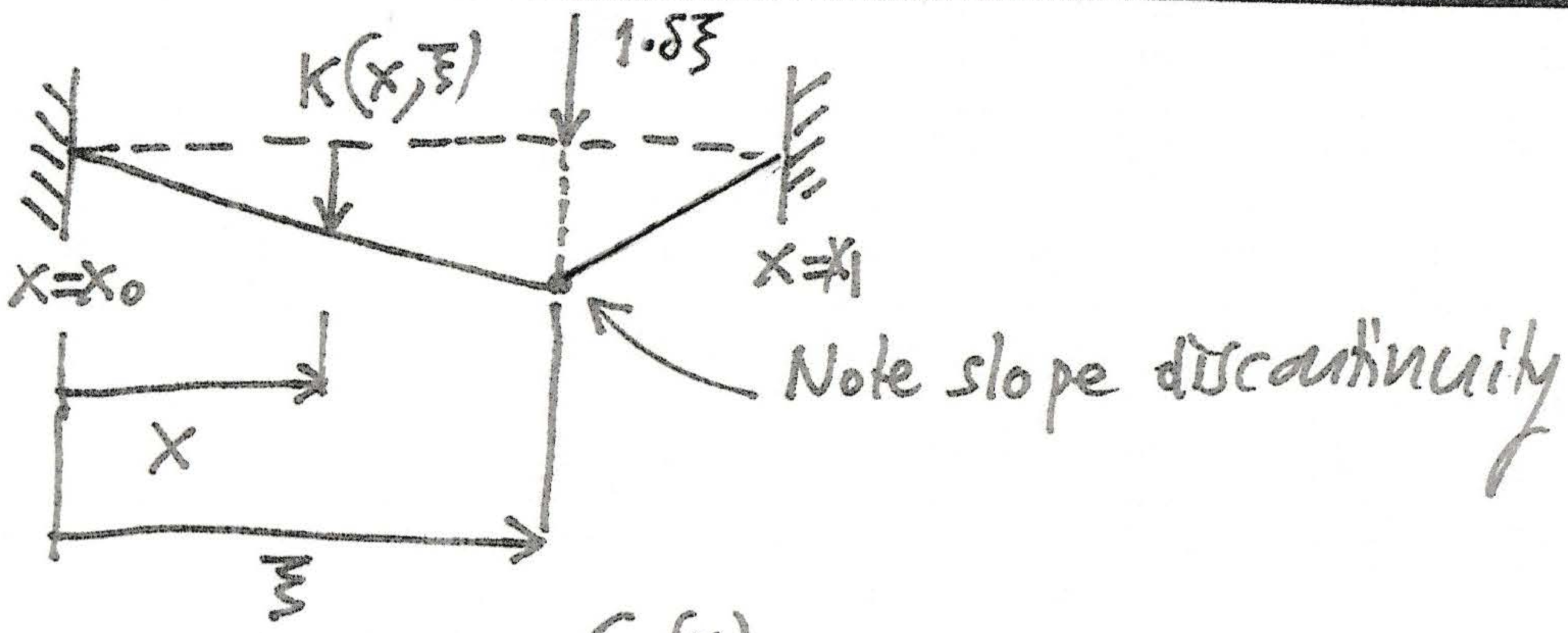
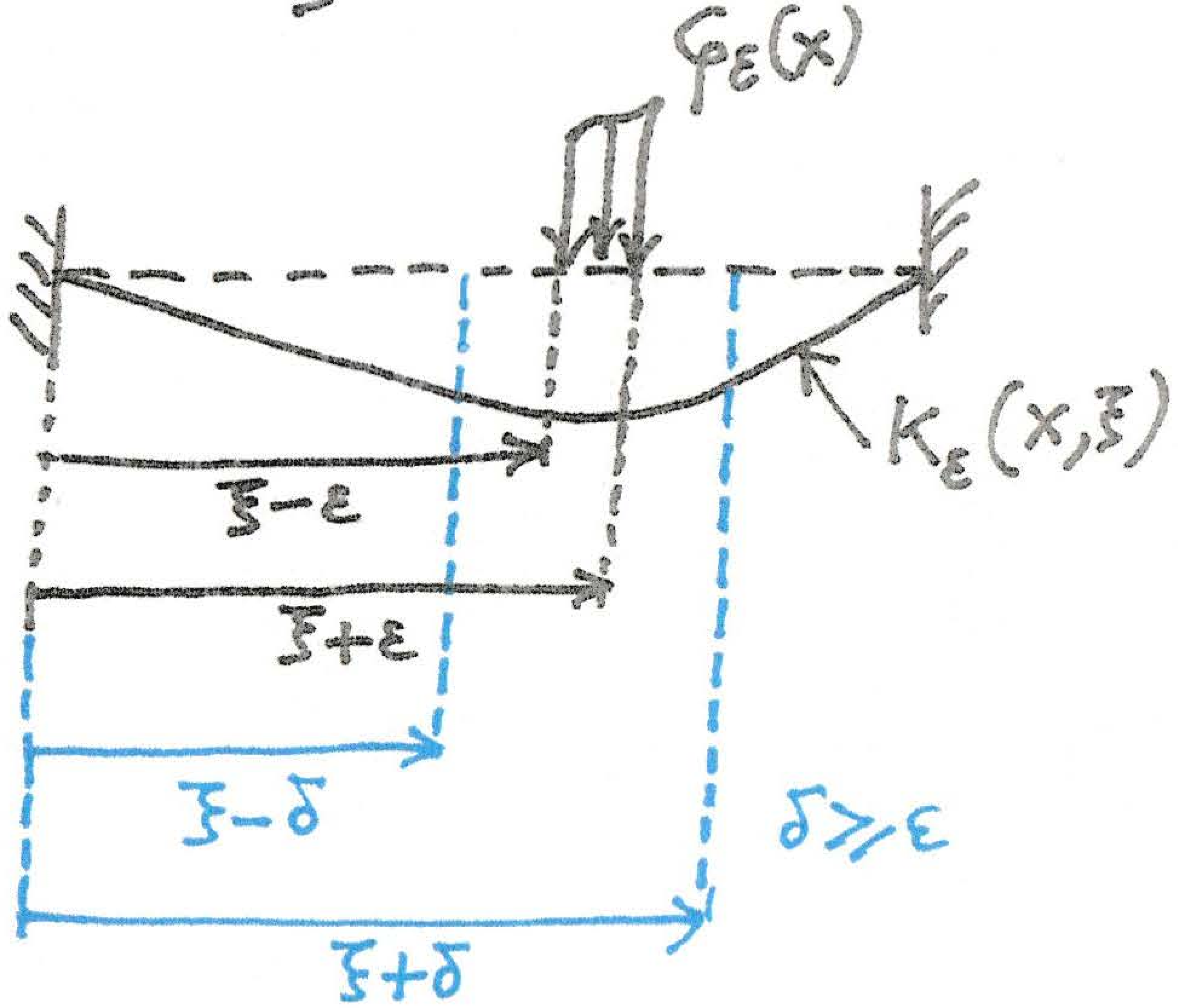


