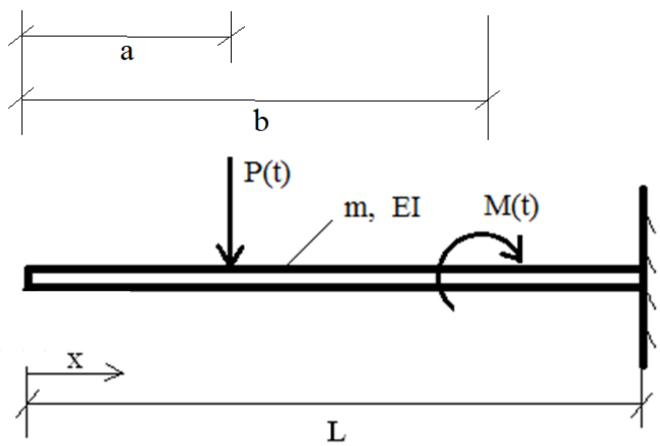
TAM 514 /AE 551 HOMEWORK 4

Distributed: **3/26/2025**
Due: **4/9/2025 in class (for on-line students, the deadline for submission by email is 1pm CST on the due date)**

1 (100 pts). Consider the following beam forced by a point force $P(t)$ applied at $x = x_1$ and a point moment $M(t)$ at $x = x_2$. The initial conditions are $u(x,0) = 0$ and $u_t(x,0) = 0$.

(i) Compute the orthonormalized vibration modes of this system and then compute analytically the response $u(x,t)$ of the beam using modal analysis
(ii) Now suppose that we want to avoid exciting the second mode of the beam. Find the necessary and sufficient conditions for this to happen. Find analytically the response $u(x,t)$ of the beam in that case.
(iii) Under what conditions can resonance occur in this system?



2 (100 pts). Consider the following beam with viscous damping:

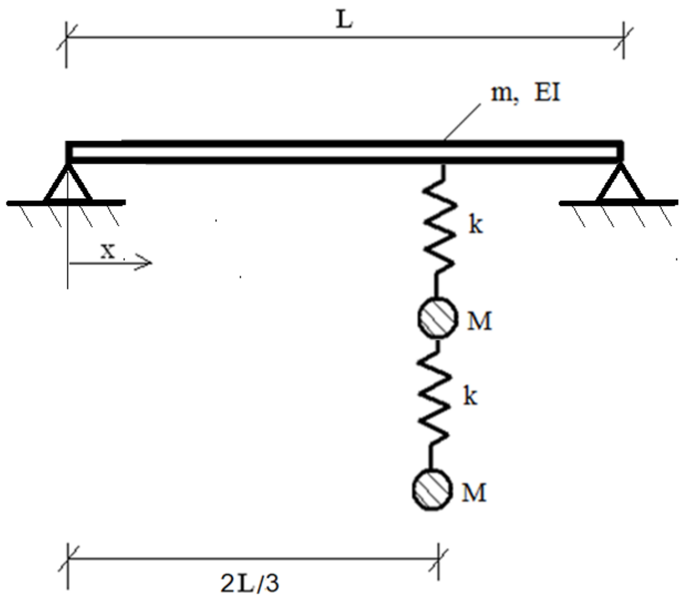
$$u_{xxxx} + 4u_{xx} + 0.05u_t + u_{tt} = 0, \quad 0 \le x \le \pi$$
$$u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) = 0, \; u(x,0) = f(x), \; u_t(x,0) = g(x)$$

(i) Show that the following Ansatz can be used to compute the response of this system,

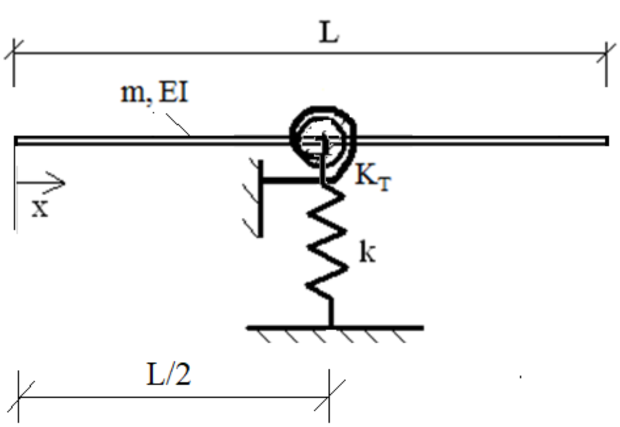
$$u(x,t) = \sum_{k=1}^{\infty} a_k(t) \sqrt{\frac{2}{\pi}} \sin kx$$

and find the equations governing the modal amplitudes $a_k(t)$, $k = 1, 2, \ldots$.
(ii) Are all modes stable? What is the physical mechanism that could cause instability? Does the damping term affect the stability of the modes?
(iii) Even if some of the modes prove to be unstable, is it possible that the overall response of the system to still be stable? Under what conditions can this happen, and what is the expression of the stable response?

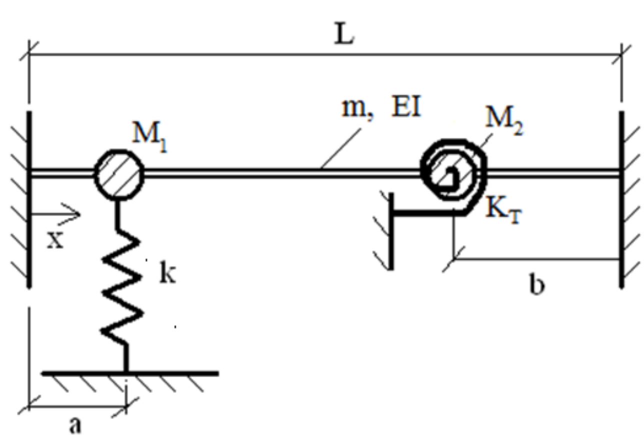
2. (200 pts) As a first step for computing the dynamical response of the following uniform simply supported beam with an attached two-DOF spring-mass oscillator, formulate and solve the eigenvalue problem and provide the orthonormality conditions for the eigenfunctions. Then show how you can reduce the dynamics to an infinite set of uncoupled modal oscillators using modal analysis. Discuss the local effect of the attachment, by considering the limiting cases $k \to 0$ and $k \to \infty$.



4 (200 points). Formulate Rayleigh's quotient for these two beam systems, labelled System I and System II. Then develop a discretized numerical methodology (either Rayleigh-Ritz or Gallerkin) to approximately solve the resulting eigenvalue problems. Assign your own (reasonable!) numerical values in SI metric system to the system parameters and find approximations for the first three modes of these systems (i.e., approximations to the natural frequencies and mode shapes). Under what conditions the dynamics of System I can resemble a two-DOF system?



System I



System II