

# ELEN30011 EDM Task

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## 1.1

For rectangular coordinates and cylindrical coordinates,

$$x = r\cos\phi$$

$$y = r\sin\phi$$

$$z = z$$

(a)

$$(1, 0, 0)$$

(b)

$$(-1, 0, 0)$$

(c)

$$(0, -1, 3)$$

(d)

$$(0, 0, -2)$$

(e)

$$(-1, 0, 0)$$

## 1.2

For spherical coordinates and rectangular coordinates,

$$x = r\cos\phi\sin\theta$$

$$y = r\sin\phi\sin\theta$$

$$z = r \cos \theta$$

It can be noted that,  $r$  is the modulus of a vector  $\sqrt{x^2 + y^2 + z^2}$ ,  $\phi$  is the angle with the x-axis in x-y plane,  $\theta$  is the angle with the z-axis.

(a)

$$(1, 0, \pi/2)$$

(b)

$$(1, \pi/2, \pi/2)$$

(c)

$$(1, 0, 0)$$

- $\phi$  can be any real number here.

(d)

$$(\sqrt{2}, \pi/2, \pi/4)$$

(e)

$$(0, 0, 0)$$

- $r = 0$ ,  $\phi$  and  $\theta$  can be any real number here.

## 2.1

(a) Let

$$x = r \cos \phi, y = r \sin \phi, z = z$$

Jacobin Matrix:

$$J(r, \phi, z) = \begin{pmatrix} \cos \phi & -r \sin \phi & 0 \\ \sin \phi & r \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = J \begin{pmatrix} dr \\ d\phi \\ dz \end{pmatrix}$$

Hence find expressions for  $\hat{r}$  and  $\hat{\phi}$  in terms of  $\hat{x}$  and  $\hat{y}$ .

Since  $\hat{r}$  and  $\hat{\phi}$  are unit vectors, let  $r = 1$

$$T(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Or, considering

$$\begin{aligned}\hat{r} &= \cos\phi\hat{x} + \sin\phi\hat{y} \\ \hat{\phi} &= -\sin\phi\hat{x} + \cos\phi\hat{y} \\ v &= v_r\hat{r} + v_\phi\hat{\phi} + v_z\hat{z}\end{aligned}$$

$$\begin{aligned}v_x &= v \cdot \hat{x} = v_r\cos\phi - v_\phi\sin\phi \\ v_y &= v \cdot \hat{y} = v_r\sin\phi + v_\phi\cos\phi \\ v_z &= v \cdot \hat{z} = v_z\end{aligned}$$

In order to make

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = T(\phi) \begin{pmatrix} v_r \\ v_\phi \\ v_z \end{pmatrix}$$
$$T(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b)

We've got that

$$T(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Hence,

$$(T^*(\phi))^T = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^*(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det[T(\phi)] = 1$$

Based on Cramer's rule,

$$T^{-1}(\phi) = \frac{1}{\det} T^*(\phi)$$

Hence,

$$T^{-1}(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c)

$$r = \sqrt{x^2 + y^2}$$

$$\cos(\phi) = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\sin(\phi) = \frac{y}{\sqrt{x^2 + y^2}}$$

(d)

$$T^{-1}(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S(P) = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 0 \\ -\frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## 2.2

In Question 2.1(d), we've got

$$S(P) = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 0 \\ -\frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a)  $P = (0, -1, 0)$

$$S(P) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b)  $P = (1, 0, 0)$

$$S(P) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c)  $P = (-1, 0, 0)$

$$S(P) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(d)  $P = (1, -1, 0)$

$$S(P) = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(e)  $P = (0, 0, 0)$

$$S(P) = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## 3.1

(a)

```
x = -2:.1:2;
y = -2:.1:2;
[xx, yy] = meshgrid(x, y);

size(xx)
size(yy)
```

