School of Engineering and Applied Science (SEAS)
Ahmedabad University

BTech(ICT) Digital Signal Processing (Section 1)

Laboratory Assignment-6

Enrollment No: AU1841145 **Name: Samarth Shah**

AIM : Understand different concepts of Z-transform.

1. Solution Problem-1

   (a) Approach: In this question, syms command was used to construct symbolic variables. All the function are attached by the unit step function. So it was created with the help of the heaviside function. Then it was multiplied with the corrosponding equations and then ztrans command was used to obtain z-transform.

   (b) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab6 (Question_1) Create single matlab script and obtain Symbolic Z-Transform
syms z n a w_sinusoidal;

%Function1
function1=heaviside(sym(1)); %UnitStep Function
output1 = ztrans(function1,z);
disp (" Ztransform of Function 1 is :") ;
disp ( output1 ) % displaying result in command window

%Function2
function2=n.*function1;
output2 = ztrans(function2,z);
disp (" Ztransform of Function 2 is :") ;
disp ( output2 ) % displaying result in command window

%Function3
function3=(1+n).*function1;
output3 = ztrans(function3,z);
disp (" Ztransform of Function 3 is :") ;
disp ( output3 ) % displaying result in command window

%Function4
function4 = (cos((w_sinusoidal).*n)).*(function1);
output4 = ztrans(function4,z);
disp (" Ztransform of Function 4 is :") ;
disp ( output4 ) % displaying result in command window

%Function5
function5 = (sin((w_sinusoidal).*n)).*(function1);
output5 = ztrans(function5,z);
disp (" Ztransform of Function 5 is :") ;
disp ( output5 ) % displaying result in command window

%Function6
function6 = (a^n).*(cos((w_sinusoidal).*n)).*(function1);
output6 = ztrans(function6,z);
disp (" Ztransform of Function 6 is :") ;
disp ( output6 ) % displaying result in command window

%Function7
```

```
function7 = (a^n).*(sin((w_sinusoidal).*n)).*(function1);
output7 = ztrans(function7,z);
disp (" Ztransform of Function 7 is :") ;
disp ( output7 ) % displaying result in command window

%Function8
function8 = (a^n).*(n).*(function1);
output8 = ztrans(function8,z);
disp (" Ztransform of Function 8 is :") ;
disp ( output8 ) % displaying result in command window

%Function9
function9 = -(a^n).*(n).*(function1);
output9 = ztrans(function9,z);
disp (" Ztransform of Function 9 is :") ;
disp ( output9 ) % displaying result in command window

%Function10
function10 = ((-1)^n).*(n).*(function1);
output10 = ztrans(function10,z);
disp (" Ztransform of Function 10 is :") ;
disp ( output10 ) % displaying result in command window

