School of Engineering and Applied Science (SEAS)
Ahmedabad University

BTech(ICT) Digital Signal Processing (Section 1)

Laboratory Assignment-7 Question 2

Enrollment No: AU1841145 **Name: Samarth Shah**

AIM : Design an IIR filter using impulse in-variance method using Chebyshev

1. Solution Problem-1

   (a) Handwritten Analysis:

① Samarth Shah
AU1841145

In the question, It is given that the order of the filter is 4.
Also, some more details are provided.

ω_L = lower CutOff frequency = 0.3π
ω_H = Higher Cutoff frequency = 0.6π
T = Sampling Time = 0.1
δ_p = Passband Ripple = 0.11

Step-1 Peek-to-Peek Passband Ripple $= 1 - \frac{1}{\sqrt{1+\epsilon^2}}$

$\therefore \delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}}$ $\quad \therefore \epsilon^2 = \frac{1}{(1-\delta_p)^2} - 1$

$\therefore \epsilon^2 = \frac{1}{(1-0.11)^2} - 1 = 0.2625$

$\boxed{\therefore \epsilon = 0.5130}$

Step-2 NOW onwards, the order will be considered as N=2, as we are initally calculating for low pass filter.

Parameter $\beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{1/N}$

$= \left[\frac{\sqrt{(1+(0.5130)^2}+1}{0.5130}\right] = \boxed{2.0347}$

② Step-3 Calculating Major & Minor Axis.

Major Axis $r_1 = \left(\frac{\beta^2+1}{2\beta}\right) = \left(\frac{(2.0347)^2+1}{2\times 2.0347}\right)$

$\boxed{\therefore r_1 = 1.2630}$

Minor Axis $r_2 = \left(\frac{\beta^2-1}{2\beta}\right) = \left(\frac{(2.0347)^2-1}{2\times 2.0347}\right)$

$\boxed{\therefore r_2 = 0.7716}$

Step-4 Finding the angular positions of poles, using this equation,

$\phi_k = \frac{(2k+N+1)\pi}{2N}$ where $k=0,1$

for $k=0$
$\phi_0 = \frac{(2+1)\pi}{4} = \frac{3\pi}{4}$

for $k=1$
$\phi_1 = \frac{(2+2+1)\pi}{4} = \frac{5\pi}{4}$

Step-5 Finding x's and y's for poles' equation, using these equations,

$x_k = r_2 \cos\phi_k \qquad y_k = r_1 \sin\phi_k$

3

For $k=0$,

$x_0 = 0.7716 \cos\left(\frac{3\pi}{4}\right)$

$y_0 = 1.2630 \sin\left(\frac{3\pi}{4}\right)$

For $k=1$

$x_1 = 0.7716 \cos\left(\frac{5\pi}{4}\right)$

$y_1 = 1.2630 \sin\left(\frac{5\pi}{4}\right)$

from the above equations,

$x_0 = -0.5456 \qquad x_1 = -0.5456$

$y_0 = 0.8931 \qquad y_1 = -0.8931$

Step-6 Equation of Poles : $S_k = x_k + j y_k$

$$\boxed{\begin{aligned} \therefore\ S_0 &= -0.5456 + j(0.8931) \\ S_1 &= -0.5456 + j(-0.8931) \end{aligned}}$$

Step-7 System function:

$$H(S) = \frac{b_0}{(S-S_0)(S-S_1)}$$

Where, $b_0 = \frac{S_0 \cdot S_1}{\sqrt{1+\varepsilon^2}}$

$$\therefore b_0 = \frac{(-0.5456 + j(0.8931)) \times (-0.5456 + j(-0.8931))}{\sqrt{1+(0.5730)^2}}$$

$$= \frac{0.2977 + j(-\cancel{0.4872}) + j(\cancel{0.4872}) + (+(0.8931)^2)}{1.1239}$$

4

$$\therefore b_0 = \frac{0.2977 + 0.7976}{1.1239} = 0.9745$$

$$\boxed{\therefore b_0 = 0.9745}$$

$$\therefore H(S) = \frac{b_0}{(S-S_0)(S-S_1)} = \frac{0.9745}{(S-S_0)(S-S_1)}$$

Simplifying the above equation,

$$H(S) = \frac{0.9745}{S^2 - (S_1+S_0)S + S_0 \cdot S_1} \quad \text{--- (1)}$$

$$S_1 + S_0 = -0.5456 + j(\cancel{0.8931}) - 0.5456 - j(\cancel{0.8931})$$

$$= -1.0912 \quad \text{--- (2)}$$

$$S_1 \cdot S_0 = (-0.5456 + j(0.8931))(-0.5456 - j(0.8931))$$

(Here's the form of $(a-b)(a+b) = a^2 - b^2$)

$$\therefore S_1 \cdot S_0 = (0.5456)^2 - (-1)(0.8931)^2$$

$$= 0.2977 + 0.7976 \quad \text{--- (3)}$$

$$= 1.0953$$

Putting Ans. of 2 & 3 into equation (1),

$$\therefore H(S) = \frac{0.9745}{S^2 + (1.0912)S + 1.0953} \quad \text{--- (4)}$$

