ELEN30011 EDM Task

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Part 1

1.1

The permittivity ϵ would be influenced by the capacitors' material.

Formular (4) (5) and (6) shows the relationship between V, $\mathbf{E}$, $\mathbf{D}$ and q.

Among them, formula (6) shows the effect of capacitive material: $\mathbf{D} = \epsilon\mathbf{E}$, which means when the electric field strength $\mathbf{E}$ is equal, these capacitors with different material can have different $\mathbf{D}$, and vice versa.

The remaining two formulas show that the voltage is the integral of the electric field along the path, and that the electric flux density is equal to the charge wrapped in the surface, respectively. Only the relationship between the electric flux density and the electric field is affected by the capacitive material.

1.2

The distance between capacitor plates and the size of the capacitor plates.

Formula (4) shows if the electric field is the same, the longer the path, the higher the voltage.

Formula (5) shows in the case of the same charge density, the larger the integral surface, the higher the electric field.

Hence, if the materials is the same, the larger the capacitor, the larger its value.

Part 2

2.1

2.1.1

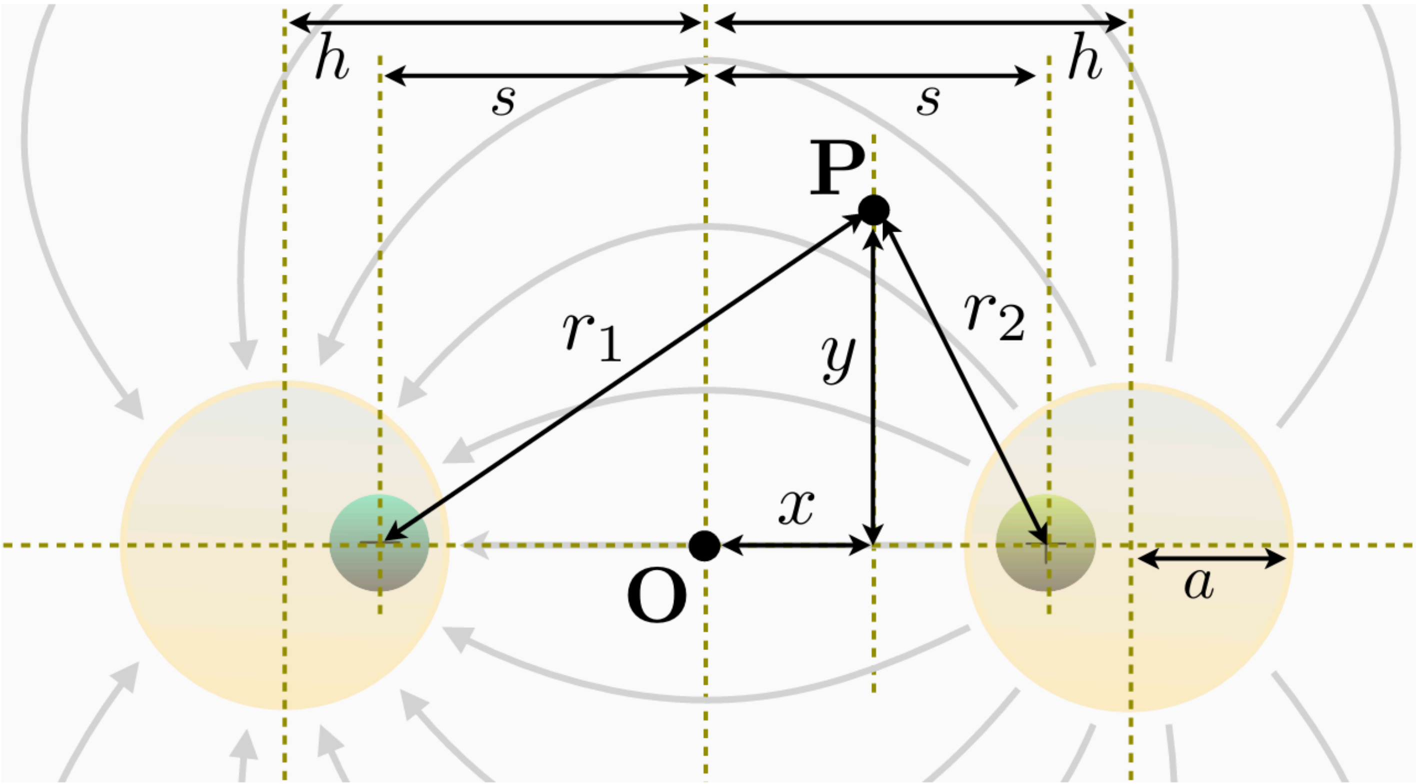


Figure 3: End view of two infinite length parallel wires. The $-$ and $+$ charges at distance s from O represent the two line charges.

In figure 3,

$$r_1 = \sqrt{(x+s)^2 + y^2} \tag{1}$$

$$r_2 = \sqrt{(s-x)^2 + y^2} \tag{2}$$

$U(\mathbf{P})$ is a constant

$$k = \frac{r_1}{r_2} = \frac{\sqrt{(x+s)^2 + y^2}}{\sqrt{(s-x)^2 + y^2}} \tag{3}$$

Hence,

$$k^2 = \frac{r_1^2}{r_2^2} = \frac{(x+s)^2 + y^2}{(s-x)^2 + y^2} \tag{4}$$

$$\Rightarrow (s+x)^2 + y^2 = k^2[(s-x)^2 + y^2] \tag{5}$$

2.1.2

Given

$$(x - s\frac{k^2+1}{k^2-1})^2 + y^2 = (\frac{2ks}{k^2-1})^2 \tag{6}$$

Replace k with x and y, the left side of the equation:

$$\begin{aligned}
(x - s\frac{(s+x)^2 + (s-x)^2 + 2y^2}{(s+x)^2 - (s-x)^2})^2 + y^2 &= (x - \frac{s^2 + x^2 + y^2}{2x})^2 + y^2 \\
&= (\frac{x^2 - y^2 - s^2}{2x})^2 + y^2 \\
&= \frac{x^4 + y^4 + s^4 + 2x^2y^2 - 2x^2s^2 + 2y^2s^2}{4x^2}
\end{aligned}$$

The right side of the equation:

$$\begin{aligned}
(\frac{2ks}{k^2-1})^2 &= (2ks\frac{(s-x)^2 + y^2}{(s+x)^2 - (s-x)^2})^2 \\
&= 4k^2(\frac{(s-x)^2 + y^2}{4x})^2 \\
&= \frac{(x+s)^2 + y^2}{(s-x)^2 + y^2}(\frac{(s-x)^2 + y^2}{2x})^2 \\
&= \frac{[(s+x)^2 + y^2][(s-x)^2 + y^2]}{4x^2} \\
&= \frac{[(s+x)^2(s-x)^2 + y^2(s-x)^2 + y^2(s+x)^2 + y^4]}{4x^2} \\
&= \frac{x^4 + y^4 + s^4 + 2x^2y^2 - 2x^2s^2 + 2y^2s^2}{4x^2}
\end{aligned}$$

Hence, the equation holds.