%Function11
function11 = ((n)^2).*(function1);
output11 = ztrans(function11,z);
disp (" Ztransform of Function 11 is :") ;
disp ( output11 ) % displaying result in command window
```

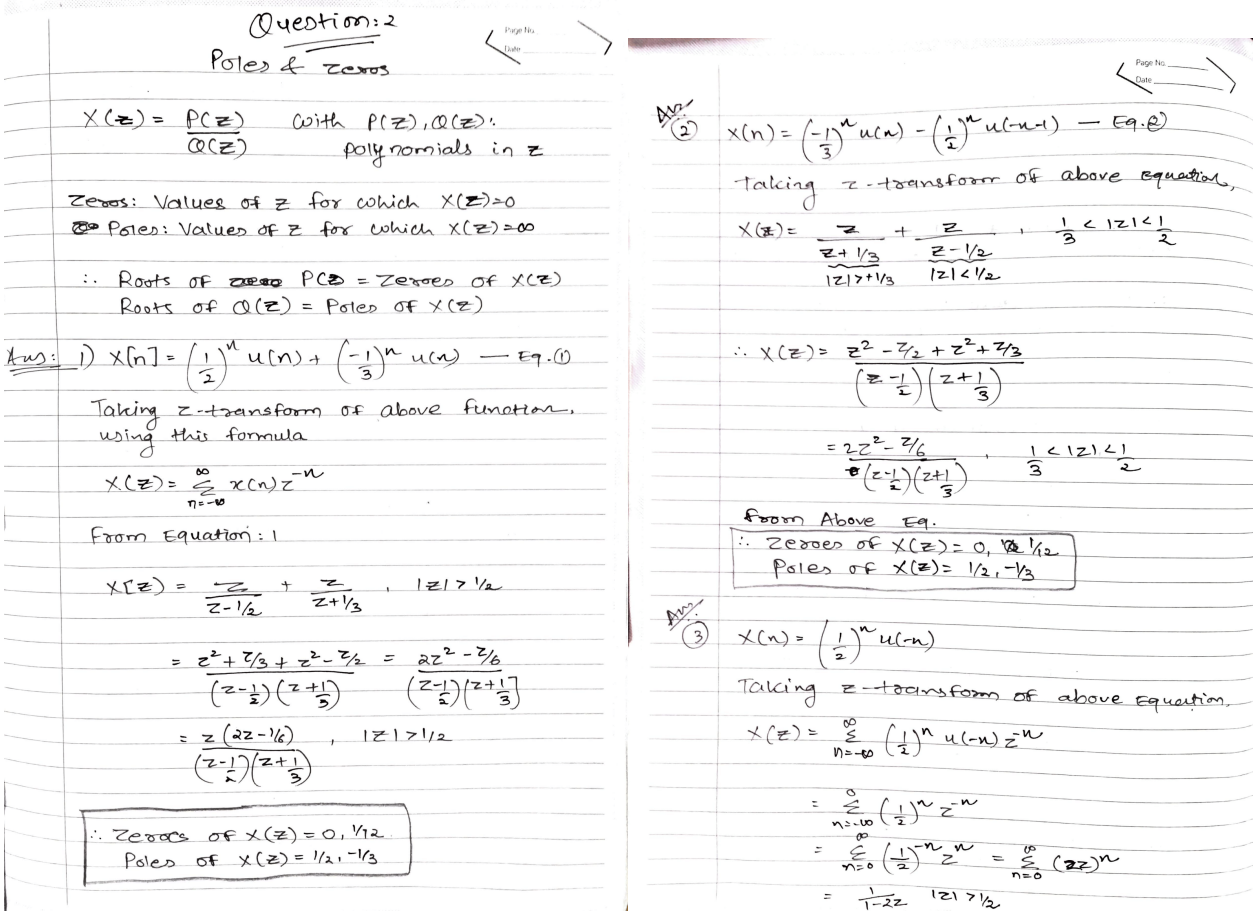
(c) Simulation Output:

```
>> Lab6_Question1
 Ztransform of Function 1 is :
z/(z - 1)

 Ztransform of Function 2 is :
z/(z - 1)^2

 Ztransform of Function 3 is :
z/(z - 1) + z/(z - 1)^2

 Ztransform of Function 4 is :
(z*(z - cos(w_sinusoidal)))/(z^2 - 2*cos(w_sinusoidal)*z + 1)

 Ztransform of Function 5 is :
(z*sin(w_sinusoidal))/(z^2 - 2*cos(w_sinusoidal)*z + 1)

 Ztransform of Function 6 is :
-(z*(cos(w_sinusoidal) - z/a))/(a*(z^2/a^2 - (2*z*cos(w_sinusoidal))/a + 1))

 Ztransform of Function 7 is :
(z*sin(w_sinusoidal))/(a*(z^2/a^2 - (2*z*cos(w_sinusoidal))/a + 1))

