Maths Assignment

3) A fair die is tossed 7 times

$n = 7$ $\quad$ p (probability of success) $= \frac{1}{3}$

$r = 0, 1, 2, 3, 4, 5, 6, 7$ $\quad$ q " " failures $= \frac{2}{3}$

distribution of no of success

$P(x=0) = {}^7C_0\, p^0 q^7 = \left(\frac{2}{3}\right)^7 = \frac{128}{2187}$

$P(r=1) = {}^7C_1\, p^1 q^6 = 7\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^6 = \frac{448}{2187}$

$P(r=2) = {}^7C_2\, p^2 q^5 = 21\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^5 = \frac{672}{2187}$

$P(r=3) = {}^7C_3\, p^3 q^4 = 35\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^4 = \frac{560}{2187}$

$P(r=4) = {}^7C_4\, p^4 q^3 = 35\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^3 = \frac{280}{2187}$

$P(r=5) = {}^7C_5\, p^5 q^2 = 21\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2 = \frac{84}{2187}$

$P(r=6) = {}^7C_6\, p^6 q^1 = 7\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^1 = \frac{14}{2187}$

$P(r=7) = {}^7C_7\, p^7 q^0 = 1\left(\frac{1}{3}\right)^7 = \frac{1}{2187}$

Probability Distribution

r	0	1	2	3	4	5	6	7
$P(r)$	$\frac{128}{2187}$	$\frac{448}{2187}$	$\frac{672}{2187}$	$\frac{560}{2187}$	$\frac{280}{2187}$	$\frac{84}{2187}$	$\frac{14}{2187}$	$\frac{1}{2187}$

$P(\text{three success}) = {}^7C_3\, p^3 q^4 = \frac{560}{2187}$

$P(\text{no success}) = {}^7C_0\, p^0 q^7 = \left(\frac{2}{3}\right)^7 = \frac{128}{2187}$

4) $P(\text{head}) = p \qquad P(\text{tail}) = 1-p = q$

$P(\text{first head at even toss}) = P(TH) + P(TTTH) + P(TTTTTH) + \cdots$

$= \cancel{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) +}$

$= pq + q^3 p + q^5 p + \cdots$

$= pq(1 + q^2 + q^4 + \cdots)$

$= pq \dfrac{1}{1-q^2}$

$= p(1-p)\left(\dfrac{1}{1-(1-p)^2}\right) = p(1-p)\dfrac{1}{\cancel{1}-\cancel{1}-p^2+2p}$

$= \cancel{p}(1-p)\dfrac{1}{\cancel{p}(2-p)}$

$$\boxed{P(\text{first head at even toss}) = \frac{1-p}{2-p}}$$

5) $P(A \text{ gives correct}) = \alpha \qquad P(B \text{ gives correct}) = \beta$

$P(A \text{ gives incorrect}) = (1-\alpha) \qquad P(B \text{ gives incorrect}) = (1-\beta)$

a) A answers the first question

$P(A \text{ wins}) = P(A) + P(\bar{A}\bar{B}A) + P(\bar{A}\bar{B}\bar{A}\bar{B}A) + \cdots$

$= \alpha + (1-\alpha)(1-\beta)\alpha + (1-\alpha)^2(1-\beta)^2\alpha + \cdots$

$= \alpha\left(1 + (1-\alpha)(1-\beta) + (1-\alpha)^2(1-\beta)^2 + \cdots\right)$

$= \alpha\left(\dfrac{1}{1-(1-\alpha)(1-\beta)}\right)$

$P(A \text{ wins}) = \alpha\left(\dfrac{1}{1-(1-\alpha)(1-\beta)}\right)$

ii) B answers the first question

$$P(A\ wins) = P(\bar{B}A) + P(\bar{B}\bar{A}\bar{B}A) + P(\bar{B}\bar{A}\bar{B}\bar{A}\bar{B}A)$$

$$= (1-\beta)\alpha + (1-\beta)^2(1-\alpha)\alpha + (1-\beta)^3(1-\alpha)^2\alpha + \cdots$$

$$= (1-\beta)\alpha \left(1 + (1-\beta)(1-\alpha) + (1-\beta)^2(1-\alpha)^2 + \cdots\right)$$

$$P(A\ wins) = (1-\beta)\alpha\left(\frac{1}{1-(1-\beta)(1-\alpha)}\right) = \frac{\alpha(1-\beta)}{1-(1-\beta)(1-\alpha)}$$

