ELEN30011 EDM Task

- Xiufu SUN 1372750
- Wenyang SUN 1354302

1.1

$\nabla \cdot \mathbf{F}$ (div $\mathbf{F}$) is a scalar, $\nabla \times \mathbf{F}$ is a vector field.

Explaination: $\Downarrow$

1.2

Let $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field with

$$\mathbf{F}(x, y, z) = F_x(x, y, z)\hat{\mathbf{x}} + F_y(x, y, z)\hat{\mathbf{y}} + F_z(x, y, z)\hat{\mathbf{z}}$$

The divergence of $\mathbf{F}$ (div $\mathbf{F}$) is

$$\begin{aligned} \nabla \cdot \mathbf{F} &= (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \cdot (F_x, F_y, F_z) \\ &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \end{aligned}$$

$\nabla \cdot \mathbf{F}$ is a scalar.

$$\begin{aligned} \nabla \times \mathbf{F} &= (\frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}) \times (F_x(x, y, z)\hat{\mathbf{x}} + F_y(x, y, z)\hat{\mathbf{y}} + F_z(x, y, z)\hat{\mathbf{z}}) \\ &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \\ &= (\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z})\hat{\mathbf{x}} + (\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x})\hat{\mathbf{y}} + (\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x})\hat{\mathbf{z}} \end{aligned}$$

$\nabla \times \mathbf{F}$ is a vector field.

1.3

(a)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$$

(b)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = 1\hat{\mathbf{x}} + z\hat{\mathbf{y}} + y\hat{\mathbf{z}}$$

(c)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = x\hat{\mathbf{x}} + (y + \frac{1}{2}z^2 siny)\hat{\mathbf{y}} - zcosy\hat{\mathbf{z}}$$

(d)

$$grad f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}} = \frac{2x}{x^2+y^2+z^2}\hat{\mathbf{x}} + \frac{2y}{x^2+y^2+z^2}\hat{\mathbf{y}} + \frac{2z}{x^2+y^2+z^2}\hat{\mathbf{z}}$$

1.4

(a)

$$div\mathbf{F} = \boldsymbol{\nabla} \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0$$

$$curl\mathbf{F} = \boldsymbol{\nabla} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 0 \end{vmatrix} = 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$$

(b)

$$div\mathbf{F} = \boldsymbol{\nabla} \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = -1 + 1 = 0$$

$$curl\mathbf{F} = \boldsymbol{\nabla} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x & 0 & z \end{vmatrix} = 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$$

(c)

$$div\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0$$

$$curl\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & 0 & -x \end{vmatrix} = [1-(-1)]\hat{\mathbf{y}} = 2\hat{\mathbf{y}}$$

(d)

$$div\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y} + \frac{\partial^2 f}{\partial z^2}$$

$$\begin{aligned} curl\mathbf{F} = \nabla \times \mathbf{F} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix} \\ &= (\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y})\hat{\mathbf{x}} + (\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z})\hat{\mathbf{y}} + (\frac{\partial^2 f}{\partial y \partial x} - \frac{\partial^2 f}{\partial x \partial y})\hat{\mathbf{z}} \\ &= 0\hat{\mathbf{x}} + 0\hat{\mathbf{y}} + 0\hat{\mathbf{z}} \end{aligned}$$