 Ztransform of Function 8 is :
(a*z)/(a - z)^2

 Ztransform of Function 9 is :
-(a*z)/(a - z)^2

 Ztransform of Function 10 is :
-z/(z + 1)^2

 Ztransform of Function 11 is :
(z*(z + 1))/(z - 1)^3
```

2. Solution Problem-2

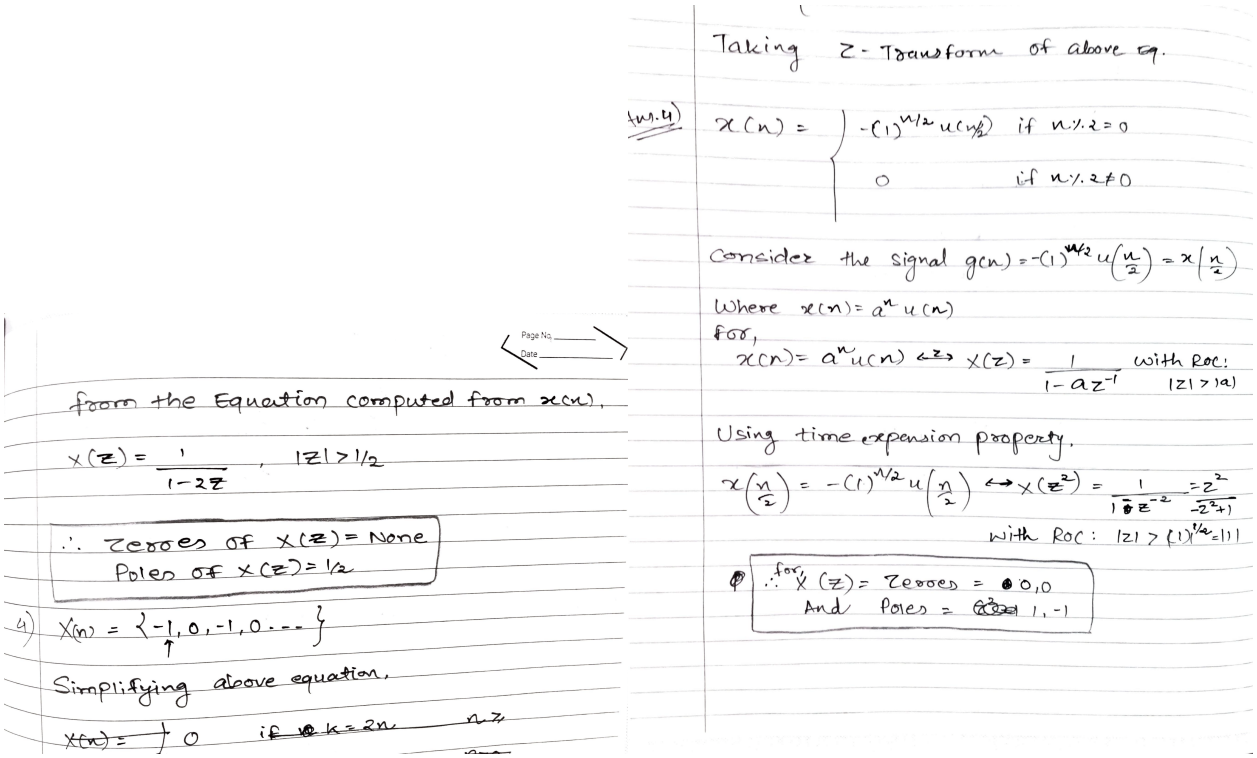
(a) Approach: In this question, from input function x(n), z-transform was calculated. The z-transform will be in the p/q format. If we take p=0 then it will give all the zeroes of transform function and if we take q=0 then it will give all the poles associated with the transform function.

(b) Hand-written Analysis:



(c) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab6 (Question_2) Plot poles and zeros of the Z-transform obtained for signals
clc ;
close all ;
clear all ;

%Function-1
figure ;
zplane ([0;1/12] ,[ -1/3;1/2]) ;% zeros and poles of 1st function
title('Function 1');

%Function-2
figure ;
zplane ([0;1/12] ,[ -1/3;1/2]) ; % zeros and poles of 2nd function
title('Function 2');

%Function-3
```



Taking Z-Transform of above eq.

fun.4) $x(n) = \begin{cases} -(1)^{n/2} u(n/2) & \text{if } n\%2 = 0 \\ 0 & \text{if } n\%2 \neq 0 \end{cases}$

Consider the signal $g(n) = -(1)^{n/2} u\left(\frac{n}{2}\right) = x\left(\frac{n}{2}\right)$

Where $x(n) = a^n u(n)$

for,

$x(n) = a^n u(n) \leftrightarrow X(z) = \frac{1}{1 - az^{-1}}$ with ROC: $|z| > |a|$

Using time expension property.

