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Experimental design for product and process design and development

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Summary. The design of new products, or the improvement of existing ones, as well as the design and development of manufacturing processes to produce them, are crucial activities in most industrial organizations. Engineers and scientists play a critical role in these activities, and the efficiency and effectiveness with which the development process is performed are often a key factor in organizational success. Statistically designed experiments are an important component of product and process design and development. The last 20 years have seen many new developments in experimental design, accompanied by a significant growth of applications in engineering. Typical experimental design application areas include process and product characterization, achieving variability reduction, control and stability, process optimization and designing processes and products to achieve robustness. This presentation focuses on these applications, surveying and discussing contemporary methodology. Examples are given that illustrate applying these techniques in practice.

Keywords: Combined array designs; Multiple-response optimization; Response model; Response surfaces; Robust parameter design

1. Introduction and background

Experiments are performed by investigators in virtually all fields of inquiry. Generally, experiments are used in studying or evaluating the performance of systems. The system under study can be represented by the general model of Fig. 1. We visualize the system as a combination of components, materials, people, equipment, processes and other resources that function collectively to transform a set of inputs into outputs described by one or more response variables. There are $p = k + r$ variables that potentially affect the performance of the system. Variables $x_1, x_2, \ldots, x_k$ are controllable variables that can be adjusted to and held at specific target levels, whereas the variables $z_1, z_2, \ldots, z_r$ are either difficult to control or uncontrollable in the field performance of the system, although they may be controlled for performing a specific experiment.

In many industrial environments, the system of Fig. 1 is a production process or a product, and the objectives of the experiment may include the following:

(a) to determine which variables are most influential on the response(s),
(b) to determine where to set the influential xs so that the response or responses are almost always near their desired target values,
(c) to determine where to set the influential xs so that the variability in the response(s) is small or

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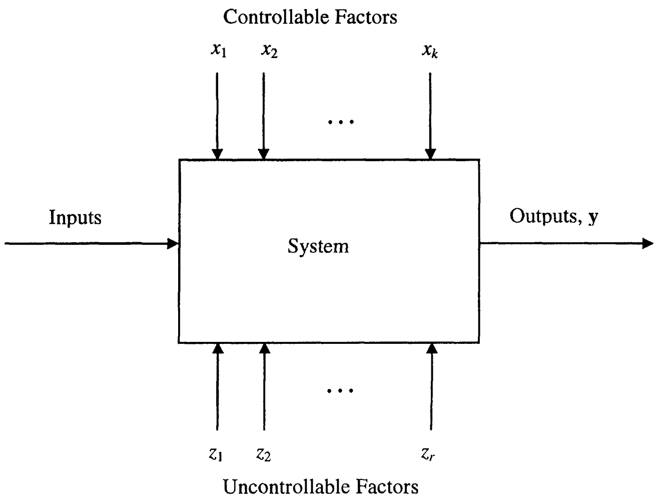

Fig. 1. General model of a system

(d) to determine where to set the influential xs so that the effects of the uncontrollable variables on the response(s) are small.

Statistically designed experiments are extremely important in these activities. This paper discusses some aspects of how statistical design is used in developing new processes or improving the performance of existing processes. Designed experiments also play a major role in engineering design activities, where new products are designed and developed and existing products are modified to enhance their features, characteristics or operation.

2. A perspective on the development of statistical design

There have been four eras in the modern development of statistical experimental design. The agricultural era was led by the pioneering work of R. A. Fisher in the 1920s and 1930s. Fisher recognized that the analysis of data from systems (in this case agricultural systems) was often hampered by flaws in the way the experiment that generated the data had been performed. By interacting with scientists and researchers in many fields he developed the insights that led to the three basic principles of experimental design: randomization, replication and blocking. Fisher systematically introduced statistical thinking and principles into designing experimental investigations, including the factorial design concept, the basic building-block of much subsequent work in the field.

Although applications of statistical design in industrial settings certainly began in the 1930s, the second or industrial era was catalysed by the development of response surface methodology (RSM) by Box and Wilson (1951). They recognized and exploited the fact that many industrial experiments are fundamentally different from their agricultural counterparts in two ways:

(a) the response variable can usually be observed (nearly) immediately and
(b) the experimenter can quickly learn crucial information from a small group of runs that can be used to plan the next experiment.

Over the next 30 years, RSM and other design techniques spread throughout the chemical and the process industries, although mostly in research and development work. The application of statistical design at the plant or manufacturing process level was still not extremely widespread. Some of the reasons for this include inadequate training in basic statistical concepts and methods for engineers and other process specialists, and the lack of computing and software resources to support the application of statistically designed experiments.

The increasing interest of Western industry in quality improvement that began in the late 1970s ushered in the third era of statistical design. The work of Genichi Taguchi (Taguchi and Wu, 1980; Kackar, 1985; Taguchi, 1987, 1991) had a significant effect on expanding the interest in and use of designed experiments. Taguchi advocated using designed experiments for what he termed robust parameter design, or

(a) making processes insensitive to environmental factors or other factors that are difficult to control,
(b) making products insensitive to variation transmitted from components and
(c) finding levels of the process variables that force the mean to a desired value while simultaneously reducing variability around this value.

Taguchi suggested highly fractionated factorial designs and other orthogonal arrays along with some novel statistical methods to solve these problems. The resulting methodology generated much discussion and controversy. Part of the controversy arose because Taguchi's methodology was advocated in the West initially (and primarily) by entrepreneurs, and the underlying statistical science had not been adequately peer reviewed. By the late 1980s the results of peer review indicated that, although Taguchi's engineering concepts and objectives were well founded, there were substantial problems with his experimental strategy and methods of data analysis. For specific details of these issues, see Box (1988), Box *et al.* (1988), Hunter (1985, 1989), Montgomery (1992, 1997), Myers and Montgomery (1995) and Pignatiello and Ramberg (1992). Many of these concerns are also summarized in the extensive panel discussion in the May 1992 issue of *Technometrics* (see Nair (1992)).

There were two positive outcomes of the Taguchi controversy. First, designed experiments became more widely used in the discrete parts industries, including automotive and aerospace manufacturing, electronics and semiconductors, and many other industries that had previously made little use of the technique. Second, the fourth era of statistical design began. This era has included a renewed interest in statistical design by both researchers and practitioners, and the development of alternatives to Taguchi's technical methods that allow his engineering concepts to be carried into practice efficiently and effectively. Some of these alternatives will be discussed and illustrated in subsequent sections.

