

**School of Engineering and Applied Science (SEAS)
Ahmedabad University**

BTech(ICT) Digital Signal Processing (Section 1)

Laboratory Assignment-6

Enrollment No: AU1841145

Name: Samarth Shah

AIM : Understand different concepts of Z-transform.

1. Solution Problem-1

- (a) Approach: In this question, syms command was used to construct symbolic variables. All the function are attached by the unit step function. So it was created with the help of the heaviside function. Then it was multiplied with the corresponding equations and then ztrans command was used to obtain z-transform.
- (b) Matlab Script:

```
1 % Name : Samarth Shah
2 % Roll No: AU1841145
3 % Lab6 (Question_1) Create single matlab script and obtain Symbolic Z-Transform
4 syms z n a w_sinusoidal;
5
6 %Function1
7 function1=heaviside(sym(1)); %UnitStep Function
8 output1 = ztrans(function1,z);
9 disp (" Ztransform of Function 1 is :") ;
10 disp ( output1 ) % displaying result in command window
11
12 %Function2
13 function2=n.*function1;
14 output2 = ztrans(function2,z);
15 disp (" Ztransform of Function 2 is :") ;
16 disp ( output2 ) % displaying result in command window
17
18 %Function3
19 function3=(1+n).*function1;
20 output3 = ztrans(function3,z);
21 disp (" Ztransform of Function 3 is :") ;
22 disp ( output3 ) % displaying result in command window
23
24 %Function4
25 function4 = (cos((w_sinusoidal).*n)).*(function1);
26 output4 = ztrans(function4,z);
27 disp (" Ztransform of Function 4 is :") ;
28 disp ( output4 ) % displaying result in command window
29
30 %Function5
31 function5 = (sin((w_sinusoidal).*n)).*(function1);
32 output5 = ztrans(function5,z);
33 disp (" Ztransform of Function 5 is :") ;
34 disp ( output5 ) % displaying result in command window
35
36 %Function6
37 function6 = (a^n).*(cos((w_sinusoidal).*n)).*(function1);
38 output6 = ztrans(function6,z);
39 disp (" Ztransform of Function 6 is :") ;
40 disp ( output6 ) % displaying result in command window
41
42 %Function7
```

```

43 function7 = (a^n).*(sin((w_sinusoidal).*n)).*(function1);
44 output7 = ztrans(function7,z);
45 disp (" Ztransform of Function 7 is :") ;
46 disp ( output7 ) % displaying result in command window
47
48 %Function8
49 function8 = (a^n).*(n).*(function1);
50 output8 = ztrans(function8,z);
51 disp (" Ztransform of Function 8 is :") ;
52 disp ( output8 ) % displaying result in command window
53
54 %Function9
55 function9 = -(a^n).*(n).*(function1);
56 output9 = ztrans(function9,z);
57 disp (" Ztransform of Function 9 is :") ;
58 disp ( output9 ) % displaying result in command window
59
60 %Function10
61 function10 = ((-1)^n).*(n).*(function1);
62 output10 = ztrans(function10,z);
63 disp (" Ztransform of Function 10 is :") ;
64 disp ( output10 ) % displaying result in command window
65
66 %Function11
67 function11 = ((n)^2).*(function1);
68 output11 = ztrans(function11,z);
69 disp (" Ztransform of Function 11 is :") ;
70 disp ( output11 ) % displaying result in command window

```

(c) Simulation Output:

```

Ztransform of Function 4 is :
(z*(z - cos(w_sinusoidal)))/(z^2 - 2*cos(w_sinusoidal)*z + 1)

Ztransform of Function 5 is :
(z*sin(w_sinusoidal))/(z^2 - 2*cos(w_sinusoidal)*z + 1)

Ztransform of Function 6 is :
-(z*(cos(w_sinusoidal) - z/a))/(a*(z^2/a^2 - (2*z*cos(w_sinusoidal))/a + 1))

Ztransform of Function 7 is :
(z*sin(w_sinusoidal))/(a*(z^2/a^2 - (2*z*cos(w_sinusoidal))/a + 1))

Ztransform of Function 8 is :
(a*z)/(a - z)^2

Ztransform of Function 9 is :
-(a*z)/(a - z)^2

Ztransform of Function 10 is :
-z/(z + 1)^2

Ztransform of Function 11 is :
(z*(z + 1))/(z - 1)^3