1.5

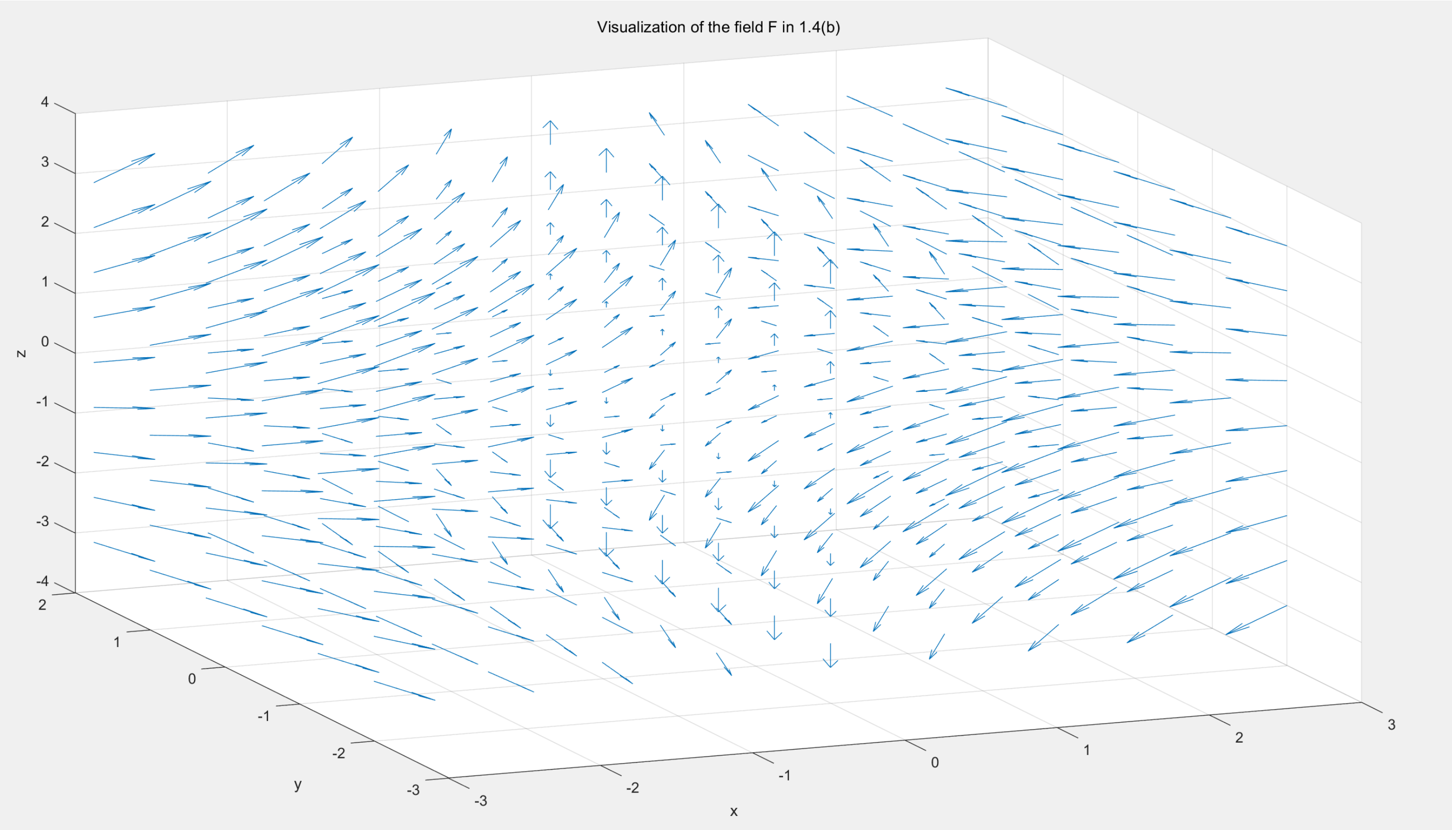
```
close all
clear
clc

x = -3:.75:3;
y = -2:.75:2;
z = -3:.75:3;

[X, Y, Z] = meshgrid(x, y, z);

FX = -X;
FY = 0.*Y;
FZ = Z;

figure(1);
quiver3(X,Y,Z,FX,FY,FZ)
xlabel("x");
ylabel("y");
zlabel("z");
title("Visualization of the field F in 1.4(b)")
```