2.1.3

The coordinates of the center of the circle are:

$$(s\frac{k^2+1}{k^2-1},0) \tag{7}$$

Its radius is:

$$R=\frac{2ks}{k^2-1} \tag{8}$$

2.1.4

Based on figure 3,

$$a=R=\frac{2ks}{k^2-1} \tag{9}$$
$$h=s\frac{k^2+1}{k^2-1} \tag{10}$$

2.1.5

$$\begin{aligned}\frac{h}{a}&=\frac{k^2+1}{2k}\\ 0&=k^2-2(\frac{h}{a})k+1\\ [k-(\frac{h}{a})]^2&=(\frac{h}{a})^2-1\\ \Rightarrow k_{1,2}&=\frac{h}{a}\pm\sqrt{(\frac{h}{a})^2-1}\end{aligned}$$

$$k^-=\frac{h}{a}-\sqrt{(\frac{h}{a})^2-1},\quad k^+=\frac{h}{a}+\sqrt{(\frac{h}{a})^2-1} \tag{11}$$

$$k^-k^+=(\frac{h}{a}-\sqrt{(\frac{h}{a})^2-1})(\frac{h}{a}+\sqrt{(\frac{h}{a})^2-1})=1 \tag{12}$$

If $h>a$, $\frac{h}{a}>1$, $(\frac{h}{a})^2-1>0$

$$k^+=\frac{h}{a}+\sqrt{(\frac{h}{a})^2-1}>1 \tag{13}$$

$$k^-=\frac{1}{k^+}\Rightarrow k^-\in(0,1) \tag{14}$$

2.1.6

In Question 2.1.4, $a = \frac{2ks}{k^2-1} \Rightarrow s = \frac{k^2-1}{k}\frac{a}{2}$.

For $k^+ > 1$,

$$s^+ = (\frac{(k^+)^2 - 1}{k^+})\frac{a}{2} > 0 \tag{15}$$

For $k^- \in (0,1)$,

$$s^- = (\frac{(k^-)^2 - 1}{k^-})\frac{a}{2} < 0 \tag{16}$$

Since $k = \frac{r_1}{r_2}$, if it is located at the surface of the positively charged wires $r_1 > r_2$, $k^+ > 1$. Hence, (k^+, s^+) correspond to the surface of positively charged wires.

If $r_1 < r_2$, $k^- \in (0,1)$, $s^- < 0$. Hence, (k^-, s^-) correspond to the surface of negative charged wires.

2.2 Potential and the electric field

2.2.1

$U(P) = \frac{\lambda}{2\pi\epsilon}\log(\frac{r_1}{r_2}) + U(0)$

$r_1^2 = (s+x)^2 + y^2$

$r_2^2 = (s-x)^2 + y^2$

So that: $\frac{r_1}{r_2} = \left(\frac{(s+x)^2+y^2}{(s-x)^2+y^2}\right)^{\frac{1}{2}}$

$U(P) = \frac{\lambda}{2\pi\epsilon}\log\left(\frac{(s+x)^2+y^2}{(s-x)^2+y^2}\right)\cdot\frac{1}{2} + U(0)$

$= \frac{\lambda}{4\pi\epsilon}\log\left(\frac{(s+x)^2+y^2}{(s-x)^2+y^2}\right) + U(0)$

2.2.2:

Symmetry around the origin: the potential surface is symmetric with respect to the y-axis, reflecting the symmetrical placement of the two parallel wires around the origin. This symmetry corresponds to the equal but opposite charges on the wires.

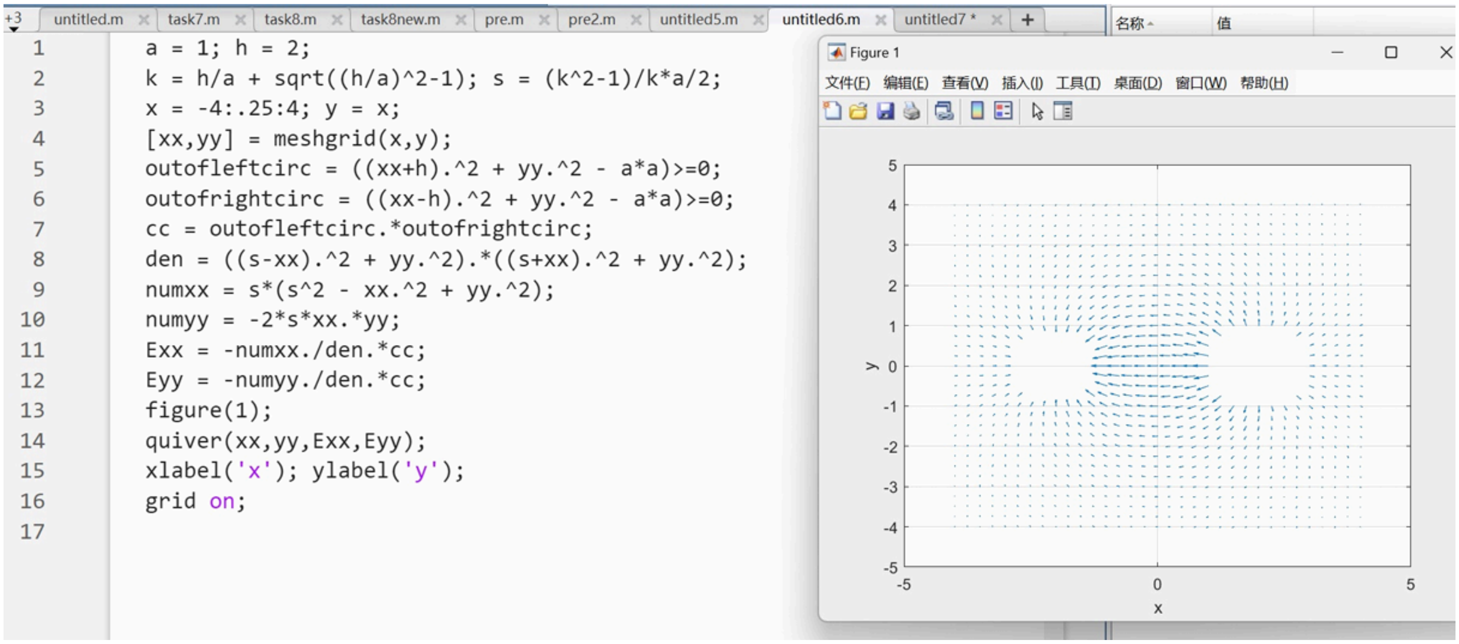
Potential Wells corresponds to the Wire Positions: The two distinct "wells" or depressions in the potential surface correspond to the locations of the wires at $x = \pm h$. These wells represent the regions where the potential is most negative due to the proximity to the line charges.

Steep gradient between the wires: the steep gradient in the potential between the two wells indicates a strong electric field between the wires. This field is directed from the positively charged wire to the negatively charged wire, consistent with the physical configuration shown in figure 3.

Flattening of the potential far from the wires: As you move away from the wires, the potential surface flattens, indicating that the influence of the electric field diminishes with distance from the wires. This corresponds to the weaker field far from the source of the charges.

Zero potential at the origin: The origin is set at zero potential, serving as the reference point. The point difference of 2 V between the wires is visualized by the height difference between the two potential wells, reflecting the energy required to move a charge between the two wires.

