8.2 Red-Black Trees

A *red-black tree* is a binary search tree with one extra attribute for each node: the *colour*, which is either red or black. We also need to keep track of the parent of each node, so that a red-black tree's node structure would be:

```
struct t_red_black_node {
    enum { red, black } colour;
    void *item;
    struct t_red_black_node *left,
                     *right,
                     *parent;
    }
```

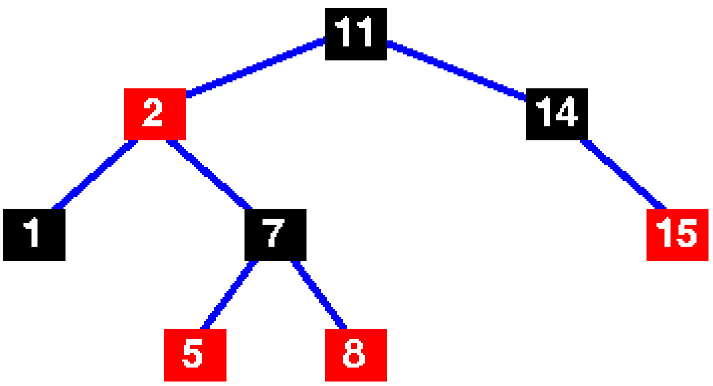
For the purpose of this discussion, the NULL nodes which terminate the tree are considered to be the leaves and are coloured black.

Definition of a red-black tree

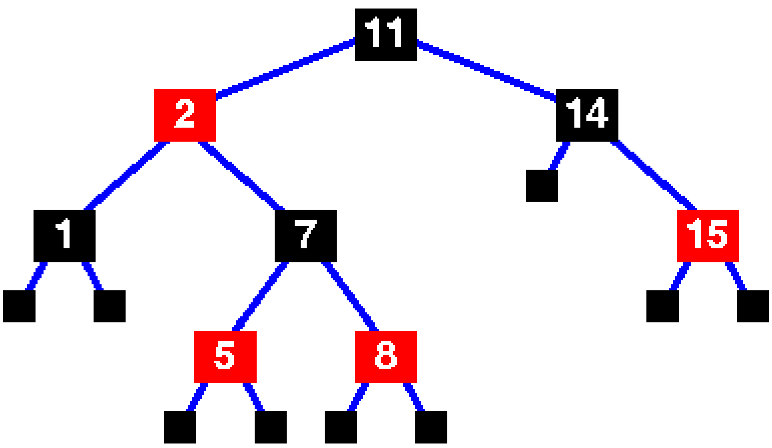
A red-black tree is a binary search tree which has the following *red-black properties*:

1. Every node is either red or black.
2. Every leaf (NULL) is black.
3. If a node is red, then both its children are black.
4. Every simple path from a node to a descendant leaf contains the same number of black nodes.

3. implies that on any path from the root to a leaf, red nodes must not be adjacent. However, any number of black nodes may appear in a sequence.



A basic red-black tree



Basic red-black tree with the **sentinel** nodes added. Implementations of the red-black tree algorithms will usually include the sentinel nodes as a convenient means of flagging that you have reached a leaf node.

They are the NULL black nodes of property 2.

The number of black nodes on any path from, but not including, a node ***x*** to a leaf is called the *black-height* of a node, denoted **bh(x)**. We can prove the following lemma:

Lemma

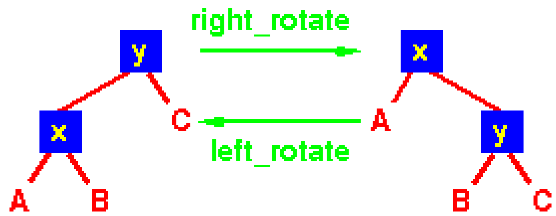
A red-black tree with ***n*** internal nodes has height at most 2**log(*n*+1)**.
(For a proof, see Cormen, p 264)

This demonstrates why the red-black tree is a good search tree: it can always be searched in **O(log n)** time.

As with heaps, additions and deletions from red-black trees destroy the red-black property, so we need to restore it. To do this we need to look at some operations on red-black trees.

Rotations

A rotation is a local operation in a search tree that preserves *in-order* traversal key ordering.



Note that in both trees, an in-order traversal yields:

```
A x B y C
```

The left_rotate operation may be encoded:

```
left_rotate( Tree T, node x ) {
    node y;
    y = x->right;
    /* Turn y's left sub-tree into x's right sub-tree */
    x->right = y->left;
    if ( y->left != NULL )
        y->left->parent = x;
    /* y's new parent was x's parent */
    y->parent = x->parent;
    /* Set the parent to point to y instead of x */
    /* First see whether we're at the root */
    if ( x->parent == NULL ) T->root = y;
    else
        if ( x == (x->parent)->left )
            /* x was on the left of its parent */
            x->parent->left = y;
        else
            /* x must have been on the right */
            x->parent->right = y;
    /* Finally, put x on y's left */
    y->left = x;
    x->parent = y;
    }
```

Insertion

Insertion is somewhat complex and involves a number of cases. Note that we start by inserting the new node, x, in the tree just as we would for any other binary tree, using the `tree_insert` function. This new node is labelled red, and possibly destroys the red-black property. The main loop moves up the tree, restoring the red-black property.

```
rb_insert( Tree T, node x ) {
    /* Insert in the tree in the usual way */
    tree_insert( T, x );
    /* Now restore the red-black property */
    x->colour = red;
    while ( (x != T->root) && (x->parent->colour == red) ) {
       if ( x->parent == x->parent->parent->left ) {
           /* If x's parent is a left, y is x's right 'uncle' */
           y = x->parent->parent->right;
           if ( y->colour == red ) {
               /* case 1 - change the colours */
               x->parent->colour = black;
               y->colour = black;
               x->parent->parent->colour = red;
```

```
            /* Move x up the tree */
            x = x->parent->parent;
            }
        else {
            /* y is a black node */
            if ( x == x->parent->right ) {
                /* and x is to the right */
                /* case 2 - move x up and rotate */
                x = x->parent;
                left_rotate( T, x );
                }
            /* case 3 */
            x->parent->colour = black;
            x->parent->parent->colour = red;
            right_rotate( T, x->parent->parent );
            }
        }
    else {
        /* repeat the "if" part with right and left
           exchanged */
        }
    }
  /* Colour the root black */
  T->root->colour = black;
  }
```

Here's an example of the insertion operation.

Animation

Red-Black Tree Animation This animation was written by Linda Luo, Mervyn Ng, Anita Lee, John Morris and Woi Ang.	Please email comments to: John Morris j.morris@auckland.ac.nz

Examination of the code reveals only one loop. In that loop, the node at the root of the sub-tree whose red-black property we are trying to restore, x, may be moved up the tree *at least one level* in each iteration of the loop. Since the tree originally has **O(log n)** height, there are **O(log n)** iterations. The `tree_insert` routine also has **O(log n)** complexity, so overall the `rb_insert` routine also has **O(log n)** complexity.

Key terms

Red-black trees
: Trees which remain **balanced** - and thus guarantee **O(logn)** search times - in a dynamic environment. Or more importantly, since any tree can be re-balanced - but at considerable cost - can be re-balanced in **O(logn)** time.

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