```

>> Lab6_Question1

Ztransform of Function 1 is :

$$z/(z - 1)$$

Ztransform of Function 2 is :

$$z/(z - 1)^2$$

Ztransform of Function 3 is :

$$z/(z - 1) + z/(z - 1)^2$$

2. Solution Problem-2

- (a) Approach: In this question, from input function $x(n)$, z-transform was calculated. The z-transform will be in the p/q format. If we take $p=0$ then it will give all the zeroes of transform function and if we take $q=0$ then it will give all the poles associated with the transform function.
- (b) Hand-written Analysis:

Question 2

Poles & zeros

$x(z) = \frac{P(z)}{Q(z)}$ with $P(z), Q(z)$: polynomials in z

Zeroes: Values of z for which $x(z)=0$

Poles: Values of z for which $x(z)=\infty$

\therefore Roots of $P(z) = \text{Zeroes of } x(z)$
 Roots of $Q(z) = \text{Poles of } x(z)$

Aus: 1) $x(n) = \left(\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{3}\right)^n u(n)$ — Eq. (1)

Taking z-transform of above function, using this formula

$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

From Equation: 1

$$x(z) = \frac{z}{z-1/2} + \frac{z}{z+1/3}, |z| > 1/2$$

$$= \frac{z^2 + z/3 + z^2 - z/6}{(z-1/2)(z+1/3)} = \frac{2z^2 - z/6}{(z-1/2)(z+1/3)}$$

$$= \frac{z(2z-1/6)}{(z-1/2)(z+1/3)}, |z| > 1/2$$

\therefore Zeros of $x(z) = 0, 1/12$
 Poles of $x(z) = 1/2, -1/3$

Ans: 2) $x(n) = \left(-\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n-1) — \text{Eq. 2}$

Taking z-transform of above equation,

$x(z) = \frac{z}{z+1/3} + \frac{z}{z-1/2}, \frac{1}{3} < |z| < \frac{1}{2}$

$\therefore x(z) = \frac{z^2 - z/2 + z^2 + z/3}{(z-1/2)(z+1/3)}$
 $= \frac{2z^2 - z/6}{(z-1/2)(z+1/3)}, \frac{1}{3} < |z| < \frac{1}{2}$

From Above Eq.

\therefore Zeros of $x(z) = 0, 1/12$
 Poles of $x(z) = 1/2, -1/3$

Ans: 3) $x(n) = \left(\frac{1}{2}\right)^n u(-n)$

Taking z-transform of above Equation,

$x(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(-n) z^n$
 $= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$
 $= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{-n} z^n = \sum_{n=0}^{\infty} (2z)^n$
 $= \frac{1}{1-2z}, |z| > 1/2$

(c) Matlab Script:

```

1 % Name : Samarth Shah
2 % Roll No: AU1841145
3 % Lab6 (Question_2) Plot poles and zeros of the Z-transform obtained for signals
4 clc ;
5 close all ;
6 clear all ;
7
8 %Function-1
9 figure ;
10 zplane ([0;1/12] , [ -1/3;1/2]) ; % zeros and poles of 1st function
11 title('Function 1');
12
13 %Function-2
14 figure ;
15 zplane ([0;1/12] , [ -1/3;1/2]) ; % zeros and poles of 2nd function
16 title('Function 2');
17
18 %Function-3

```

Taking Z-Transform of above eq.

$$x(n) = \begin{cases} (-1)^{n/2} u\left(\frac{n}{2}\right) & \text{if } n \cdot 2 = 0 \\ 0 & \text{if } n \cdot 2 \neq 0 \end{cases}$$

$$\text{Consider the signal } g(n) = (-1)^{\frac{n}{2}} u\left(\frac{n}{2}\right) = x\left(\frac{n}{2}\right)$$

$$\text{Where } x(n) = a^n u(n)$$

$$\text{for, } x(n) = a^n u(n) \Leftrightarrow X(z) = \frac{1}{1 - az^{-1}} \text{ with ROC: } |z| > |a|$$

