式 (7.29) を確認する。

そもそも、(7.19),(7.20),(3.20) を考慮すると、

$$p(Y|F) = \prod_i Bern(y_i|Sig(f(x_i))) = \prod_i \{Sig(f(x_i))^{y_i}(1 - Sig(f(x_i)))^{1-y_i}\} \tag{1}$$

(7.29) ではこれの対数を、f_i で微分したものを考える。

$$\begin{aligned}\frac{\partial}{\partial f_i} ln\, p(Y|F) &= \frac{\partial}{\partial f_i} ln \prod_i \{Sig(f(x_i))^{y_i}(1 - Sig(f(x_i)))^{1-y_i}\} = \frac{\partial}{\partial f_i} ln\, \{Sig(f(x_i))^{y_i}(1 - Sig(f(x_i)))^{1-y_i}\} \\ &= \frac{y_i}{Sig(f(x_i))}\frac{\partial}{\partial f_i}Sig(f_i) - \frac{1-y_i}{1-Sig(f(x_i))}\frac{\partial}{\partial f_i}Sig(f_i) = \frac{\partial}{\partial f_i}Sig(f_i)(\frac{y_i}{Sig(f(x_i))} - \frac{1-y_i}{1-Sig(f(x_i))}) \\ &= \frac{\partial}{\partial f_i}Sig(f_i)\frac{y_i - Sig(f(x_i))}{Sig(f(x_i))(1-Sig(f(x_i)))} = y_i - Sig(f(x_i))\end{aligned} \tag{2}$$

なお、(2.23) より、

$$Sig(f_i) = \frac{1}{1+e^{-f_i}} \tag{3}$$

となり、

$$Sig(f_i)(1 - Sig(f_i)) = \frac{1}{1+e^{-f_i}}(1 - \frac{1}{1+e^{-f_i}}) = \frac{1}{1+e^{-f_i}}\frac{e^{-f_i}}{1+e^{-f_i}} = \frac{e^{-f_i}}{(1+e^{-f_i})^2} \tag{4}$$

を考えると、

$$\frac{\partial}{\partial f_i}Sig(f_i) = \frac{\partial}{\partial f_i}\frac{1}{1+e^{-f_i}} = (-\frac{1}{(1+e^{-f_i})^2})(-e^{-f_i}) = \frac{e^{-f_i}}{(1+e^{-f_i})^2} = Sig(f_i)(1 - Sig(f_i)) \tag{5}$$

となっていることに注意する。

念のために (7.30) も確認すると、$i \neq j$ のときは、

$$\frac{\partial^2}{\partial f_j \partial f_i} ln\, p(Y|F) = \frac{\partial}{\partial f_j}(y_i - Sig(f(x_i))) = 0 \tag{6}$$

$i = j$ のときは、

$$\frac{\partial^2}{\partial {f_i}^2} ln\, p(Y|F) = \frac{\partial}{\partial f_i}(y_i - Sig(f(x_i))) = \frac{\partial}{\partial f_i}(-Sig(f(x_i))) = -Sig(f_i)(1 - Sig(f_i)) \tag{7}$$

となる。