Which output is:

```
ans =
```

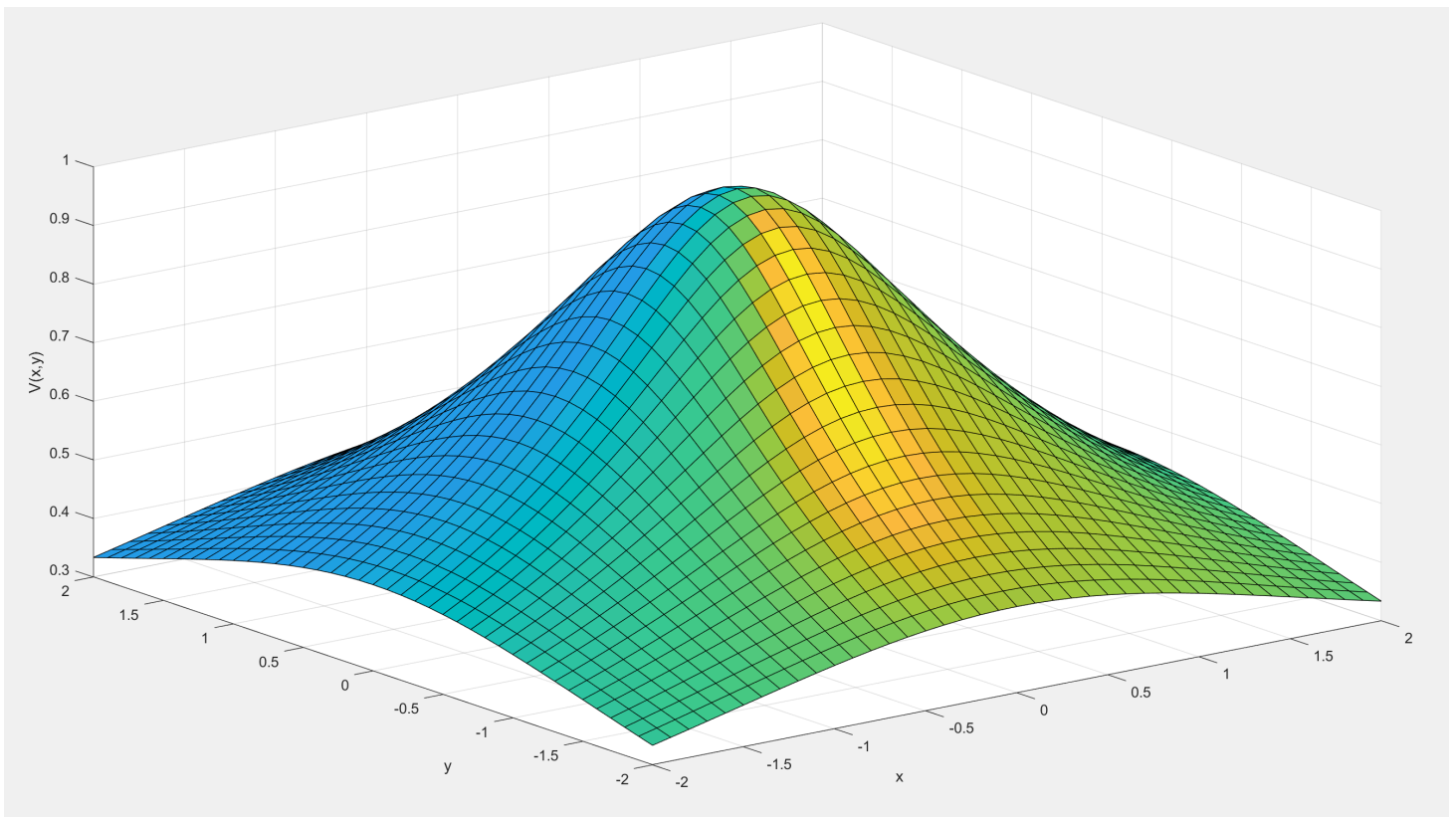
```
    41    41
```

```
ans =
```

```
    41    41
```

(b)

```
x = -2:.1:2;  
y = -2:.1:2;  
[xx, yy] = meshgrid(x, y);  
  
size(xx)  
size(yy)  
  
zz = 1./sqrt(1 + xx.^2 + yy.^2);  
figure(1);  
surf1(xx, yy, zz);  
xlabel('x');  
ylabel('y');  
zlabel('V(x,y)');  
grid on;
```



Based on the picture above, it has been shown that the surface exhibit a maximum.

After checked the value "zz" in workspace, we get the maximum point is  $(0, 0, 1)$ .

For certain plane, origin can always be the point with the highest electrostatic potential. If a charge moves in any direction on its x-y plane, the electric field does positive work on it.

