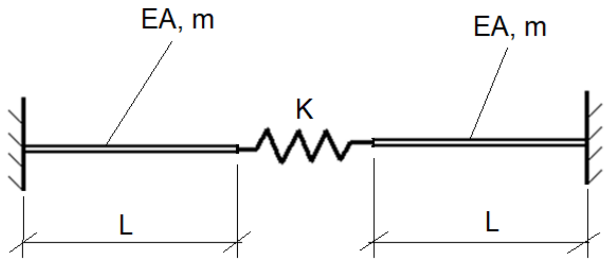
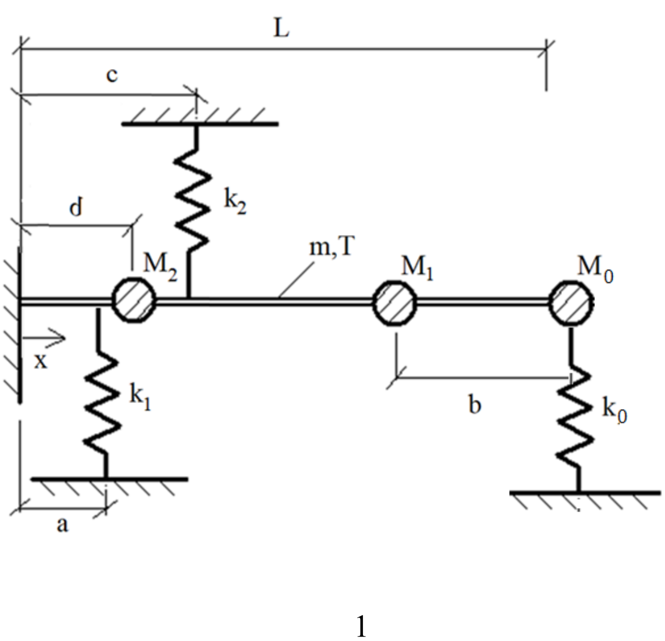
TAM 514 /AE 551 HOMEWORK 3

Distributed: 3/5/2025
Due: 3/24/2025 in class (for on-line students, the deadline for submission by email is 1pm CST on the due date)

1 (100 pts). Compute the vibration modes (natural frequencies and mode shapes) of the following system of uniform axial rods that are coupled by a axial linear spring of constant K. Write the orthogonality conditions that are satisfied by these modes and mass-orthonormalize them. Use reasonable values for the system parameters in SI units. Study the vibration modes of the limiting systems as $K \rightarrow 0$ and $K \rightarrow \infty$.



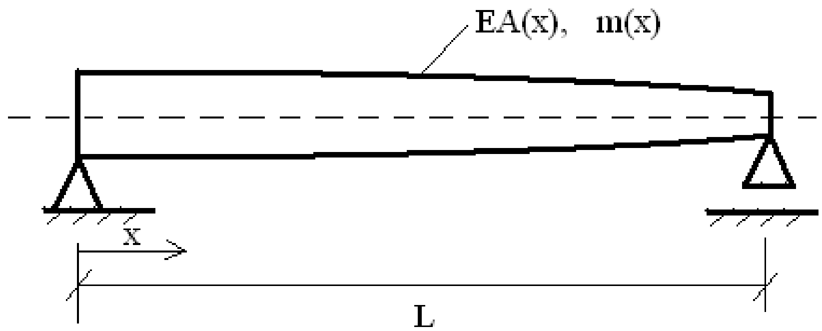
2 (100 pts). Consider a uniform elastic string undergoing transverse vibration with the grounded vertical springs and the concentrated masses as shown below (disregard gravity). Assign your own numerical values to the system parameters (use reasonable SI units).
(i) Formulate Rayleigh's quotient for this system and estimate the first natural frequency. Using at least three different test functions perform a convergence study to study the convergence of the Rayleigh quotient to the first natural frequency.
(ii) Based on the RQ that you developed in (i) develop a Rayleigh-Ritz methodology for discretizing the eigenvalue problem of this system. Show all steps in detail and using appropriate trial functions compute approximations for the leading three modes of this system.



3 (100 points). Consider the following nonuniform rod in axial vibration, with elastic and inertial properties $EA(x)$ and $m(x)$ varying as:

$$EA(x) = EA\left[1 - sin^{3/2}(.5x/L)\right] \text{ and } m(x) = m\left[1 - sin^{3/2}(.5x/L))\right]$$

Use the Rayleigh-Ritz method to approximate the leading three normal modes of this system. Assume a steel rod with $L = 1m$. Compare your results with the first three normal modes of the uniform rod with properties $EA(x) = EA$ and $m(x) = m$. Do the differences make sense? Justify the differences using mechanics arguments.



4 (100 points). Consider the following (normalized) elastic string undergoing transverse vibrations governed by the classical wave equation:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial^2 u(x,t)}{\partial x^2}, \quad u(x,0) = 0, \qquad \frac{\partial u(x,0)}{\partial t} = 0, \qquad 0 \leq x \leq 1, \quad t \geq 0$$

The string has fixed-free supports which undergo prescribed motions as follows:

$$u(0,t) = u_{g1}(t) \equiv 2(1 - cos\omega t), \qquad u(1,t) = u_{g2}(t) \equiv cos2\omega t - 1, \qquad t \geq 0$$

Compute the transverse vibrations of the string. Can resonance occur in this system? If yes, under what condition(s) can this occur?

5 (100 points). We reconsider problem 5 of HW2. Assign reasonable numerical values to the system parameters in SI units.

(i) Compute the fundamental natural frequency and the corresponding first eigenfunction by (numerically) solving the exact eigenvalue problem.

(ii) Provide three different estimates for the fundamental natural frequency using Rayleigh's quotient and compare them with the exact value. In each case the test function that you'll use approximates the first eigenfunction, so compare the approximations of approximate eigenfunctions with exact first eigenfunction as well.