Using time expansion property,

$$x\left(\frac{n}{2}\right) = -(-1)^{\frac{n}{2}} u\left(\frac{n}{2}\right) \Leftrightarrow X(z^2) = \frac{1}{1 - z^{-2}} = \frac{z^2}{z^2 - 1}$$

with ROC: $|z| > |(-1)^{\frac{1}{2}}| = 1$

for $X(z) = \text{Zeros} = 0, 0$
And Poles = ~~0, 1~~ 1, -1

from the Equation computed from $x(n)$,

$$X(z) = \frac{1}{1 - 2z}, \quad |z| > 1/2$$

∴ Zeros of $X(z) = \text{None}$
Poles of $X(z) = 1/2$

$$i) X(n) = \{-1, 0, -1, 0, \dots\}$$

Simplifying above equation,

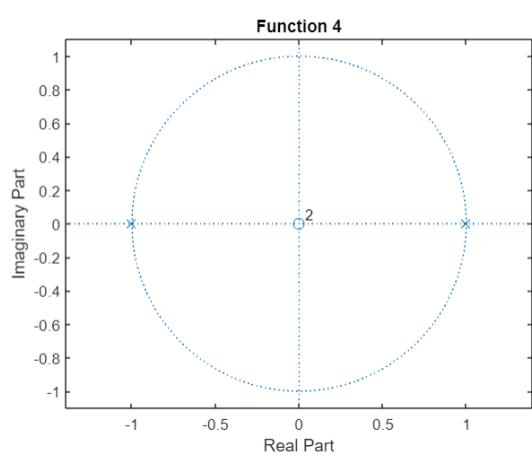
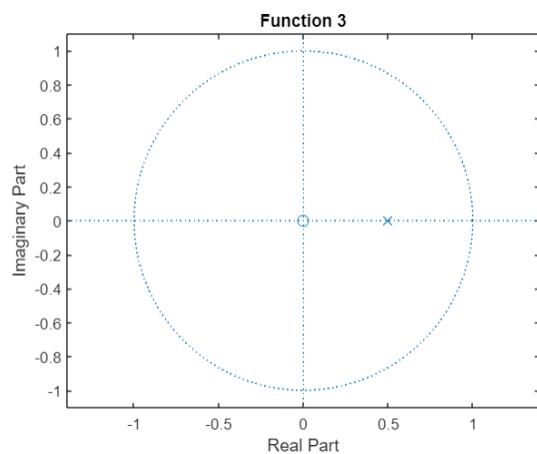
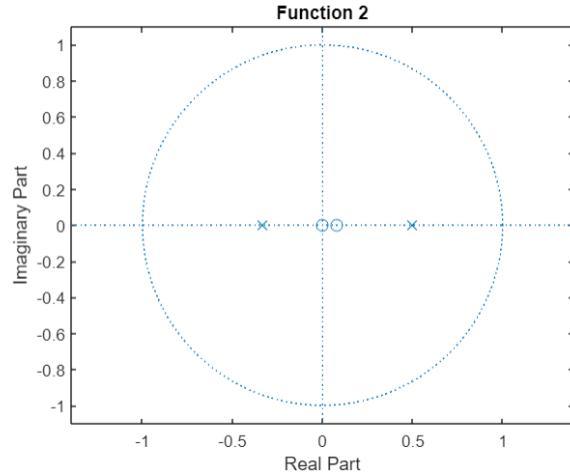
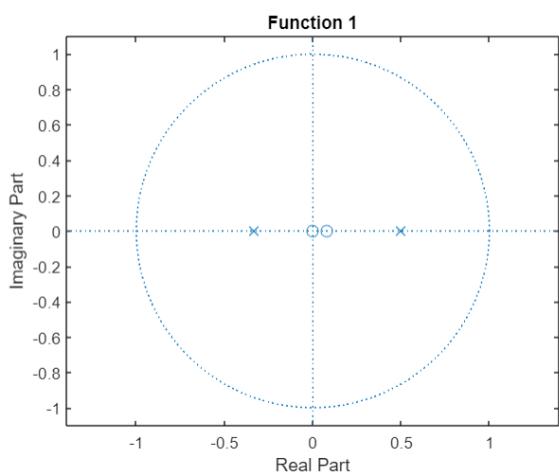
$$x(n) = \begin{cases} 0 & \text{if } k \neq 2n \\ n/2 & \text{else} \end{cases}$$

```

19 figure ;
20 zplane (0, 1/2); % zeros and poles of 3rd function
21 title('Function 3');
22
23 %Function-4
24 figure ;
25 zplane ([0;0], [-1;1]) % zeros and poles of 4th function
26 title('Function 4');