(c)

A circle.

$$V = \frac{1}{\sqrt{1 + x^2 + y^2}}$$

$$\sqrt{1 + x^2 + y^2} = \frac{1}{V}$$

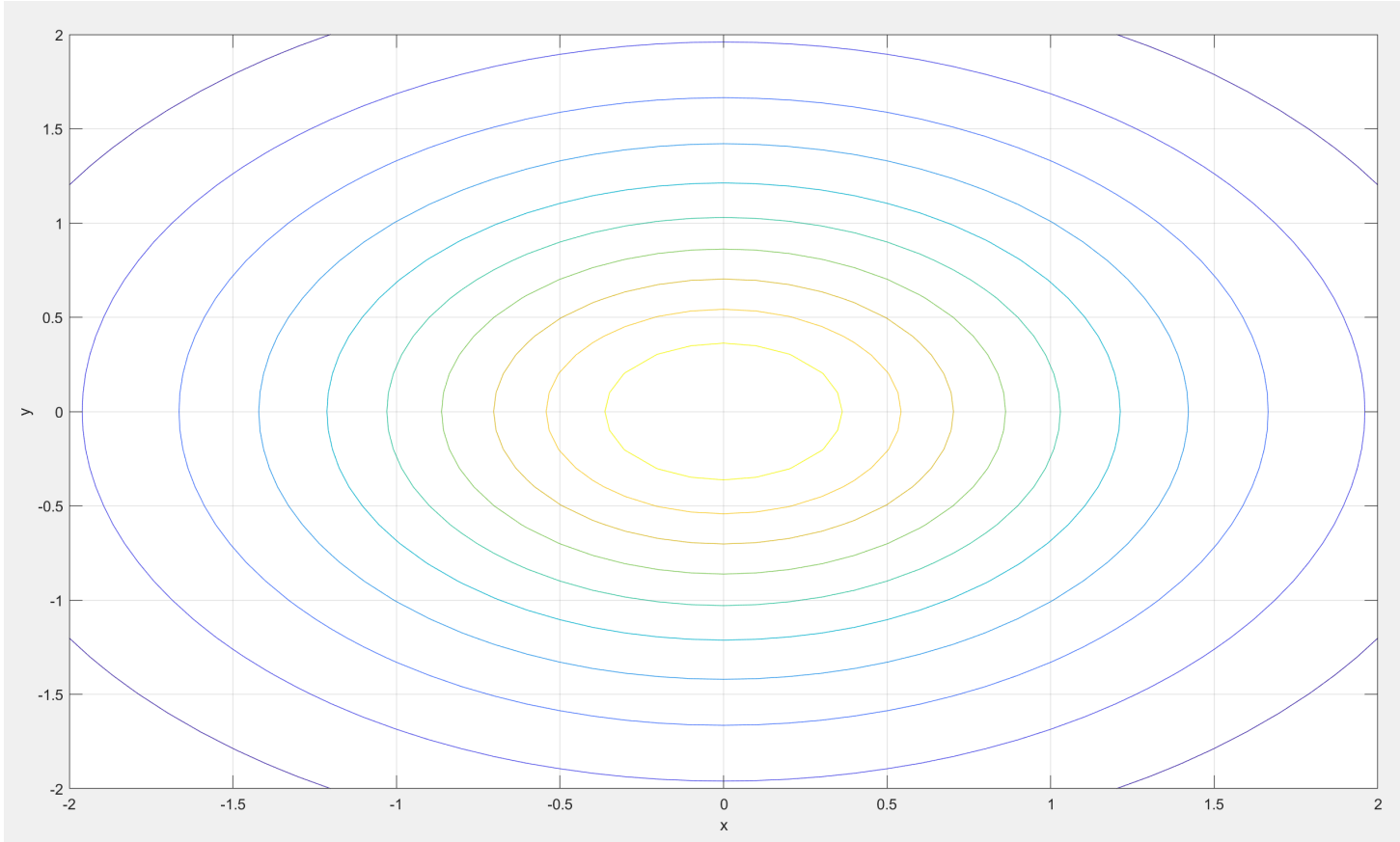
$$x^2 + y^2 = \frac{1}{V^2} - 1$$

Its radius is  $\sqrt{\frac{1}{V^2} - 1}$

If  $V = c > 1$ , radius will be an imaginary number, which is impossible here. Hence, c will never be greater than 1.