2.2.3


untitled.m
task7.m
task8.m
task8new.m
pre.m
pre2.m
untitled5.m
untitled6.m
untitled7 *
名称
值
a = 1; h = 2;
k = h/a + sqrt((h/a)^2-1); s = (k^2-1)/k*a/2;
x = -4:.25:4; y = x;
[xx,yy] = meshgrid(x,y);
outofleftcirc = ((xx+h).^2 + yy.^2 - a*a)>=0;
outofrightcirc = ((xx-h).^2 + yy.^2 - a*a)>=0;
cc = outofleftcirc.*outofrightcirc;
den = ((s-xx).^2 + yy.^2).*((s+xx).^2 + yy.^2);
numxx = s*(s^2 - xx.^2 + yy.^2);
numyy = -2*s*xx.*yy;
Exx = -numxx./den.*cc;
Eyy = -numyy./den.*cc;
figure(1);
quiver(xx,yy,Exx,Eyy);
xlabel('x'); ylabel('y');
grid on;
Figure 1
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
x
y

2.2.4

1.Symmetry Around the Origin: The electric field is symmetric about the y-axis, similar to the potential in Figure 4. This symmetry arises because the two wires are identical and positioned symmetrically about the origin. This symmetry reflects the distribution of the electric potential, where both the field and potential exhibit symmetric behavior centered along the axis between the wires.
2. Field Lines Between the Wires: The field lines originate from the positively charged wire and terminate at the negatively charged wire. The density and direction of the field lines reflect the strength and direction of the electric field, which corresponds to the gradient of the potential shown in Figure 4. The field lines' behavior indicates that the electric field is strongest near the wires, where the potential gradient is steepest.
3.Strong Field Near the Wires: The electric field is strongest near the wires, where the potential changes most rapidly. This is represented by the steeper regions in Figure 4. where the potential wells are located. The high density of field lines near the wires further emphasizes this strong electric field, which is consistent with the steep potential gradients observed
4.Field Lines Curving Outward: The electric field lines curve outward as they move away4from the wires, indicating a decrease in field strength with distance. This relates to the flattening of the potential surface in Figure 4 as you move farther from the wires. The outward curvature of the field lines also illustrates the effect of the wire geometry and spacing on the field distribution.
5.Electric Field and Potential Gradient: The electric field represents the negative gradient of the potential U(P). This means that the electric field vectors in Figure 5 point in the direction where the potential decreases most rapidly. The steepness of the potential surface in Figure 4 corresponds directly to the strength of the electric field in Figure 5.

2.3

2.3.1

$$U(\vec{P}) = \frac{\lambda}{2\pi\epsilon}\log(K) + U(\vec{O})$$

Because of $K^+, K^- = \frac{h}{a} \pm \sqrt{(\frac{h}{a})^2 - 1}$

$$\therefore U(\vec{P}^+) = \frac{\lambda}{2\pi\epsilon}\log\left(\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}\right) + U(\vec{O})$$

$$U(\vec{P}^-) = \frac{\lambda}{2\pi\epsilon}\log\left(\frac{h}{a} - \sqrt{(\frac{h}{a})^2 - 1}\right) + U(\vec{O})$$

2.3.2

$$V = U(\vec{P^+}) - U(\vec{P^-})$$

$$= \frac{\lambda}{2\pi\epsilon}\left(\log\left(\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}\right) - \log\left(\frac{h}{a} - \sqrt{(\frac{h}{a})^2 - 1}\right)\right)$$

$$= \frac{\lambda}{2\pi\epsilon}\log\left(\frac{\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}}{\frac{h}{a} - \sqrt{(\frac{h}{a})^2 - 1}}\right)$$

$$= \frac{\lambda}{2\pi\epsilon}\log\left(\left(\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}\right)^2\right)$$

$$= \frac{\lambda}{\pi\epsilon}\log\left(\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}\right)$$

2.3.3

$$\because C = \frac{Q}{V}$$

Because of λ is the charge change per unit length

$$\therefore \hat{C} = \frac{\lambda}{V} = \frac{\lambda}{\frac{\lambda}{\pi\epsilon}\log\left(\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}\right)}$$

$$= \frac{\pi\epsilon}{\log\left(\frac{h}{a} + \sqrt{(\frac{h}{a})^2 - 1}\right)}\ F\,m^{-1}$$

Part 3 Capacitance in CAT5 UTP network cables

3.1 Preliminary calculations

3.1.1

the characteristic impedance and propagation delay per unit length for CAT5 UTP cable are $Z_0 = 100\Omega$, $D = 5ns/m$.

$$Z_0 = \sqrt{\frac{\hat{L}}{\hat{C}}} = 100$$
$$D = \sqrt{\hat{L}\hat{C}} = 5 \times 10^{-9}$$

Hence,

$$\hat{L} = 5 \times 10^{-7} H/m \tag{17}$$
$$\hat{C} = 5 \times 10^{-11} F/m \tag{18}$$

each twisted pair consists of two 0.51 mm diameter copper wires (American Wire Gauge AWG 24), each with an insulated diameter of 0.88 mm. High density polyethylene (HDPE) is used as the insulator, which has a nominal relative permittivity of $\epsilon_r \doteq \epsilon/\epsilon_0 \doteq 2.3$. The nominal propagation delay per unit length for CAT5e as $D \doteq 5.2\,\mathrm{ns\,m^{-1}}$.

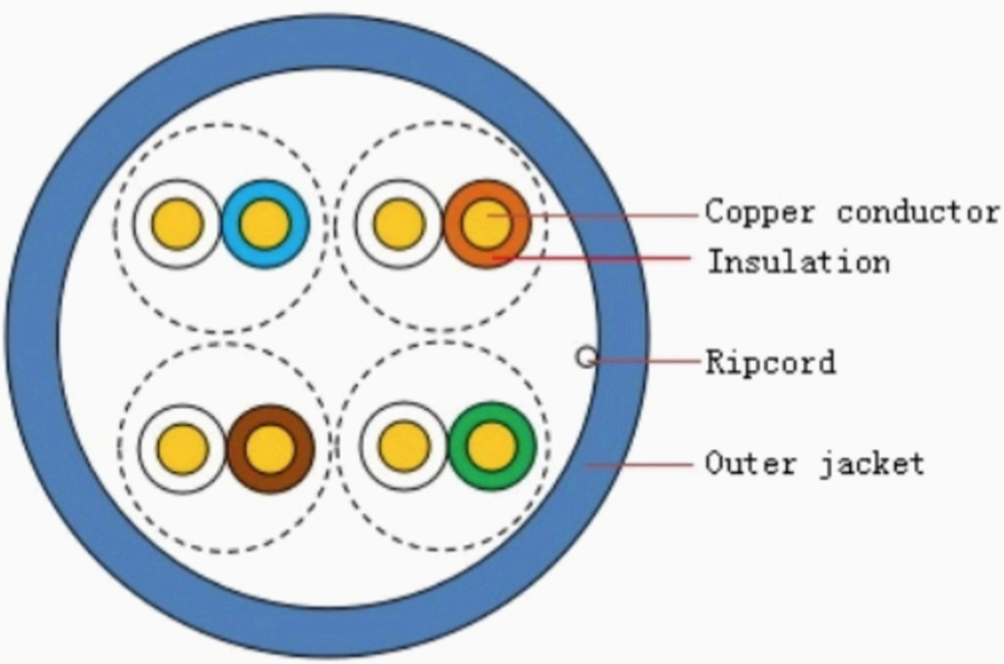


Figure 7: CAT5e UTP ethernet network cable [Section 3.1, image downloaded from `https://hongan-group.en.made-in-china.com`].

