

# CS 584: Machine Learning

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In 2014, Allstate provided the data on Kaggle.com for the Allstate Purchase Prediction Challenge which is open. The data contain transaction history for customers that ended up purchasing a policy. For each Customer ID, you are given their quote history and the coverage options they purchased.

The data is available on the Blackboard as Purchase\_Likelihood.csv.

1. It contains 665,249 observations on 97,009 unique Customer ID.
2. The nominal target variable is **insurance** which has these categories 0, 1, and 2
3. The nominal features are (categories are inside the parentheses):
  - a. **group\_size**. How many people will be covered under the policy (1, 2, 3 or 4)?
  - b. **homeowner**. Whether the customer owns a home or not (0 = No, 1 = Yes)?
  - c. **married\_couple**. Does the customer group contain a married couple (0 = No, 1 = Yes)?

## Question 1 (35 points)

You will build a multinomial logistic model with the following model specifications.

1. Enter the six effects to the model in this sequence:
  - a. group\_size
  - b. homeowner
  - c. married\_couple
  - d. group\_size \* homeowner
  - e. group\_size \* married\_couple
  - f. homeowner \* married\_couple
2. Include the Intercept term in the model
3. The optimization method is Newton
4. The maximum number of iterations is 100
5. The tolerance level is 1e-8.
6. Use the `sympy.Matrix().rref()` method to identify the non-aliased parameters

Please answer the following questions based on your model.

- a) (5 points) List the aliased columns that you found in your model matrix.  
Answer : List of Aliased Columns for the model :

group\_size\_4  
 homeowner\_1  
 married\_couple\_1  
 group\_size\_1 \* homeowner\_1  
 group\_size\_2 \* homeowner\_1  
 group\_size\_3 \* homeowner\_1  
 group\_size\_4 \* homeowner\_0  
 group\_size\_4 \* homeowner\_1  
 group\_size\_1 \* married\_couple\_1  
 group\_size\_2 \* married\_couple\_1  
 group\_size\_3 \* married\_couple\_1  
 group\_size\_4 \* married\_couple\_0  
 group\_size\_4 \* married\_couple\_1  
 homeowner\_0 \* married\_couple\_1  
 homeowner\_1 \* married\_couple\_0  
 homeowner\_1 \* married\_couple\_1

b) (5 points) How many degrees of freedom does your model have?

**Degree of Freedom = 2**

c) (20 points) After entering each model effect, calculate the Deviance test statistic, its degrees of freedom, and its significance value between the current model and the previous model. List your Deviance test results by the model effects in a table.

Step	Effect Entered	# Free Parameter	Log-Likelihood	Deviance	Degrees of Freedom	Significance
0	Intercept	2	-595406.761884422	Not Applicable		
1	group_size	8	-594912.973584159	987.576600525993	6	4.34787038953134E-210
2	homeowner	10	-591979.082833983	5867.78150035347	2	0
3	married_couple	12	-591936.793832791	84.5780023836996	2	4.30645721853696E-19
4	group_size * homeowner	18	-591809.754770109	254.078125363449	6	5.51210596856643E-52
5	group_size * married_couple	24	-591118.483588268	1382.54236368276	6	1.45970012121037E-295

Step	Effect Entered	# Free Parameter	Log-Likelihood	Deviance	Degrees of Freedom	Significance
6	homeowner * married_couple	26	-591105.493177193	25.980822149897	2	0.0000022821077850016

- d) (5 points) Calculate the Feature Importance Index as the negative base-10 logarithm of the significance value. List your indices by the model effects.

Effect Entered	Importance
Intercept	Not Applicable
group_size	209.361723410756
homeowner	Infinity
married_couple	18.3658798628204
group_size * homeowner	51.2586824418404
group_size * married_couple	294.835736355914
homeowner * married_couple	5.64166384750502

## Question 2 (25 points)

Please answer the following questions based on your multinomial logistic model in Question 1.

