式 (6.45) の展開を考えてみる。

$$\mathcal{L}(\psi,\xi) = \ln p(X) - D_{KL}[q(Z,\theta;X,\psi,\xi)|p(Z,\theta|X)]$$

$$= \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln p(X)] + \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln p(Z,\theta|X)] - \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln q(Z,\theta;X,\psi,\xi)](\text{ここまで式 (6.10)})$$

$$= \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln (p(X)p(Z,\theta|X))] - \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln q(Z,\theta;X,\psi,\xi)]$$

$$= \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln p(X,Z,\theta)] - \mathbb{E}_{q(Z,\theta;X,\psi,\xi)}[\ln q(Z,\theta;X,\psi,\xi)]$$

$$= \mathbb{E}_{\prod_n q_\psi(z_n|x_n,\theta)q_\xi(\theta)}[\ln p(X,Z,\theta)] - \mathbb{E}_{\prod_n q_\psi(z_n|x_n,\theta)q_\xi(\theta)}[\ln q(Z,\theta;X,\psi,\xi)](\text{式 (6.44)})$$

$$= \mathbb{E}_{\prod_n q_\psi(z_n|x_n,\theta)q_\xi(\theta)}[\ln (p(\theta)\prod_n(p(x_n|z_n,\theta)p(z_n,\theta)))] - \mathbb{E}_{\prod_n q_\psi(z_n|x_n,\theta)q_\xi(\theta)}[\ln (q_\xi(\theta)\prod_n q_\psi(z_n|x_n.\theta))](\text{式 (6.39), (6.44)})$$

$$= \mathbb{E}_{\prod_n q_\psi(z_n|x_n,\theta)q_\xi(\theta)}[\ln (p(\theta)\prod_n p(x_n,z_n|\theta))] - \mathbb{E}_{\prod_n q_\psi(z_n|x_n,\theta)q_\xi(\theta)}[\ln (q_\xi(\theta)\prod_n q_\psi(z_n|x_n.\theta))](\text{式 (6.39), (6.44)})$$

$$= \mathbb{E}_{q_\xi(\theta)}[\ln p(\theta)] + \mathbb{E}_{q_\xi(\theta)\prod_n q_\psi(z_n|x_n,\theta)}[\ln (\prod_n p(x_n,z_n|\theta))] - \mathbb{E}_{q_\xi(\theta)}[\ln q_\xi(\theta)] - \mathbb{E}_{q_\xi(\theta)\prod_n q_\psi(z_n|x_n,\theta)}[\ln (\prod_n q_\psi(z_n|x_n.\theta))]$$

$$(\text{変数が}\theta\text{のみの部分は } q_\psi \text{によらない。} z_n\text{で期待値を取ると } 1 \text{になるので、} q_\psi\text{は消える。})$$

$$= \mathbb{E}_{q_\xi(\theta)}[\ln p(\theta) - \ln q_\xi(\theta)] + \mathbb{E}_{q_\xi(\theta)\prod_n q_\psi(z_n|x_n,\theta)}[\ln \prod_n p(x_n,z_n|\theta) - \ln \prod_n q_\psi(z_n|x_n.\theta)]$$

$$= \mathbb{E}_{q_\xi(\theta)}[\ln p(\theta) - \ln q_\xi(\theta)] + \sum_n \mathbb{E}_{q_\xi(\theta)\prod_m q_\psi(z_m|x_m,\theta)}[\ln p(x_n,z_n|\theta) - \ln q_\psi(z_n|x_n.\theta)]$$

$$= \mathbb{E}_{q_\xi(\theta)}[\ln p(\theta) - \ln q_\xi(\theta)] + \sum_n \mathbb{E}_{q_\xi(\theta)q_\psi(z_n|x_n,\theta)}[\ln p(x_n,z_n|\theta) - \ln q_\psi(z_n|x_n.\theta)]$$

$$(1)$$

となり、(6.45) が求まる。