School of Engineering and Applied Science (SEAS)
Ahmedabad University

BTech(ICT) Digital Signal Processing (Section 1)

Laboratory Assignment-8 Question 1

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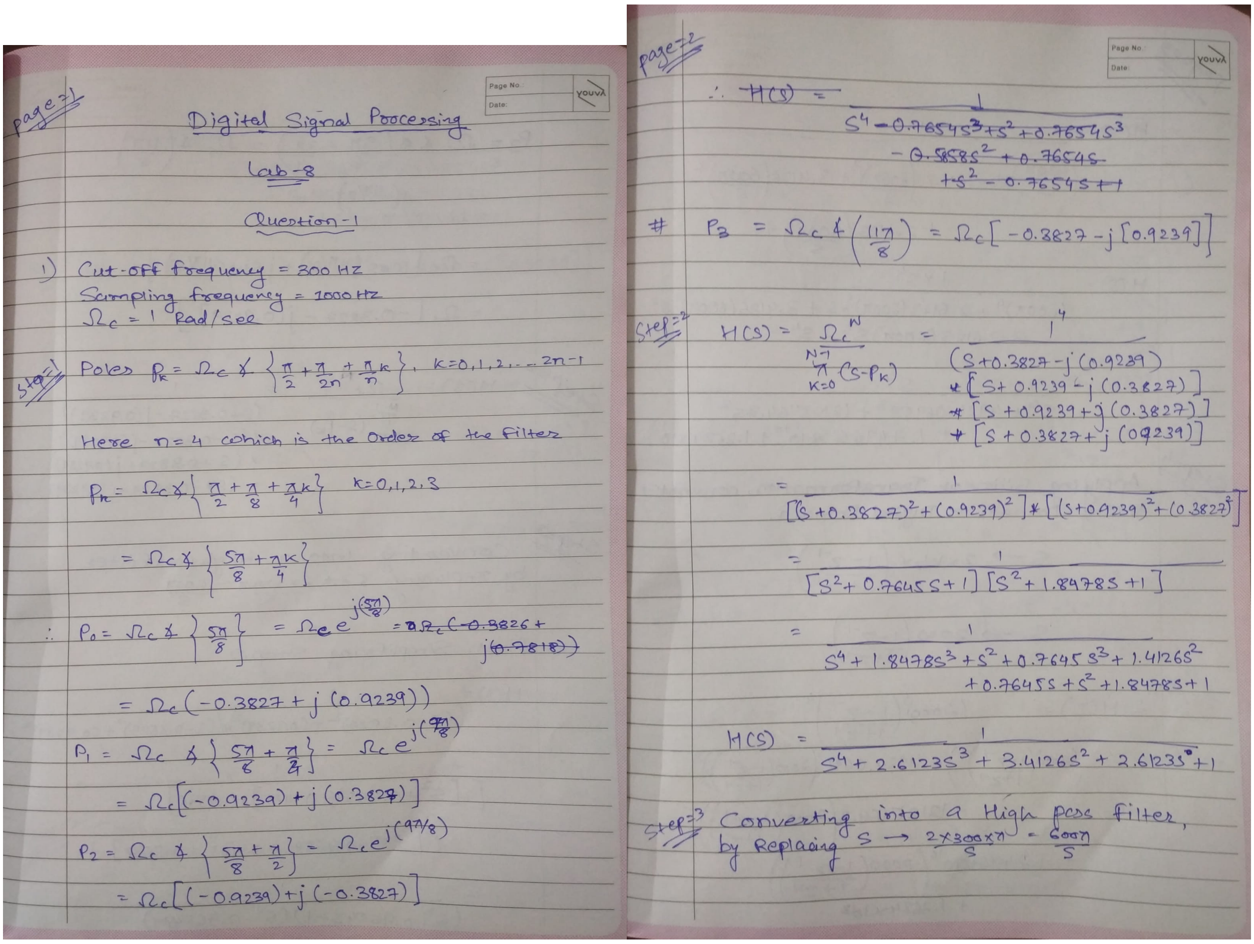
AIM : Design an IIR filter using bilinear transformation using Butterworth Filter design approximations.

