$$\frac{d}{dx}\left[A(x)\frac{d\varphi_r}{dx}\right] = -\omega_r^2 B(x)\varphi_r \Rightarrow \int_0^L \frac{d}{dx}\left[A(x)\frac{d\varphi_r}{dx}\right]\varphi_s\,dx = -\omega_r^2\int_0^L B(x)\varphi_r\varphi_s\,dx \Rightarrow$$

$$\Rightarrow \underbrace{A(x)\frac{d\varphi_r}{dx}\varphi_s\Big|_0^L}_{A(L)\frac{d\varphi_r(L)}{dx}\varphi_s(L) - A(0)\frac{d\varphi_r(0)}{dx}\varphi_s(0)} - \int_0^L A(x)\frac{d\varphi_r}{dx}\frac{d\varphi_s}{dx}\,dx = -\omega_r^2\int_0^L B(x)\varphi_r\varphi_s\,dx$$

Now, let's consider the second equation satisfied by $\varphi_s(x)$, and do the same operations $\Rightarrow$

$$A(L)\frac{d\varphi_s(L)}{dx}\varphi_r(L) - A(0)\frac{d\varphi_s(0)}{dx}\varphi_r(0) - \int_0^L A(x)\frac{d\varphi_s}{dx}\frac{d\varphi_r}{dx}\,dx =$$

$$= -\omega_s^2\int_0^L B(x)\varphi_r\varphi_s\,dx$$

$(-) \Rightarrow$

$$\Rightarrow \underbrace{A(L)\varphi_r'(L)}_{-(K_2-\omega_r^2M_2)\varphi_r(L)}\varphi_s(L) - \underbrace{A(0)\varphi_r'(0)}_{(K_1-\omega_r^2M_1)\varphi_r(0)}\varphi_s(0) - \underbrace{A(L)\varphi_s'(L)}_{-(K_2-\omega_s^2M_2)\varphi_s(L)}\varphi_r(L) + \underbrace{A(0)\varphi_s'(0)}_{(K_1-\omega_s^2M_1)\varphi_s(0)}\varphi_r(0) =$$

$$= -(\omega_r^2-\omega_s^2)\int_0^L B(x)\varphi_r\varphi_s\,dx \Rightarrow$$