3.1.2

According to (18), $\hat{C} = \frac{\pi E}{\log(\frac{h}{a} + (\sqrt{(\frac{h}{a})^2 - 1})}$ Fm^{-1}

and the Note on CAT5e UTP cable given that

the copper wires diameter $a = 0.51mm$,

and each with an insulated diameter $h = 0.88mm$,

Converts to meters: $a = 0.51 \times 10^{-3}m$; $h = 0.88 \times 10^{-3}m$

and the dielectric constant of HDPE = 2.3

So $\varepsilon = \varepsilon_r \cdot \varepsilon_0 = 2.3 \times 8.854 \times 10^{-12} = 2.03642 \times 10^{-11}$ Fm^{-1}

So $\hat{C} = \frac{\pi(2.03642 \times 10^{-11})}{\log\left(\frac{0.88 \times 10^{-3}}{0.51 \times 10^{-3}} + \sqrt{\left(\frac{0.88 \times 10^{-3}}{0.51 \times 10^{-3}}\right)^2 - 1}\right)} \approx 5.604 \times 10^{-11}$ Fm^{-1}

It clearly showed that $\hat{C}$ have some differences compare to the $\hat{C}$ in (3.1.1), the values of Z_0 and D inaccurate may cause this differences., and the values of a and h in certain types of cables may vary, and different manuals might provide different values. and it the error in ε_r may also have error.

3.1.3

Given $L = 60m$, $D = 5ns/m$, hence

$$T_d = 5ns/m \times 60m = 300ns \tag{19}$$

$$\Rightarrow f_{RING} = \frac{1}{2\pi T_d} \approx 530516.5Hz \tag{20}$$

$$f_{KNEE} = \frac{0.5}{t_{sw}} = \frac{0.5}{10^{-8}} = 5 \times 10^7 Hz \tag{21}$$

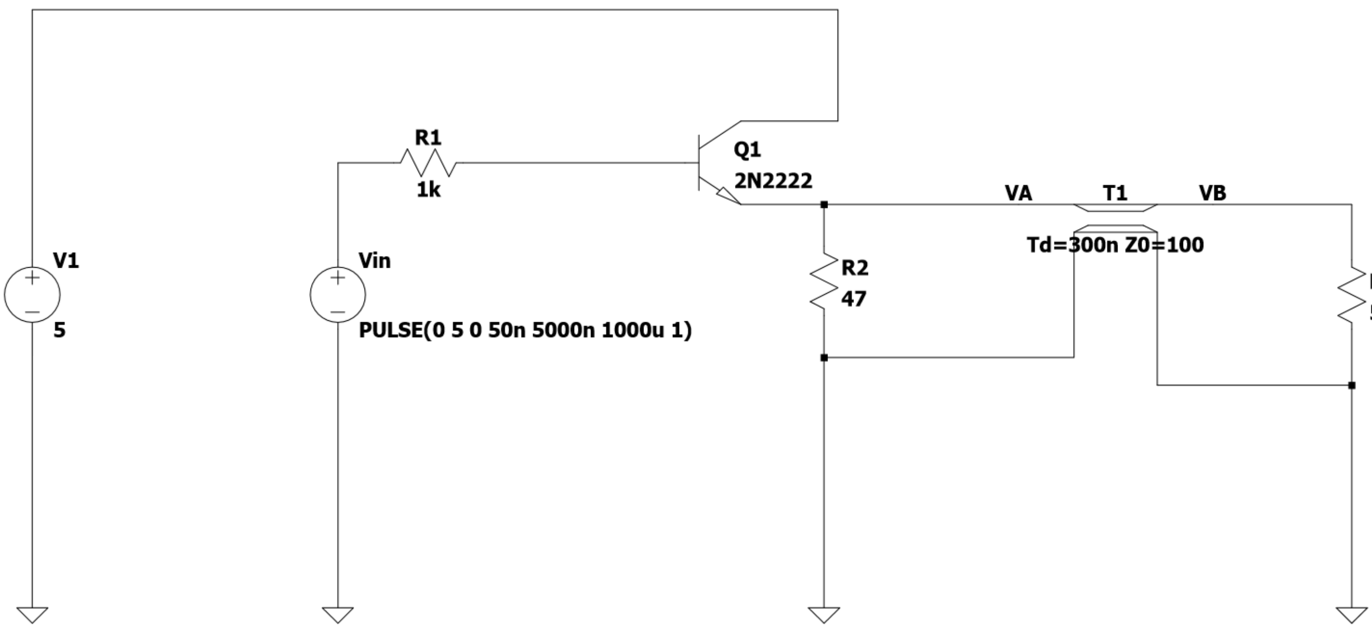
3.1.4

Since $f_{KNEE} >> f_{RING}$, the signal distortion will be a significant issue for this combination of signal and cable.

