PHYC90045 Introduction to Quantum Computing

Assignment 1

Due: 5pm Thursday 19th September, 2019
Submission: place in the assignment box "PHYC90045 Introduction to Quantum Computing" outside Tutorial Room 207 (Physics Podium, David Caro Building)

Welcome to Assignment 1 for PHYC90045 *Introduction to Quantum Computing.*

Instructions: Work on your own, attempt all questions. Hand in your completed written work (with name and student number on the front) on or before the due date as per instructions above. The QUI circuits you create for this project must be saved with the indicated filenames (including your student number as specified).

Total marks = 40

1. [1+2+1+2 = 6 marks] For all of the following, use the matrix representation.

(a) Starting from the $|0\rangle$ state, write out the state obtained when you apply a Hadamard (H) operation.

(b) Starting from $|0\rangle$, find an operation (matrix form) that produces the state $\frac{|0\rangle-\sqrt{3}|1\rangle}{2}$. Plot the resulting state on the Bloch sphere.

(c) Starting from $\frac{|0\rangle-\sqrt{3}|1\rangle}{2}$, find an operation (matrix form) that produces the state $\frac{|0\rangle+i|1\rangle}{\sqrt{2}}$.

(d) Consider a single qubit rotation operation around the Z-axis is an angle θ_R which is given to you in matrix form as:

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta_R} \end{bmatrix}$$

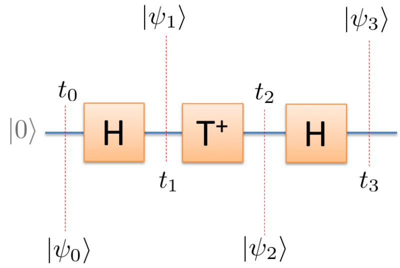
Using the expression for the general R-gate $R_Z(\theta_R)$ as implemented in the QUI find the appropriate setting for the global phase valid for a given rotation angle θ_R.

2. [2 marks] Consider the single qubit operators R1 = THTH and R2 = HTHT. Starting from the state $|0\rangle$, Bob is doing a QC assignment where he has to design a sequence using exactly four operators from the set {R1,R2} to produce the state $|\psi\rangle = a_0|0\rangle + a_1|1\rangle$ where $a_0 = 0.757\, e^{i0.294\pi}$ and $a_1 = 0.653\, e^{i0.625\pi}$.

(a) Find the sequence of H and T gates Bob is looking for (QUI might help you).

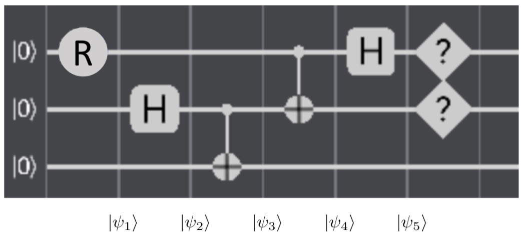
(b) By definition Bob's solution involves 16 H and T gates in total. Alice says she can do it in 14. From Bob's solution find Alice's sequence of H and T gates.

3. [2 marks] Consider the sequence of gates shown (where T^{+} is the Hermitian conjugate of T). Compute by hand the states at each time step in the matrix representation, and convert to ket representation.



4. [3 marks] Consider the following entangled two-qubit state: $|\psi\rangle = a_{00}|00\rangle + a_{11}|11\rangle$, where $a_{00} = i/2$ and $a_{11} = -\sqrt{3}/2$. Find the circuit containing one R-gate around a single cartesian axis (indicating the axis, rotation angle, and global phase) and one two-qubit gate that produces this state from $|00\rangle$.

