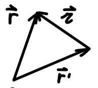
PHYSICS 110A Final Sihong Xu



Chapter 2. Electrostatics

Coulomb's Law: $\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\imath^2}\hat{\imath}$, $\vec{\imath}=\vec{r}-\vec{r}'$

Electric Field: $\vec{F}=Q\vec{E}$. $\vec{E}(\vec{r})=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\imath_i^2}\hat{\imath}$

Continuous: $\vec{E}(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int\frac{1}{\imath^2}\hat{\imath}\,dq$, $dq\sim\lambda dl\sim\sigma da\sim\rho d\tau$

Gauss' Law: $\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho \Longleftrightarrow \nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho$.

Curl of $\vec{E}$: $\nabla\times\vec{E}=\vec{0}\Longleftrightarrow\oint\vec{E}\cdot d\vec{l}=0$.

Electric potential: $V(\vec{r})=-\int_{O}^{\vec{r}}\vec{E}\cdot d\vec{l}$.

$V(b)-V(a)=-\int_a^b\vec{E}\cdot d\vec{l}$. $\vec{E}=-\nabla V$.

Poisson's equation: $\nabla^2V=-\frac{\rho}{\varepsilon_0}\xrightarrow{\rho=0}\nabla^2V=0$.

With $O=\infty$: $V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\sum_i\frac{q}{\imath_i}$, $V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int\frac{1}{\imath}dq$.

Boundary Conditions:

$E_\perp^{(2)}-E_\perp^{(1)}=\frac{\sigma}{\varepsilon_0}$. $E_\parallel^{(1)}=E_\parallel^{(2)}\Rightarrow\vec{E}^{(2)}-\vec{E}^{(1)}=\frac{\sigma}{\varepsilon_0}\hat{n}$

$V^{(2)}=V^{(1)}$, $\frac{\partial V^{(2)}}{\partial n}-\frac{\partial V^{(1)}}{\partial n}=-\frac{\sigma}{\varepsilon_0}$, $\frac{\partial V}{\partial n}=\nabla V\cdot\hat{n}$

Point on a surface charge: $\vec{E}_{other}=\frac{1}{2}(\vec{E}_{above}+\vec{E}_{below})$

Work to move a charge $a\to b$: $V(b)-V(a)=\frac{W}{Q}$

Bring charge from ∞: $W=QV(\vec{r})$.

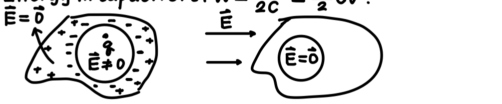
Energy: $W=\frac{1}{2}\sum_{i=1}^{n}q_iV(\vec{r}_i)$, $W=\frac{\varepsilon_0}{2}\int E^2d\tau$ (all space)

Conductors: 1. $\vec{E}=\vec{0}$ inside 2. $\rho=0$ inside 3. Net charge resides on the surface. 3. Conductor is equipotential 4. $\vec{E}$ is $\perp$ to the surface.

Force on conductor: $\vec{f}=\frac{1}{2\varepsilon_0}\sigma^2\hat{n}$.

Capacitors: $C=\frac{Q}{V}$, $V=V_+-V_-=-\int_-^+\vec{E}\cdot d\vec{l}$

Energy in capacitors: $W=\frac{Q^2}{2C}=\frac{1}{2}CV^2$.



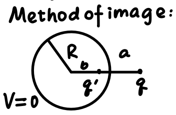
Chapter 3. Potentials

Laplace: $\nabla^2V=0$. Solution: 1. $V(\vec{r})=\frac{1}{4\pi R^2}\oint V dS$.

2. Extreme values occur at boundary.

Uniqueness Theorems: 1. if V is specified on boundary S 2. (a) ρ (b) V at boundaries are satisfied 3. Total charge on each conductors is given.

Method of image: All the same except for the energy



$b=R^2/a$, $q'=-\frac{R}{a}q$ $\Big\}$ $V(R)=0$

Separation of variables

1. Cartesian: $\frac{d^2X}{dx^2}=C_1X$, $\frac{d^2Y}{dy^2}=C_2Y$, $\frac{d^2Z}{dz^2}=C_3Z$.

$\int_0^a\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n'\pi x}{a}\right)dx=\frac{a}{2}\delta_{nn'}$.