(d)

```
figure(2);
contour(xx, yy, zz, 10);
xlabel('x');
ylabel('y');
grid on;
```



"10" means: Display 10 contour lines at automatically chosen levels (heights).

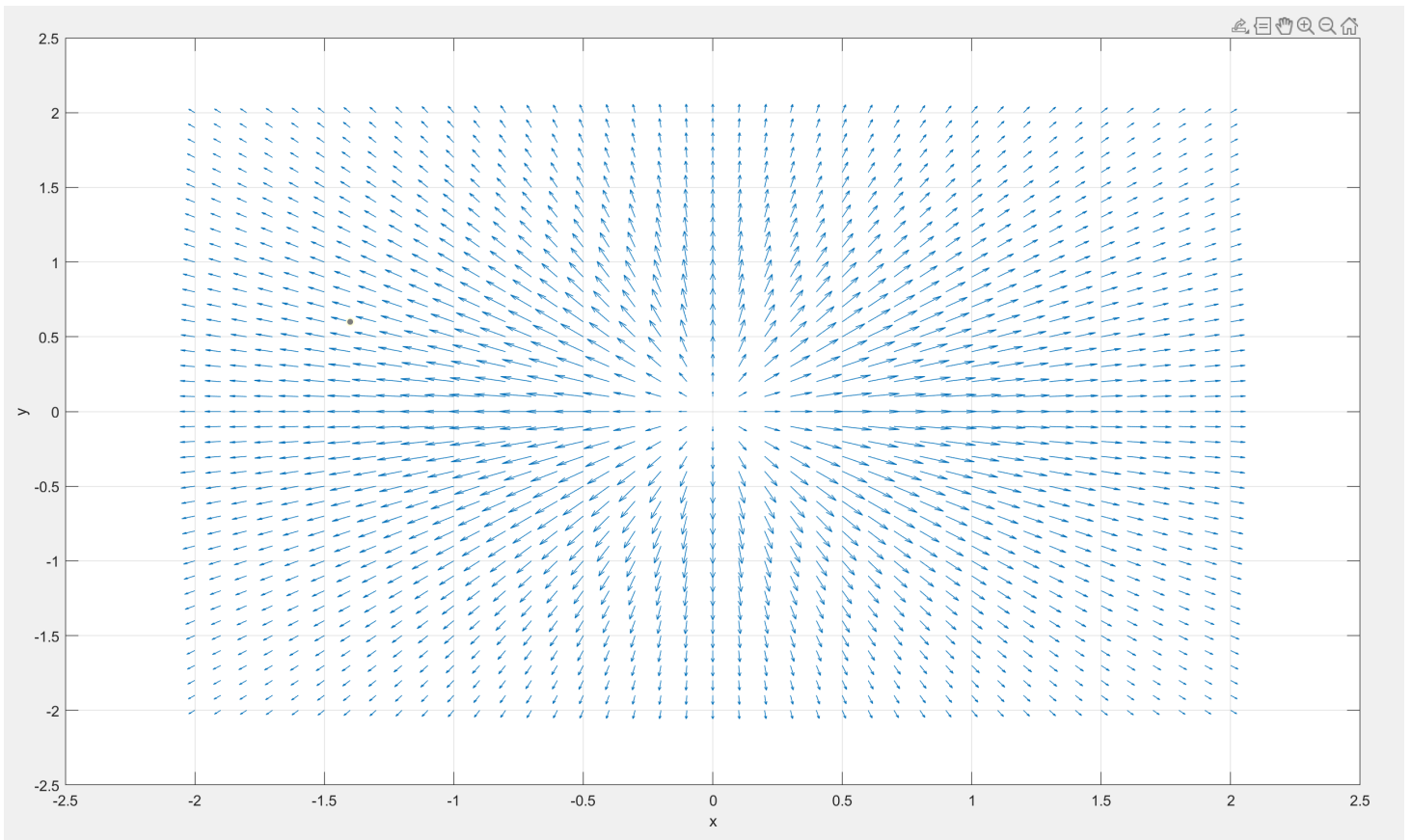
