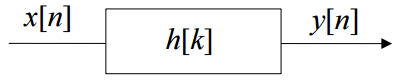
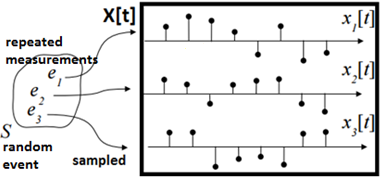
* **Description of system: x[n], h[k], y[n]**

Input is **white noise:**



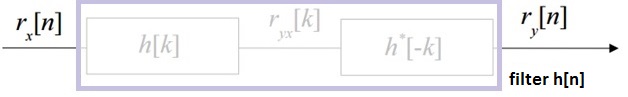
**Figure 1** Definition of a Discrete-Time Linear System

* **Uncorrelated Ergodic WSS Stochastic Random Process**
* **Random** Process**:**
* 1st moment µ (mean), 2nd moment 𝜎 (covariance) are sufficient to model signal
* **Stochastic** Process:
  + Repeated measurements e1, e2, e3 of random process S, sampled in **time**



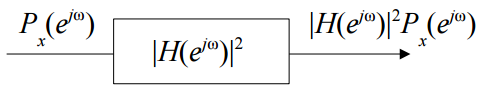
**Figure 2** Definition of a Stochastic Process

* ex) spike train, thermal noise in amplifier
* **Stationary** Process: is independent of time
* **Wide Sense Stationary**: µ and 𝜎 are independent of time
  + **Ergodic**: sample mean converges to ensemble mean:
  + **Uncorrelated (white)**: 𝜎 is constant
* Since white noise is **uncorrelated ergodic WSS**, only 𝜎 is needed to model signal!
* **Estimate Autocorrelation**
* Input is equidistantly sampled white noise
  + 2nd moment = 𝜎 =
* Estimate:
  + direct estimate method:
  + use **xcorr( )**
* **Estimate Power spectrum** 
  + Oscillations of 𝜎 best viewed in **frequency** domain
  + Estimate
* Estimate improves in µ with increasing N
* Estimate inconsistent (𝜎 does not converge to 0) with increasing N
* Use **periodogram** to average over K windows: **psd(x)**; **pwelch(x)**;
* **Filtering :**
* Filter is defined as the convolution of autocorrelation



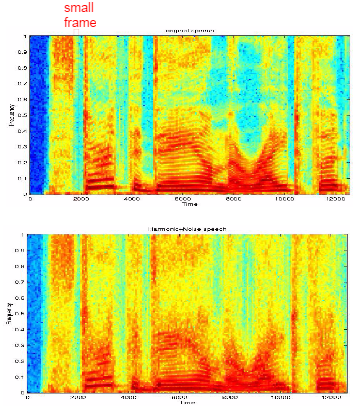
**Figure 3** Definition of Filter h[n] for

* **Filtering :**
* Filter shapes the power spectrum



**Figure 4** Definition of Filter for

* Use **abs(fft(h,N)).^2**
* **Wold Decomposition Theorem**
* Any WSS input can be decomposed into a sum of:
  + Regular process
  + Predictive process
* **IF** regular process and predictive process are orthogonal:
* Power spectrum of WSS input is made of:
  + Continuous component with invertible H(z) (from )
  + Discrete lines (from )
* Power spectrum is additive:
* Application: Modeling Speech using Spectrogram
  + Power spectrum with strong spectral component
  + Assuming WSS, estimate power spectrum for a short window
  + Add windows together over time to create spectrogram
  + Use specgram(x)



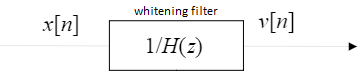
**Figure 5** Modeling Speech using Wold Decomposition

* **Regular Process**
* Input: white noise v[n]
* Filter: h[n] is causal, invertible
* Output: colored noise x[n]

**D:\Documents\Steven\regular process.png**

**Figure 6** Definition of Regular Process

* is the regular process
* the corresponding innovation process



**Figure 7** Definition of Whitening Filter

* Inverse filter 1/H(z)is **whitening filter** of x[n]
* **Power Spectrum of Regular Process**

**D:\Documents\Steven\regularpower.png**

**Figure 8** Power Spectrum of Regular Process

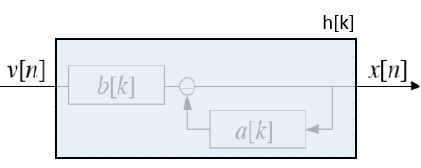
* Power spectrum is **continuous**

|  |
| --- |
| * **Predictive Process** * Predicted exactly from linear combination of past * **Power Spectrum of Predictive Process** * Power spectrum is **discrete lines**   more in Lecture 7! – Harmonic Analysis |

* **AR modeling**
* **Find a[k] by AR modeling**

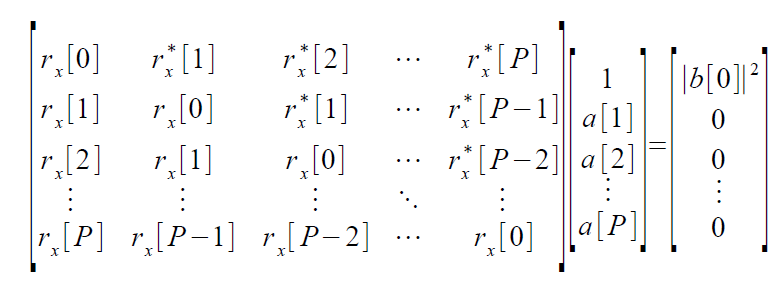
White noise v[n] filtered by a rational transfer function

AR modeling approximate Px(e^jw) for a given x[n]

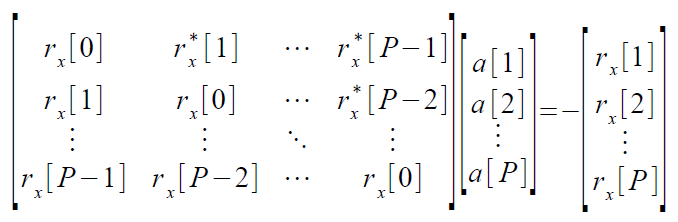


Since input is WSS, need to derive a statistical AR model

normal equations



Can be rewritten



**Linear Prediction**

Predict sample x[n] from its past P samples