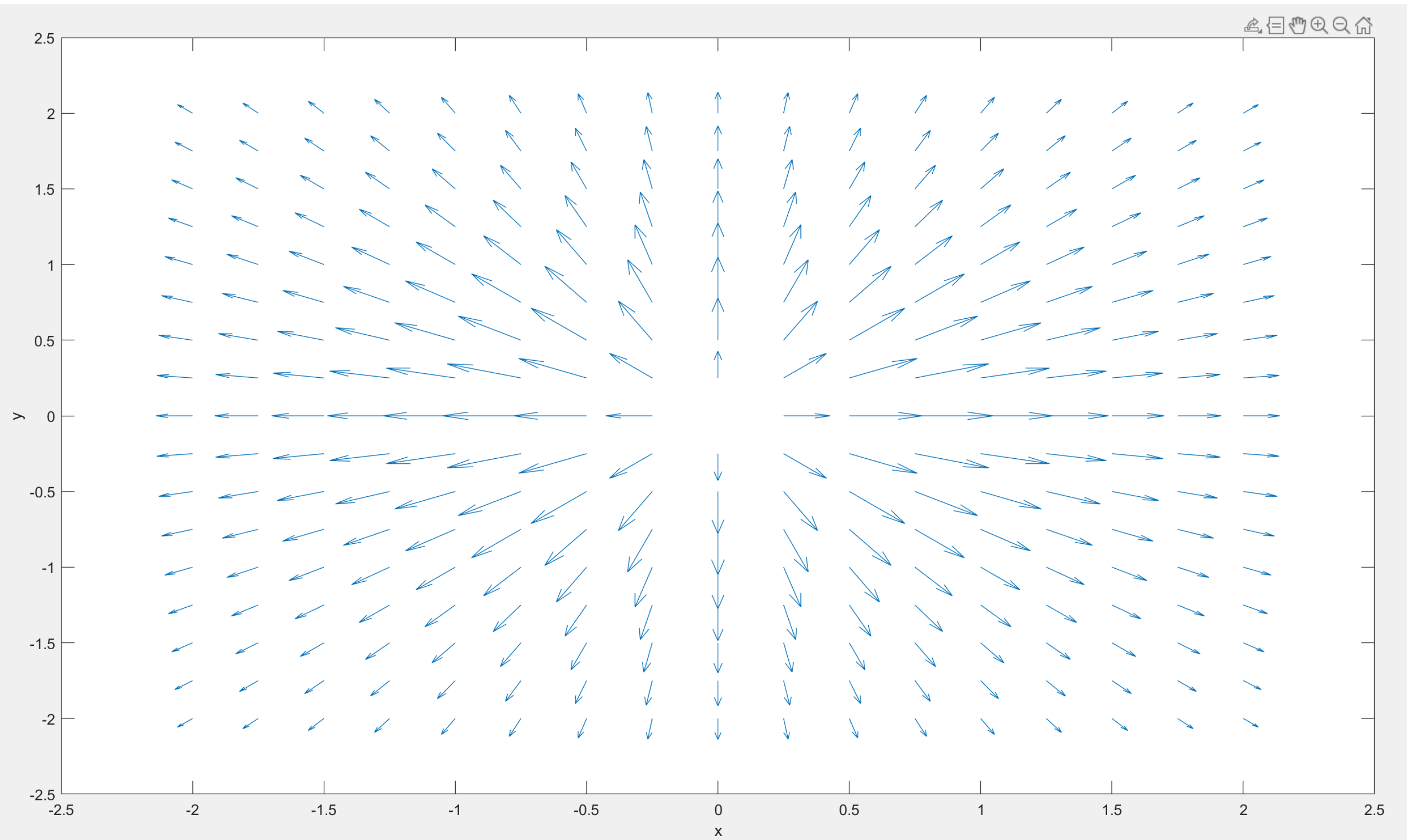
(a)

```
exx = xx./(1 + xx.^2 + yy.^2).^(3/2);
eyy = yy./(1 + xx.^2 + yy.^2).^(3/2);

figure(3);
quiver(xx,yy,exx,eyy);
xlabel('x');
ylabel('y');
grid on;
```



