Student Number:

THE UNIVERSITY OF MELBOURNE
Department of Electrical and Electronic Engineering

ELEN90061 Communication Networks

Mid-Semester Test 2017

Time allowed: 45 minutes
This paper has 12 pages

Authorised materials:

Approved electronic calculators are permitted.

Mobile phones, tablets, or any other computing devices are NOT permitted!

This is a closed book and notes test!

Instructions to students:

Please write your answers clearly and legibly to the provided boxes. *Show all your work to receive full credit!* You can use the back of the pages for intermediate steps when needed.

The marks for each question are indicated in brackets after the question.

The total marks for this test is **30**.

Question 1 [12 marks]:

Explain your answers for full marks. No marks if there is no explanation or derivation!

10 children play a game of cops and robbers (the rules of the game are not important) within a **circular playground of radius r**. Each child has a **walkie-talkie** (*handheld wireless radio*) which allows them to talk to others by broadcasting their message to the entire circular area when they keep pressing the send button. When the send button is not pressed, which is the default case, all the radios listen to all the broadcasters in the area. When two or more walkie-talkies broadcast at the same time, their transmissions interfere with each other and get garbled, i.e. nobody understands the message sent. **Assume for simplicity** that all walkie-talkies use the same channel, the transmissions cover the entire circular area, and the children do not leave the playground.

1.1) Are these walkie-talkies, operating as described above, simplex, half-duplex, or full-duplex communication devices? Briefly explain your answer. **[1 mark]**

Half-duplex since they can both send & receive but not at the same time.

1.2) The children decide to adopt a random media access protocol to communicate better. The only rules they agree on are the following:

(i) Each child will only talk (broadcast) for 10 seconds in each attempt.

(ii) They will only start talking in the beginning of 10sec intervals, i.e. the start times of t=0sec, t=10secs, t=20secs and so on.

(iii) They will attempt to talk with **probability p** in each 10 second interval.

What is the probability that any child succeeds in communicating successfully (talking in a legible way) in any 10 second interval under this scheme? Explain your answer. **[2 marks]**

Hint: assume that each child has always something to say during the game!

This is a slotted ALOHA scheme.

$P_{success} = N \cdot p(1-p)^{N-1}$ for any node. $N=10$ here, so

$P_{success} = 10 \cdot p(1-p)^{9}$

1.3) The children agree on flipping a coin to decide broadcast or not broadcast in each interval, i.e. probability **p=0.5**. Is this the optimal probability? If yes, justify your answer with a derivation. Otherwise, derive the optimal probability. **[3 marks]**

See the solution of Question 3 of Week 2!

$$\max_p Np(1-p)^{N-1} = \max_p f(N,p)$$

to find p^*, set derivative to zero since objective is strictly concave (as discussed in class!)

$$\frac{\partial f}{\partial p} = N(1-p)^{N-1} - N(N-1)p(1-p)^{N-2} \quad \text{from calculus}$$

$$= N(1-p)^{N-2}\underbrace{\left[1-p-(N-1)p\right]}_{=0 \text{ for } p^*}$$

$$\Rightarrow 1-p-Np+p=0 \Rightarrow p^* = 1/N = 1/10 \text{ here}$$

Therefore $p=1/2$ is suboptimal, they should choose $p^* = \frac{1}{10}$ to maximise success probability.

1.4) Suggest two different improved communication protocols that the children can immediately use. Discuss the advantages and disadvantages of your proposals. **[6 marks]**
Hint: feel free to suggest any type of media access protocol that will work with the walkie-talkies they have. However, new hardware purchases or modifications are not allowed.

There are multiple options.

1) One obvious extension is using a CSMA variant, i.e. children check if anyone is talking in that slot before attempting to talk. Similarly CSMA/CD or CSMA/CA schemes can be designed. Min contention window size is 2r/c, with c speed of light and 2r maximum distance in playground circle. Saying their names in 1sec or so should take longer.
2) They can use taking turns or token-based schemes if the children assign an order to themselves, e.g. each child having a number.

