School of Engineering and Applied Science (SEAS)
Ahmedabad University

BTech(ICT) Digital Signal Processing (Section 1)

Laboratory Assignment-5

Enrollment No: AU1841145 **Name: Samarth Shah**

AIM : Understand different concepts of Fourier transform along with its applications.

1. Solution Problem-1

   (a) Approach: In this question, cutoff frequency =1/6, high frequency = 1/3 and low frequency =1/9 were given. From the handwritten analysis, equation of the low pass filter were derived. The procedure is mentioned in the analysis. High pass filter is nothing but the complement of the low pass filter. So basically, low pass filter equation was multiplied with -1. Equation of h(n) at low and high frequency were found by exchanging the cutoff frequency with low and high frequency respectively. Band pass filter can be found using low pass and high pass filter. Band pass filter = h(n) at high frequency - h(n) at lower frequency. Band stop filter is a complement of the band pass filter.

   (b) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab5 (Question_1) Create single matlab script and write h(n) equation for each filter type
clc;
close all;
clear all;

range_x_axis = -20:0.01:20; %Range of X-Axis
%In the question, 3 Frequencies are given
%1) Cutoff Frequency
cutOff_Frequency = 1/6;
%2) High_freuqency
high_frequency = 1/3;
%3) Low_Frequency
low_frequency = 1/9;

%for Ideal low-pass filter and High pass-filter at cutoff frequency
h_lowPass= 2*cutOff_Frequency*sinc(2*cutOff_Frequency*range_x_axis); %Equation h(n)= 2*pi*f*sinc(2*pi*f/pi)/pi. Here,2*PI*f= W(Omega)
h_highPass= -2*cutOff_Frequency*sinc(2*cutOff_Frequency*range_x_axis); %Complement of Highpass

%for Idea Band-pass filter and band stop filter at low and high frquencies
h_lowfreq= 2*low_frequency*sinc(2*low_frequency*range_x_axis); %Equation h(n)= 2*pi*f*sinc(2*pi*f/pi)/pi. Here,2*PI*f= W(Omega)
h_highfreq= 2*high_frequency*sinc(2*high_frequency*range_x_axis); %Complement of Highpass

%Ideal Band-pass filter
h_bandpass = h_highfreq - h_lowfreq; %Bandpass filter = H_High - H_Low
%Ideal Band-stop filter
h_bandstop = h_lowfreq - h_highfreq; %Bandstop filter = H_Low - H_High