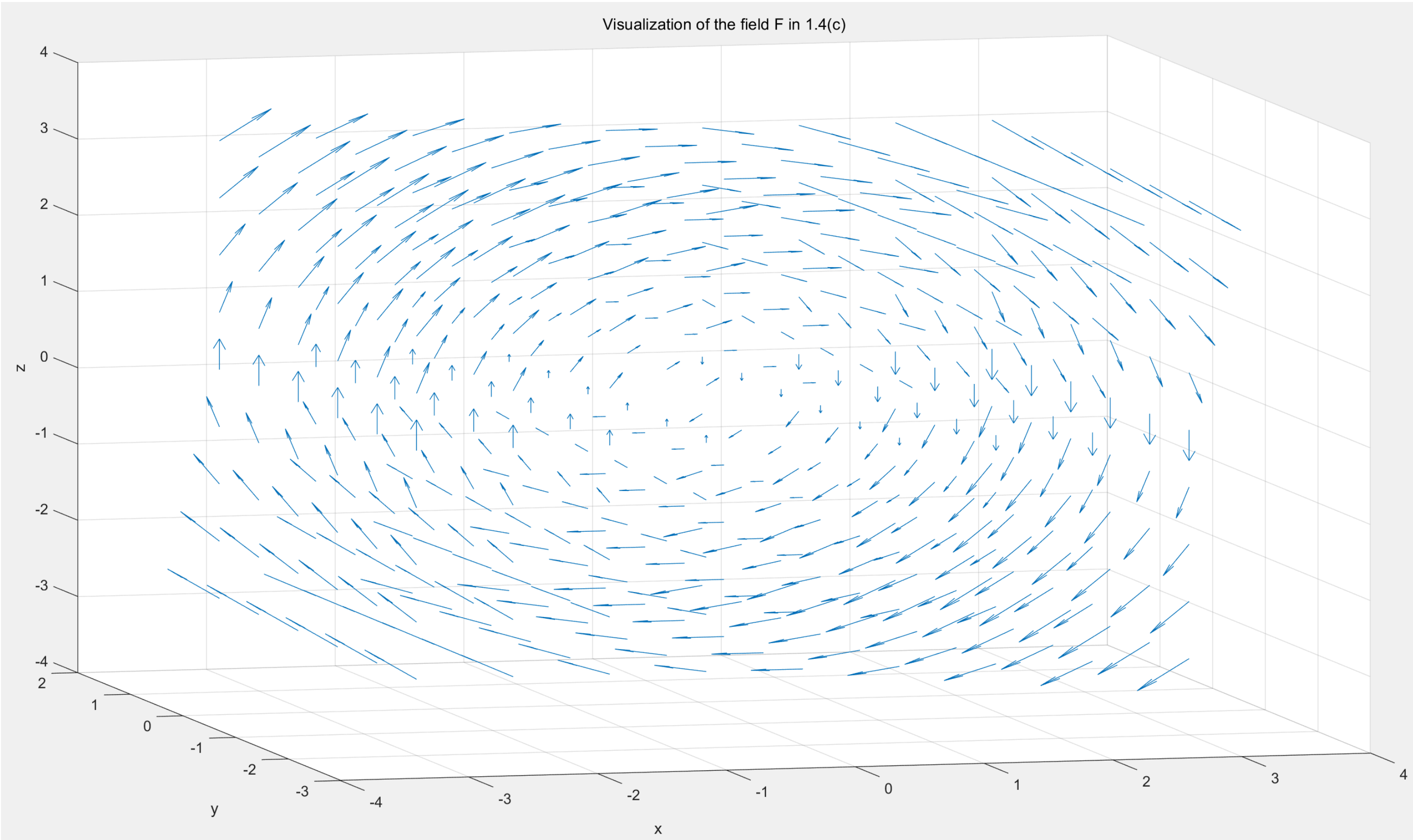

```
close all
clear
clc

x = -3:.75:3;
y = -2:.75:2;
z = -3:.75:3;

[X, Y, Z] = meshgrid(x, y, z);

FX = Z;
FY = 0.*Y;
FZ = -X;

figure(1);
quiver3(X,Y,Z,FX,FY,FZ)
xlabel("x");
ylabel("y");
zlabel("z");
title("Visualization of the field F in 1.4(c)")
```



2.1

2.1.1

$\mathbf{l_1}$ is a straight line.

$$\mathbf{l_1}(s) = (1-s)\mathbf{P_1} + s\mathbf{P_2}$$

$\mathbf{l_2}$ is circular arc of radius 1.

Let $x = cos(\frac{\pi s}{4})$, $y = sin(\frac{\pi s}{4})$.

$$x^2 + y^2 = cos^2(\frac{\pi s}{4}) + sin^2(\frac{\pi s}{4}) = 1$$

In cartesian coordinates, $x^2 + y^2 = 1$ is describing a circle with a radius of 1.

Plus, since $s \in [0, 1]$, $x \in [\frac{\sqrt{2}}{2}, 1]$, $y \in [0, \frac{\sqrt{2}}{2}]$.

When s increases, $\mathbf{l_2}$ moves from $(1, 0)$ to $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ along this circle, anticlockwise.