3.2 Simulation

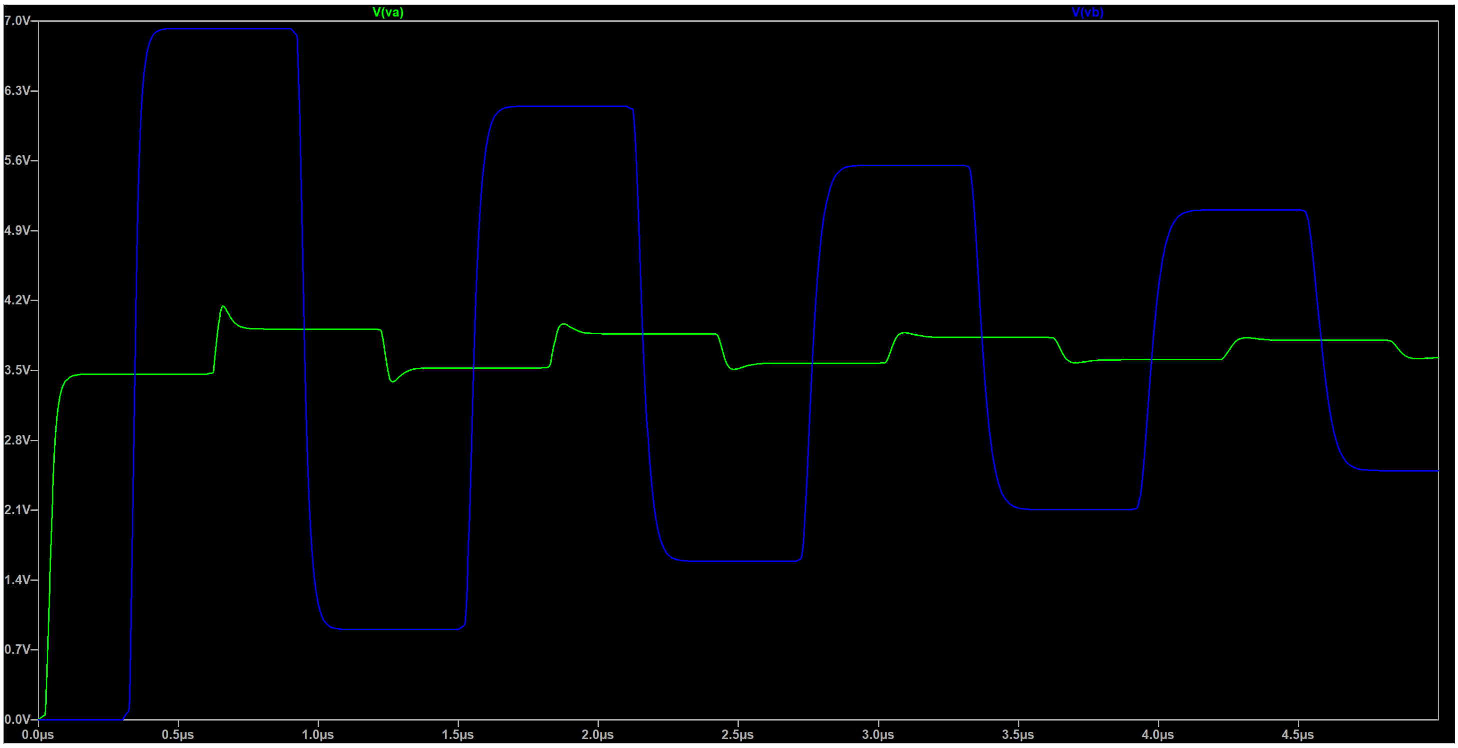
3.2.1

$T_d = 300ns$ and $Z_0 = 100\Omega$:

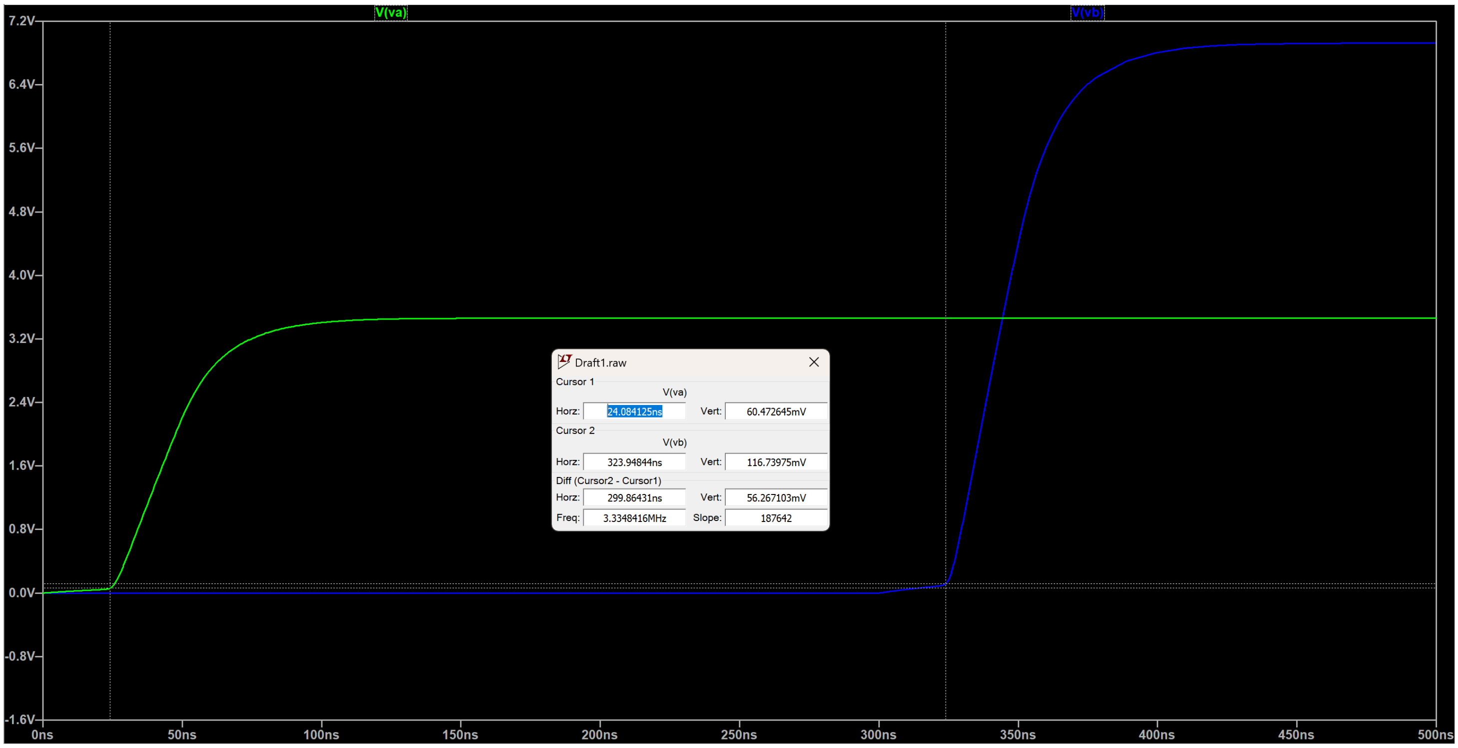


3.2.2

The simulation waveform is as follows (green-Va,blue-Vb):