Page No.: Date:

Step-4: Conversion of lowpass filter to Bandpass filter.

from the question,

(1) $\Omega_L = \frac{\omega_L}{T} = \frac{0.3\pi}{0.1} = 3\pi$ rad/sec

(2) $\Omega_H = \frac{\omega_H}{T} = \frac{0.6\pi}{0.1} = 6\pi$ rad/sec

Replacing S to $\left(\frac{S^2+\Omega_L\Omega_H}{S(\Omega_H-\Omega_L)}\right)$ in the equation (4).

$$= \frac{S^2+18\pi^2}{3\pi S}$$

$$= \frac{S^2+177.6529}{9.425S}$$

$$\therefore H(S) = \frac{0.9745}{\left(\frac{S^2+177.6529}{9.425S}\right)^2 + (1.0912)\left(\frac{S^2+177.6529}{9.425S}\right) + 1.0953}$$

$$= \frac{0.9745 \times (9.425S)^2}{(S^2+177.6529)^2+(1.0912)(S^2+177.6529)(9.425S) + (9.425S)^2(1.0953)}$$

$$= \frac{0.9745 \times 88.8306 S^2}{S^4+355.3058S^2+31560.5529 + 10.2846S^3+1827.0890S + 97.2962S^2}$$

Page No.: Date:

$$\therefore \boxed{H(S) = \frac{86.5654 S^2}{S^4+10.2846S^3+452.602S^2+1827.0890S + 31560.5529}}$$

$$= \frac{86.5654 S^2}{(S^2+3.5696S+94.4377)(S^2+6.715S+334.194)}$$

$$= \frac{86.5654S^2}{\left(\left[S+1.785\right]^2+91.25\right)\left(\left[S+3.36\right]^2+322.9\right)}$$

Note: As I'm not able to find H(Z) from this way I have tried 2nd Approach here.

OR from equation (1)

$$H(S) = \left[\frac{0.9745}{(S-S_0)(S-S_1)}\right]$$

$$= 0.9745\left[\frac{A}{S-S_0} + \frac{B}{S-S_1}\right]$$

Where, $\frac{A}{S-S_0} + \frac{B}{S-S_1} = \frac{1}{(S-S_0)(S-S_1)}$

$$\therefore (S-S_1)A + B(S-S_0) = 1$$

When $S = S_1$

$$B = \frac{1}{S_1 - S_0} \quad \& \quad A = \frac{1}{S_0 - S_1}$$

$\therefore A = -B$

$$A = \frac{1}{2 \times (0.8931)j} = -0.5598j$$

$\therefore B = 0.5598j$

$$\therefore H(S) = 0.9745 \times 0.5598j \left[\frac{1}{S - S_1} - \frac{1}{S - S_0} \right]$$

$$= 0.5455j \left[\frac{1}{S - S_1} - \frac{1}{S - S_0} \right]$$

Replacing S to $\frac{S^2 + 177.6529}{9.425 \times S}$

This $S = S_0 \cdot S_1$

$$= \left[\frac{(1.0953)^2 + 177.6529}{9.425 \times S} \right]$$

from eq 4

$$= \left[\frac{18.9764}{S} \right]$$

$$\therefore H(S) = 0.5455j \left[\frac{S}{18.9764 - SS_1} - \frac{S}{18.9764 - SS_0} \right]$$

⇒ This also seems can't solvable for H(z). (manually)

(b) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab7 ( Question_2 ) From the given transfer function co-efficients, get a plot of the
% IIR bandpass Chebyshev Prototype-1 Filter
clc ;
clear all ;
close all ;
%Cutoff low and high frequencies
Low_cut=0.3*pi;
High_cut=0.6*pi;
%Sampling Time T
S_time=0.1;
%Order of the filter, was given in the question
order = 4;
%Passband Ripple
p_ripple= 0.11;
%Converting Passband Ripple to db values
p_ripple_db=-20*log10(1-p_ripple);

W=sqrt(Low_cut * High_cut)/S_time;
Bandwidth=(High_cut - Low_cut)/S_time;
%Using Inbuilt function of Chebyshev Type I filter design. This will return
%Transfer function co-efficients a and b of H(s).
[b,a]=cheby1(order,p_ripple_db,1,'s');
%Converting Low-pass filter to band-pass filter
[d,c]=lp2bp(b,a,W,Bandwidth);
%Converting analog to digital domain (S->Z)
[f,e]=impinvar(d,c,10);
figure;
freqz(f,e); %Plotting the Magnitude and Phase response
figure;
zplane(f,e); %Plotting Zeros and poles