2. Spherical with azimuthal symmetry.

$V(r,\theta)=\sum_{l=0}^{\infty}\left(A_lr^l+\frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$

$\int_0^\pi P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta d\theta=\begin{cases}0, & l'\neq l\\ \frac{2}{2l+1}, & l'=l\end{cases}$

$P_0(x)=1$, $P_1(x)=x$, $P_2(x)=(3x^2-1)/2$.

Multiple expansion:

$V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int(r')^nP_n(\cos\alpha)\rho(\vec{r}')d\tau'$

$V_{mon}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$, $Q=\int\rho d\tau$.

$V_{dip}=\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$, $\vec{p}=\int\vec{r}'\rho(\vec{r}')d\tau'$ dipole moment

Change origin: $\vec{p}_{new}=\vec{p}-Q\vec{a}\leftarrow$ displacement of O.

For a perfect dipole: $V_{dip}(r,\theta)=\frac{p\cos\theta}{4\pi\varepsilon_0r}$

$z \uparrow \vec{p}$ $\vec{E}_{dip}=\frac{p}{4\pi\varepsilon_0r^3}(2\cos\theta\hat{r}+\sin\theta\hat{\theta})$.

Chapter 4. Electric Fields in Matter

Alignment of polar molecules: $\vec{F}=(\vec{p}\cdot\nabla)\vec{E}$, $\vec{N}=\vec{p}\times\vec{E}$

$\vec{P}$ = dipole moment per volume.

Bound charges: $\sigma_b=\vec{P}\cdot\hat{n}$, $\rho_b=-\nabla\cdot\vec{P}$

$V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\oint\frac{\sigma_b}{\imath}da'+\frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho_b}{\imath}d\tau'$

Electric displacement: $\vec{D}=\varepsilon_0\vec{E}+\vec{P}$

Gauss' Law: $\nabla\cdot\vec{D}=\rho_f\Longleftrightarrow\oint\vec{D}\cdot d\vec{a}=Q_{fenc}$

Boundary conditions:

$D^\perp_{above}-D^\perp_{below}=\sigma_f$. $\vec{D}^\parallel_{above}-\vec{D}^\parallel_{below}=\vec{P}^\parallel_{above}-\vec{P}^\parallel_{below}$

Energy: $W=\frac{1}{2}\int\vec{D}\cdot\vec{E}d\tau$.

Linear dielectrics: $\vec{P}=\varepsilon_0\chi_e\vec{E}$, $\vec{D}=\varepsilon\vec{E}$, $\varepsilon=\varepsilon_0(1+\chi_e)$

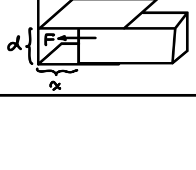
Boundary conditions for linear dielectric $\varepsilon_r=\frac{\varepsilon}{\varepsilon_0}=1+\chi_e$

$\nabla\cdot\vec{D}=\rho_f$, $\nabla\times\vec{D}=\vec{0}$, $\rho_b=-\nabla\cdot\vec{P}=-\frac{\chi_e}{1+\chi_e}\rho_f$.

$\varepsilon_{above}\underbrace{E^\perp_{above}}_{\partial V_{above}/\partial n}-\varepsilon_{below}\underbrace{E^\perp_{below}}_{\partial V_{below}/\partial n}=\sigma_f$. $V_{above}=V_{below}$

Forces on dielectric:

$F=\frac{1}{2}V^2\frac{d^2C}{dx^2}$



Chapter 5. Magnetostatics

Lorentz force: $\vec{F}_{mag}=Q(\vec{v}\times\vec{B})$, $\vec{F}=Q[\vec{E}+\vec{v}\times\vec{B}]$

Current. $\vec{I}=\lambda\vec{v}$, $\vec{K}=\frac{d\vec{I}}{dl_\perp}=\sigma\vec{v}$, $\vec{J}=\frac{d\vec{I}}{da_\perp}=\rho\vec{v}$

$\vec{F}_{mag}=\int I(d\vec{l}\times\vec{B})=\int\vec{K}\times\vec{B}\,da=\int\vec{J}\times\vec{B}\,d\tau$.