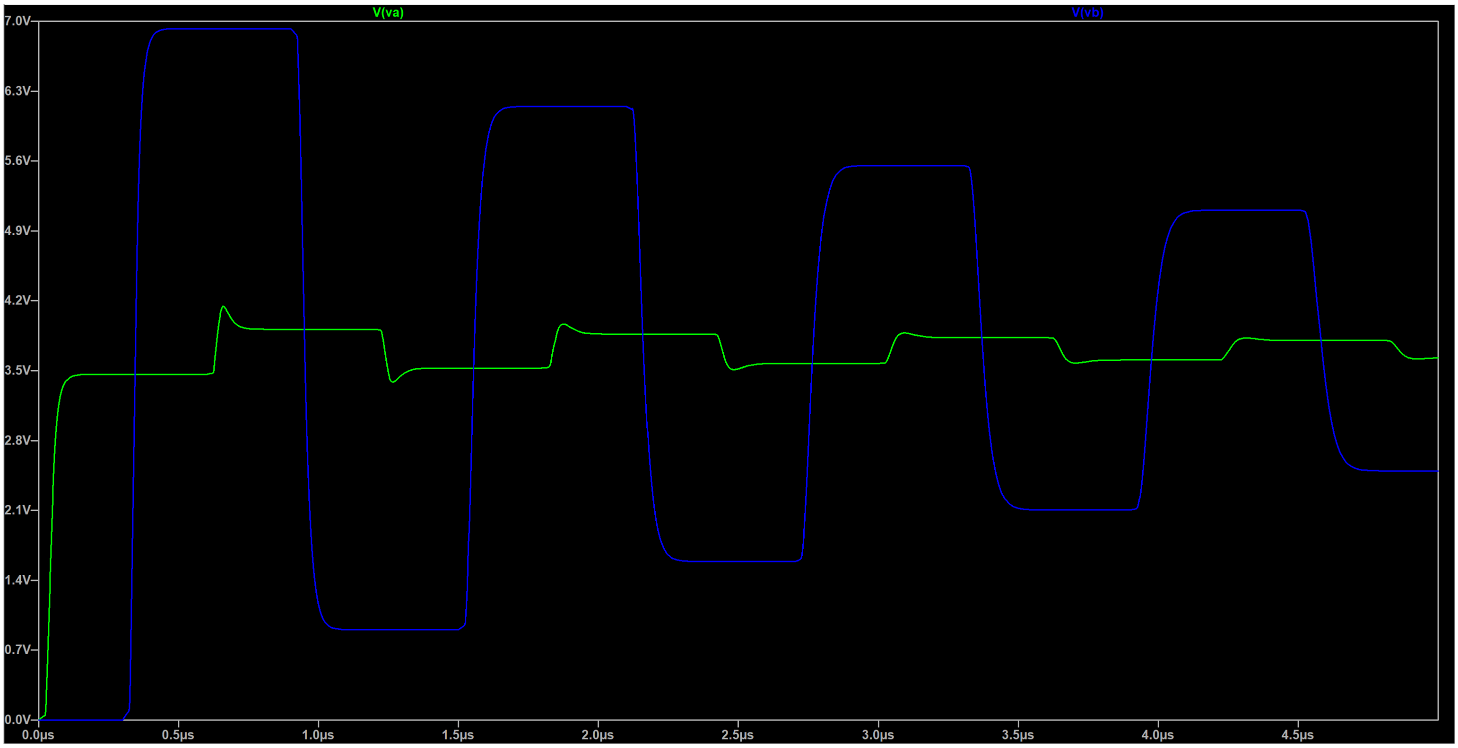


Use cursors to measure the delay:



We measure $T_d \approx 300ns$.

3.2.3



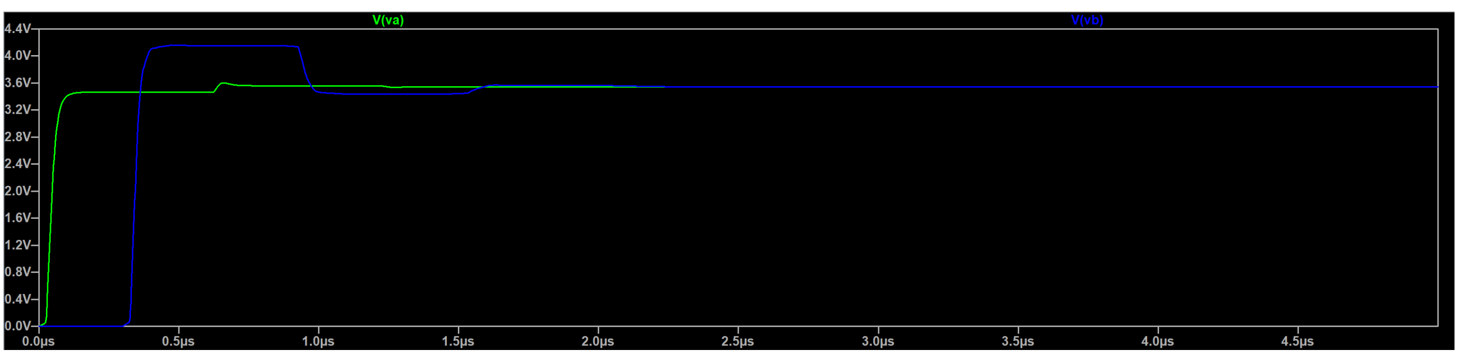
Yes, signal distortion is a significant issue for this network interconnection.

In low frequency signals, this problem of signal distortion may not be a significant issue. However, in high frequency circuits, we need to consider the problem of impedance matching. The reflected signal superimposed on the original signal will change the shape of the original signal and cause distortion. This distortion could lead to signal integrity problems, potentially affecting the accuracy of data transmission.

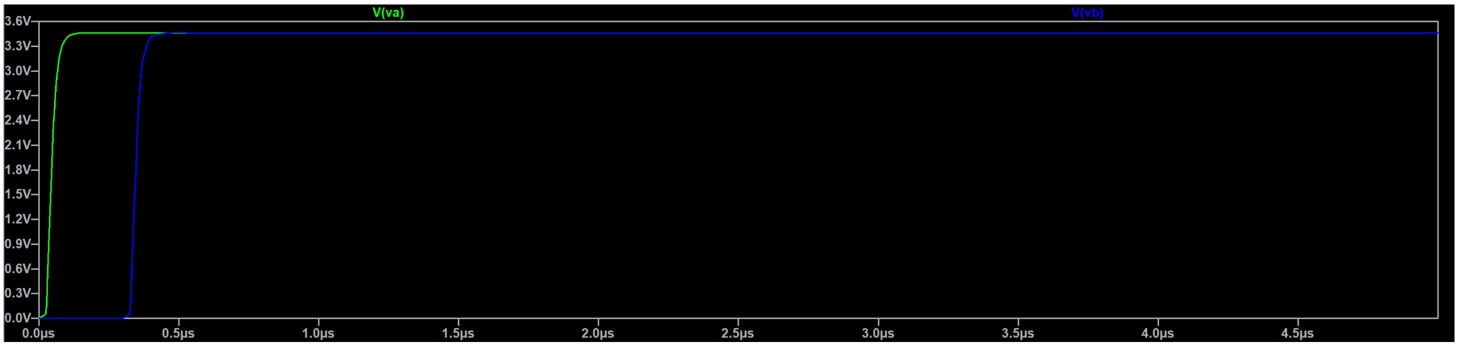
(We can use Smith's chart to deal with this problem.)

3.2.4

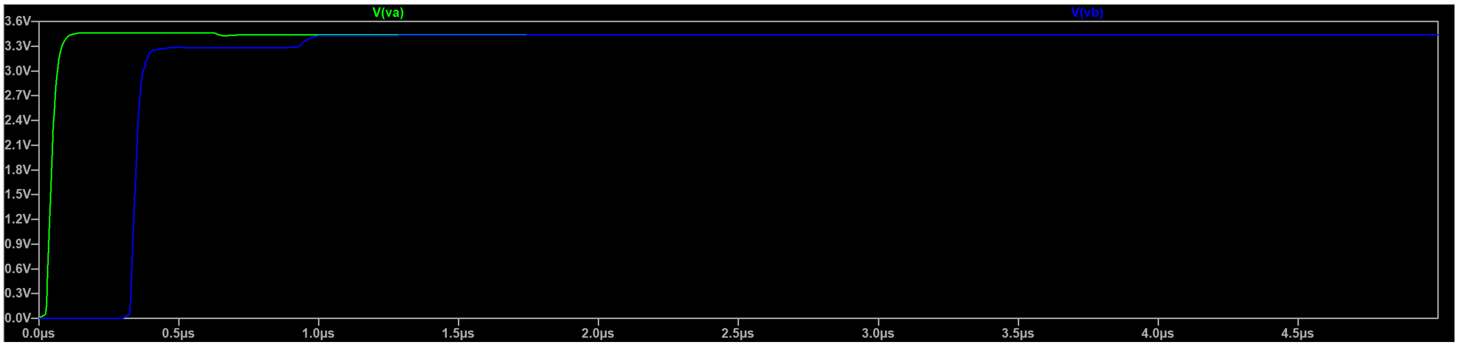
$R = 150\Omega$:



$R = 100\Omega$:



$R = 90\Omega$:



After adjusting the resistance value of the resistor several times, we believe that there is no distortion when $R = 100\Omega$ (impedance matching). It equals to Z_0.