$x\left(\frac{n}{2}\right) = -(1)^{n/2} u\left(\frac{n}{2}\right) \leftrightarrow X(z^2) = \frac{1}{1 + z^{-2}} = \frac{z^2}{z^2 + 1}$

with ROC: $|z| > (1)^{1/2} = |1|$

$\therefore$ for $X(z)$ = zeroes = 0,0
And poles = 1, -1

from the Equation computed from x(n),

$X(z) = \frac{1}{1 - 2z}$, $|z| > 1/2$

$\therefore$ zeroes of $X(z)$ = None
Poles of $X(z) = 1/2$

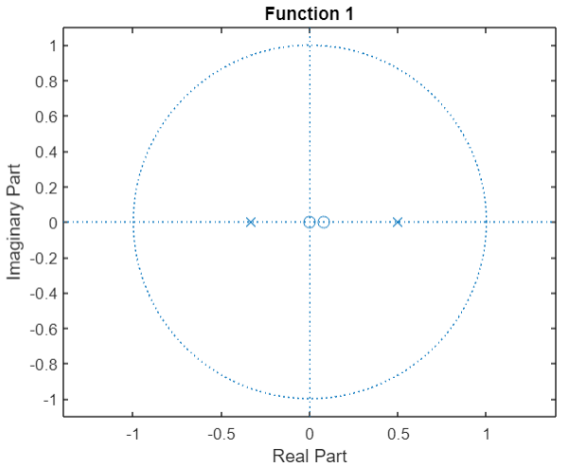
4) $x(n) = \{-1, 0, -1, 0 \ldots\}$

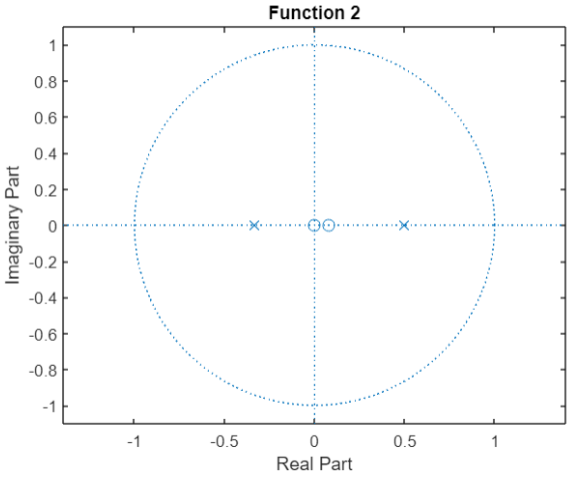
Simplifying above equation,

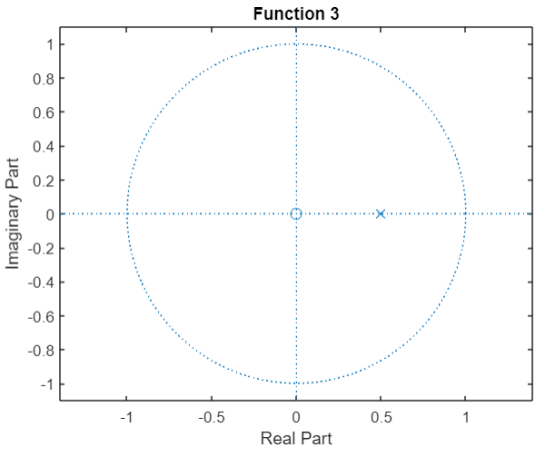
$x(n) = \begin{cases} 0 & \text{if } k = 2n \quad n \in \end{cases}$