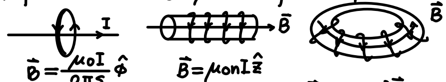
Continuity: $\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$. Steady currents: $\frac{\partial\rho}{\partial t}=0$, $\frac{\partial J}{\partial t}=0$.

Biot-Savart law:

$B(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{I}(\vec{r}')\times\hat{\imath}}{\imath^2}dl'$, $\frac{\mu_0}{4\pi}\int\frac{\vec{K}(\vec{r}')\times\hat{\imath}}{\imath^2}da'$, $\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')\times\hat{\imath}}{\imath^2}d\tau'$

Divergence: $\nabla\cdot\vec{B}=0$

Ampère's law: $\nabla\cdot\vec{B}=\mu_0J\Longleftrightarrow\oint\vec{B}\cdot d\vec{l}=\mu_0I_{enc}$.



$\vec{B}=\frac{\mu_0I}{2\pi s}\hat{\phi}$ $\vec{B}=\mu_0nI\hat{z}$ $\vec{B}=\frac{\mu_0NI}{2\pi s}\hat{\phi}$

Vector potential: $\vec{B}=\nabla\times\vec{A}$. Let $\nabla\cdot\vec{A}=0$. $\nabla^2\vec{A}=-\mu_0\vec{J}$.

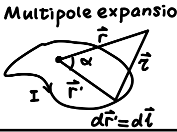
$\xrightarrow{J\to 0\text{ at }\infty}\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')}{\imath}d\tau'=\frac{\mu_0}{4\pi}\int\frac{\vec{I}}{\imath}dl'=\frac{\mu_0}{4\pi}\int\frac{\vec{K}}{\imath}da'$

Boundary conditions: $\mu_0(\vec{K}\times\hat{n})$.

$B^\perp_{above}=B^\perp_{below}$. $\underbrace{B^\parallel_{above}-B^\parallel_{below}}_{\perp\text{ to }K}=\mu_0K$. $\vec{B}_{above}-\vec{B}_{below}=$

$\vec{A}_{above}=\vec{A}_{below}$ $\frac{\partial\vec{A}_{above}}{\partial n}-\frac{\partial\vec{A}_{below}}{\partial n}=-\mu_0\vec{K}$.

Multipole expansion: $\vec{A}(\vec{r})=\frac{\mu_0I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint(r')^nP_n(\cos\alpha)d\vec{l}'$



$\vec{A}_{dip}(\vec{r})=\frac{\mu_0I}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$, $\underset{\downarrow}{\vec{m}}=I\int d\vec{a}$

magnetic dipole moment

Chapter 6. Magnetic Fields Matter

Magnetic dipoles in field: $\vec{N}=\vec{m}\times\vec{B}$, $\vec{F}=\nabla(\vec{m}\cdot\vec{B})$.

$\vec{M}$ = magnetic dipole moment per unit volume.

Bound currents: $\vec{J}_b=\nabla\times\vec{M}$, $\vec{K}_b=\vec{M}\times\hat{n}$.

In magnetized medium, $\vec{J}=\vec{J}_b+\vec{J}_f$, $\vec{H}=\frac{1}{\mu_0}\vec{B}-\vec{M}$, $\nabla\cdot\vec{H}=-\nabla\cdot\vec{M}$

Ampère's law: $\nabla\times\vec{H}=\vec{J}_f\Longleftrightarrow\oint\vec{H}\cdot d\vec{l}=I_{fenc}$

Boundary conditions: $H^\perp_{above}-H^\perp_{below}=-(M^\perp_{above}-M^\perp_{below})$

$\vec{H}^\parallel_{above}-\vec{H}^\parallel_{below}=\vec{K}_f\times\hat{n}$.

Linear media: $\vec{M}=\chi_m\vec{H}$, $\vec{B}=\mu_0(1+\chi_m)\vec{H}=\mu\vec{H}$, $\mu=\mu_0(1+\chi_m)$.