3.3 Hardware investigation

3.3.1

- Green - Green: 5.566Ω
- White - White: 5.522Ω
- Black - Black: 1.934Ω
- Red - Red: 1.954Ω

3.3.2

We use Green and White line. The blue line on the bottom is Va and the yellow line on the top is Vb.

3.3.3

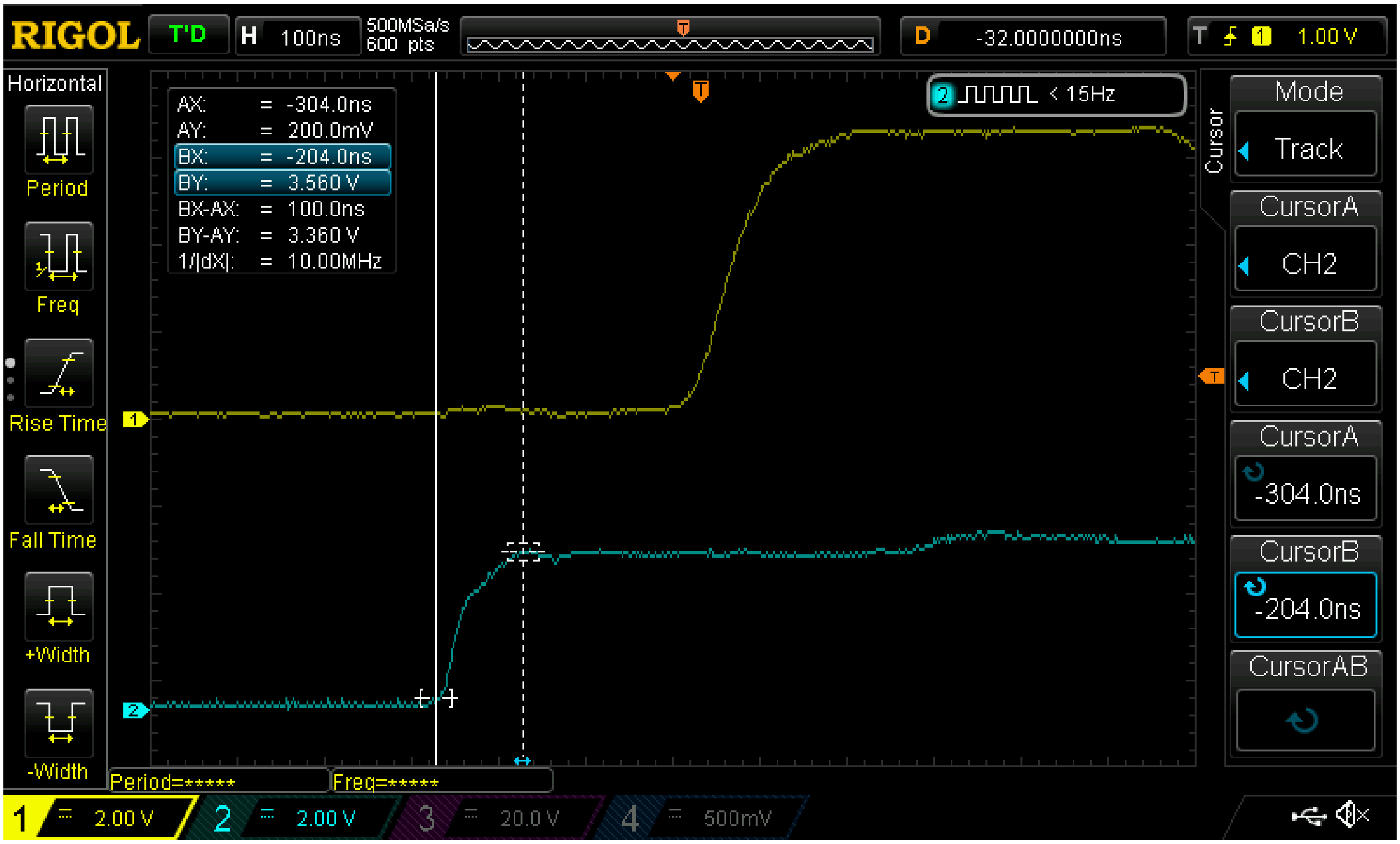
Yes. As shown in 3.2.3, we think it is an impedance mismatch problem. The reflected signal superimposed on the original signal will change the shape of the original signal and cause distortion. This distortion could lead to signal integrity problems, potentially affecting the accuracy of data transmission.

Moreover, we notice that the measured signal is even more distorted:

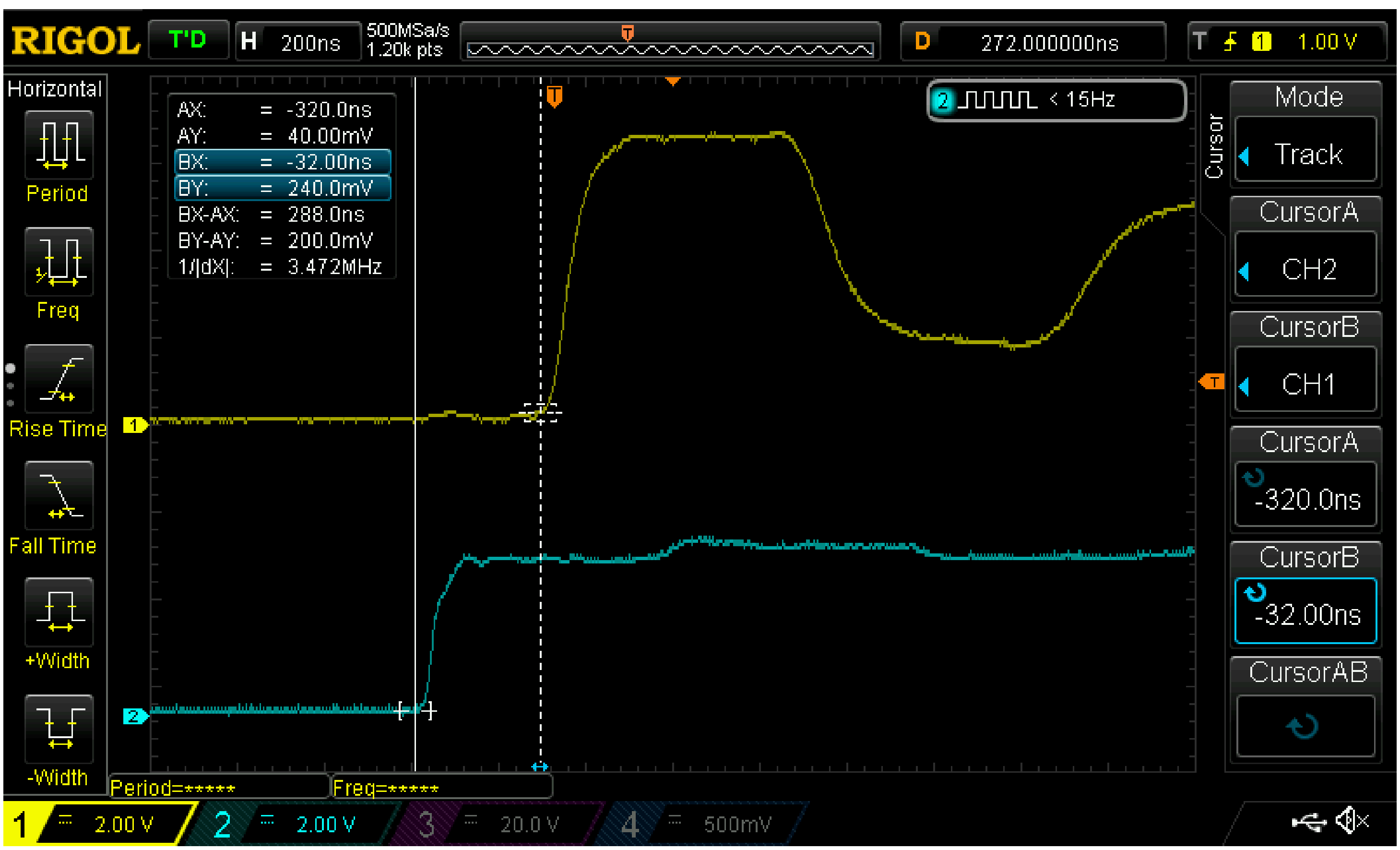
- Breadboards, wires and the environments may introduce more noise.
- Measurements are inevitably subject to error.
- Because of Gibbs phenomenon, the input square wave will overshoot(9%).