```

(d) Simulation Output:



3. Solution Problem-3

(a) Approach: We have taken X(z) and then using inverse ztransform mathematical equation we have calculated x(n) using iztrans function and displayed it in command window .

(b) Matlab Script:

```
1 % Name : Samarth Shah
2 % Roll No: AU1841145
3 % Lab6 (Question_3) Create single matlab script and obtain Symbolic Z-Transform
4 clc ;
5 clear all ;
6 close all ;
7 syms z % symbolically declared value
8
9 %First Function
10 Funct1 = (z^2 + 3*z)/(z^2 - 3*z + 2); % first function
11 invz_funct1 = iztrans (Funct1) ; % Inverse z transform
12 disp (" Inverse Z transform of Function 1 is :") ;
13 disp ( invz_funct1 ) ; % displaying value in command
14
15 %Second Function
16 Funct2 = ((z^2 + 2*z)/(z^2 + 1)); % second function
17 invz_funct2 = iztrans (Funct2); % Inverse z transform
18 disp (" Inverse Z transform of Function 2 is :") ;
19 disp ( invz_funct2 ) ; % displaying value in command
20
21 %Third Function
22 Funct3 = ((z ^3) /((z -1) ^2 * (z -2))); % third function
23 invz_funct3 = iztrans (Funct3); % Inverse z transform
24 disp (" Inverse Z transform of Function 3 is :") ;
25 disp ( invz_funct3 ) ; % displaying value in command
```

(c) Simulation Output:

Inverse Z transform of Function 1 is :

$5*2^n - 4$

Inverse Z transform of Function 2 is :

$(-1i)^{n-1}*(1 - 1i/2) + 1i^{n-1}*(1 + 1i/2)$

Inverse Z transform of Function 3 is :

$4*2^n - n - 3$

>> |

4. Solution Problem-4

(a) Approach: In this question, first z-transform was taken from the input $y(n)$ function. Then from the ratio of $y(z)$ and $x(z)$, we get a transfer function. After simplifying the transfer function, it will be in p/q form. If we put $p=0$, we get zeros of transform function and if we put $q=0$, then we will get poles of transform function. Impulse response of the z-transform was plotted using impz function. Impulse response length was considered $n = 16$ to plot impulse response.

(b) Hand-written Analysis:

Question: 4

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A) $y(n) = \frac{3}{4}y(n-1) - \frac{5}{4}y(n-2) + x(n)$

Taking the z-transform of the diff. eq. we obtain

$$Y(z) \left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} \right] = X(z)$$

$\therefore \frac{Y(z)}{X(z)} = \text{System function } H(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

$$\therefore \frac{Y(z)}{X(z)} = \frac{8z^2}{8z^2 - 6z + 1} = \frac{8z^2}{8z^2 - 4z - 2z + 1}$$

$$= \frac{8z^2}{(4z+1)(2z-1)} = \frac{8z^2}{(4z+1)(2z-1)}$$

$$\therefore \frac{y(z)}{x(z)} = \frac{8z^2}{(4z+1)(2z-1)}$$

\therefore Poles of above function = $-1/4, 1/2$
Zeros of above function = $0, 0$