%From Handwritten analysis, defining co-efficients a and b of H(s):
%Transfer function co-efficients a and b.
a=[1,1.0912,1.0953];
b=[0,0,0.9745];
%Converting Low-pass filter to band-pass filter
[d,c]=lp2bp(b,a,W,Bandwidth);
%Converting analog to digital domain (S->Z)
[f,e]=impinvar(d,c,10);
figure;
freqz(f,e); %Plotting the Magnitude and Phase response
figure;
zplane(f,e); %Plotting Zeros and poles
```

(c) Simulation Output:

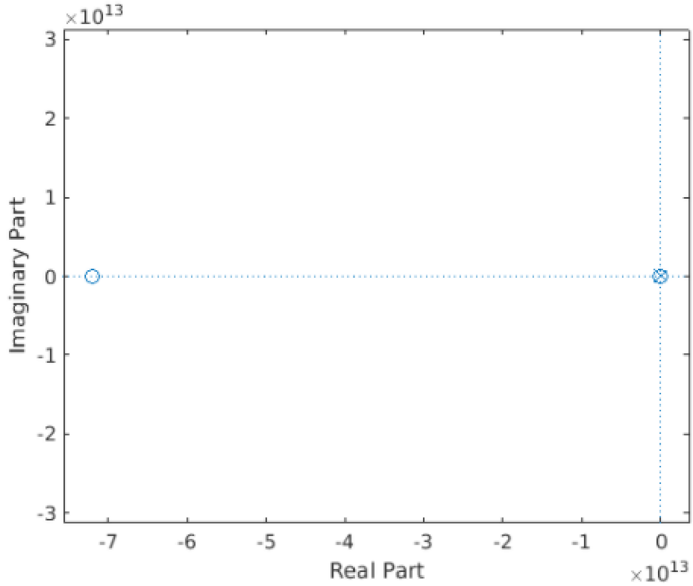

Figure 1: The image shows the value of poles.

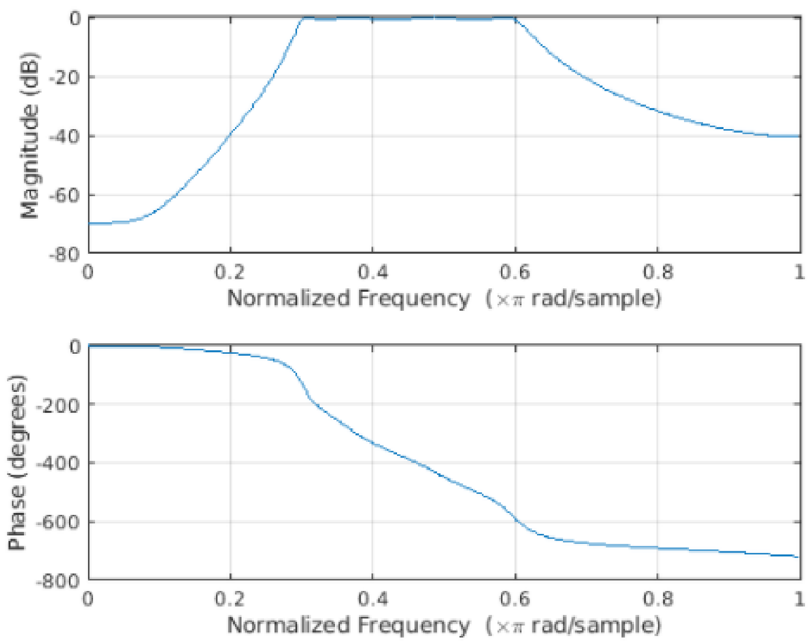

Figure 2: IIR lowpass Chebyshev Prototype-1 Filter (Magnitude and Phase Response)

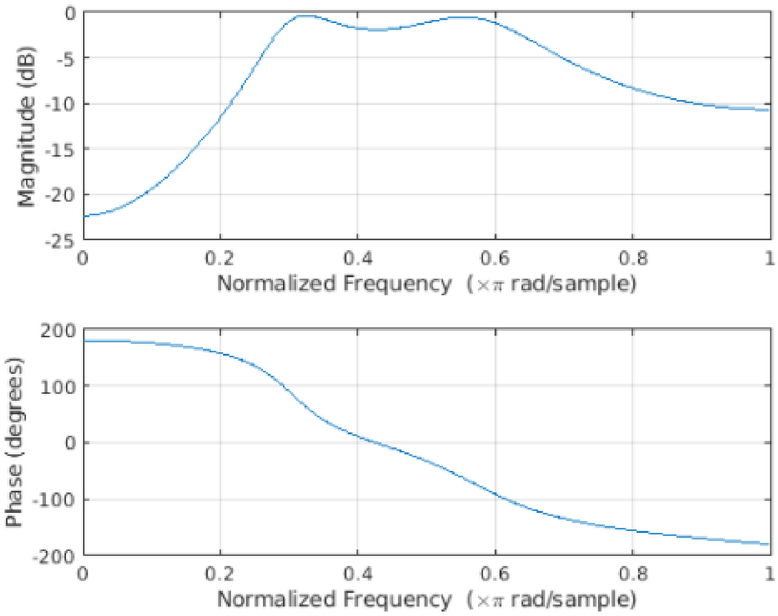

Figure 3: IIR lowpass Chebyshev Prototype-1 Filter from Handwritten Analysis (Magnitude and Phase Response)