%plotting functions
```

```
figure(1);
subplot(211)
plot(range_x_axis,h_lowPass,'r');
xlabel ('Range ');
ylabel ('Lowpass');
title ('Lowpass filter');
subplot(212)
plot(range_x_axis,h_highPass,'b');
xlabel ('Range ');
ylabel ('Highpass');
title ('Highpass filter');
figure(2);
subplot(211)
plot(range_x_axis,h_bandpass,'r');
xlabel ('Range ');
ylabel ('BandPass');
title ('Bandpass filter');
subplot(212)
plot(range_x_axis,h_bandstop,'b');
xlabel ('Range ');
ylabel ('BandStop');
title ('Bandstop filter');
```

(c) Hand-written Analysis:

Bandlimited DTFT (Handwritten Analysis)

⇒ Ideal bandlimited signal is a function that is nonzero in the low frequency band $|\hat{\omega}| \le \hat{\omega}_b$ and 0 in high freq. band $\hat{\omega}_b \le \hat{\omega} \le \pi$.

$$\therefore X(e^{j\hat{\omega}}) = \begin{cases} 1 & |\hat{\omega}| \le \hat{\omega}_b \\ 0 & \hat{\omega}_b \le |\hat{\omega}| \le \pi \end{cases}$$

$$\therefore x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\hat{\omega}})e^{j\hat{\omega}n}\,d\hat{\omega}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\hat{\omega}_b} 0\cdot e^{j\hat{\omega}n}\,d\omega + \frac{1}{2\pi}\int_{-\hat{\omega}_b}^{\hat{\omega}_b} 1\cdot e^{j\hat{\omega}n}\,d\omega + \frac{1}{2\pi}\int_{\hat{\omega}_b}^{\pi} 0\cdot e^{j\hat{\omega}n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\hat{\omega}_b}^{\hat{\omega}_b} e^{j\hat{\omega}n}\,d\omega$$

$$= \frac{e^{j\hat{\omega}_b n} - e^{-j\hat{\omega}_b n}}{(2j\pi n)} = \frac{\sin(\hat{\omega}_b n)}{\pi n} \quad \text{—(1)}$$

$$-\infty < n < \infty$$

Here, $\mathrm{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$ is a defination of sinc function. If we expressed $x(n)$ in Eq. (1) in terms of this defination of sinc function, we would write

$$x[n] = \frac{\hat{\omega}_b}{\pi}\,\mathrm{sinc}\left(\frac{\hat{\omega}_b n}{\pi}\right)$$

For cutoff frequency $= f_c$

$$x[n] = \frac{2\pi f_c}{\pi}\,\mathrm{sinc}\left(\frac{2\pi f_c n}{\pi}\right)$$

$$x[n] = 2f_c\,\mathrm{sinc}(2f_c n) \quad \text{— Eq. (2)}$$

Above Equation is for ~~band~~ low pass filter.

for, Highpass filter.

$$x[n] = -2f_c\,\mathrm{sinc}(2f_c n) \quad \text{— Eq. (3)}$$

If f_H = High frequency and f_L = low frequency