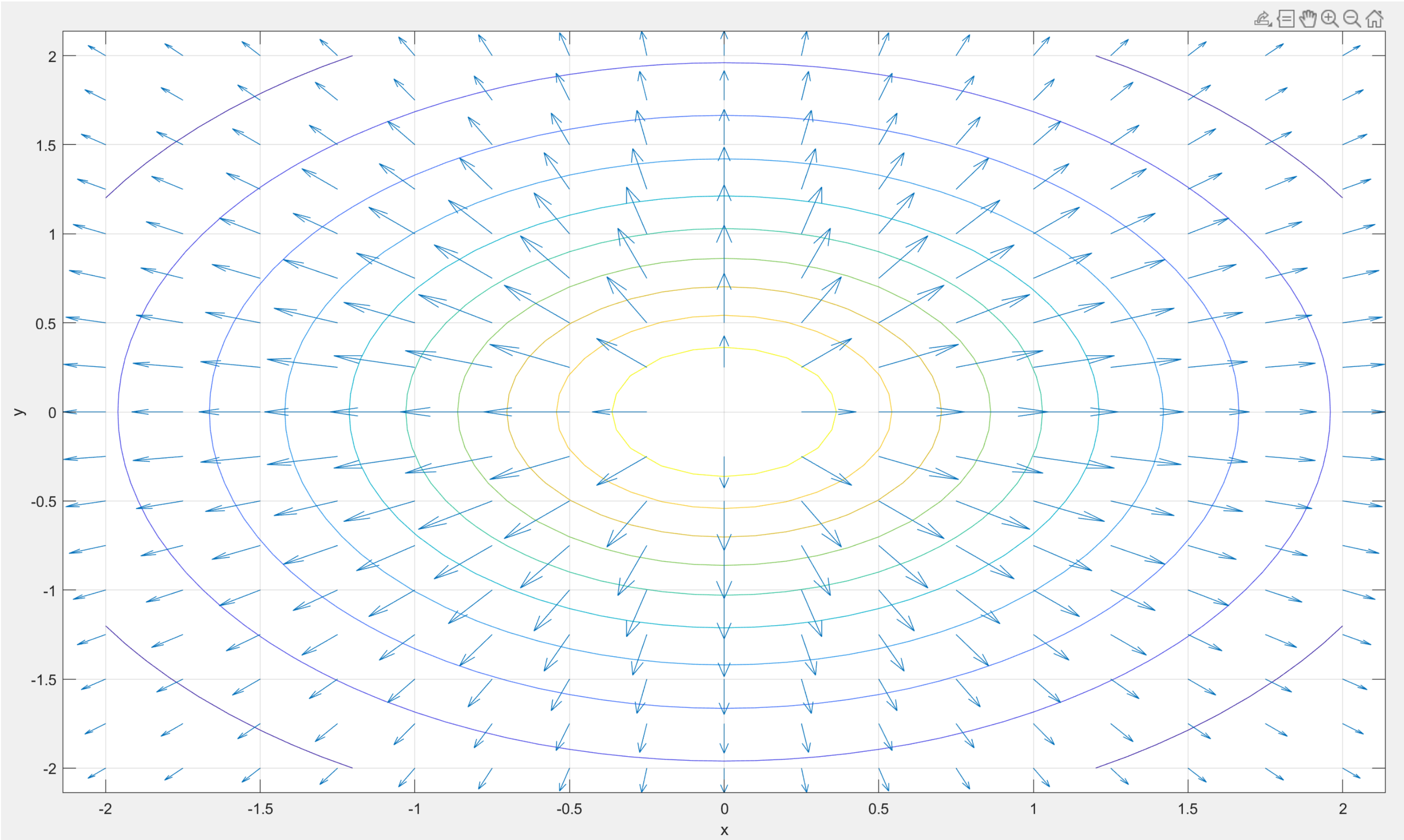
(b)

```
xnew = -2:.25:2;
ynew = xnew;
[xxnew, yynew] = meshgrid(xnew, ynew);
exxnew = xxnew./(1 + xxnew.^2 + yynew.^2).^(3/2);
eyynew = yynew./(1 + xxnew.^2 + yynew.^2).^(3/2);

figure(4);
quiver(xxnew, yynew, exxnew, eyynew);
hold on;
```



(c)

```
figure(2);
hold on;
quiver(xxnew, yynew, exxnew, eyynew);
```



Perpendicular to each other.

3.3

(a)