## 3.2

(a)

```
exx = xx./(1 + xx.^2 + yy.^2).^(3/2);
eyy = yy./(1 + xx.^2 + yy.^2).^(3/2);

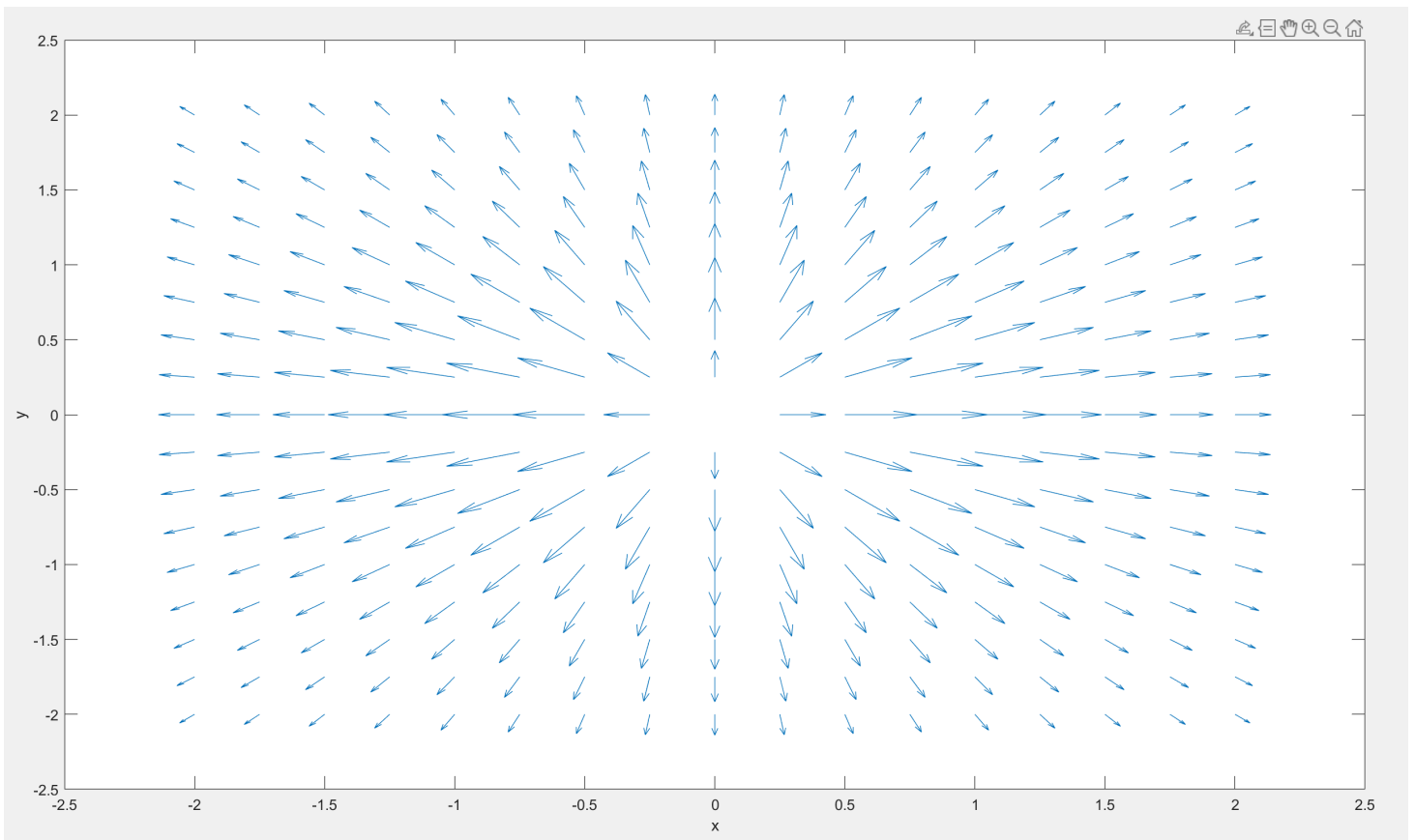
figure(3);
quiver(xx,yy,exx,eyy);
xlabel('x');
ylabel('y');
grid on;
```