3.3.4

Move the cursors to measure rise time and delay:



As shown in the figure, rise time t_{SW} of V_A is 100ns.



The measured value $T_d = 288ns$.

Hence,

$$D = \frac{288ns}{60m} = 4.8ns/m \tag{22}$$

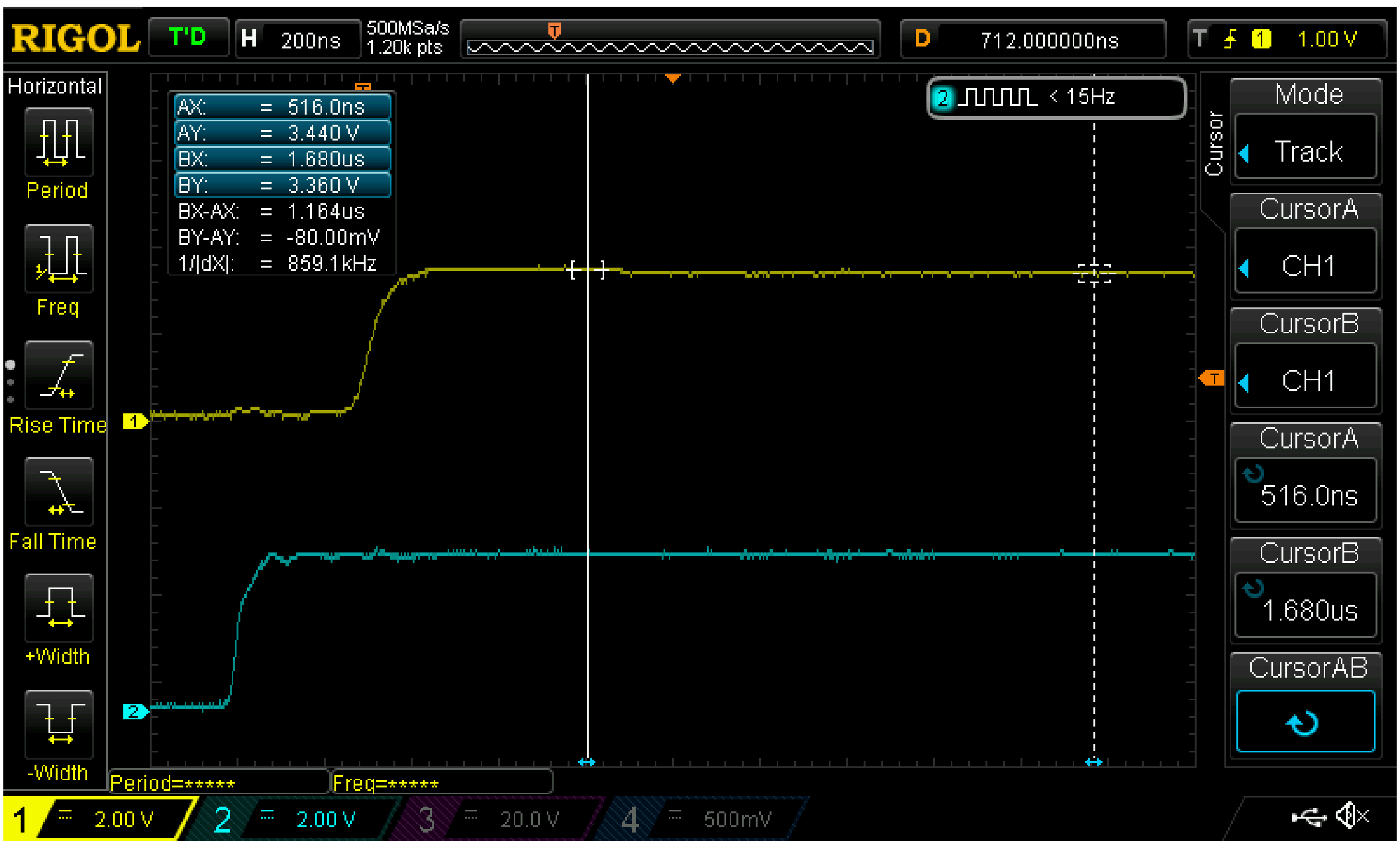
$$f_{RING} = \frac{1}{2\pi T_d} \approx 552621.33Hz \tag{23}$$

$$f_{KNEE} = \frac{0.5}{t_{sw}} = \frac{0.5}{10^{-7}} = 5 \times 10^6 Hz \tag{24}$$

f_{KNEE} is 10 times larger than f_{RING}, the distortion still exists.

3.3.5

By adjusting the variable resistance, we make the output waveform as follows:



It is measured that the resistance at this time is $R = 112\Omega$:

We've got $D = 4.8ns/m$, $Z_0 = R = 112\Omega$ (Impedance matching).

$$Z_0 = \sqrt{\frac{\hat{L}}{\hat{C}}} = 112$$
$$D = \sqrt{\hat{L}\hat{C}} = 4.8 \times 10^{-9}$$

Hence,

$$\hat{L} = 5.3 \times 10^{-7} H/m \tag{25}$$
$$\hat{C} = 4.3 \times 10^{-11} F/m \tag{26}$$

So we estimate from our measurements $\hat{L} = 5.3 \times 10^{-7} H/m$ and $\hat{C} = 4.3 \times 10^{-11} F/m$.