$\vec{J}_b=\chi_m\vec{J}_f$

Chapter 7. Electrodynamics

Ohm's law. $\vec{J}=\sigma\vec{f}=\sigma\vec{E}$, $V=IR$.

Joule heating law: $P=VI=I^2R$.

Current around circuit: $\vec{f}=\vec{f}_s+\vec{E}$. $\vec{f}_s=\vec{0}$ outside source

emf: $\mathcal{E}=\oint\vec{f}\cdot d\vec{l}=\oint\vec{f}_s\cdot d\vec{l}$.

Faraday's law: changing $\vec{B}$ induces $\vec{E}$: $\Phi=\int\vec{B}\cdot d\vec{a}$. $\mathcal{E}=-\frac{d\Phi}{dt}$.

$\xrightarrow{\rho=0}\nabla\cdot\vec{E}=0$, $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\Rightarrow\oint\vec{E}\cdot d\vec{l}=-\frac{d\Phi}{dt}$.

Mutual inductance: $\Phi_2=M_{21}I_1$. $M=M_{12}=M_{21}=\frac{\mu_0}{4\pi}\oint\oint\frac{d\vec{l}_1\cdot d\vec{l}_2}{\imath}$

Self inductance: $\Phi=LI$. $\mathcal{E}=-L\frac{dI}{dt}$. back emf.

Energy in B field: $W=\frac{1}{2}LI^2$, $W=\frac{1}{2\mu_0}\int B^2d\tau$ (all space).

Ampère's law. $\nabla\times\vec{B}=\mu_0(\vec{J}+\vec{J}_d)$, $\vec{J}_d=\varepsilon_0\frac{\partial\vec{E}}{\partial t}$.

Maxwell's equations

$$\begin{cases}\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho\\ \nabla\cdot\vec{B}=0\\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\end{cases}$$

Maxwell's equations in Matter

$$\begin{cases}\nabla\cdot\vec{D}=\rho_f\\ \nabla\cdot\vec{B}=0\\ \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}\end{cases}$$

Integral forms:

$$\begin{cases}\oint_S\vec{D}\cdot d\vec{a}=Q_{fenc}\\ \oint_S\vec{B}\cdot d\vec{a}=0\\ \oint_P\vec{E}\cdot d\vec{l}=-\frac{d}{dt}\int_S\vec{B}\cdot d\vec{a}\\ \oint_P\vec{H}\cdot d\vec{l}=I_{fenc}+\frac{d}{dt}\int_S\vec{D}\cdot d\vec{a}\end{cases}$$

Boundary conditions

$D_1^\perp-D_2^\perp=\sigma_f$. $B_1^\perp-B_2^\perp=0$

$\vec{E}_1^\parallel-\vec{E}_2^\parallel=\vec{0}$, $\vec{H}_1-\vec{H}_2=\vec{K}_f\times\hat{n}$

Linear media:

$\varepsilon_1E_1^\parallel-\varepsilon_2E_2^\parallel=0$

$\frac{1}{\mu_1}\vec{B}_1^\parallel-\frac{1}{\mu_2}\vec{B}_2^\parallel=\vec{K}_f\times\hat{n}$.

Chapter 8. Conservation Laws

Countinuity equation: $\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{J}$

Energy density: $u=\frac{1}{2}\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)$

Poynting's theorem

$\underbrace{\frac{dW}{dt}}_{\text{work done on charges by EM force}}=-\frac{d}{dt}\underbrace{\int_V u\,d\tau}_{\text{energy remain in fields}}-\underbrace{\oint_S\vec{S}\cdot d\vec{a}}_{\text{energy flow out surface}}$, $\vec{S}=\frac{1}{\mu_0}(\vec{E}\times\vec{B})$ Poynting vector.

Maxwell stress tensor $(\vec{a}\cdot\overleftrightarrow{T})_j=\sum_i a_iT_{ij}$, $(\overleftrightarrow{T}\cdot\vec{a})_j=\sum_i T_{ji}a_i$

$T_{ij}=\varepsilon_0(E_iE_j-\frac{1}{2}\delta_{ij}E^2)+\frac{1}{\mu_0}(B_iB_j-\frac{1}{2}\delta_{ij}B^2)$

EM force on charges in V:

$\vec{F}=\oint_S\overleftrightarrow{T}\cdot d\vec{a}-\varepsilon_0\mu_0\frac{d}{dt}\int_V\vec{S}d\tau$. $\xrightarrow[\text{case}]{\text{static}}\vec{F}=\oint_S\overleftrightarrow{T}\cdot d\vec{a}$

Conservation of momentum: $\frac{d\vec{p}_{mech}}{dt}=-\varepsilon_0\mu_0\frac{d}{dt}\int_V\vec{S}d\tau+\oint_S\overleftrightarrow{T}\cdot d\vec{a}$

Momentum stored in fields: $\vec{p}=\varepsilon_0\mu_0\int_V\vec{S}d\tau$.

Momentum density: $\vec{g}=\varepsilon_0\mu_0\vec{S}=\varepsilon_0(\vec{E}\times\vec{B})$

If $dp_{mech}/dt=0\Rightarrow\frac{\partial g}{\partial t}=\nabla\cdot\overleftrightarrow{T}$

Angular momentum: $\vec{l}=\vec{r}\times\vec{g}=\varepsilon[\vec{r}\times(\vec{E}\times\vec{B})]$

Chapter 9 Electromagnetic Waves

The wave equation: $\frac{\partial^2f}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2f}{\partial t^2}$

Sinusoidal waves: $f(z,t)=A\cos[k(z-vt)+\delta]$

k: wavenumber. λ: wavelength: $\lambda=\frac{2\pi}{k}$

T: period $T=\frac{2\pi}{kv}$, $\nu=\frac{1}{T}=\frac{v}{\lambda}$ frequency

$\omega=2\pi\nu=kv$: angular frequency.

$f(z,t)=A\cos(kz-\omega t+\delta)$ travel to right

$f(z,t)=A\cos(-kz-\omega t+\delta)$ left.

Complex notation: $\tilde{f}(z,t)=\tilde{A}e^{i(kz-\omega t)}$, $\tilde{A}=Ae^{i\delta}$

$f(z,t)=\mathrm{Re}[\tilde{f}(z,t)]$

Reflection & Transmission

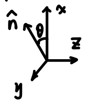
$\tilde{f}(z,t)=\begin{cases}\tilde{A}_Ie^{i(k_1z-\omega t)}+\tilde{A}_Re^{i(-k_1z-\omega t)} & z<0\\ \tilde{A}_Te^{i(k_2z-\omega t)} & z>0\end{cases}$

At knot

$\tilde{f}(0^-,t)=\tilde{f}(0^+,t)$, $\frac{\partial\tilde{f}}{\partial z}\Big|_{0^-}=\frac{\partial\tilde{f}}{\partial z}\Big|_{0^+}$

$\Rightarrow A_R=\frac{v_2-v_1}{v_2+v_1}A_I$, $A_T=\frac{2v_2}{v_2+v_1}A_I$

Polarization



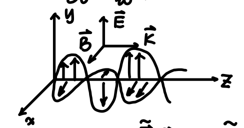
$\tilde{\vec{f}}(z,t)=\tilde{A}e^{i(kz-\omega t)}\hat{n}$

$\hat{n}=\cos\theta\hat{x}+\sin\theta\hat{y}$

Monochromatic Plane waves:

$\tilde{\vec{E}}(z,t)=\tilde{\vec{E}}_0e^{i(kz-\omega t)}$, $\tilde{\vec{B}}=\tilde{\vec{B}}_0e^{i(kz-\omega t)}$, $\omega=ck$.

$\tilde{\vec{B}}_0=\frac{k}{\omega}(\hat{z}\times\tilde{\vec{E}}_0)$, $B_0=\frac{1}{c}E_0$.