3. A strategy for product and process development

A fundamental approach to process and product design and development consists of three phases: characterization, control and optimization. *Characterization* is the process of discovering the specific process variables that are responsible for the variability in the system's output responses. Systems (product and processes) are often described by many variables, particularly in the early stages of design and development work, when the level of scientific and engineering knowledge may be low. Thus identifying the most important factors early is critical to successful eventual development of the system. Without proper characterization, a considerable amount of guesswork about which variables are important and the effect of various factors on the responses of interest will typically occur. This contributes to long development lead times, missed deadlines for releasing products and process certification, and subsequent loss of competitive advantage.

Factorial and fractional factorial designs are very useful in characterizing systems. In particular, the 2^k- and 2^{k-p}-designs, often augmented with centre points (see Box *et al.* (1978) and Montgomery (1997)) are typical choices. Statisticians usually refer to many of the activities that are performed during characterization as factor screening. The use of two-level factors with centre points allows both the efficient estimation of effects and protection against pure quadratic curvature.

Control refers to process stability, i.e. to obtaining a consistent performance from the system. One of Taguchi's important contributions was his observation that some process variables may affect the mean of the response variable, whereas others may affect the response variance. These are typically referred to as location effects and dispersion effects respectively. Taguchi's approach to this problem was based on the use of signal-to-noise ratios. Although this approach was flawed (see Box (1988), Box *et al.* (1988), Montgomery (1997) and Myers and Montgomery (1995)), it spurred much research and the development of several useful alternatives, including the estimation of dispersion effects through the analysis of residuals from conventional factorial and fractional designs, and separate modelling of the mean and variance. Some useful references are Carroll and Ruppert (1988), Bartlett and Kendall (1946), Box and Meyer (1986), Myers and Montgomery (1995), Vining and Schaub (1996) and Vining and Bohn (1997).

Optimization refers to manipulating the most important process variables to levels or settings that result in the best obtainable set of operating conditions for the system. In most industries, it is not unusual to have several system or process outputs that need to be jointly optimized. For example, in a chemical process, it may be necessary to consider responses such as process yield, product molecular weight, viscosity, concentration and environmental parameters ('green' manufacturing considerations). This will usually require the simultaneous optimization of these (often conflicting) responses. RSM is an extremely useful framework for modelling and optimizing systems. For an introduction to RSM, see Box and Draper (1987), Myers and Montgomery (1995) and Khuri and Cornell (1996).

Considerable effort has been devoted to developing response surface alternatives to Taguchi's robust design. For details of the approaches and examples, refer to Myers (1991), Myers *et al.* (1992), Lucas (1994) and Myers and Montgomery (1995). Mixture experiments, a variation of RSM, are also highly useful for robust product development and formulation.

The characterization–control–optimization strategy places a strong emphasis on sequential experimentation, in which we move from a state of relatively low knowledge about the system to a state of advanced knowledge. The objectives of determining which factors produce important effects, the role that each factor plays in driving the mean and/or variance of each response and the determination of optimum conditions are most easily and efficiently accomplished through a series of small interrelated experiments. Taguchi's approach to robust parameter design typically used relatively large comprehensive experiments. This strategy is often ineffective in practice, because at the earliest stages of a development problem the engineers and scientists simply do not know enough to plan a good comprehensive experiment. Furthermore, as pointed out by Coleman and Montgomery (1993), large comprehensive experiments are difficult to complete successfully as planned.

4. Making products and processes robust

The original robust parameter design strategy proposed by Taguchi consists of first classifying the p system variables as k controllable variables and r noise variables (the uncontrollable variables in Fig. 1). Then an $n_x \times k$ 'control (or inner) array' is selected for the controllable variables. As the name suggests, the control array contains the settings of the controllable variables to be used in the experiment. Each row of this array consists of settings for the x_i for a particular experimental run. Then an $n_z \times r$ 'noise (or outer) array' is used to define the levels of the uncontrollable variables z_i. This array specifies how the noise variables will be varied during the experiment. The complete experimental design then consists of evaluating each row in the control array over all the conditions in the noise array. This leads to a crossed array design structure with $n_x \times n_z$ trials.

Because the number of trials in the crossed array structure is potentially large, the control and noise arrays are usually small (often saturated) fractional factorials. For example, in the connector pull-off force example described in Byrne and Taguchi (1987), the control array is a 3^{4-2}-design (called an L9 orthogonal array by Taguchi) and the noise array is a 2^3-factorial (called an L8 orthogonal array). The crossed array has 72 experimental trials. This is a large experiment indeed, prohibitively large for many industrial settings (such as chemical and process plants or semiconductor factories).

The motivation for the crossed array strategy is straightforward. It allows all interactions between control variables and noise variables to be estimated. If such interactions exist, it may be possible to find settings for the controllable variables that minimize the variability transmitted from the noise variables. In fact, without such interactions there will be no robust parameter design problem.

The weaknesses of the crossed array strategy have been well documented (see Montgomery (1990), Shoemaker *et al.* (1991), Pignatiello and Ramberg (1992) and Myers and Montgomery (1995), for example). For instance, despite the large number of trials, these designs will often not provide information on interactions between controllable factors. Furthermore, many crossed array designs were not run in random order. The usual method of running the experiment produced a split-plot structure in the noise array. The resulting dual error structure was typically not accounted for in the statistical analysis.

We now give an example of a robust process development problem. The example illustrates some of the problems described above and some alternative approaches that are more efficient.

5. An example of robust process development

An experiment was performed in a semiconductor factory to determine how five factors influence the transistor gain for a particular device. It is desirable to hold the gain as close as possible to a target value of 200 (the specifications are actually 200 $\pm$ 20). Three of the variables are relatively easy to control: x_1 (implant dose), x_2 (drive-in time) and x_3 (vacuum level). Two variables are difficult to control in routine manufacturing and are considered as noise factors: z_1 (oxide thickness) and z_2 (temperature). The experimenters set up a crossed array design, using a 2^3-

Table 1. Crossed array for the semiconductor experiment

Controllable variables			*Noise variables for the following values of z_1 and z_2:*				*Response average*	*Response standard deviation*	*Type T signal-to-noise ratio*
x_1	x_2	x_3	$z_1=-1$, $z_2=-1$	$z_1=+1$, $z_2=-1$	$z_1=-1$, $z_2=+1$	$z_1=+1$, $z_2=+1$			
−1	−1	−1	118.9	65.7	95.3	92.4	93.08	21.77	29.06
+1	−1	−1	153.7	229.4	119.9	251.5	188.63	62.07	31.35
−1	+1	−1	196.7	170.9	234.2	166.6	192.10	31.06	36.44
+1	+1	−1	211.1	245.7	241.0	252.6	237.60	18.30	51.28
−1	−1	+1	145.2	132.2	167.1	137.9	145.60	15.29	45.07
+1	−1	+1	125.3	201.6	185.5	267.3	194.93	58.36	24.11
−1	+1	+1	283.0	251.1	263.4	190.4	246.98	39.94	36.44
+1	+1	+1	184.2	279.5	247.2	259.2	242.53	41.11	35.50

factorial for the control factors and a 2^2-factorial for the noise factors. The resulting 32-run experiment, employing the usual ± 1 notation, is shown in Table 1.