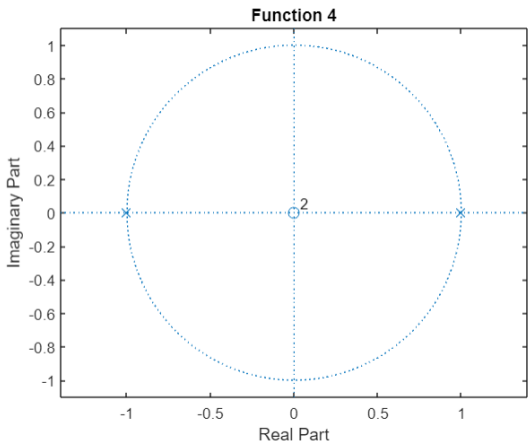
```
figure ;
zplane (0 ,1/2) ; % zeros and poles of 3rd function
title('Function 3');

%Function-4
figure ;
zplane ([0;0] ,[ -1;1]) % zeros and poles of 4th function
title('Function 4');
```

(d) Simulation Output:









3. Solution Problem-3

   (a) Approach: We have taken X(z) and then using inverse ztransform mathematical equation we have calculated x(n) using iztrans function and displayed it in command window .

   (b) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab6 (Question_3) Create single matlab script and obtain Symbolic Z-Transform
clc ;
clear all ;
close all ;
syms z % symbolically declared value

%First Function
Funct1 = (z^2 + 3*z)/(z^2 - 3*z + 2); % first function