1. Solution Problem-1

   (a) Handwritten Analysis:



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Digital Signal Processing

Lab-8

Question-1

1) Cut-off frequency = 300 HZ
Sampling frequency = 1000 HZ
$\Omega_c = 1$ Rad/sec

step-1 Poles $P_k = \Omega_c \measuredangle \left\{ \frac{\pi}{2} + \frac{\pi}{2n} + \frac{\pi k}{n} \right\}$, $k = 0, 1, 2, \ldots 2n-1$

Here $n = 4$ which is the Order of the Filter

$P_k = \Omega_c \measuredangle \left\{ \frac{\pi}{2} + \frac{\pi}{8} + \frac{\pi k}{4} \right\}$ $k = 0, 1, 2, 3$

$= \Omega_c \measuredangle \left\{ \frac{5\pi}{8} + \frac{\pi k}{4} \right\}$

$\therefore P_0 = \Omega_c \measuredangle \left\{ \frac{5\pi}{8} \right\} = \Omega_c e^{j\left(\frac{5\pi}{8}\right)} = \cancel{\Omega_c(-0.3826 + j(0.7818))}$

$= \Omega_c(-0.3827 + j(0.9239))$

$P_1 = \Omega_c \measuredangle \left\{ \frac{5\pi}{8} + \frac{\pi}{4} \right\} = \Omega_c e^{j\left(\frac{7\pi}{8}\right)}$

$= \Omega_c[(-0.9239) + j(0.3827)]$

$P_2 = \Omega_c \measuredangle \left\{ \frac{5\pi}{8} + \frac{\pi}{2} \right\} = \Omega_c e^{j(9\pi/8)}$

$= \Omega_c[(-0.9239) + j(-0.3827)]$

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$\cancel{\therefore H(s) = \frac{1}{s^4 - 0.7654s^3 + s^2 + 0.7654s^3 - 0.5858s^2 + 0.7654s + s^2 - 0.7654s + 1}}$

$P_3 = \Omega_c \measuredangle \left(\frac{11\pi}{8}\right) = \Omega_c[-0.3827 - j[0.9239]]$

Step-2 $H(s) = \frac{\Omega_c^N}{\prod_{k=0}^{N-1} (s - P_k)} = \frac{1^4}{(s + 0.3827 - j(0.9239)) * [s + 0.9239 - j(0.3827)] * [s + 0.9239 + j(0.3827)] * [s + 0.3827 + j(0.9239)]}$

$= \frac{1}{[(s + 0.3827)^2 + (0.9239)^2] * [(s + 0.9239)^2 + (0.3827)^2]}$

$= \frac{1}{[s^2 + 0.7645s + 1][s^2 + 1.8478s + 1]}$

$= \frac{1}{s^4 + 1.8478s^3 + s^2 + 0.7645s^3 + 1.4126s^2 + 0.7645s + s^2 + 1.8478s + 1}$

$H(s) = \frac{1}{s^4 + 2.6123s^3 + 3.4126s^2 + 2.6123s + 1}$

step-3 Converting into a High pass filter, by Replacing $s \rightarrow \frac{2 \times 300 \times \pi}{s} = \frac{600\pi}{s}$

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$$H(S) = \frac{1}{\left(\frac{600\pi}{s}\right)^4 + 2.6123\left(\frac{600\pi}{s}\right)^3 + 3.4126\left(\frac{600\pi}{s}\right)^2 + 2.6123\left(\frac{600\pi}{s}\right) + 1}$$

$$H(S) = \frac{1 \times S^4}{(600\pi)^4 + 2.6123(600\pi)^3 \times S + 3.4126(600\pi)^2 S^2 + 2.6123(600\pi)S^3 + S^4}$$

$$= \frac{S^4}{S^4 + 4924.0695S^3 + 12125164.3S^2 + 1.7496S * 10^{10} + 1.2624 \times 10^{13}}$$

Step=4

Applying Bilinear Transformation, converting Analog Domain to Digital Domain.

$$S \rightarrow 2 \times W_c \times \left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$S \rightarrow 2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$\therefore H(z) = \frac{(2000)^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^4}{\left(2000^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^4\right) + 4924.0695\left(2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^3 + 12125164.3\left(2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 1.7496 \times 10^{10}\left(2000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) + 1.2624 \times 10^{13}}$$