The last three columns of Table 1 present the summary statistics for the response variable, computed across the levels of the noise variables. The last column contains the 'target-is-best' signal-to-noise ratio

$$\mathrm{SN} = 10\ln(\bar{y}^2/s^2).$$

The usual analysis of a crossed array (Byrne and Taguchi, 1987) consists of constructing marginal means plots of the signal-to-noise ratio and the response average. Then values of the controllable factors are selected so that the signal-to-noise ratio is maximized and the response average takes on the desired value. Figs 2 and 3 present these marginal means plots. Although we shall not give the full Taguchi analysis, note that the signal-to-noise ratio is maximized when implant dose x_1 is at the low level and drive-in time x_2 is at the high level. Both of these variables also affect the gain, but since vacuum level x_3 has little effect on the signal-to-noise ratio it may be possible to manipulate it to a satisfactory level to make the gain near the desired target of 200. The Taguchi approach to analysis also occasionally examines certain interactions between controllable factors and noise factors. The only such interaction that is significant here is between implant dose and oxide thickness. This interaction is depicted graphically in Fig. 4. It is not obvious that there is a

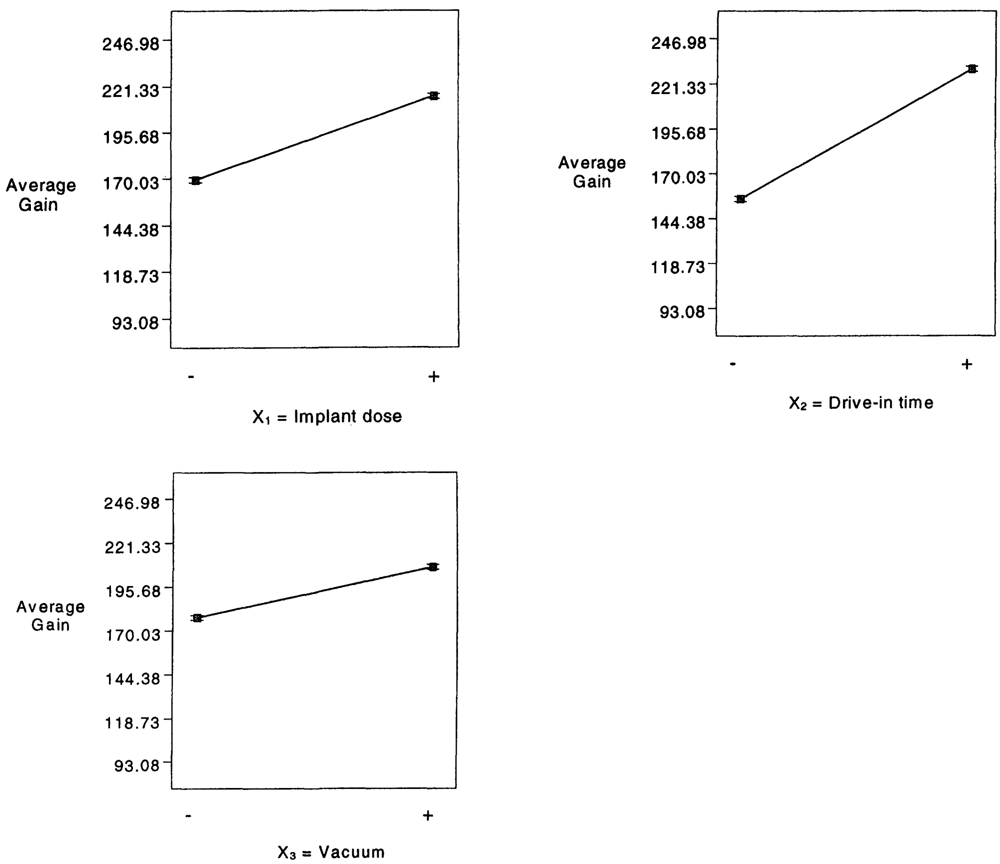

Fig. 2. Marginal means plots of the average response

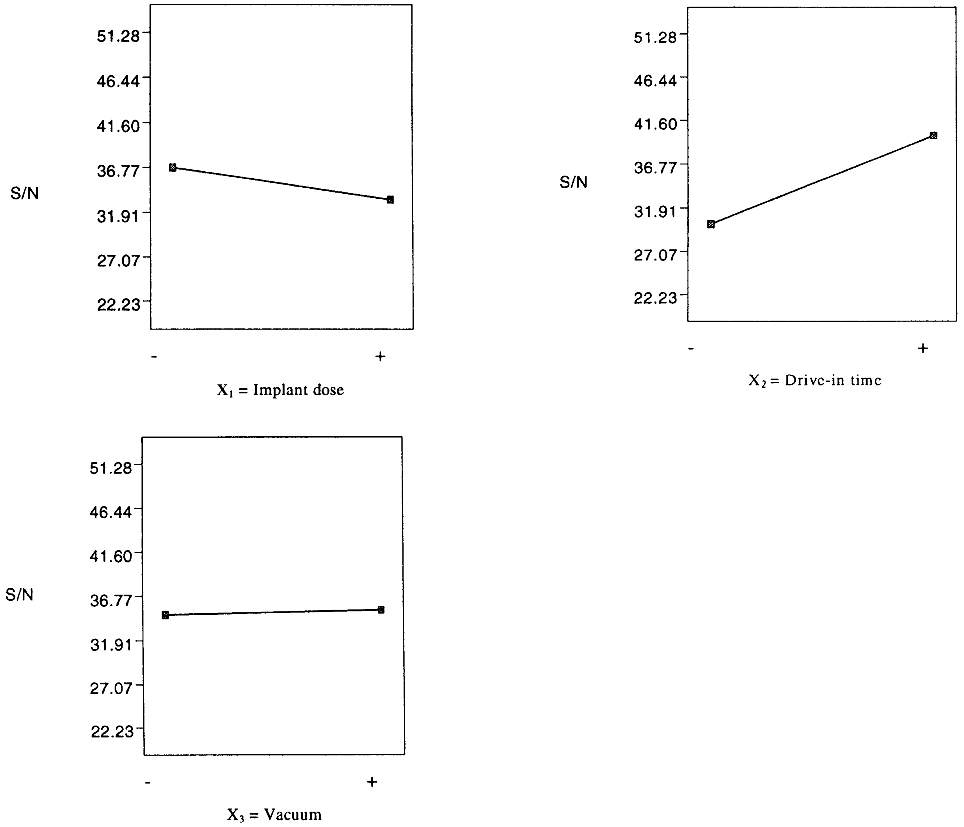

Fig. 3. Marginal means plots of the signal-to-noise ratio

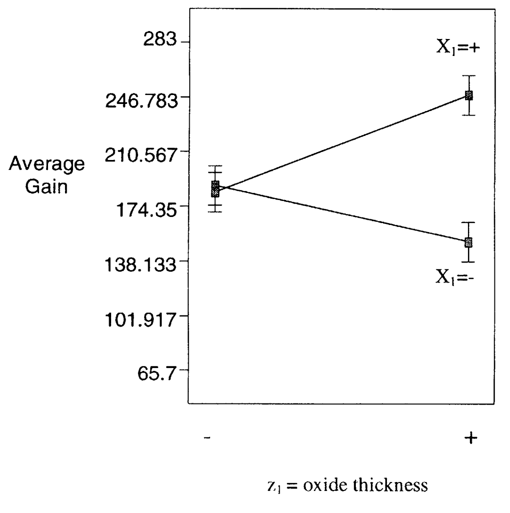

Fig. 4. Implant dose–oxide thickness ($x_1 - z_1$) interaction plot

setting for the implant dose that will minimize the variability transmitted from the oxide thickness.

We now present an alternative analysis of the experiment in Table 1. Since the design is really a full 2^5-factorial, we could fit the model

$$y = \beta_0 + \sum_{i=1}^{3} \beta_i x_i + \sum_{i<} \sum_{j=2}^{3} \beta_{ij} x_i x_j + \sum_{j=1}^{2} \gamma_j z_j + \sum_{i=1}^{3} \sum_{j=1}^{2} \delta_{ij} x_i z_j + \epsilon. \tag{1}$$

Model (1) contains the main effects and interactions of the control variables, the main effects of the noise variables and the interactions between the control and noise factors. The interactions between the noise variables could be included, but they are generally not necessary. Equation (1) is usually called the *response model*.

Following a standard analysis for the 2^k-design, the following fitted model for gain is obtained:

$$\hat{y} = 192.68 + 23.24x_1 + 37.12x_2 + 14.83x_3 - 12.98x_1x_2 - 12.03x_1x_3 + 6.95z_1 + 25.48x_1z_1.$$

Note that the noise factor temperature has no effect, and that there is a large interaction between the control factor implant dose x_1 and the noise factor oxide thickness z_1. We assume that in the process the noise factor z_1 is a random variable with mean 0 and variance σ_z^2. Also assume that the low and high levels of the noise factors are at ± 1 standard deviation around their means, so that in coded form $\sigma_z^2 = 1$. Finally, assume that the true process model is

$$y = \beta_0 + \sum_{i=1}^{3} \beta_i x_i + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \epsilon. \tag{2}$$

Now it follows that a model for the process mean is given by

$$E_z(y) = \beta_0 + \sum_{i=1}^{3} \beta_i x_i + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3.$$

Consequently, an estimate of the mean model is found by substituting estimates for the parameters, resulting in

$$\hat{E}_z(y) = 192.68 + 23.24x_1 + 37.12x_2 + 14.83x_3 - 12.98x_1x_2 - 12.03x_1x_3.$$

