

9.1 Z-Test and Confidence intervals for a difference between two population mean.

- Basic Assumption

- $X_1 \dots X_m$ has μ_1, σ_1^2

- $Y_1 \dots Y_n$ has μ_2, σ_2^2

- X, Y are independent.

- Proposition of $\bar{X} - \bar{Y}$

- Expected value: $\mu_1 - \mu_2$

- Standard Deviation: $\sigma_{\bar{X} - \bar{Y}} = \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}$

- Test Procedure:

- ① $H_0: \mu_1 - \mu_2 = \Delta_0$

- $H_a: \mu_1 - \mu_2 > \Delta_0$

- ②
$$Z = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$$

- ③ P-value from Z

- ④ Compare with α .

- ⑤ Conclude:

- β and choice of Sample size.

$$\begin{aligned}\beta(\Delta) &= P(\text{Not rejecting } H_0 \text{ when } \mu_1 - \mu_2 = \Delta) \\ &= P(\text{Type II error})\end{aligned}$$

• large sample test.

When $m > 40$ & $n > 40$, we can apply CLT and assume normal distribution.

• CI for $\mu_1 - \mu_2$.

$$\bar{x} - \bar{y} \pm z_{\alpha/2} \sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}.$$