We consider the point force at $x=\xi$ as the limiting case of a $\varphi_\varepsilon(x)$ which vanishes in G for $|x-\xi|>\varepsilon$, and its total intensity is

$$\int_{\xi-\varepsilon}^{\xi+\varepsilon}\varphi_\varepsilon(x)\,dx=1$$



We denote the corresponding deflection of the string by $K_\varepsilon(x,\xi)\Rightarrow$

$$\Rightarrow L[K_\varepsilon]=-\varphi_\varepsilon(x)\Rightarrow(pK_\varepsilon')'-qK_\varepsilon=-\varphi_\varepsilon(x)$$

$$\Rightarrow\int_{\xi-\delta}^{\xi+\delta}(\cdot),\quad\delta\geqslant\varepsilon\Rightarrow\int_{\xi-\delta}^{\xi+\delta}\left\{\frac{d}{dx}\left(p\frac{dK_\varepsilon}{dx}\right)-qK_\varepsilon\right\}dx=$$

$$=-\int_{\xi-\delta}^{\xi+\delta}\varphi_\varepsilon(x)\,dx=-1\Rightarrow$$

$$\Rightarrow\int_{\xi-\delta}^{\xi+\delta}\left\{\frac{d}{dx}\left(p\frac{dK_\varepsilon}{dx}\right)-qK_\varepsilon\right\}dx=-1\Rightarrow$$

$\Rightarrow$ first let $\varepsilon\to0$, and assume $\lim_{\varepsilon\to0}K_\varepsilon(x,\xi)=K(x,\xi)$ <u>continuously dif. except at $x=\xi$</u> $\Rightarrow$

$$\underset{\varepsilon\to 0}{\Rightarrow} \int_{\xi-\delta}^{\xi+\delta} \left\{ \frac{d}{dx}\left(p\frac{dK}{dx}\right) - \underbrace{qK}_{\text{continuous, } x\in G} \right\} dx = -1 \Rightarrow \text{As } \delta\to 0 \text{ we get,}$$

$$\lim_{\delta\to 0} \int_{\xi-\delta}^{\xi+\delta} \frac{d}{dx}\left(p\frac{dK}{dx}\right) dx = -1 \Rightarrow \lim_{\delta\to 0} \left.\frac{dK(x;\xi)}{dx}\right|_{\xi-\delta}^{\xi+\delta} = -\frac{1}{p(\xi)} \Rightarrow$$

This "jump" condition distinguishes $K(x,\xi)$ from any other homogeneous solution

$$\Rightarrow \left[\frac{dK(x,\xi)}{dx}\right]_{\xi-}^{\xi+} = -\frac{1}{p(\xi)}$$

This heuristic argument computes the jump in the slope of $K(x,\xi)$ at the singularity $x=\xi$.

This holds only since $L[u]$ was of second order in x

Based on the above discussion we can define the Green's function $K(x,\xi)$ as the solution of $L[u]=\delta(x-\xi)$, so that it satisfies the following two conditions:

i) It solves the homogeneous equation $L[K]=0$, $x\in G-\{\xi\}$ satisfying the boundary conditions.

ii) At the point of singularity $x=\xi$ it satisfies $\left[K'(x,\xi)\right]_{\xi-}^{\xi+} = -\frac{1}{p(\xi)}$