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$$H(z) = \frac{(2000)^4(1-z^{-1})^4}{(2000 \times (1-z^{-1}))^4 + 4924.0695(1+z^{-1})[2000(1-z^{-1})]^3 + 12125164.3(1+z^{-1})^2[2000[1-z^{-1}]]^2 + 1.7496 \times 10^{10}(1+z^{-1})^3[2000[1-z^{-1}]] + 1.2643 \times 10^{13}[1+z^{-1}]^4}$$

$$H(z) = \frac{1.6 \times 10^{13}(1-z^{-1})^2(1-z^{-1})^2}{(1.6 \times 10^{13} \times (1-z^{-1})^2(1-z^{-1})^2) + [3.9392 \times 10^{13}(1+z^{-1})(1-z^{-1})^3] + [4.85 \times 10^{13}(1+z^{-1})^2(1-z^{-1})^2] + [3.4992 \times 10^{13}(1+z^{-1})^3[1-z^{-1}]] + [1.2643 \times 10^{13}[1+z^{-1}]^2[1+z^{-1}]^2]}$$

$$\Rightarrow (1-z^{-1})^2(1-z^{-1})^2 = [1-2z^{-1}+z^{-2}][1-2z^{-1}+z^{-2}]$$
$$= [1-2z^{-1}+z^{-2}-2z^{-1}+4z^{-2} - 2z^{-3}+z^{-2} - 2z^{-3}+z^{-4}]$$
$$= [1-4z^{-1}+6z^{-2}-4z^{-3}+z^{-4}]$$

$$\Rightarrow (1+z^{-1})^2(1+z^{-1})^2 = 1+4z^{-1}+6z^{-2}+4z^{-3}+z^{-4}$$

$$\Rightarrow (1+z^{-1})(1-z^{-1})^3 = (1+z^{-1})(1-z^{-3}-3z^{-1}(1-z^{-1}))$$
$$= 1-z^{-3}-3z^{-1}+3z^{-2} + z^{-1}-z^{-4}-3z^{-2}+3z^{-3}$$
$$= 1-2z^{-1}+2z^{-3}-z^{-4}$$

$$\Rightarrow (1+z^{-1})^2(1-z^{-1})^2 = [1-2z^{-1}+z^{-2}][1+2z^{-1}+z^{-2}]$$
$$= 1+\cancel{2z^{-1}}+z^{-2}-\cancel{2z^{-1}}-4z^{-2} - \cancel{2z^{-3}}+z^{-2}+\cancel{2z^{-3}}+z^{-4}$$
$$= 1-2z^{-2}+z^{-4}$$

$$\Rightarrow [1+z^{-1}]^3[1-z^{-1}] = [1-z^{-1}][1+z^{-3}+3z^{-1}(1+z^{-1})]$$
$$= 1+z^{-3}+3z^{-1}+\cancel{3z^{-2}}-z^{-1}-z^{-4} - \cancel{3z^{-2}}-3z^{-3}$$
$$= 1+2z^{-1}-2z^{-3}-z^{-4}$$

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$$H(z) = \frac{1.6(1-4z^{-1}+6z^{-2}-4z^{-3}+z^{-4}]}{\begin{aligned}&[1.6(1-4z^{-1}+6z^{-2}-4z^{-3}+z^{-4})]\\ &+[3.9393(1-2z^{-1}+2z^{-3}-z^{-4})]\\ &+[4.85(1-2z^{-2}+z^{-4})]\\ &+[3.4992(1+2z^{-1}-2z^{-3}-z^{-4})]\\ &+[1.2643(1+4z^{-1}+6z^{-2}+4z^{-3}+z^{-4})]\end{aligned}}$$

$$H(z) = \frac{1.6-6.4z^{-1}+9.6z^{-2}-6.4z^{-3}+1.6z^{-4}}{\begin{aligned}&1.6-6.4z^{-1}+9.6z^{-2}-6.4z^{-3}+1.6z^{-4}\\ &+3.9393-7.8786z^{-1}+7.8786z^{-3}\\ &-3.9393z^{-4}\\ &+4.85-9.7z^{-2}+4.85z^{-4}+3.4992+6.9984z^{-1}\\ &-6.9984z^{-3}-3.4992z^{-4}\\ &+1.2643+5.0572z^{-1}+7.5858z^{-2}\\ &+5.0572z^{-3}+1.2643z^{-4}\end{aligned}}$$

$$H(z) = \frac{1.6-6.4z^{-1}+9.6z^{-2}-6.4z^{-3}+1.6z^{-4}}{15.1528-2.223z^{-1}+7.4858z^{-2}-0.4626z^{-3}-0.276z^{-4}}$$