Now apply a conditional variance operator across equation (2), giving

$$\mathrm{var}_z(y) = \gamma_1^2\sigma_z^2 + \delta_{11}^2 x_1^2 \sigma_z^2 + 2\delta_{11}\gamma_1\sigma_z^2 + \sigma^2.$$

This is a model for the process variance. Replacing parameters by estimates, we obtain

$$\widehat{\mathrm{var}}_z(y) = 6.95^2\sigma_z^2 + 25.48^2 x_1^2 \sigma_z^2 + 2(25.48)(6.95)x_1\sigma_z^2 + 639.09$$

where we have used the residual mean square as an estimate of σ^2. Since $\sigma_z^2 = 1$, we may write the model for the process variance as

$$\widehat{\mathrm{var}}_z(y) = (6.95 + 25.48x_1)^2 + 639.09.$$

Note that the process variance is a function of the control factor implant dose. This has resulted directly from the interaction between implant dose and oxide thickness ($x_1 z_1$).

By applying standard model building and analysis techniques to the response model (1) we have, in effect, obtained two response surfaces: one for the process mean and one for the process variance. Recall that our objective is to find settings for the controllable variables that result in a mean gain of 200, and we would like the variability transmitted from the noise factors to be minimized. The transmitted variability is minimized directly by solving $\mathrm{d}\{\widehat{\mathrm{var}}(y)\}/\mathrm{d}x_1 = 0$. This results in

$$x_1 = -6.95/25.48 = -0.27.$$

Fig. 5 presents a response surface contour plot of the mean model with implant dose x_1 held constant at -0.27. An inspection of this contour plot reveals that there are many combinations of the factors x_2 (drive-in time) and x_3 (vacuum level) that will produce an on-target process.

The solution given by the Taguchi analysis resulted in a setting for the implant dose that is near the optimum setting; however, this approach does not necessarily reveal that there are many combinations of drive-in time and vacuum level that will produce gain near the target of 200. This could be of considerable practical significance, since there will probably be several other responses to optimize simultaneously. For example, operating the process with relatively low levels of drive-in time and high levels of vacuum could reduce the throughput time. A model-based approach to the analysis of the experiment will generally be necessary to reveal this type of flexibility. This is the principal advantage of the RSM framework that we have advocated.

In concluding this example, we note that the response model (1) could have been fitted with a much simpler design than was actually used. The 2^{5-1}-design, a 16-run resolution V fraction, is adequate to support the model. When both controllable factors and noise factors are included in the same design matrix, the resulting design is usually called a combined array (see Myers and Montgomery (1995) for more details). These combined arrays are usually much more efficient than the crossed array approach.

6. Generalization of the response model approach

It is possible to generalize the mean and variance modelling approach of the previous section. In many situations, the model in the control variables could be a second-order model. This would probably be the case if previous experiments with the system have focused on characterization or factor screening and a region that is likely to contain the optimum conditions has been found, perhaps by steepest ascent or some other first-order optimization technique. We continue to assume that first-order interactions between the controllable factors and the noise factors must also be included in the model. Under these assumptions, a useful generalization of the response model (1) is