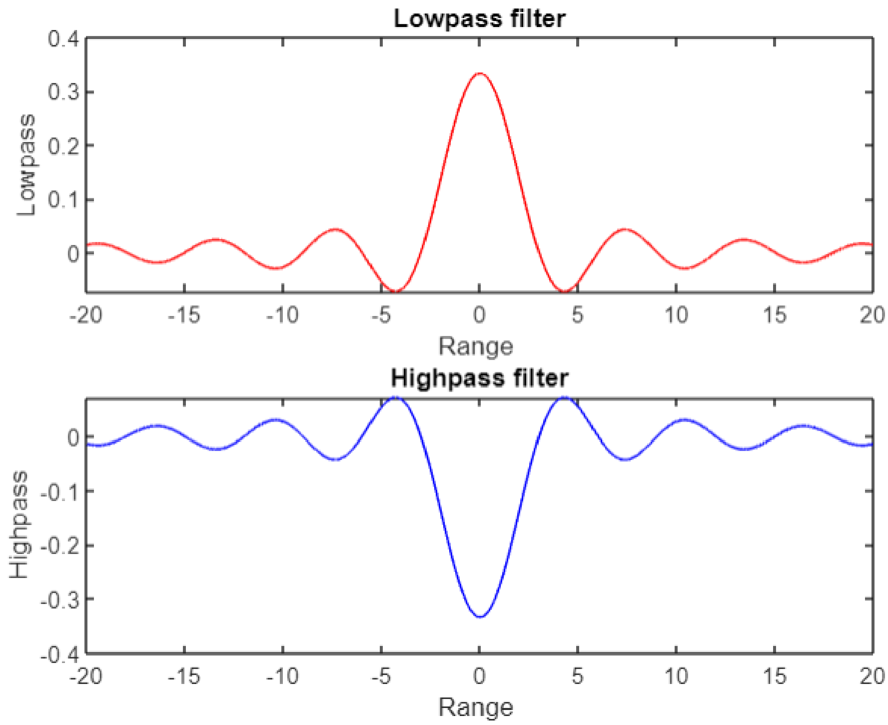
$$\therefore X_{High}[n] = 2f_H\,\mathrm{sinc}(2f_H n)$$

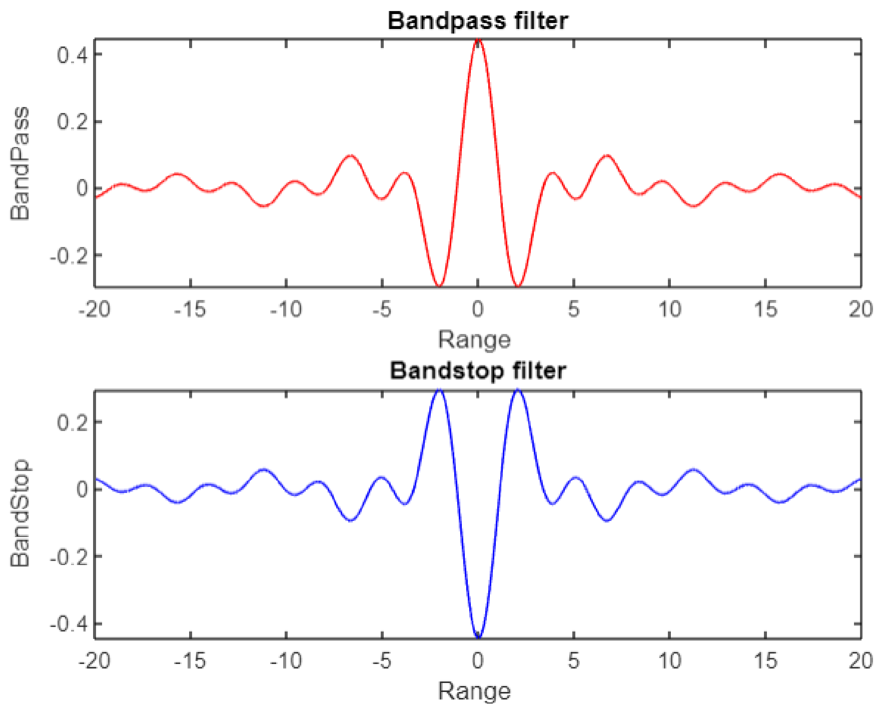
$$X_{low}[n] = 2f_L\,\mathrm{sinc}(2f_L n)$$

$\therefore$ Band pass filter $x[n] = X_{High}[n] - X_{low}[n]$ —(4)

Band Stop Filter $x[n] = X_{low}[n] - X_{High}[n]$ —(5)

(d) Simulation Output:





2. Solution Problem-2

   (a) Approach: In this question, the cut-off frequency=1/12 was given. Then the input function was defined as per question. The equation of the law pass filter and high pass filter was defined. The process of the equation is already mentioned in the first question. Using conv(inbuilt function), convolution between input signal and low pass and also with the high pass filter were done. Initially the range was defined between -20 to 20. Out put of the convolution was a double sized vector of the range. Therefor new range for plotting the output of the convolution was defined which was double size of the initial range.

   (b) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab5 (Question_2) input signal x(n) passes though ideal low pass and ideal high
    pass filter by determining and .
clc;
close all;
clear all;

range_x_axis = -20:1:20; %Range of X-Axis
range_2 = -40:1:40; %Range of convolution
%In the question, 1 Frequency is given
%1) Cutoff Frequency
cutOff_Frequency = 1/12;

%Input Function
input_func=10*cos(2*pi*range_x_axis/20)+5*cos(2*pi*range_x_axis/8); %Input Signal

%Low pass filter
lowPass = 2*cutOff_Frequency*sinc(2*cutOff_Frequency*range_x_axis);
%High pass filter
highPass= -lowPass;

conv_lowPass= conv(input_func,lowPass); %Convolution of main signal with low pass
    filter
conv_HighPass= conv(input_func,highPass); %Convolution of main signal with high
    pass filter
%plotting the functions
%Main function
figure;
stem(range_x_axis,input_func,'r');
xlabel ('Range ');
ylabel ('Input Function');
title ('Input Function');
figure;
stem(range_2,conv_lowPass,'b');
xlabel ('Range ');
ylabel ('Low pass');
title ('Linear Convolution of low pass and input function');
figure;
stem(range_2,conv_HighPass,'v');
xlabel ('Range ');
ylabel ('High pass');
title ('Linear Convolution of High pass and input function');
```

(c) Simulation Output: