

$$\frac{d}{dx} \left[A(x) \frac{d\phi_r}{dx} \right] = -\omega_r^2 B(x) \phi_r \Rightarrow \int_0^L \frac{d}{dx} \left[A(x) \frac{d\phi_r}{dx} \right] \phi_s dx = -\omega_r^2 \int_0^L B(x) \phi_r \phi_s dx \Rightarrow$$

$$\Rightarrow \underbrace{A(x) \frac{d\phi_r}{dx} \phi_s \Big|_0^L}_{A(L) \frac{d\phi_r(L)}{dx} \phi_s(L) - A(0) \frac{d\phi_r(0)}{dx} \phi_s(0)} - \int_0^L A(x) \frac{d\phi_r}{dx} \frac{d\phi_s}{dx} dx = -\omega_r^2 \int_0^L B(x) \phi_r \phi_s dx$$

Now, let's consider the second equation satisfied by $\phi_s(x)$ and do the same operation \Rightarrow

$$A(L) \frac{d\phi_s(L)}{dx} \phi_r(L) - A(0) \frac{d\phi_s(0)}{dx} \phi_r(0) - \int_0^L A(x) \frac{d\phi_s}{dx} \frac{d\phi_r}{dx} dx =$$

$$\underbrace{-(k_2 - \omega_r^2 M_2) \phi_r(L)}_{A(L) \phi_r'(L)} \underbrace{(k_1 - \omega_r^2 M_1) \phi_r(0)}_{-A(0) \phi_r'(0)} = -\omega_s^2 \int_0^L B(x) \phi_r \phi_s dx$$

$$\Rightarrow A(L) \phi_r'(L) \phi_s(L) - A(0) \phi_r'(0) \phi_s(0) - \underbrace{A(L) \phi_s'(L) \phi_r(L)}_{-(k_2 - \omega_s^2 M_2) \phi_s(L)} + \underbrace{A(0) \phi_s'(0) \phi_r(0)}_{(k_1 - \omega_s^2 M_1) \phi_s(0)} =$$

$$= -(\omega_r^2 - \omega_s^2) \int_0^L B(x) \phi_r \phi_s dx \Rightarrow$$