Question 2 [8 marks]:

<u>Explain your answers for full marks. No marks if there is no explanation or derivation!</u>

Traffic arrives to a network router as a Poisson process with a mean of **λ** packets per second. The router has two parallel cores for processing packets. Each one processes the packets with a mean rate of **μ** packets/sec according to an exponential distribution. After processed by the router, each packet of size **B** bits is transmitted over an outbound fibre-optic link of length **L** km and bit rate of **R** bps. Assume a speed of light of **c** km/sec.

Clearly identify the components contributing to aggregate delay and find the aggregate average delay experienced by a packet when passing through the router and the link.

2) $d_{total} = d_{queue} + d_{service} + d_{transmission} + d_{propagation}$

* This is an M/M/2 queue.

– $d_{service} = \bar{X} = 1/\mu$, $\rho = \lambda/\mu$

* From Little's Law: $\bar{N_q} = \lambda \cdot W$; $W = d_{queue}$, here!

See formulas for M/M/m with m=2 for $\bar{N_q}$

– Then $d_{queue} = W = \bar{N_q}/\lambda$

– $d_{trans} = B/R$ See Week 3 slides!

– $d_{prop} = L/c$

$$\Rightarrow d_{total} = \frac{\bar{N_q}}{\lambda} + \frac{1}{\mu} + \frac{B}{R} + \frac{L}{C}$$

with $\bar{N_q}$ given for M/M/m, m=2

(P_0, $\rho = \lambda/\mu$).

Question 3 [10 marks]:

Explain your answers for full marks. No marks if there is no explanation or derivation!

The arrival of packets to a network switch is modelled as a Poisson process with rate of **λ=8** packets per second. The switch processes the packets with a mean rate of **μ=10** packets/sec. In a non-conventional way, the switch processes the packets in a Last-In, First-Out (LIFO) scheme, where the last arriving packet is processed first.

3.1) Assume that the service times are exponentially distributed. From the point of view of an arriving packet, what is the average waiting time in the queue and average number of packets in the queue? Explain your answers. **[4 marks]**

Hint: the variance of the service time of the mentioned exponential distribution is $\mathbf{1/\mu^2}$.

3.1) See Week 6 slides for info!

This is M/M/1 with LIFO.

Since arrivals are Poisson, PASTA property applies!

The P-K formula is valid for any order of servicing (as long as independent of service times) and M/M/1 is also M/G/1 by definition.

$\rho = \frac{\lambda}{\mu} = 0.8 \qquad Var(x) = \overline{x^2} - (\overline{x})^2 = \overline{x^2} - \frac{1}{\mu^2} = \frac{1}{\mu^2}$

$\Rightarrow \overline{x^2} = 2/\mu^2$

P-K: $W = \frac{\lambda \overline{x^2}}{2(1-\rho)} = \frac{\lambda \cancel{2}}{\mu^2 \cdot \cancel{2}(1-\rho)} = \underline{\frac{\rho}{\mu(1-\rho)}}$

and $\overline{N}_q = \lambda \cdot W = \underline{\frac{\rho^2}{1-\rho}}$ Same as M/M/1 formula!

(Little's Law)

* Note that M/M/1 formulas work both for LIFO and FIFO!

Numerically $W = \frac{0.8}{10(1-0.8)} = \underline{0.4 \text{ sec}}$

3.2) Assume that the service process is deterministic with the same mean service time. Find the average time a packet spends in the network switch (system) and the average number of packets in the network switch (system). **[6 marks]**

3.2) M/D/1 with LIFO. We use M/G/1, P-K formula

$\rho = 0.8$ $\quad \bar{X} = \frac{1}{\mu}$ deterministic, so $\overline{X^2} = \frac{1}{\mu^2}$

$W = \frac{\lambda}{2\mu^2(1-\rho)}$ and $T = \frac{1}{\mu} + W$

$N = \lambda \cdot T = \rho + \frac{\rho^2}{2(1-\rho)}$

Numerically, $\quad T = 0.1 + \frac{0.8}{2 \cdot 10 \cdot 0.2} = \underline{\underline{0.3}}$ secs

$N = \lambda \cdot T = 8 \cdot 0.3 = \underline{\underline{2.4}}$ packets

END OF EXAM

Do not forget to write your student number!