invz_funct1 = iztrans (Funct1) ; % Inverse z transform
disp (" Inverse Z transform of Function 1 is :") ;
disp ( invz_funct1 ); % displaying value in command

%Second Function
Funct2 = ((z^2 + 2*z)/(z^2 + 1)); % second function
invz_funct2 = iztrans (Funct2); % Inverse z transform
disp (" Inverse Z transform of Function 2 is :") ;
disp ( invz_funct2 ); % displaying value in command

%Third Function
Funct3 = ((z ^3) /((z -1) ^2 * (z -2))); % third function
invz_funct3 = iztrans (Funct3); % Inverse z transform
disp (" Inverse Z transform of Function 3 is :") ;
disp ( invz_funct3 ); % displaying value in command
```

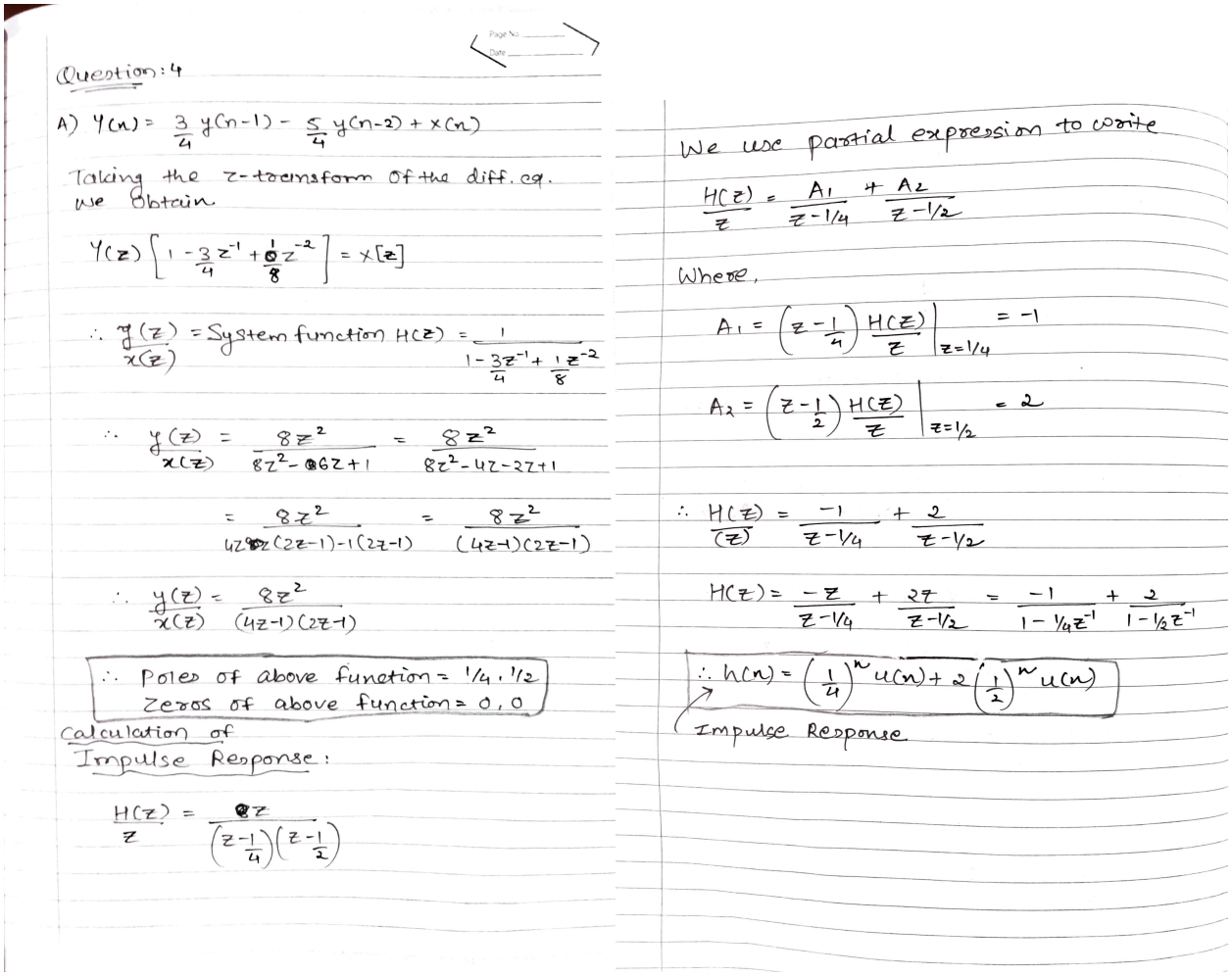
   (c) Simulation Output:

```
 Inverse Z transform of Function 1 is :
5*2^n - 4

 Inverse Z transform of Function 2 is :
(-1i)^(n - 1)*(1 - 1i/2) + 1i^(n - 1)*(1 + 1i/2)

 Inverse Z transform of Function 3 is :
4*2^n - n - 3

>>
```

4. Solution Problem-4

(a) Approach: In this question, first z-transform was taken from the input y(n) function. Then from the ration of y(z) and x(z), we get a transfer function. After simplifying the transfer function, it will be in p/q form. If we put p=0, we get zeros of transform function and if we put q=0, then we will get poles of transform function. Impulse response of the z-transform was plotted using impz function. Impulse response length was considered n = 16 to plot impulse response.

(b) Hand-written Analysis:



Question: 4

A) $y(n) = \frac{3}{4}y(n-1) - \frac{5}{4}y(n-2) + x(n)$

Taking the z-transform of the diff. eq. we obtain

$$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X[z]$$

$$\therefore \frac{y(z)}{x(z)} = \text{System function } H(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$\therefore \frac{y(z)}{x(z)} = \frac{8z^2}{8z^2 - 6z + 1} = \frac{8z^2}{8z^2 - 4z - 2z + 1}$$

$$= \frac{8z^2}{4z(2z-1) - 1(2z-1)} = \frac{8z^2}{(4z-1)(2z-1)}$$

$$\therefore \frac{y(z)}{x(z)} = \frac{8z^2}{(4z-1)(2z-1)}$$

∴ Poles of above function = 1/4, 1/2
Zeros of above function = 0, 0

Calculation of Impulse Response:

$$\frac{H(z)}{z} = \frac{8z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{2}\right)}$$

We use partial expression to write

$$\frac{H(z)}{z} = \frac{A_1}{z - 1/4} + \frac{A_2}{z - 1/2}$$

Where,

$$A_1 = \left(z - \frac{1}{4}\right)\frac{H(z)}{z}\Big|_{z=1/4} = -1$$

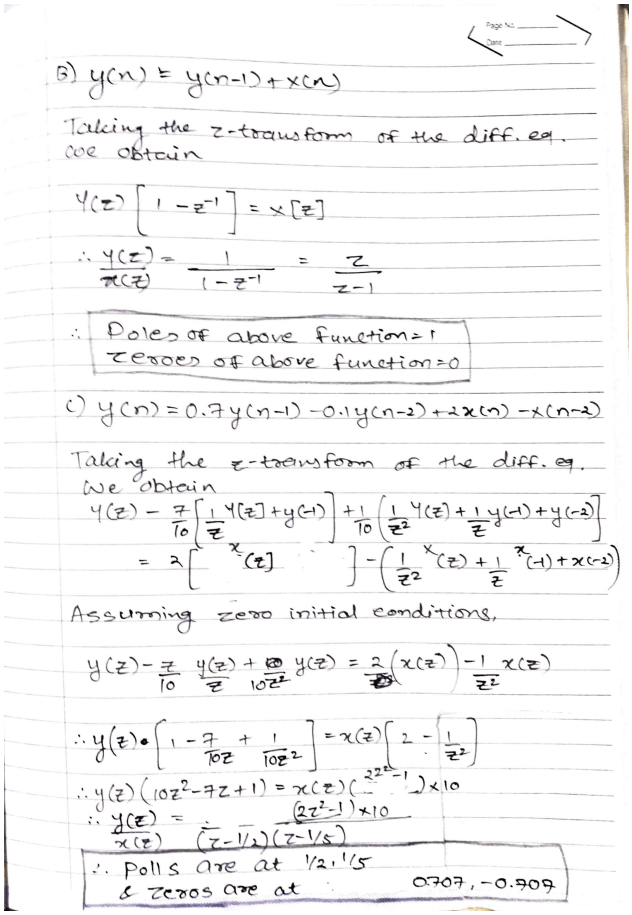
$$A_2 = \left(z - \frac{1}{2}\right)\frac{H(z)}{z}\Big|_{z=1/2} = 2$$

$$\therefore \frac{H(z)}{(z)} = \frac{-1}{z - 1/4} + \frac{2}{z - 1/2}$$

$$H(z) = \frac{-z}{z - 1/4} + \frac{2z}{z - 1/2} = \frac{-1}{1 - 1/4z^{-1}} + \frac{2}{1 - 1/2z^{-1}}$$

$$\therefore h(n) = \left(\frac{1}{4}\right)^n u(n) + 2\left(\frac{1}{2}\right)^n u(n)$$

Impulse Response



(c) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab6 (Question_4) Given equation X(n) and Y(n)we have to do z transform using
    zplane and then we have to and impulse response
clc ;
clear all ;
close all ;
%Sub-Question1
figure
subplot (211)
zplane ([0;0] ,[1/4;1/2]) ; % poles and zeros of 1st equation
subplot (212)
impz (1 ,[1;0.75;0.125] ,16) ; %Impulse Response

%Sub-Question2
figure ;
subplot (211)
zplane (0 ,1); % poles and zeros of second equation
subplot (212)
impz (1 ,[1; -1] ,16) ; %Impulse Response

%Sub-Question3
figure ;
subplot (211)
zplane ([0.707; -0.707] ,[1/5;1/2]) ; % poles and zeros of third equation
subplot (212)
impz ([2;0; -1] ,[1; -0.7;0.1] ,16) ; %Impulse Response
```

(d) Simulation Output:

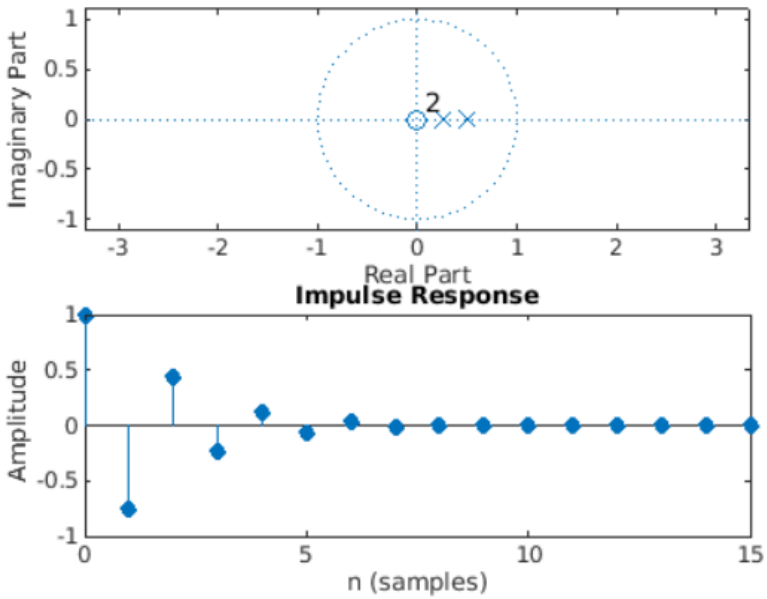


Figure 1: Function 1

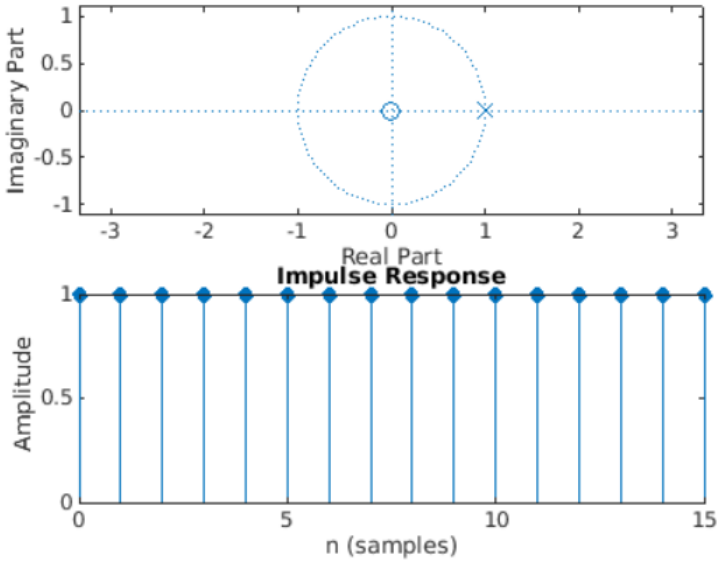


Figure 2: Function 2

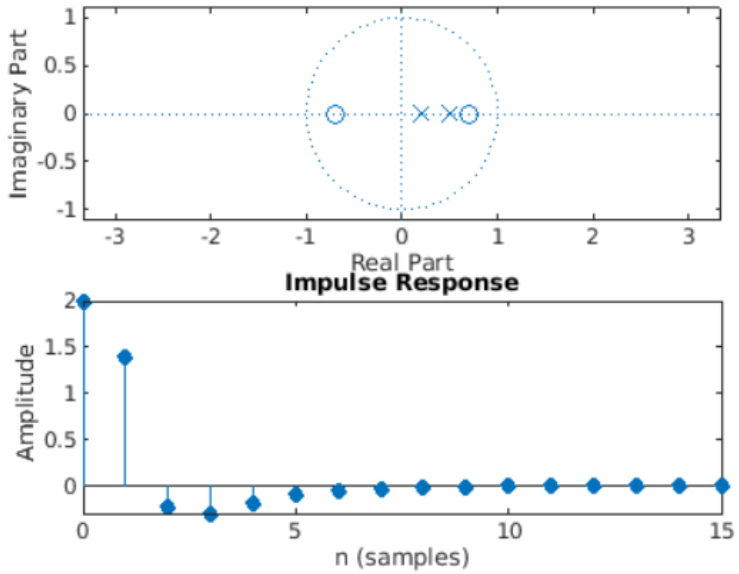


Figure 3: Function 3