$\hat{k}$: propogation vector

$\hat{n}$: polarization vector

General case: $\tilde{\vec{E}}(\vec{r},t)=\tilde{E}_0e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\tilde{\vec{B}}(\vec{r},t)=\frac{1}{c}\tilde{E}_0e^{i(\vec{k}\cdot\vec{r}-\omega t)}(\hat{k}\times\hat{n})$

$=\frac{1}{c}\hat{k}\times\tilde{\vec{E}}$

Real form: $\vec{E}(\vec{r},t)=E_0\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)\hat{n}$

$\vec{B}(\vec{r},t)=\frac{1}{c}E_0\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)(\hat{k}\times\hat{n})$

$u=\varepsilon_0E_0^2\cos^2(kz-\omega t+\delta)$, $\vec{S}=cu\hat{z}$, $\vec{g}=\frac{1}{c}u\hat{z}$

$\langle u\rangle=\frac{1}{2}\varepsilon_0E_0^2$, $\langle\vec{S}\rangle=\frac{1}{2}c\varepsilon_0E_0^2\hat{z}$, $\langle\vec{g}\rangle=\frac{1}{2c}\varepsilon_0E_0^2\hat{z}$

Intensity: $I=\langle S\rangle=\frac{1}{2}c\varepsilon_0E_0^2$ average power per unit area

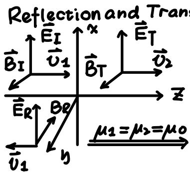
Radiation pressure on perfect absorber

$\Delta\vec{p}=\langle\vec{g}\rangle Ac\Delta t\Rightarrow P=\frac{1}{A}\frac{\Delta p}{\Delta t}=\frac{I}{c}$.

EM waves in matter: $v=\frac{1}{\sqrt{\varepsilon\mu}}=\frac{c}{n}$, $n=\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$

$\varepsilon_0\to\varepsilon$, $\mu_0\to\mu$, $c\to v$. index of refraction. $\approx\sqrt{\varepsilon_r}$

Reflection and Transmission at Normal Incidence



$\tilde{E}_{0R}=\frac{1-\beta}{1+\beta}\tilde{E}_{0I}$ $\beta=\frac{\mu_1v_1}{\mu_2v_2}=\frac{\mu_1n_1}{\mu_2n_2}$

$\tilde{E}_{0T}=\frac{2}{1+\beta}\tilde{E}_{0I}$

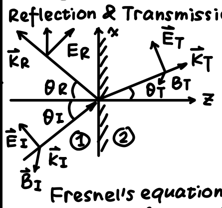
$\xrightarrow{\mu_1=\mu_2=\mu_0}\tilde{E}_{0R}=\left|\frac{v_2-v_1}{v_2+v_1}\right|\tilde{E}_{0I}$

$\tilde{E}_{0T}=\frac{2v_2}{v_2+v_1}\tilde{E}_{0I}$.

Reflection & transmission coefficients:

$R=\frac{I_R}{I_I}=\left(\frac{n_1-n_2}{n_1+n_2}\right)^2$, $T=\frac{I_T}{I_I}=\frac{4n_1n_2}{(n_1+n_2)^2}$. $R+T=1$.

Reflection & Transmission at Oblique Incidence.



First law: $k_I\sin\theta_I=k_R\sin\theta_R=k_T\sin\theta_T$.

Second law: $\theta_I=\theta_R$

Third law: $\frac{\sin\theta_T}{\sin\theta_I}=\frac{n_1}{n_2}$. (Snell's law)

Fresnel's equations.

$\tilde{E}_{0R}=\left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}$, $\tilde{E}_{0T}=\left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}$

$\alpha=\frac{\cos\theta_T}{\cos\theta_I}$, $\beta=\frac{\mu_1v_1}{\mu_2v_2}=\frac{\mu_1n_2}{\mu_2n_1}$

$R=\left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$, $T=\alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

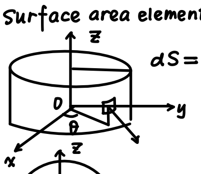
$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B}).$$

$$\nabla=\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z} \qquad \nabla T=\frac{\partial T}{\partial x}\hat{x}+\frac{\partial T}{\partial y}\hat{y}+\frac{\partial T}{\partial z}\hat{z}.$$