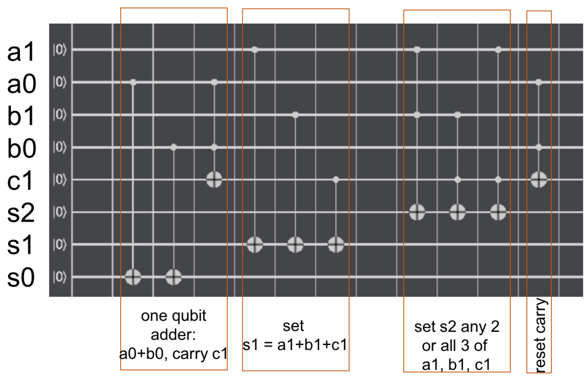
5. [4 marks] A quantum teleportation circuit is shown below:

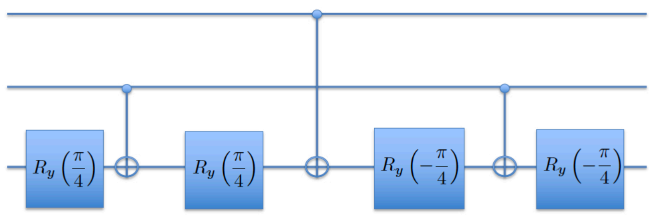


(a) Draw the circuit labelling the qubits associated with Alice and Bob and the qubit state to be teleported. For arbitrary rotation θ around the X-axis in the initial R-gate (global phase zero), determine by hand the states $|\psi_1\rangle$ to $|\psi_5\rangle$ (ket notation) as defined above.

(b) Show that after measurement by Alice and appropriate correction(s) applied by Bob the original state is teleported for all angles θ.

6. [5 marks] Consider the two-bit adder circuit we encountered in Lab 3 (below left).





The Toffoli decomposition given above (right) is efficient, but does not preserve the relative phase (as opposed to that given in Lab 2). Using this decomposition, program the adder circuit in the QUI using only CNOT gates and single qubit gates, and minimise the total number of gates and/or qubits as far as possible. The circuit must produce the correct probabilities for superposition inputs, but you may allow the relative phases in the output state to differ from that in the original circuit. Briefly describe any simplifications you have made. You will be marked on whether the circuit works, and also the degree of compression you have achieved.

Save the circuit as "<Student number>_Assignment-1_Q6".

7. [6 marks] Consider the function $f(x) = 2^x \bmod 15$. Create a circuit in the QUI which uses Grover's algorithm to search for the values of x where $f(x) = 4$. The register for x should comprise four qubits. Briefly describe how the circuit works and comment on the results.

Save the circuit as "<Student number>_Assignment-1_Q7".

8. [2+2+3 = 7 marks]

(a) Construct an oracle (4 qubits), which identifies inputs (applies a phase of π) which are prime numbers (2, 3, 5…) less than 16. Briefly describe your implementation of this circuit.

Save this circuit as "<Student number>_Assignment-1_Q8a".

(b) Use the oracle which you constructed in part (a) to implement a single iteration of Grover's algorithm, and determine the probability of measuring a prime at the output.

Save this circuit as "<Student number>_Assignment-1_Q8b".

(c) Describe, using circuit diagrams and showing your working, how the probability of measuring a prime could be increased to 100%.

Save this circuit as "<Student number>_Assignment-1_Q8c".

9. [1+4 = 5 marks] "Qutrits" are a generalisation of qubits over three basis states $\{|0\rangle, |1\rangle, |2\rangle\}$. With respect to the associated matrix representation, i.e.

$$|0\rangle = \begin{bmatrix}1\\0\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\\0\end{bmatrix}, |2\rangle = \begin{bmatrix}0\\0\\1\end{bmatrix},$$

a particular operation U on a general qutrit state is given in matrix form as:

$$U = \begin{bmatrix}\cos(\pi/3) & 0 & i\sin(\pi/3)\\ 0 & 1 & 0\\ i\sin(\pi/3) & 0 & \cos(\pi/3)\end{bmatrix}$$

(a) Show that the operation U is unitary (i.e. $UU^{\dagger} = 1$) and show explicitly how it acts on an arbitrary superposition over the qutrit basis states.

(b) Using two qubits we can implement the qutrit operation U using the states $\{|00\rangle, |01\rangle, |10\rangle\}$ to represent the qutrit states. Do this in the QUI, and comment briefly on the construction of your circuit. Save this circuit as "<Student number>_Assignment-1_Q9b".