Calculation of Impulse Response:

$$H(z) = \frac{8z}{(z+1/4)(z-1/2)}$$

We use partial expression to write

$$H(z) = \frac{A_1}{z-1/4} + \frac{A_2}{z-1/2}$$

Where,

$$A_1 = \left(z - \frac{1}{4} \right) \frac{H(z)}{z} \Big|_{z=1/4} = -1$$

$$A_2 = \left(z - \frac{1}{2} \right) \frac{H(z)}{z} \Big|_{z=1/2} = 2$$

$$\therefore H(z) = \frac{-1}{z-1/4} + \frac{2}{z-1/2}$$

$$H(z) = \frac{-z}{z-1/4} + \frac{2z}{z-1/2} = \frac{-1}{1-1/4z^{-1}} + \frac{2}{1-1/2z^{-1}}$$

$$\therefore h(n) = \left(\frac{1}{4}\right)^n u(n) + 2\left(\frac{1}{2}\right)^n u(n)$$

Impulse Response.

$$6) y(n) = y(n-1) + x(n)$$

Taking the z-transform of the diff. eq.
we obtain

$$Y(z) [1 - z^{-1}] = X(z)$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1}} = \frac{z}{z-1}$$

\therefore Poles of above function = 1
 \therefore Zeros of above function = 0

$$c) y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$$

Taking the z-transform of the diff. eq.
we obtain

$$\begin{aligned} Y(z) - \frac{7}{10} \left[\frac{1}{z} Y(z) + y(-1) \right] + \frac{1}{10} \left(\frac{1}{z^2} Y(z) + \frac{1}{z} y(-1) + y(-2) \right) \\ = 2 \left[\frac{x(z)}{z} \right] - \left(\frac{1}{z^2} x(z) + \frac{1}{z} x(-1) + x(-2) \right) \end{aligned}$$

Assuming zero initial conditions,

$$Y(z) - \frac{7}{10} \frac{Y(z)}{z} + \frac{1}{10z^2} Y(z) = 2 \left(\frac{x(z)}{z} \right) - \frac{1}{z^2} x(z)$$

$$\therefore Y(z) \left[1 - \frac{7}{10z} + \frac{1}{10z^2} \right] = X(z) \left[2 - \frac{1}{z^2} \right]$$

$$\therefore Y(z) (10z^2 - 7z + 1) = X(z) (2z^2 - 1) \times 10$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{(2z^2 - 1) \times 10}{(10z^2 - 7z + 1)}$$

\therefore Poles are at $1/2, 1/5$
& zeros are at $0.707, -0.707$

(c) Matlab Script:

```

1 % Name : Samarth Shah
2 % Roll No: AU1841145
3 % Lab6 (Question_4) Given equation X(n) and Y(n) we have to do z transform using
4 % zplane and then we have to find impulse response
5 clc ;
6 clear all ;
7 close all ;
8 % Sub-Question1
9 figure
10 subplot (211)
11 zplane ([0;0] , [1/4;1/2]) ; % poles and zeros of 1st equation
12 subplot (212)
13 impz (1 ,[1;0.75;0.125] ,16) ; % Impulse Response
14 % Sub-Question2
15 figure ;
16 subplot (211)
17 zplane (0 ,1) ; % poles and zeros of second equation
18 subplot (212)
19 impz (1 , [1; -1] ,16) ; % Impulse Response
20
21 % Sub-Question3
22 figure ;
23 subplot (211)
24 zplane ([0.707; -0.707] , [1/5;1/2]) ; % poles and zeros of third equation
25 subplot (212)
26 impz ([2;0; -1] , [1; -0.7;0.1] ,16) ; % Impulse Response

```

(d) Simulation Output:

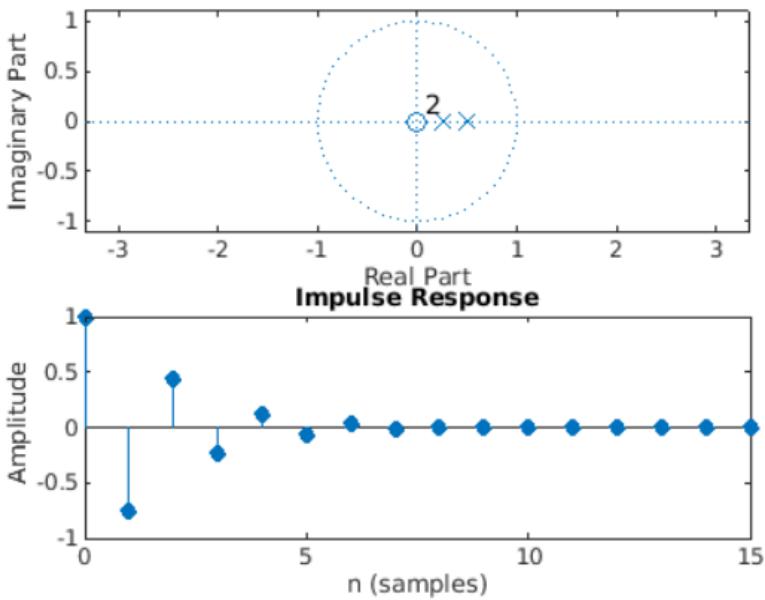


Figure 1: Function 1

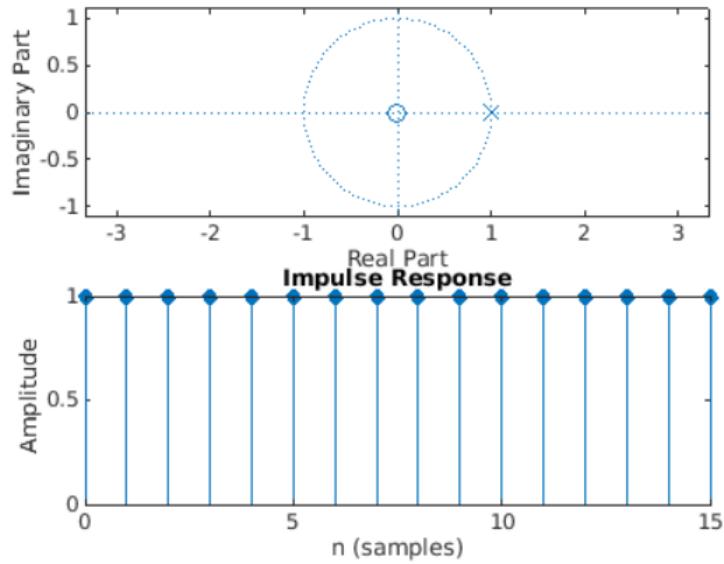


Figure 2: Function 2

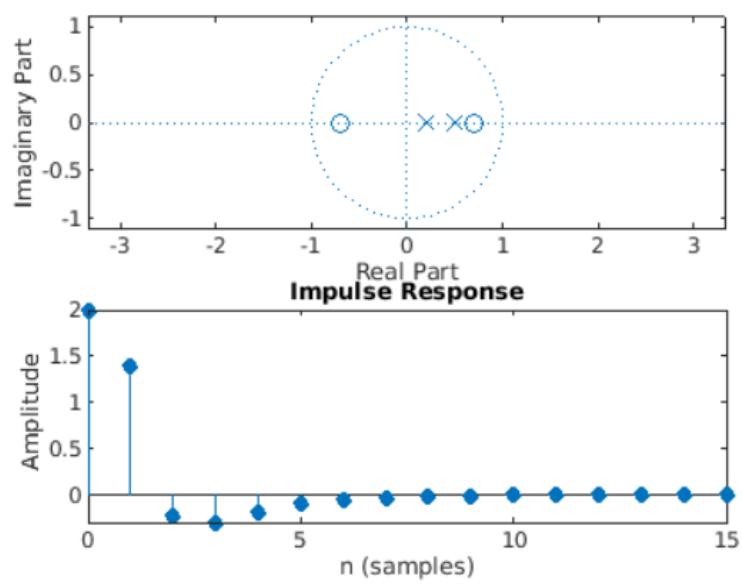


Figure 3: Function 3