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9) Money Taken = 100000 × 500

Money given in casuality = $100000 \frac{1}{1000} \times 200000$

Money loss = $5\times10^7 - 2\times10^7 = 3\times10^7$

no money is loss

a) P(company will suffer a loss) = 0

b) Profit = 30000000 = 30 million

P(profit of atleast Rs 25 million) = 1

12) Bayes Theorem is a result in probability theory that relates conditional probabilities. If A and B denote two events, P(A|B) denotes the conditional probability of A occuring, given that B occurs. The two conditional probabilities P(A|B) and P(B|A) are in general different

Proof:-

The probability of two events A and B happening, $P(A \cap B)$, is the probability of A, P(A), times the probability of B given that A has occured, P(B|A)

$$P(A \cap B) = P(A)\, P(B/A)$$

On the other hand

$$P(A \cap B) = P(B)\, P(A/B)$$

and thus

$$P(A/B) = \frac{P(A)\, P(B/A)}{P(B)}$$

13) Q. $G \to$ Genetic anomaly
$NG \to$ Not Genetic anomaly

$P(G) = \frac{1}{10^5}$ $\quad P(NG) = 1 - \frac{1}{10^5} = \frac{99999}{100000}$

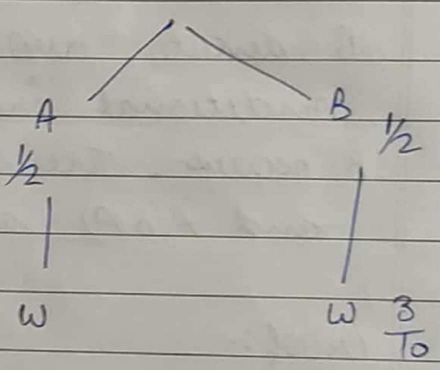
$P \to$ Tested positive

A. $P(G/P) = \dfrac{\frac{1}{10^5} \times \frac{99}{100}}{\frac{1}{10^5} \times \frac{99}{100} + \frac{99999}{10^5} \times \frac{1}{100}}$

$= \frac{99}{99 + 99999} = \frac{99}{100098}$

$= 9.89 \times 10^{-4}$

14) A Box $\quad$ B Box
4 White $\quad$ 3 white
3 Red $\quad$ 7 red



$P(\text{choosing Bag } A) = \frac{1}{2}$

$P(\text{" " } B) = \frac{1}{2}$

$P(W/A) = 4/7 \quad P(W/B) = \frac{3}{10}$

$P(W) = \frac{1}{2}\left(\frac{4}{7}\right) + \frac{1}{2}\left(\frac{3}{10}\right) = \frac{40 + 21}{140} = \frac{61}{140}$

$P(A/W) = \dfrac{\frac{1}{2} \cdot \frac{4}{7}}{\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{3}{10}} = \dfrac{\frac{4}{7}}{\frac{40+21}{70}} = \frac{40}{61}$

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15) $P(A) = 1/3 \quad P(B) = 1/3 \quad P(C) = 1/3$

$P(M/A) = \frac{1}{20} \quad P(M/B) = \frac{1}{10} \quad P(M/C) = \frac{1}{5}$

$$P(C/M) = \frac{P(C)\ P(M/C)}{P(A)\ P(M/A) + P(B) \cdot P(M/B) + P(C) \cdot P(M/C)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\frac{1}{3} \times \frac{1}{20} + \frac{1}{3} \times \frac{1}{10} + \frac{1}{3} \times \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{1}{20} + \frac{1}{10} + \frac{1}{5}}$$

$$= \frac{\frac{1}{5}}{\frac{1+2+4}{20}} = \frac{4}{7}$$

16) $P(A) = \frac{25}{100} \quad P(B) = \frac{35}{100} \quad P(C) = \frac{40}{100}$