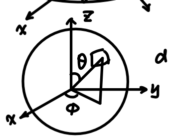
$$dT=(\nabla T)\cdot d\vec{l}. \quad \nabla\cdot\vec{v}=\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}.$$

$$\nabla\times\vec{v}=\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ v_x & v_y & v_z \end{vmatrix}=\hat{x}\left(\partial_y v_z-\partial_z v_y\right)+\hat{y}\left(\partial_z v_x-\partial_x v_z\right)+\hat{z}\left(\partial_x v_y-\partial_y v_x\right).$$

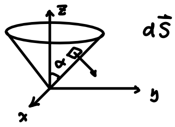
Surface area elements



$$dS=R\,dz\,d\theta$$



$$dS=R^2\sin\theta\,d\theta\,d\phi$$



$$d\vec{S}=r\sin\alpha\,dr\,d\phi\,\hat{\theta}$$

$$\vec{r}(s,t)\Rightarrow dS=\left\|\frac{\partial\vec{r}}{\partial s}\times\frac{\partial\vec{r}}{\partial t}\right\|dsdt$$

$$d\vec{S}=\left(\frac{\partial\vec{r}}{\partial s}\times\frac{\partial\vec{r}}{\partial t}\right)dsdt$$

Product rules:

$$\nabla(fg)=f\nabla g+g\nabla f$$

$$\nabla(\vec{A}\cdot\vec{B})=\vec{A}\times(\nabla\times\vec{B})+\vec{B}\times(\nabla\times\vec{A})+(\vec{A}\cdot\nabla)\vec{B}+(\vec{B}\cdot\nabla)\vec{A}.$$

$$\nabla(f\vec{A})=f(\nabla\cdot\vec{A})+\vec{A}\cdot\nabla f$$

$$\nabla\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\nabla\times\vec{A})-\vec{A}\cdot(\nabla\times\vec{B})$$

$$\nabla\times(f\vec{A})=f(\nabla\times\vec{A})-\vec{A}\times\nabla f$$

$$\nabla\times(\vec{A}\times\vec{B})=(\vec{B}\cdot\nabla)\vec{A}-(\vec{A}\cdot\nabla)\vec{B}+\vec{A}(\nabla\cdot\vec{B})-\vec{B}(\nabla\cdot\vec{A}).$$

$$\nabla^2 T=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}.$$

$$\nabla^2\vec{v}=\nabla^2 v_x\hat{x}+\nabla^2 v_y\hat{y}+\nabla^2 v_z\hat{z}.$$

$$\nabla\times(\nabla T)=\vec{0}, \quad \nabla\cdot(\nabla\times\vec{v})=0.$$

$$\nabla\times(\nabla\times\vec{v})=\nabla(\nabla\cdot\vec{v})-\nabla^2\vec{v}.$$

$$\int_C\vec{v}\cdot d\vec{l}=\int_a^b\vec{v}(\vec{r}(t))\cdot\vec{r}'(t)dt$$

$$\int_S\vec{v}\cdot d\vec{S}=\int\vec{v}(\vec{r}(s,t))\underbrace{\left(\frac{\partial\vec{r}}{\partial s}\times\frac{\partial\vec{r}}{\partial t}\right)dsdt}_{\hat{n}dS}.$$

$$\int_{\vec{a}}^{\vec{b}}(\nabla T)\cdot d\vec{l}=T(\vec{b})-T(\vec{a}).$$

Divergence theorem: $\int_V(\nabla\cdot\vec{v})dV=\oint_S\vec{v}\cdot d\vec{A}$

Stokes' theorem: $\int_S(\nabla\times\vec{v})\cdot d\vec{A}=\oint_P\vec{v}\cdot d\vec{l}$

$$\int_V f\cdot(\nabla A)dV=-\int\vec{A}\cdot(\nabla f)dV+\oint f\vec{A}\,d\vec{a}.$$

Spherical coordinates.