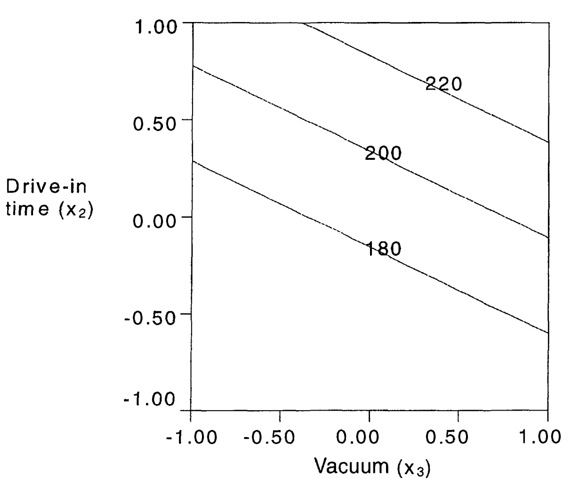

Fig. 5. Response surface for mean gain with implant dose $x_1 = -0.27$

$$y(\mathbf{x}, \mathbf{z}) = \beta_0 + \mathbf{x}'\boldsymbol{\beta} + \mathbf{x}'\mathbf{B}\mathbf{x} + \mathbf{z}'\boldsymbol{\gamma} + \mathbf{x}'\Delta\mathbf{z} + \epsilon \tag{3}$$

where $\boldsymbol{\beta}$ is a $k \times 1$ vector of the first-order coefficients associated with the controllable factors, the $k \times k$ matrix $\mathbf{B}$ contains coefficients for the controllable factor interactions and pure quadratic terms involving these variables, $\boldsymbol{\gamma}$ is an $r \times 1$ vector of the first-order noise factor coefficients and the $k \times r$ matrix Δ contains the interaction effects between the controllable factors and the noise factors. A combined array design will be used to fit equation (3).

The mean and variance response surfaces can be derived easily (see Myers *et al.* (1992) and Myers and Montgomery (1995)). The response surface for the process mean is

$$E_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\} = \beta_0 + \mathbf{x}'\boldsymbol{\beta} + \mathbf{x}'\mathbf{B}\mathbf{x} \tag{4}$$

and the response surface for the process variance is

$$\begin{aligned} \mathrm{var}_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\} &= \sigma_{\mathbf{z}}^2(\boldsymbol{\gamma}' + \mathbf{x}'\Delta)(\boldsymbol{\gamma} + \Delta'\mathbf{x}) + \sigma^2 \\ &= \sigma_{\mathbf{z}}^2\mathbf{l}(\mathbf{x})'\mathbf{l}(\mathbf{x}) + \sigma^2 \end{aligned} \tag{5}$$

where $\mathbf{l}(\mathbf{x}) = \boldsymbol{\gamma} + \Delta'\mathbf{x}$. Note that $\mathbf{l}(\mathbf{x})$ is just the vector of partial derivatives of the response model (3) with respect to the noise variables $\mathbf{z}$. Thus, variation in the response is transmitted through the slope of the response surface in the directions of the noise variables. The matrix Δ is extremely important; if $\Delta = \mathbf{0}$ the process variance does not depend on the controllable variables so it is impossible to set these variables to levels that minimize the variability transmitted to the response from the noise variables.

All the Taguchi engineering objectives for robust design can be accomplished through the use of equations (3)–(5). For example, the larger-the-better (or smaller-the-better) objectives can be realized by solving

$$\max_{\mathbf{x}}(\text{or min})[E_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\}] \quad \text{subject to} \quad \mathrm{var}_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\} \leqslant c$$

where c is a specified constant representing the maximum value allowable for the process variance. To minimize variability around a target, we would solve

$$\min[\mathrm{var}_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\}] \quad \text{subject to} \quad a \leqslant E_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\} \leqslant b$$

where a and b are bounds (perhaps based on specifications) on the process mean.

It may also be possible to minimize the process variance directly from the solution of

$$\mathbf{l}(\mathbf{x}) = \mathbf{0}.$$

This condition represents r equations in p unknowns. When $p > r$, there will be a line or a plane representing the region of minimum process variance. When $p = r$ there will be a single point $\mathbf{x}_0$ that will result in minimum variance. If $p < r$, then $\mathbf{l}(\mathbf{x}) = \mathbf{0}$ may not have a solution. See the discussion in Myers and Montgomery (1995).

We illustrate these ideas with a second example. Consider a situation where there are two controllable variables and three noise variables. The combined array design used by the experimenters is shown in Table 2. The design used is a 23-run variation of a central composite design, and it will support the response model (3). The fitted response model is

$$\begin{aligned} \hat{y} = 30.37 &- 2.92x_1 - 4.13x_2 + 2.60x_1^2 + 2.18x_2^2 + 2.87x_1x_2 + 2.73z_1 - 2.33z_2 + 2.33z_3 \\ &- 0.27x_1z_1 + 0.89x_1z_2 + 2.58x_1z_3 + 2.01x_2z_1 - 1.43x_2z_2 + 1.56x_2z_3. \end{aligned}$$

The mean and variance models are

$$\hat{E}_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\} = 30.37 - 2.92x_1 - 4.13x_2 + 2.60x_1^2 + 2.18x_2^2 + 2.87x_1x_2$$

Table 2. Combined array experiment with two controllable variables and three noise variables

Run	x_1	x_2	z_1	z_2	z_3	y
1	−1.00	−1.00	−1.00	−1.00	1.00	44.2
2	1.00	−1.00	−1.00	−1.00	−1.00	30.0
3	−1.00	1.00	−1.00	−1.00	−1.00	30.0
4	1.00	1.00	−1.00	−1.00	1.00	35.4
5	−1.00	−1.00	1.00	−1.00	−1.00	49.8
6	1.00	−1.00	1.00	−1.00	1.00	36.3
7	−1.00	1.00	1.00	−1.00	1.00	41.3
8	1.00	1.00	1.00	−1.00	−1.00	31.4
9	−1.00	−1.00	−1.00	1.00	−1.00	43.5
10	1.00	−1.00	−1.00	1.00	1.00	36.1
11	−1.00	1.00	−1.00	1.00	1.00	22.7
12	1.00	1.00	−1.00	1.00	−1.00	16.0
13	−1.00	−1.00	1.00	1.00	1.00	43.2
14	1.00	−1.00	1.00	1.00	−1.00	30.3
15	−1.00	1.00	1.00	1.00	−1.00	30.1
16	1.00	1.00	1.00	1.00	1.00	39.2
17	−2.00	0.00	0.00	0.00	0.00	46.1
18	2.00	0.00	0.00	0.00	0.00	36.1
19	0.00	−2.00	0.00	0.00	0.00	47.4
20	0.00	2.00	0.00	0.00	0.00	31.5
21	0.00	0.00	0.00	0.00	0.00	30.8
22	0.00	0.00	0.00	0.00	0.00	30.7
23	0.00	0.00	0.00	0.00	0.00	31.0

and

$$\widehat{\text{var}}_{\mathbf{z}}\{y(\mathbf{x}, \mathbf{z})\} = 19.26 + 3.20x_1 + 12.45x_2 + 7.52x_1^2 + 8.52x_2^2 + 2.21x_1x_2$$

where we have substituted parameter estimates into equations (4) and (5), and as in the previous example assumed that $\sigma_{\mathbf{z}}^2 = 1$. Figs 6 and 7 present contour plots of the process mean and variance response surfaces generated from these models.

