

- $f_i(\mathbf{x})$ 为 r 次多项式函数, \mathbf{x} 为 n 维模式, 则有

$$f_i(\mathbf{x}) = x_{p_1}^{s_1} x_{p_2}^{s_2} \cdots x_{p_r}^{s_r}, p_1, p_2, \cdots, p_r = 1, 2, \cdots, n, s_1, s_2, \cdots, s_r = 0, 1$$

此时, 判别函数 $d(\mathbf{x})$ 可用以下递推关系给出:

$$\text{常数项: } d^{(0)}(\mathbf{x}) = w_{n+1}$$

$$\text{一次项: } d^{(1)}(\mathbf{x}) = \sum_{p_1=1}^n w_{p_1} x_{p_1} + d^{(0)}(\mathbf{x})$$

$$\text{二次项: } d^{(2)}(\mathbf{x}) = \sum_{p_1=1}^n \sum_{p_2=p_1}^n w_{p_1 p_2} x_{p_1} x_{p_2} + d^{(1)}(\mathbf{x})$$

$$\text{r 次项: } d^{(r)}(\mathbf{x}) = \sum_{p_1=1}^n \sum_{p_2=p_1}^n \cdots \sum_{p_r=p_{r-1}}^n w_{p_1 p_2 \cdots p_r} x_{p_1} x_{p_2} \cdots x_{p_r} + d^{(r-1)}(\mathbf{x})$$

- $d(\mathbf{x})$ 总项数的讨论: 对于 n 维 \mathbf{x} 向量, 若用 r 次多项式, $d(\mathbf{x})$ 的权

$$\text{系数的总项数为: } N_w = C_{n+r}^r = \frac{(n+r)!}{r!n!}$$

$$\text{当 } r=2 \text{ 时: } N_w = C_{n+2}^2 = \frac{(n+2)!}{2!n!} = \frac{(n+2)(n+1)}{2}$$

$$\text{当 } r=3 \text{ 时: } N_w = C_{n+3}^3 = \frac{(n+3)!}{3!n!} = \frac{(n+3)(n+2)(n+1)}{6}$$