$$\begin{cases} x=r\sin\theta\cos\phi \\ y=r\sin\theta\sin\phi \\ z=r\cos\theta \end{cases} \quad \begin{cases} \hat{r}=\sin\theta\cos\phi\,\hat{x}+\sin\theta\sin\phi\,\hat{y}+\cos\theta\,\hat{z}. \\ \hat{\theta}=\cos\theta\cos\phi\,\hat{x}+\cos\theta\sin\phi\,\hat{y}-\sin\theta\,\hat{z} \\ \hat{\phi}=-\sin\phi\,\hat{x}+\cos\phi\,\hat{y}. \end{cases} \quad \begin{cases} d\vec{l}=dr\,\hat{r}+r\,d\theta\,\hat{\theta}+r\sin\theta\,d\phi\,\hat{\phi} \\ da=r^2\sin\theta\,d\theta\,d\phi \\ d\tau=r^2\sin\theta\,dr\,d\theta\,d\phi. \end{cases}$$

$$\nabla T=\frac{\partial T}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial T}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial T}{\partial\phi}\hat{\phi}$$

$$\nabla\cdot\vec{v}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,v_\theta)+\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}.$$

$$\nabla\times\vec{v}=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\,v_\phi)-\frac{\partial v_\theta}{\partial\phi}\right]\hat{r}+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi}-\frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta}+\frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta)-\frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 T=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}.$$

$$\frac{d\hat{r}}{dr}=0, \quad \frac{d\hat{r}}{d\theta}=\hat{\theta}. \quad \frac{d\hat{r}}{d\phi}=\sin\theta\,\hat{\phi}$$

$$\frac{d\hat{\theta}}{dr}=0, \quad \frac{d\hat{\theta}}{d\theta}=-\hat{r}, \quad \frac{d\hat{\theta}}{d\phi}=\cos\theta\,\hat{\phi}$$

$$\frac{d\hat{\phi}}{dr}=0. \quad \frac{d\hat{\phi}}{d\theta}=0. \quad \frac{d\hat{\phi}}{d\phi}=-\sin\theta\,\hat{r}-\cos\theta\,\hat{\theta}$$

Cylindral Coordinates

$$\begin{cases} x=r\cos\phi \\ y=r\sin\phi \\ z=z \end{cases} \quad \begin{cases} \hat{r}=\cos\phi\,\hat{x}+\sin\phi\,\hat{y} \\ \hat{\phi}=-\sin\phi\,\hat{x} \\ \hat{z}=\hat{z} \end{cases} \quad \begin{aligned} d\vec{l}&=dr\,\hat{r}+r\,d\phi\,\hat{\phi}+dz\,\hat{z} \\ d\tau&=r\,dr\,d\phi\,dz \end{aligned}$$

$$\nabla T=\frac{\partial T}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial T}{\partial\phi}\hat{\phi}+\frac{\partial T}{\partial z}\hat{z}$$

$$\nabla\cdot\vec{v}=\frac{1}{r}\frac{\partial}{\partial r}(r v_r)+\frac{1}{r}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$

$$\nabla\times\vec{v}=\left(\frac{1}{r}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right)\hat{r}+\left(\frac{\partial v_r}{\partial z}-\frac{\partial v_z}{\partial r}\right)\hat{\phi}+\frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\phi)-\frac{\partial v_r}{\partial\phi}\right]\hat{z}$$

$$\nabla^2 T=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2}+\frac{\partial^2 T}{\partial z^2}.$$

Derivatives for $\hat{r}, \hat{\phi}, \hat{z}$ are all 0 except for $\frac{d\hat{r}}{d\phi}=\hat{\phi}$, $\frac{d\hat{\phi}}{d\phi}=-\hat{r}$

$$\int\tan x\,dx=-\ln|\cos x|+C \qquad \int\frac{dx}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C,\ a>0 \qquad \int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+C$$

$$\int\cot x\,dx=\ln|\sin x|+C$$

$$\int\frac{1}{\cos^2 x}dx=\tan x+C \qquad \int\frac{dx}{\sqrt{x^2+a}}=\log\left|x+\sqrt{x^2+a}\right|+C. \qquad \int\frac{dx}{x^2+a^2}=\frac{1}{a}\arctan\frac{x}{a}+C.$$