In this problem it is desirable to keep the process mean below 30. From an inspection of Figs 6 and 7 it is clear that some trade-off will be necessary if we wish to make the process variance

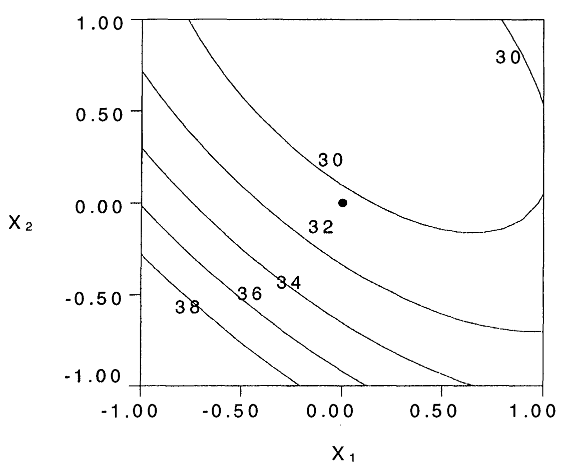

Fig. 6. Response surface for the process mean

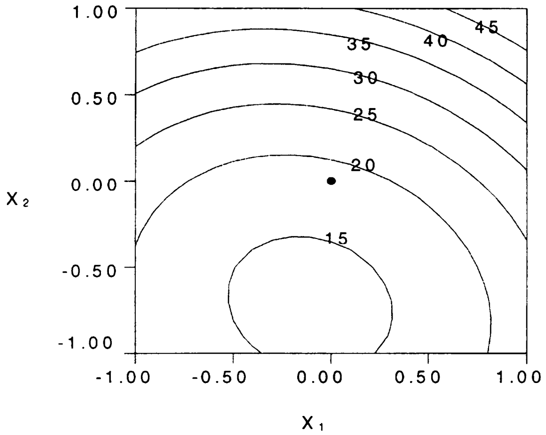

Fig. 7. Response surface for the process variance

small. Since there are only two controllable variables, a logical way to accomplish this trade-off is to overlay the contours of constant mean response and constant variance, as shown in Fig. 8. This plot shows the contours for which the process mean is less than or equal to 30 and the process variance is less than or equal to 25. The region bounded by these contours would represent a typical operating region of low mean response and low process variance.

7. Some benefits of statistical design to the engineering design and development community

Statistically designed experiments play a significant role in the industrial world. Among their most important uses are in the design and development of new products and production processes. Engineers, scientists and other technical process specialists are the primary users of designed experiments in these applications. The effective use of statistical design in process development can result in

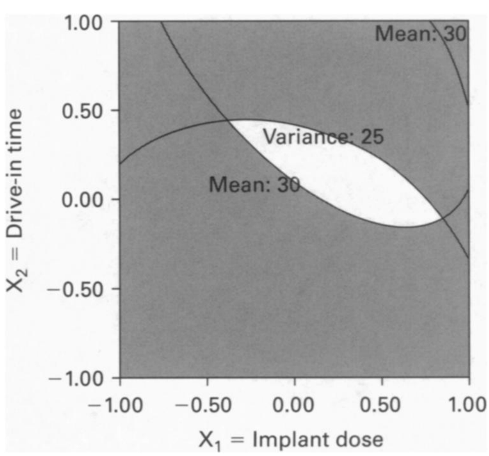

Fig. 8. Overlay of the contours with process mean 30 or less and variance 25 or less

(a) improved process performance and yield,
(b) reduced variability and closer conformance to target requirements for critical product properties,
(c) reduced development lead times for new processes and
(d) lower overall manufacturing costs.

Statistical design can also play a major role in engineering design activities, where new products are developed and existing products are improved. Some important applications of statistical design in product design include

(a) the evaluation and comparison of basic design configurations,
(b) the evaluation of material alternatives,
(c) the formulation of products,
(d) the determination of key product design parameters that affect product field performance,
(e) the selection of design parameters so that the product will work well under a wide variety of field conditions, or so that it will be insensitive to potential sources of variability transmitted from processing variables that are difficult to control, and
(f) the evaluation of product reliability and life performance.

Many of these applications can be thought of as a variation of robust design. We have reviewed some aspects of the robust design problem, beginning with the original contributions of Taguchi, and pointed out that response surface methods provide a more logical and straightforward framework in which to implement his engineering concepts.

8. Concluding remarks

As noted earlier, interest in and application of Taguchi's robust parameter design methodology in the early 1980s ushered in the third era in the use of statistical design. The eventual peer review of his work, and other events, led to the beginning of the fourth era in the late 1980s. This fourth era has seen the use of statistical design techniques expand dramatically, both in the breadth of industries using the techniques and in the extent to which engineering, scientific and operating personnel are familiar with the approach. Several things have fostered this expanded use of statistical design, including the growing availability of good and inexpensive computer software supporting applications and the incorporation of designed experiments as a formal part of the process characterization. For example, the semiconductor industry has been a leader in this regard.

The development of effective and efficient methods for robust design based on separate mean and variance modelling and the response model approach has been a highlight of the fourth era. Many important problems remain, however, so these aspects of statistical design will continue to be an active area of research. For example, information (such as we have for standard response surface designs) that would allow an experimenter to select easily an appropriate design for the combined array is incomplete. Modified central composite designs, such as we illustrated, and computer-generated alphabetically optimal designs are logical candidates, but the properties and performance of these designs have not been thoroughly studied. There is also a need for tolerance and prediction intervals based on the response model approach.

There will continue to be expanded research and development in design optimality and its applications. Mixture designs, mixture designs with process variables, non-standard models and other design problems involving constraints on process variables and irregular regions are obvious applications for alphabetically optimal designs. Another area of potential application for these techniques is experiments with deterministic computer models. These computer models are used in mechanical and electrical design problems. Design problems involving these models often have a large number of factors and many responses. Many of the analysis and modelling issues for these types of experiments are fertile areas for research.

As experimental design is more widely applied, situations where the underlying statistical assumptions are not satisfied will be more frequently encountered. For example, in the semiconductor industry it is relatively common to use defects and yield as response variables. Furthermore, many electrical parameters are strictly positive variables, often with many values near 0, so the assumption of normality is probably inappropriate. The generalized linear model is a logical framework in which to analyse these types of experiment. The generalized linear model has been widely used in the biopharmaceutical field. It is now starting to find application in more general industrial experiments; for more information and some interesting examples, see Grego (1993), McCullagh and Nelder (1989), Myers and Montgomery (1997), Hamada and Nelder (1997) and Engel and Huele (1996).