$\boxed{\textbf{Useful formulas}}$

$$\begin{aligned}
General: \quad & N = \lambda T \\
& P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \ \ (Poisson) \\
& f_X(x) = \lambda e^{-\lambda x}, \ x \geq 0 \ \ (Exponential) \\
& p_k = p_0 \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}} \ \ (General\ balance) \\
M/M/1: \quad & \rho = \frac{\lambda}{\mu} \\
& p_k = (1-\rho)\rho^k \\
& \bar{N}_q = \frac{\rho^2}{1-\rho} \\
& P[\geq k\ in\ system] = \rho^k \\
M/M/m: \quad & \bar{N}_q = \left(\frac{(m\rho)^m \rho}{m!(1-\rho)^2}\right) p_0 \\
& p_0 = \left[\sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!}\frac{1}{1-\rho}\right]^{-1} \\
& \rho = \frac{\lambda}{m\mu} \\
& p_k = p_0 \frac{(m\rho)^k}{k!}, \quad k \leq m, \ \rho < 1 \\
& p_k = p_0 \frac{(m\rho)^k}{m! m^{k-m}}, \quad k > m, \ \rho < 1.
\end{aligned}$$

$$P[queueing] = \frac{\frac{(m\rho)^m}{m!}\frac{1}{1-\rho}}{\left[\sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!}\frac{1}{1-\rho}\right]} \; (Erlang\, C)$$

$$M/M/\infty: \quad \bar{N} = \frac{\lambda}{\mu}$$

$$M/M/1/K: \quad \bar{N}_q = \frac{\lambda/\mu}{1-\lambda/\mu} - \frac{(\lambda/\mu)\left[K(\lambda/\mu)^K + 1\right]}{1-(\lambda/\mu)^{K+1}}$$

$$M/M/1/K: \quad p_k = \frac{1-\rho}{1-\rho^{K+1}}\rho^k, \; 0 \le k \le K, \rho \neq 1.$$

$$p_k = 0, \; \text{otherwise.}$$

$$M/M/m/m: \quad p_m = \frac{(\lambda/\mu)^m/m!}{\sum_{k=0}^{m}(\lambda/\mu)^k/k!} \; (Erlang\, B)$$

$$M/G/1: \quad W = \frac{\lambda\overline{X^2}}{2(1-\rho)} = \frac{\lambda\overline{X^2}}{2(1-\lambda\bar{X})}$$

$$W = \frac{\lambda\overline{X^2}}{2(1-\rho)} + \frac{\overline{V^2}}{2\overline{V}} \; (with\ vacations)$$

$$\text{Non-preemptive priority}: \quad W_k = \frac{R}{(1-\rho_1-\cdots-\rho_{k-1})(1-\rho_1-\cdots-\rho_k)}$$

$$R = \frac{1}{2}\sum_{i=1}^{n}\lambda_i\overline{X_i^2}$$

$$\text{Preemptive resume priority}: \quad T_k = \frac{\frac{1}{\mu_k}(1-\rho_1-\cdots-\rho_k) + R_k}{(1-\rho_1-\cdots-\rho_{k-1})(1-\rho_1-\cdots-\rho_k)}$$

$$R_k = \frac{1}{2}\sum_{i=1}^{k}\lambda_i\overline{X_i^2}$$

$$G/G/1: \quad W \le \frac{\lambda(var^2_{intarrival} + var^2_{service})}{2(1-\rho)}$$