$P(D/A) = \frac{5}{100} \quad P(D/B) = \frac{4}{100} \quad P(D/C) = \frac{2}{100}$

$$P(A|D) = \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}} = \frac{125}{125 + 140 + 80}$$

$$= \frac{125}{345} = \frac{25}{69}$$

$$P(B|D) = \frac{140}{125 + 140 + 80} = \frac{28}{69}$$

$$P(C|D) = \frac{80}{125 + 140 + 80} = \frac{16}{69}$$

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17) P(Diagonose correctly) = 0.6

P(Not Diagonose correctly) = 0.4

P(Died / correct Diagonose) = 0.4

P(Died / Incorrect Diagonose) = 0.7

$$P(\text{Correct Diagonoses} / \text{Died}) = \frac{0.6 \times 0.4}{0.6 \times 0.4 + 0.4 \times 0.7}$$

$$= \frac{0.24}{0.24 + 0.28} = \frac{24}{52} = \frac{6}{13}$$

18) P(Man) = p   P(woman) = 1-p

$P(\text{Stomach Ulcer} / \text{Man}) = \frac{2}{100}$ $P(\text{Stomach Ulcer} / \text{Women}) = \frac{1}{100}$

$$P(\text{Man} / \text{Stomach Ulcer}) = \frac{p\,\frac{2}{100}}{p\,\frac{2}{100} + (1-p)\,\frac{1}{100}} = \frac{2p}{2p+1-p} = \frac{2p}{1+p}$$

19) P(Faulty) = 0.1   P(Non Faulty) = 0.9

P(House collapse / Faulty) = 0.95   P(House collapse / Non faulty) = 0.45

$$P(\text{Faulty} / \text{House collapse}) = \frac{0.1 \times 0.95}{0.1 \times 0.95 + 0.9 \times 0.45} = \frac{95}{95 + 405}$$

$$= \frac{95}{500} = \frac{19}{100} = 0.19$$

20) $P(Def) = \frac{2}{100}$ $\quad P(NDef) = \frac{98}{100}$

~~P(Actually Def | Def) = 0.94 & P(Actually good/Def) = 0.5~~

$P(Def / Actually\ Def) = 0.94 \quad P(Def / Actually\ good) = 0.5$

$$P(Actually\ Def / Def) = \frac{\frac{2}{100} \times \frac{94}{100}}{\frac{2}{100} \times \frac{94}{100} + \frac{98}{100} \times \frac{5}{100}} = \frac{188}{188 + 490}$$

$$= \frac{94}{339} = 0.27728$$

21) $P(sufferer) = \frac{5}{1000}$ $\quad P(Non\ sufferer) = \frac{995}{1000}$

$P(+ve\ test / sufferer) = 0.95 \quad P(+ve\ test / Non\ sufferer) = 0.10$

a) $$P(+ve\ test) = \frac{5}{1000} \times \frac{95}{100} + \frac{995}{1000} \times \frac{1}{10} = \frac{475 + 9950}{100000}$$

$$= 0.10425$$

b) $$P(sufferer / +ve\ test) = \frac{\frac{5}{1000} \times \frac{95}{100}}{\frac{5}{1000} \times \frac{95}{100} + \frac{995}{1000} \times \frac{1}{10}} = \frac{475}{475 + 9950}$$

$$= \frac{475}{10425} = 0.0455$$

c) $P(-ve\ test / sufferer) = 0.05 \quad P(-ve\ test / Non\ sufferer) = 0.90$

$$P(non\ sufferer / -ve\ test) = \frac{\frac{995}{1000} \times \frac{90}{100}}{\frac{5}{1000} \times \frac{5}{100} + \frac{995}{1000} \times \frac{90}{100}} = \frac{89550}{25 + 89550}$$

$$= 0.9997$$

d) $P(\text{person will be misclassified}) = \frac{5}{1000} \times \frac{5}{100} + \frac{995}{1000} + \frac{10}{100}$