$$H(z) = \frac{\frac{1.6}{15.1528}-\frac{6.4z^{-1}}{15.1528}+\frac{9.6z^{-2}}{15.1528}-\frac{6.4z^{-3}}{15.1828}+\frac{1.6z^{-4}}{15.158}}{\frac{15.1528}{15.1528}-\frac{2.223z^{-1}}{15.1528}+\frac{7.4858z^{-2}}{15.1528}-\frac{0.4826z^{-3}}{15.1528}-\frac{0.276z^{-4}}{15.1528}}$$

$$H(z) = \frac{0.1056-0.4223z^{-1}+0.6335z^{-2}-0.4223z^{-3}+0.1056z^{-4}}{1-0.1467z^{-1}+0.4940z^{-2}-0.0305z^{-3}-0.0018z^{-4}}$$

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$$\therefore H(z) = \frac{0.1056-0.4223z^{-1}+0.6335z^{-2}-0.4223z^{-3}+0.1056z^{-4}}{1-0.1467z^{-1}+0.4940z^{-2}-0.0305z^{-3}-0.0018z^{-4}}$$

(b) Matlab Script:

```
% Name : Samarth Shah
% Roll No: AU1841145
% Lab8 ( Question_1 ) Design the 4th Order Butterworth high pass filter having cut
    -off frequency 300Hz andsampling frequency 1 KHz using bilinear transformation
    .
clc ;
clear all ;
close all ;
%Designing Filter using in-built functions
% Here, 's' designs a lowpass, highpass, bandpass, or bandstop analog
% Butterworth filter with cutoff angular frequency Wn. 4 and 1 is the order
% of the filter and cut-off frequency respectively.
[b,a] = butter (4,1,'s');
%Transform lowpass analog filters to highpass. Cutoff angular frequency =
%2*pi*300
[b_highpass, a_highpass] = lp2hp (b,a,2*pi*300);
%Bilinear transformation method for analog-to-digital filter conversion
[numd,dend] = bilinear (b_highpass,a_highpass,1000) ;
%Plotting Frequency Response
figure;
[numd1,dend1] = freqz(numd,dend);
plot(abs(numd1)); %Magnitude of frequency Response
title('Using Inbuilt Function')
xlabel('Frequency')
ylabel('Magnitude')
%Plotting Zeroes and poles
figure;
zplane (numd,dend);

%From Hand-written Analysis
%Transfer Function Co-efficients
b1=[0.1056,-0.4223,0.6335,-0.4223,0.1056];
a1=[1,-0.1467,0.4940,-0.0305,-0.0182];
%Ploting Frequency Response
figure;
[numd2,dend2] = freqz(b1,a1);
plot(abs(numd2)); %Magnitude of frequency Response
title('From Hand-Written Analysis')
xlabel('Frequency')
ylabel('Magnitude')
```

(c) Simulation Output:

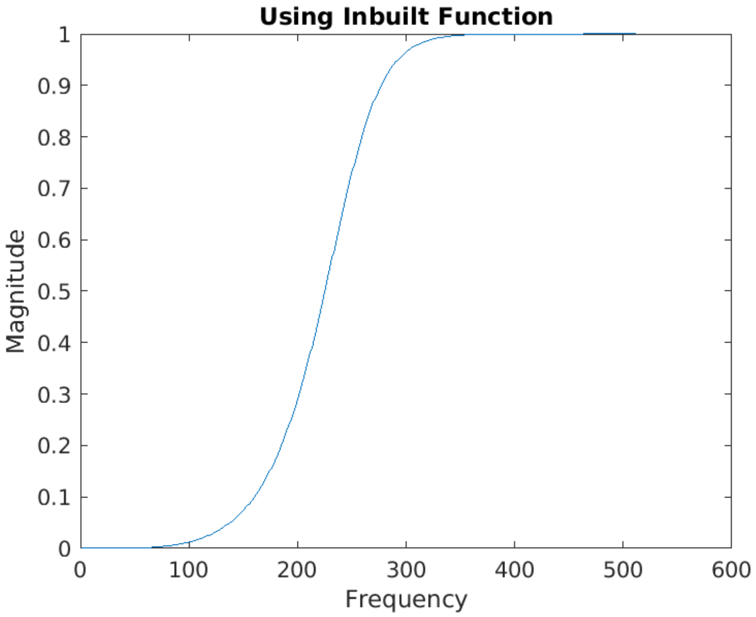

Figure 1: The 4th Order Butterworth high pass filter from Inbuilt Function (Magnitude Response)

