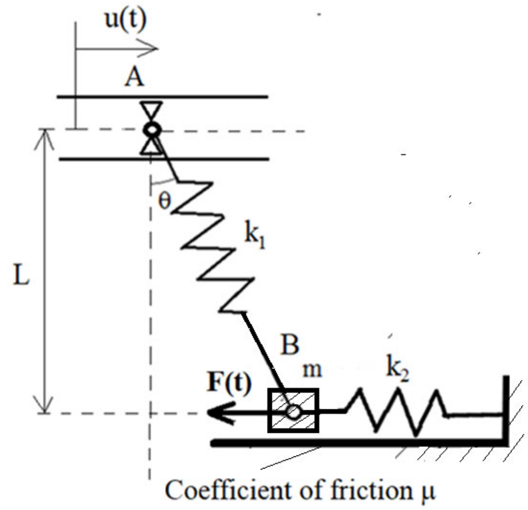
TAM 514 /AE 551 HOMEWORK 1

Distributed: 1/29/2025
Due: 2/12/2025 in class (for the on-line students, the deadline for submission by email is 1pm CST on the due date)

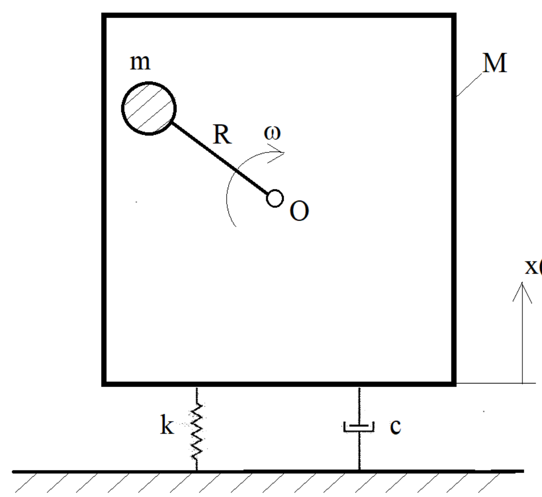
1 (50 pts). Consider the mechanical system shown. Pivot A slides horizontally with a prescribed displacement $u(t)$, and mass B is also restricted to more horizontally on a rough base. The system is in static equilibrium (and the springs are unstretched) when $\theta(t) = u(t) = 0$. The only mass of the system is m, and the coefficient of friction of the rough base where mass m is sliding is μ. A horizontal force $F(t)$ is applied to the mass. Gravity is included.
(i) How many degrees of freedom (DOFs) does this system have?
(ii) Derive the equation(s) of motion of this system using Lagrange's equations



2 (50 pts). Consider a machine of mass M with a rotating imbalance produced by a mass m rotating with frequency ω about an axis orthogonal to the plane and positioned at a radius R from the axis of rotation. At $t = 0$ the mass m is at its top vertical position. We assume that the machine is suspended by a spring k in parallel to a viscous damper c, and is constrained to move in the vertical direction. Gravity is ignored.
(i) Show that the vertical response of the machine with respect to an inertial frame is governed by the equation:

$$(M + m)\ddot{x}(t) + c\dot{x}(t) + kx(t) = mR\omega^2 cos\omega t$$

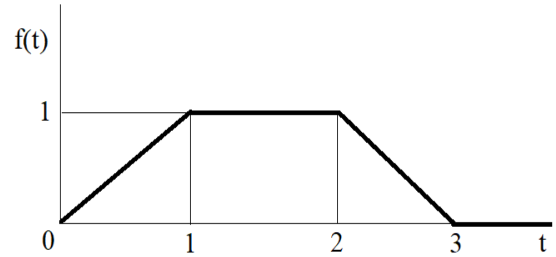
(ii) Compute the steady state response $x(t)$ of the machine.
(iii) If there is no damping ($c = 0$) when will resonance occur?



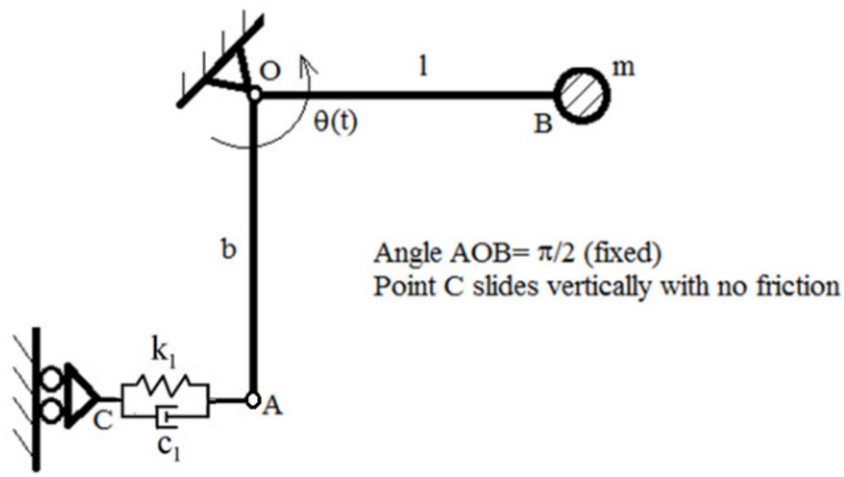
3 (50 pts). Consider the following forced oscillator,

$$\ddot{x}(t) + x(t) = f(t), \quad x(0) = 1, \dot{x}(0) = -2$$

where $f(t)$ is shown in the figure. Compute the response using Laplace transform.



4 (50 pts). Consider the system shown below. Gravity is ignored, the only mass is the mass m at position B, and the system is shown at the position of equilibrium. We consider small rotations of this system about the pivot O.
(i) Find the equation of motion.
(ii)Determine the natural frequency ω_n and viscous damping ratio ζ of this system.
(iii) Now select the relation that the system parameters should satisfy for the response to be overdamped, and in that case find the expression for the response for initial conditions $\theta(0) = \theta_0$ and $\dot{\theta}(0) = \omega_0$.



5 (50 pts). The rectangular frame has mass M and translates horizontally under the action of the horizontal force $F(t)$. Inside the frame there is a pendulum of mass m. The supporting spring and damper are at their natural lengths when $x = \theta = 0$ and gravity should be considered.
(i) Derive the equations of motion using Lagrange's equations without assuming small angles.
(ii) Now assume that $F(t) = 0$ and show that $x = \theta = 0$ is an equilibrium position of your system. Then linearize the equations of motion close to this equilibrium position.