$= 0.09975$

23) ~~Sum~~ Sum can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

X	P(X)	XP(X)	$X^2P(X)$
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
		$\frac{252}{36}$	$\frac{1974}{36}$

$E(x) = \Sigma x P(x)$

$= \frac{256}{36} = 7$

$\text{Variance} = \Sigma x^2 P(X) - (\Sigma x P(X))^2$

$= \frac{1974}{36} - 49$

$= 5.833$

24) $P(\text{guess}) = 1/3 \quad P(\text{copying}) = 1/6 \quad P(\text{skill}) = \frac{1}{2}$

$P(\text{correct}/\text{guess}) = \frac{1}{4} \quad P(\text{correct}/\text{copy}) = \frac{1}{8} \quad P(\text{correct}/\text{skill}) = 1$

$P(\text{skill}/\text{correct}) = \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} = \frac{\frac{1}{2}}{\frac{1}{12} + \frac{1}{48} + \frac{1}{2}}$

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$$= \frac{\frac{1}{2}}{\frac{4+1+24}{48}} = \frac{24}{29}$$

25) $P(0T) = 0.4 \qquad P(1T) = 0.6$

$P(0R/0T) = 0.95 \qquad P(1R/1T) = 0.90$

$P(1R/0T) = 0.05 \qquad P(0R/1T) = 0.10$

i) $P(1R) = 0.4 \times 0.05 + 0.6 \times 0.90$

$$= \frac{20}{1000} + \frac{540}{1000} = \frac{560}{1000} = 0.56$$

ii) $P(1T/1R) = \dfrac{0.6 \times 0.90}{0.4 \times 0.05 + 0.6 \times 0.90} = \dfrac{540}{20+540}$

$$= \frac{54\cancel{0}}{56\cancel{0}} = \frac{27}{28}$$

26) $P(\text{Truth}) = \frac{2}{5} \qquad P(\text{Lie}) = \frac{3}{5}$

$P(4/\text{Truth}) = \frac{1}{6} \qquad P(4/\text{Lie}) = \frac{5}{6}$

$$P(\text{Truth}/4) = \frac{\frac{2}{\cancel{5}} \times \frac{1}{\cancel{6}}}{\frac{2}{\cancel{5}} \times \frac{1}{\cancel{6}} + \frac{3}{\cancel{5}} \times \frac{5}{\cancel{6}}} = \frac{2}{2+5} = \frac{2}{7}$$

27) i) 5 defective 20 good

$$P(\text{he will accept}) = \frac{{}^{20}C_5 \; {}^{5}C_0}{{}^{25}C_5} = \frac{\frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1}}{\frac{25 \times 24 \times 23 \times 22 \times 21}{5 \times 4 \times 3 \times 2 \times 1}}$$

$$= \frac{\overset{4}{\cancel{20}} \times 19 \times \overset{3}{\cancel{18}} \times 17 \times \overset{8}{\cancel{16}}}{\underset{5}{\cancel{25}} \times \underset{4}{\cancel{24}} \times 23 \times \underset{11}{\cancel{22}} \times \overset{7}{\cancel{21}}}$$

$$= \frac{2584}{8855}$$

ii) 1 defective 24 good

$P(\text{he will reject}) = \frac{{}^{1}C_{4}\ {}^{24}C_{4}}{{}^{25}C_{5}} = \dfrac{\dfrac{24\times23\times22\times21}{4\times3\times2\times1}}{\dfrac{25\times\cancel{24}\times\cancel{23}\times\cancel{22}\times\cancel{21}}{5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}}$

$= \frac{5}{25} = \frac{1}{5}$