2.1.2

Fomular (8) shows that,

$$\mathbf{E}(x,y,z) = -y\hat{\mathbf{x}} - x\hat{\mathbf{y}}$$

For $\mathbf{l_1}$, $x = 1 - s + \frac{s}{\sqrt{2}}$, $y = \frac{s}{\sqrt{2}}$

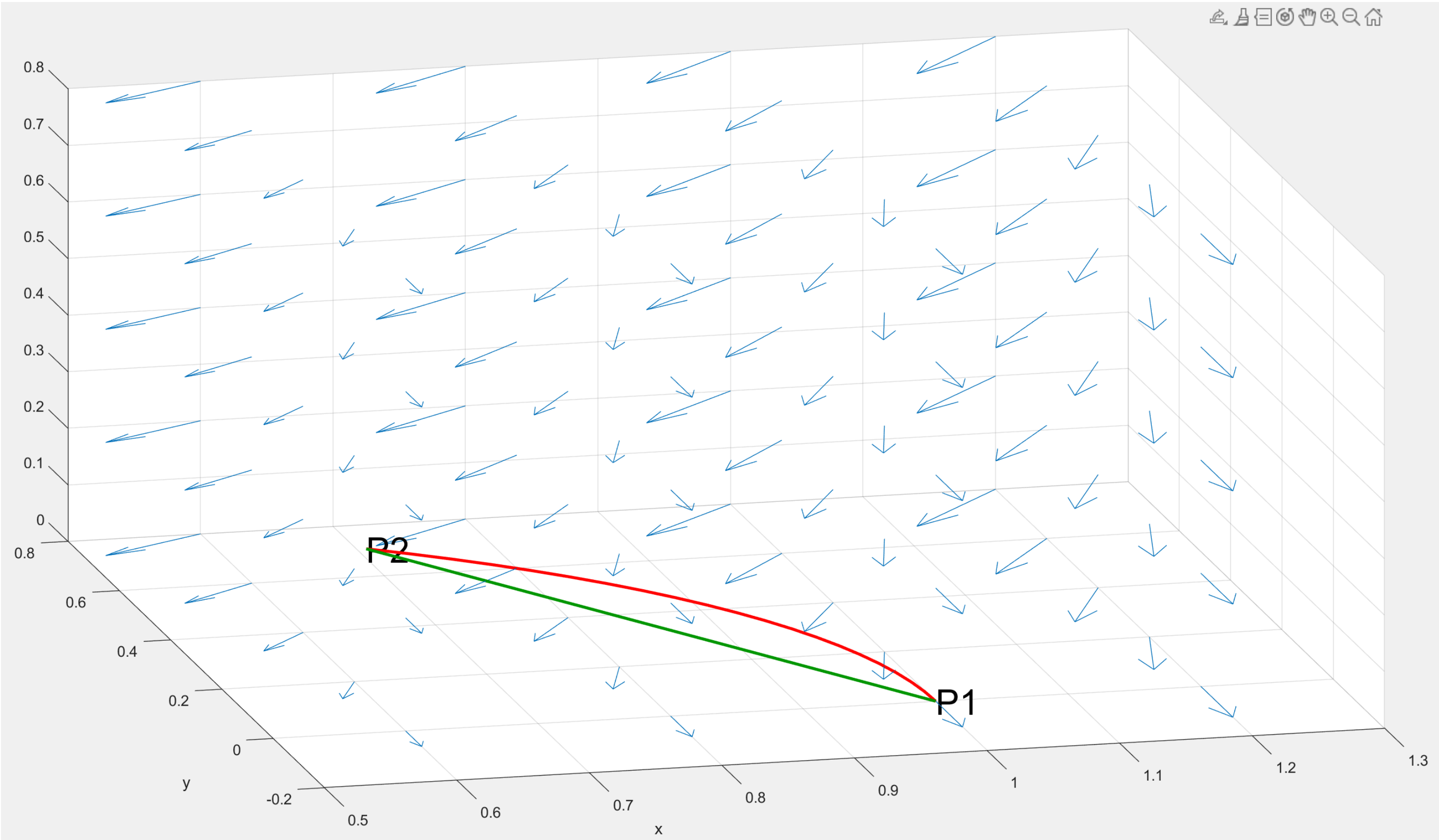
$$\mathbf{E}(\mathbf{l_1}(s)) = -\frac{s}{\sqrt{2}}\hat{\mathbf{x}} - (1 - s + \frac{s}{\sqrt{2}})\hat{\mathbf{y}}$$

For $\mathbf{l_2}$, $x = cos(\frac{\pi s}{4})$, $y = sin(\frac{\pi s}{4})$

$$\mathbf{E}(\mathbf{l_2}(s)) = -sin(\frac{\pi s}{4})\hat{\mathbf{x}} - cos(\frac{\pi s}{4})\hat{\mathbf{y}}$$

2.1.3

```
x = 0.6:.2:1.2; y = 0:.2:.8; z = 0:.2:.8;
[xx,yy,zz] = meshgrid(x,y,z);
Exx =-yy; Eyy =-xx; Ezz = 0*xx;
figure(2);
quiver3(xx,yy,zz,Exx,Eyy,Ezz);
grid on; hold on;
xhat = [1;0;0]; yhat = [0;1;0];
s = 0:.05:1;
L1 = (1- s).*xhat + s.*(xhat + yhat)/sqrt(2);
L2 = cos(pi .*s ./4).*xhat + sin(pi .*s ./4).*yhat;
plot3(L1(1,:),L1(2,:),L1(3,:),'Color',[0 .6 0],'LineWidth',2);
plot3(L2(1,:),L2(2,:),L2(3,:),'Color',[1 0 0],'LineWidth',2);
xlabel("x")
ylabel("y")
text(1,0,0,"P1","FontSize",24)
text(1/sqrt(2), 1/sqrt(2), 0, "P2", "FontSize",24)
```



2.1.4

$$
\begin{aligned}
V_{l_1} &= -\int_{l_1} \mathbf{E} \cdot dl_1 \\
&= -\int_{l_1} \mathbf{E}(\mathbf{l_1}(\mathbf{s})) \cdot \frac{d\mathbf{l_1}}{ds}(s)ds \\
&=
\end{aligned}
$$