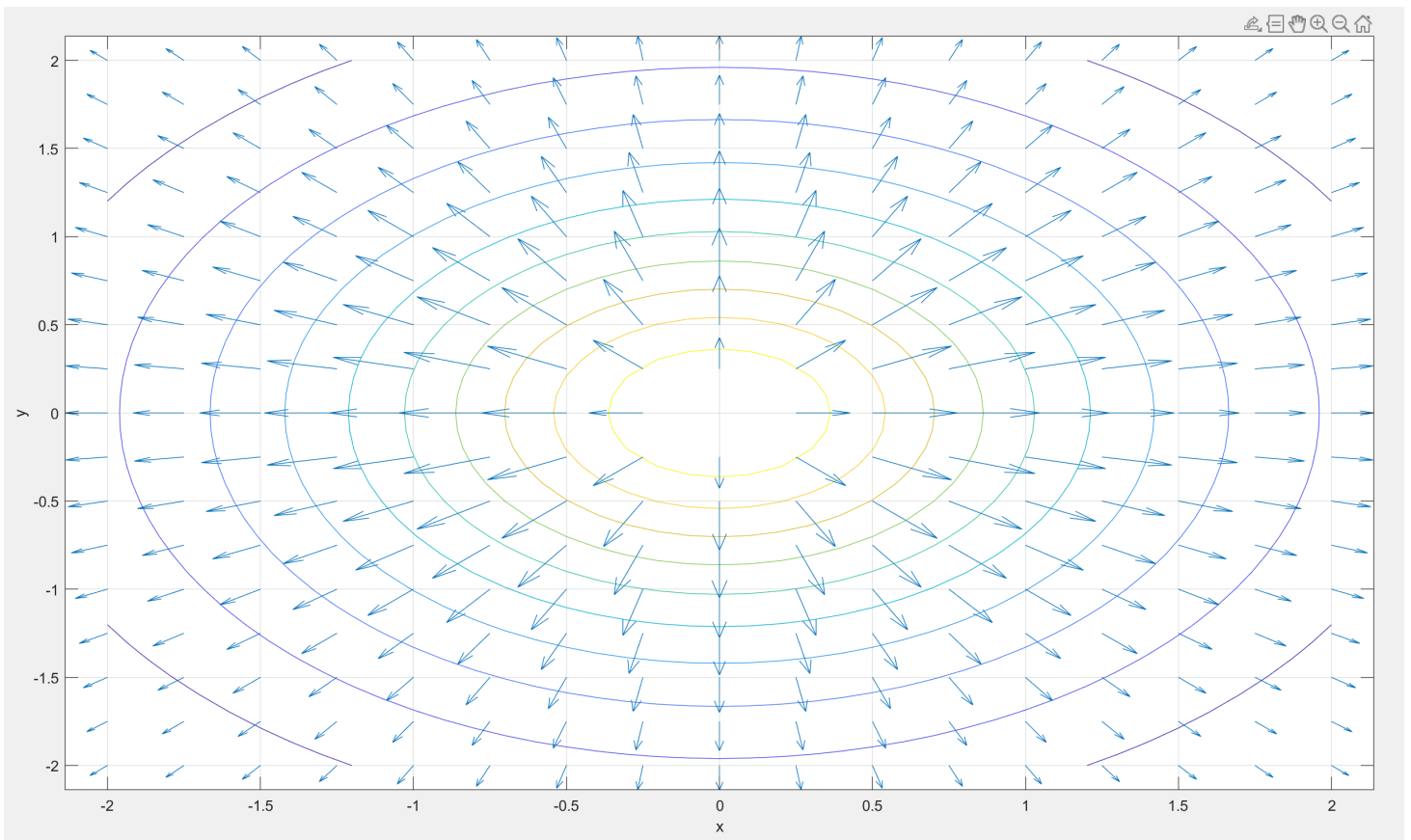
(b)

```
xnew = -2:.25:2;  
ynew = xnew;  
[xxnew, yynew] = meshgrid(xnew, ynew);  
exxnew = xxnew./(1 + xxnew.^2 + yynew.^2).^(3/2);  
eyynew = yynew./(1 + xxnew.^2 + yynew.^2).^(3/2);  
  
figure(4);  
quiver(xxnew, yynew, exxnew, eyynew);  
hold on;
```



(c)

```
figure(2);  
hold on;  
quiver(xxnew, yynew, exxnew, eyynew);
```



Perpendicular to each other.

## 3.3

(a)