Lemma 1. *The objective function Equation (10) is convex.*

Proof. We divide the proof of the objective function into two parts, namely one for the differentiable part f and the other for non-differentiable part g mentioned above. For the f part, we adopt the second order condition for the convexity proof. For the g part, we adopt the convex definition joint proof.

$$\nabla_x^2 f(x) = \lambda_2 \geq 0 \tag{1}$$

$$\begin{aligned} g(\alpha x + (1-\alpha)y) &= w^T \left[\alpha(Hx - g) + (1-\alpha)(Hy - g)\right]_+ \\ &\leq \alpha g(x) + (1-\alpha)g(y) \end{aligned} \tag{2}$$

because

$$[a+b]_+ \leq [a]_+ + [b]_+ \tag{3}$$

So the objective function is convex. □