Methods for handling multiple responses will continue to be of interest both to practitioners and to researchers. Since most experiments involve several responses, the need for improved analysis, model building and optimization methodology is apparent. Most of the current methods for multiple-response optimization assume that the responses are independent, which is obviously unrealistic. The response model approach produces two responses (the mean and variance) for each original response, and since these response surfaces are derived from the same model they are not independent. There are numerous approaches to multiple-response optimization, incorporating techniques such as goal programming, genetic algorithms and simulated annealing. These techniques are widely used in the operations research field, but they have not been adopted to any significant degree by the statistical design community.

Acknowledgements

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Comments on the paper by Montgomery

Dan Grove (*Independent Consultant, Fareham*)

This is a competent survey of where we currently stand in industrial experimentation, with many useful insights. However, from the perspective of a consultant working mostly with engineers, there are several issues and problems which turn up regularly in practice but do not receive attention here.

Variance modelling

There is now quite an impressive literature on variance modelling in relation to robust engineering design. The paper gives details of one type of model, derived by taking a fixed effects model such as equation (3) and allowing the noise variables (the zs) to vary randomly, giving equation (5). Engel and Huele (1996) generalized equation (5) by allowing σ to be a function of the controllable factors $\mathbf{x}$, either via a generalized linear model or by using the ideas in Chan and Mak (1995).

Other generalizations of equation (5) are clearly feasible, for example to cater for a multilevel error structure, but before we are carried away with our enthusiasm for modelling we need to ask how often the noise factors can sensibly by thought of as varying randomly 'in the real world'. Characteristics of system components in mass-production, and noisy signals from one subsystem to another, are obvious candidates for this type of model. For others, such as climate, this formalization is not useful and if we want to reduce the variation in output we must work directly with the *system sensitivities*, measured by the change in response when z_j is varied (for $j = 1, \ldots, r$).

'Dynamic' robustness problems

Dynamic robustness is Taguchi's name for an engineering scenario in which the aim is to engineer a robust *relationship* between an input and an output. In this context robustness means that the relationship remains almost constant for different settings of the noise factors. Miller and Wu (1996) reviewed Taguchi's approach and proposed an alternative method of achieving the same goal. Their method is, statistically speaking, similar in spirit but less daunting in detail than the proposals in chapter 12 of Grove and Davis (1992). The latter discussion links the modelling to Taguchi's general engineering philosophy.

Because of its relative complexity, and the huge experiments which it can give rise to, this is an area of robust design where large potential gains are to be had from a sequential approach to experimentation, combined with the systematic use of engineering knowledge. The losses from an unthinking application of signal-to-noise ratios can be correspondingly large.

I was sorry not to see this topic mentioned in Doug Montgomery's survey because many of the engineers whom I talk to find Taguchi's 'dynamic' formulation very appealing.

Response surface methodology and accuracy in prediction

Many engineers are very demanding when it comes to prediction accuracy! Sometimes the classic second-order model does a sufficiently good job for them, but sometimes it does not, even after response or factor transformation, or the inclusion of selected higher order interaction terms (as in Cornell and Montgomery (1996)). The well-known text-books on response surface methodology have little else to offer, and this paper does not try to go beyond the standard assumptions.

One possibility is two-stage modelling. If the response is particularly ill behaved in one dimension, we fit a relatively complicated model in just that dimension, identify key features of this 'local' fit and model them as a function of other factors. This 'repeated measures' idea has been applied very successfully to a petrol engine by Holliday *et al.* (1998). Related ideas appear in Bisgaard and Steinberg (1997).

In favourable situations, some or all of the first-stage features have a direct engineering interpretation. In the application of Holliday *et al.* (1998), this technique also helped to solve the problem of strange-shaped engineering constraints on the factor space, because the design of the data collection for the local modelling could be separated from the design in the remaining dimensions.

Another possibility that stays fairly close to standard response surface methodology is the piecewise fitting of (say) a pair of quadratic models with a smoothish join. This does not necessarily have to involve non-linear fitting because the position of the knot or join may be well established by experience.

Run order

In engineering experiments, a random run order is rarely feasible, and there is considerable interest in the following simple problem: given a set of factor combinations, in which order should they be run, taking account of cost and of possible trends and step changes in the response? Provided that we have a tentative model of the trend or the step changes (block effects), we can

(a) expand the X-matrix with one or more columns that correspond to the trend or block parameters and
(b) rearrange the runs to 'minimize' the correlations between these new columns and the original columns representing factor effects, subject to cost constraints.

There are several norms that can be used in this 'minimization', including

(a) the maximum correlation, pioneered by Draper and Stoneman (1968), and
(b) the determinant of $X'X$, pioneered by Joiner and Campbell (1976).

As these rather old references show, the necessary theory has been around for a while, but very few commercial packages offer to choose a run order.

The Taguchi controversy

The controversy about Taguchi methods is addressed in the paper, but the *historical* tone—suggesting that the controversy is over—does not fit my experience. In some firms at least, the argument over signal-to-noise ratios is rumbling on. Doug Montgomery is right to put some of the blame on the entrepreneurial manner in which Taguchi's ideas have been disseminated, but we also need to recognize the belief among many engineers that conventional statistics is obscure and difficult. They are then told in Taguchi-style training that they can stop worrying about statistics because all that they need to do is to select the appropriate signal-to-noise ratio for their type of data. It is not surprising that many of them find this approach very attractive.

If I am right, statisticians must accept a share of the blame. Although we would all like to see more statistics taught to engineers, many of them do encounter a course or module of some kind—the unfortunate thing is how many of them look back and say that they could not see the point of it. It has been said many times before, but we must put more emphasis on data collection, data analysis and modelling, and much less on formal inference.

Interactions and robustness

Finally, I am puzzled by the author's comment on Fig. 4. This diagram makes it clear that if we assume 'smooth' factor effects there must be a setting of x_1, fairly close to 0, which will completely eliminate the effect of z_1. This is confirmed by the subsequent modelling exercise.

Martin Gibson (*Motorola, Hitchin*)
I have the following comments on the paper.

(a) There is no mention of the ideal function highlighted by Grove and Davis (1992) (also not in the references). Experimenters need to find response measures for what they (or customers) want rather than what they do not want. This needs to be made more explicit.
(b) Taguchi provided a family network of designs but, as Diamond showed, Hadamaard matrices have done much the same thing.
(c) Doug mentions characterization, control and optimization and I still wonder whether these phases can be connected to Taguchi's system–parameter–tolerance principles.
(d) The concluding remarks do not expand on the operational research techniques of goal programming, genetic algorithms and simulated annealing. I only have a scant appreciation of these and it is necessary to inform the reader of the paper about their suitability and capability. I would have liked to have seen some simple examples.
(e) Other areas of research would include mixtures inside fractional factorials *versus* fractional factorials inside mixtures. Which is the preferred way forward?
(f) What happens in simultaneous optimization of categorical and continuous variables?
(g) Another method is the analysis of means for process optimization. Who is using it? What new research has been going on?
(h) What about the random strategy approach (cf. Diamond) for computer simulation applications?