- a) (10 points) For each of the sixteen possible value combinations of the three features, calculate the predicted probabilities for insurance = 0, 1, 2 based on your multinomial logistic model. List your answers in a table with proper labeling.

group_size	homeOwner	Married_couple	Prob(insurance = 0)	Prob(insurance = 1)	Prob(insurance = 2)
1	0	0	0.257582126	0.591652732	0.150765142
1	0	1	0.328059586	0.510687364	0.16125305
1	1	0	0.180463562	0.686084615	0.133451823
1	1	1	0.217257163	0.628227986	0.154514851
2	0	0	0.279424514	0.550952763	0.169622723
2	0	1	0.203284285	0.647446276	0.149269438
2	1	0	0.249383261	0.597778464	0.152838276
2	1	1	0.161437112	0.701504087	0.137058801
3	0	0	0.237433786	0.654601188	0.107965026
3	0	1	0.240406278	0.597961267	0.161632455
3	1	0	0.282650629	0.603586178	0.113763193

3	1	1	0.260167227	0.562520929	0.177311843
4	0	0	0.304007517	0.595211185	0.100781298
4	0	1	0.193714219	0.673257043	0.133028738
4	1	0	0.505939166	0.406205722	0.087855112
4	1	1	0.332065587	0.531138818	0.136795594

- b) (5 points) Based on your answers in (a), what value combination of group\_size, homeowner, and married\_couple will maximize the odds value  $\text{Prob}(\text{insurance} = 1) / \text{Prob}(\text{insurance} = 0)$ ? What is that maximum odd value?

**Answer :**

**Value of Combination**

**group\_size     2**

**homeowner     1**

**married\_couple 1**

**Max Odd Value: 4.345370642504375**

- c) (5 points) Based on your model, what is the odds ratio for group\_size = 3 versus group\_size = 1, and insurance = 2 versus insurance = 0?

(Hint: The odds ratio is this odds ( $\text{Prob}(\text{insurance} = 2) / \text{Prob}(\text{insurance} = 0) \mid \text{group\_size} = 3$ ) divided by this odds ( $\text{Prob}(\text{insurance} = 2) / \text{Prob}(\text{insurance} = 0) \mid \text{group\_size} = 1$ ).)

**Answer :**

Odd Ratio:  $\frac{(\text{Prob}(\text{insurance} = 2) / \text{Prob}(\text{insurance} = 0) \mid \text{group\_size} = 3)}{(\text{Prob}(\text{insurance} = 2) / \text{Prob}(\text{insurance} = 0) \mid \text{group\_size} = 1)}$

Odds Ratios Group=3 Vs Group=1

```
In [105]: result = pandas.concat([gz_mc_ho, odds], axis=1)
...: print(result.iloc[:, 1:])
```

	homeOwner	Married_couple	Odds_ratio
0	0	0	0.776883
1	0	1	1.367816
2	1	0	0.544273
3	1	1	0.958273

- d) (5 points) Based on your model, what is the odds ratio for homeowner = 1 versus homeowner = 0, and insurance = 0 versus insurance = 1?

Answer :

Odd Ratio=

$\frac{\text{Prob}(\text{insurance} = 0) / \text{Prob}(\text{insurance} = 1) \mid \text{homeowner} = 1)}{\text{Prob}(\text{insurance} = 0) / \text{Prob}(\text{insurance} = 1) \mid \text{homeowner} = 0)}$

$\frac{\text{Prob}(\text{insurance} = 0) / \text{Prob}(\text{insurance} = 1) \mid \text{homeowner} = 1)}{\text{Prob}(\text{insurance} = 0) / \text{Prob}(\text{insurance} = 1) \mid \text{homeowner} = 0)}$

Odd Ratio : HomeOwner 1 Vs HomeOwner 0

```
In [129]: print(result1)
```

	group_size	homeOwner	Married_couple	0
0	1	1	0	1.655149
1	1	1	1	1.857551
2	2	1	0	1.215691
3	2	1	1	1.364353
4	3	1	0	0.774560
5	3	1	1	0.869278
6	4	1	0	0.410073
7	4	1	1	0.460219

### Question 3 (40 points)

You will build a Naïve Bayes model without any smoothing. In other words, the Laplace/Lidstone alpha is zero. Please answer the following questions based on your model.