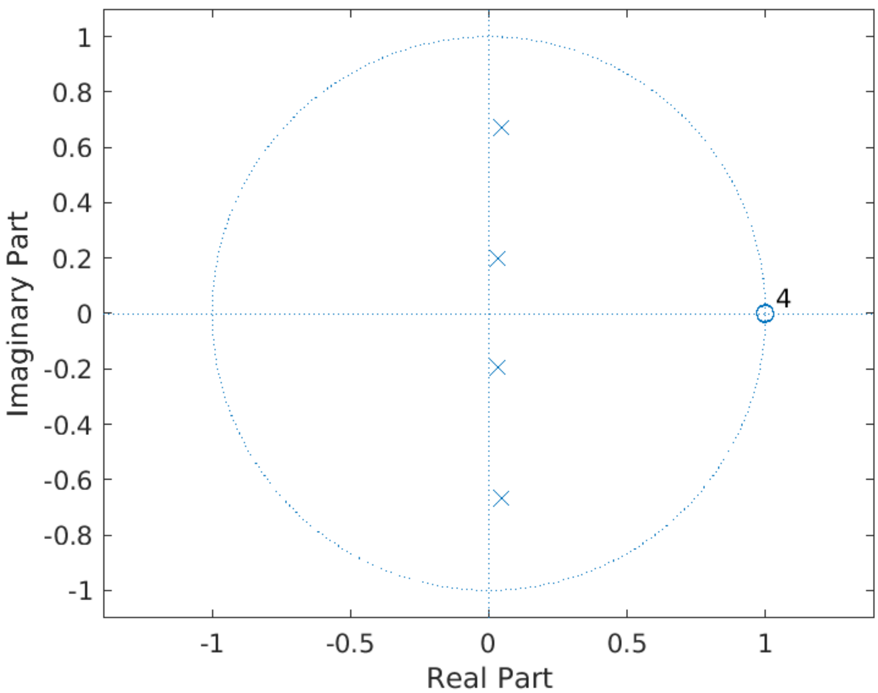

Figure 2: Plot of poles and zeros

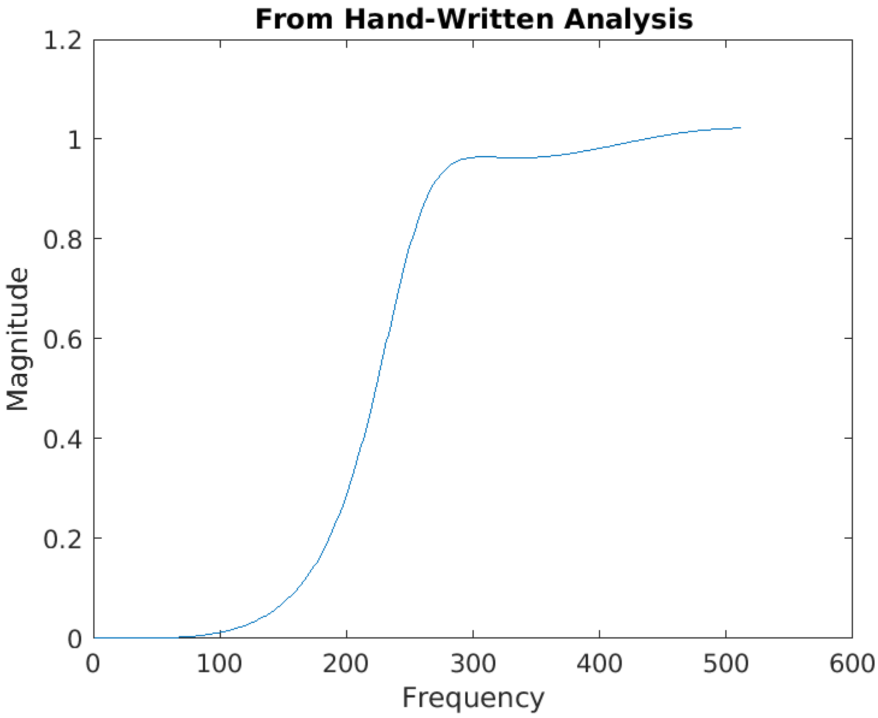

Figure 3: The 4th Order Butterworth high pass filter from Handwritten Analysis (Magnitude Response)