Overall the paper has potential and is interesting though it would be enhanced with answers to these points.

Tim Davis (*Ford Werke, Cologne*)
Within the Ford Motor Company, we work with a generalization of Taguchi's original three categories of noise factors, as follows:

(1) piece-to-piece variability due to mass-production;
(2) product changes over time and mileage, such as wear;
(3) customer usage patterns and duty cycles;
(4) external environmental effects, such as climate and road conditions;
(5) internal environmental effects (unwanted effects from one component to another), such as vibration and heat transfer, and noisy signals (called 'inputs' in Montgomery's Fig. 1).

It is not clear to me whether all these types of noise fit into the modelling framework proposed by Montgomery. Nevertheless it would be a mistake to rely entirely on parameter design to achieve robust designs, important though this methodology is. Several robustness strategies are available to the design engineer to deal with noise factors, among them the following:

(A) change the technology of the product (e.g. change from a mechanical to an electronic speedometer) to achieve better performance;
(B) make the current design assumption insensitive to the noise factors identified, through

    (i) discovery of control-by-noise factor interactions (parameter design) or
    (ii) strengthening the current design;

(C) change the magnitude of the noise factor;
(D) insert a compensation device for the noise factor (e.g. a heat shield);
(E) disguise the problem from the end-user.

The successful achievement of a robust design involves combining strategies (A)–(E) to deal with the identified noises (1)–(5). Within Ford, we call this approach 'noise factor management'. For example, in an automobile, it is extremely important to minimize unwanted cross-talk between components (noise factor (5)) by good package design (strategy (C)) rather than by relying on parameter design (strategy (B)(i)) or compensation devices (strategy (D)) applied to the components.

The first case-study presented by Montgomery is interesting from the noise factor management point of view, since one of the noise factors identified, z_2 (temperature), turned out to have no effect on the response within the range over which it was investigated. This is a major discovery! I could easily imagine a situation where the next round of experiments would be aimed at seeing how far temperature could be allowed to vary before it had an effect; this might result in no longer requiring an expensive control mechanism for temperature. The investment thus saved could be diverted into a better control mechanism for the other noise factor z_1 (oxide thickness) to reduce even further the variability of the response.

I hope to publish a more detailed discussion of noise factor management in the near future.

Author's reply

I would like to thank the discussants for their interesting comments. They have identified several important issues that time and space prohibited me from presenting formally in the paper, and I shall take this opportunity to offer a brief reply to some of their remarks.

I believe that the characterization–control–optimization paradigm is an effective alternative framework to the Taguchi approach of system–parameter–tolerance design. It logically promotes sequential experimentation by separating the objectives of system characterization (in which we attempt to isolate the important factors first) and optimization (where we take the best advantage of this information). The Taguchi approach always seemed to confuse these objectives of experimentation, or to attempt to accomplish both objectives in a single experiment.

The variance modelling aproach illustrated in the paper can be extended to various categories or types of noise variables. All that is required is that the noise variables be controllable for the purposes of the experiment, and that enough be known about the variability of a noise factor so that its standard deviation can be estimated. This could be done from a rough idea of the range over which the noise factor will vary. There have been some extensions of this idea; for example, Pledger (1996) showed how observable but uncontrollable noise factors may be incorporated in the general response model framework. Wolfinger and Tobias (1998) have presented methodology for simultaneously modelling three components of a general mixed model formulation of the robust design problem: location (fixed) effects, dispersion effects and random effects.

Both Dan Grove and Martin Gibson comment on dynamic robustness problems, and, of course, experiments where the response is a performance curve are widely encountered in practice. My feeling has always been that the Taguchi approach of creating a 'dynamic' signal-to-noise ratio is generally flawed, just as the usual 'static' signal-to-noise ratios are problematic in most applications. For example, the static smaller-is-best and larger-is-best signal-to-noise ratios actually confound location and dispersion, and turn out to be better measures of location in most cases. Furthermore, it is obvious on inspection of these quantities that they are not proper signal-to-noise ratios, because they are not dimensionless. Consider the dynamic signal-to-noise ratio

$$\mathrm{SN} = 10 \log_{10}(\beta/\sigma)^2$$

where β is usually called the sensitivity measure and the response of interest y is connected to a signal factor M through the relationship

$$y = \beta M + \epsilon$$

and ϵ has standard deviation σ. Obviously, this signal-to-noise ratio is not dimensionless, and so its usefulness as a widely applicable summary of the performance characteristics of interest in the response variable is, in my mind, questionable. Indeed, Lunani *et al.* (1997) have recently shown that Taguchi's dynamic signal-to-noise ratio is only appropriate when the variance of the response is proportional to the square of the sensitivity measure. This is analogous to the limitation of the usual static signal-to-noise ratio for the nominal-is-best case, which is really only appropriate when the variance is proportional to the square of the mean (see Box (1988)). Thus, when a performance curve is of interest, my preference would be an analysis along the lines of the proposals in Bisgaard and Steinberg (1997).

Taguchi has always emphasized the need to design experiments to reduce greatly or to eliminate interactions, and the signal-to-noise ratios are often claimed to assist in this, along with other techniques such as the use of sliding factor levels, the proper choice of a response variable and so forth. Furthermore, Taguchi has said that, if we cannot design an experiment by using these tools such that additivity (or absence of interaction) is attained, then we do not understand the engineering system sufficiently well to conduct efficient experimentation. This argument to me is analogous to one that claims that we should not use a control chart on a process because the process is too variable to use statistical process control! Now there may be some systems in which the underlying engineering science is sufficiently well understood that we could design the experiment to obtain the desired additivity, such as relatively simple mechanical systems where Newton's laws directly and often fairly obviously apply. However, there are many systems in the semiconductor, chemical and biotechnology fields where the underlying mechanisms are much more obscure and less well understood. It is just these types of system where the use of small sequential experiments and the characterization–control–optimization philosophy will have a high pay-off.

As Tim Davis points out, many strategies are available to the engineer to achieve robustness. I classify them into two general categories: *statistical* strategies, of which the variance modelling technique presented in my paper is an example, and *technological* strategies, such as selecting a different material alternative, or changing some aspect of the design, such as the mechanical-to-electronic speedometer example.

Robust design is an important problem to both the engineering and the statistics communities. Many important aspects of this problem have not been well studied, and for which good, easy-to-apply, standard solution methodologies are not available. I hope that the discussants and I have stimulated others to work in this challenging and useful area.

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