- a) (5 points) Show in a table the frequency counts and the Class Probabilities of the target variable.

insurance	0	1	2
Frequency Count	<b>143691</b>	<b>426067</b>	<b>95491</b>
Class Probability	<b>0.215996</b>	<b>0.640462</b>	<b>0.143542</b>

- b) (5 points) Show the crosstabulation table of the target variable by the feature group\_size. The table contains the frequency counts.

group_size	insurance		
	0	1	2
1	<b>115460</b>	<b>329552</b>	<b>74293</b>
2	<b>25728</b>	<b>91065</b>	<b>19600</b>
3	<b>2282</b>	<b>5069</b>	<b>1505</b>
4	<b>221</b>	<b>381</b>	<b>93</b>

- c) (5 points) Show the crosstabulation table of the target variable by the feature homeowner. The table contains the frequency counts.

homeowner	insurance		
	0	1	2

0	<b>78659</b>	<b>183130</b>	<b>46734</b>
1	<b>65032</b>	<b>242937</b>	<b>48757</b>

- d) (5 points) Show the crosstabulation table of the target variable by the feature married\_couple. The table contains the frequency counts.

Married_couple	insurance		
	0	1	2
0	<b>117110</b>	<b>333272</b>	<b>75310</b>
1	<b>26581</b>	<b>92795</b>	<b>20181</b>

- e) (5 points) Calculate the Cramer's V statistics for the above three crosstabulations tables. Based on these Cramer's V statistics, which feature has the largest association with the target insurance?

Ans:

Cramer's V statistics:

```
Cramers statistics for group_size 0.027102014055820786
Cramers statistics for homeowner 0.09708641964781962
Cramers statistics for married_couple 0.03242164583520746
the largest association value 0.09708641964781962
```

Homeowner has the highest association with target insurance.

- f) (10 points) For each of the sixteen possible value combinations of the three features, calculate the predicted probabilities for insurance = 0, 1, 2 based on the Naïve Bayes model. List your answers in a table with proper labeling.

group_size	homeowner	married_couple	Prob(insurance = 0)	Prob(insurance = 1)	Prob(insurance = 2)
1	0	0	0.269722	0.580133	0.150145
1	0	1	0.232789	0.614219	0.152992
1	1	0	0.194038	0.669659	0.136303
1	1	1	0.164935	0.698278	0.136787
2	0	0	0.231143	0.616518	0.152338
2	0	1	0.198016	0.647907	0.154078
2	1	0	0.163628	0.700288	0.136085
2	1	1	0.138274	0.725955	0.135771
3	0	0	0.308219	0.515924	0.175856
3	0	1	0.268311	0.550951	0.180738
3	1	0	0.226972	0.609612	0.163416
3	1	1	0.19437	0.64041	0.165221

group_size	homeowner	married_couple	Prob(insurance = 0)	Prob(insurance = 1)	Prob(insurance = 2)
4	0	0	0.37549	0.48781	0.1367
4	0	1	0.330743	0.527098	0.142158
4	1	0	0.282173	0.588196	0.129631
4	1	1	0.24393	0.623766	0.132304

- g) (5 points) Based on your model, what value combination of group\_size, homeowner, and married\_couple will maximize the odds value  $\text{Prob}(\text{insurance} = 1) / \text{Prob}(\text{insurance} = 0)$ ? What is that maximum odd value?

Ans: The value of combination of group size, homeowner and married couple is (2, 1, 1).

The maximum odd value = 5.250112589270714