

# Uncertain multi-objective optimal model of oilfield development planning and its algorithm

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**Abstract** In this paper, we discuss the formulation of the oilfield development plan in case of significant nondeterminacy in oilfield development. And the plan needs to ensure production and achieve minimum cost and maximum new recoverable reserves. The uncertain factors in oilfield development are analyzed in this paper, and we consider the uncertain nature of the stimulation effect of measures and new recoverable reserves per well and quantify them. On this basis, an uncertain multi-objective optimal model of oilfield development planning is constructed. The model aims to minimize the expectation of development cost and maximize the expectation of new recoverable reserves, and optimizes the workload of each measure under the constraints including the oil production and the resources limitation. Based on uncertainty theory, the model is transformed into a deterministic model. And a nondominated sorting genetic algorithm with elite strategy is developed to solve the model and get the Pareto solution set. Then the multi-attribute decision-making is applied to select the multiple development plans, which provides the basis for the decision-making of the oilfield development plan. Finally, a numerical example is given to verify the effectiveness of the model and algorithm, and their practical application values under the background of oilfield development planning.

**Keywords** Oilfield development planning · Uncertain programming · Uncertain multi-objective programming · Multi-objective genetic algorithm

## 1 Introduction

Oilfield development planning is a piece of work to guide the overall deployment of oilfield development. It determines the workload of each measure to achieve the optimization of enterprises' economic benefits, on the basis of fully grasping the status of oilfield resources and development potential before the planning period, according to the production goal and actual needs. The scientificity of planning directly affects the completion of the stable production task and the economic benefits. Therefore, before the development, the relevant managers of oilfield enterprises, experts and scholars need to combine their knowledge and use scientific methods to determine the best plan, so that the planning can improve the development effect and the oil recovery, achieve the oilfield enterprise production requirement with minimizing the cost of all aspects.

For decades, various kinds of mathematical programming methods were widely used in oilfield development, and were recognized by oil companies. Lee and Aronofsky (1958) applied linear programming to solve the finite number of homogeneous reservoirs' planning problem that is aimed at maximizing production benefit. The research started the exploratory research of oilfield development planning optimization model. After that, many scholars tried to apply mathematical programming methods to oilfield development. Dong et al. (2001) constructed a goal programming model according to the actual needs of the oilfield medium-to-long term development planning, and achieved the coordination of the planning objectives by setting priorities. Shakhisi-Niaei et al. (2014) considered the formulation of the plan from a strategic perspective, including the identification of oilfield development projects, scheduling, production planning and upstream transport planning, emphasized the long-term economic

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performance, and then established linear programming models to analyze development strategies. Zhang et al. (2014) considered the problem of too large workload in the traditional arrangement of oilfield development. So this article established a multi-objective optimization model and designed quantum hybrid leapfrog algorithm to solve the model. Tavallali and Karimi (2016) developed a non-convex and multiperiod mixed-integer nonlinear programming for locating production wells, installing and connecting manifolds, planning capacity expansions of surface processing facilities, scheduling all of these activities, and determining the best oil production and water injection flows.

The specific objectives for oilfield development are generally multifaceted. The objectives considered in the previous studies can be divided into two types, namely, the goal of stabilizing oil production and the economic objective of reducing the cost and increasing the return. Zhang et al. (2001) divided the oil and gas production into maintenance production, measures production and new capacity production according to different measures, and the oil and gas field development production planning model was established. Hou et al. (2015) combined with the characteristics of medium-to-long term development planning, aiming at maximizing benefit, minimizing investment and minimizing cost, established the optimization model of new composition and oilfield benefit to realize all-in-one optimization. Li et al. (2016) used a mixed integer nonlinear programming model to optimize the production time and capacity scale of the gas field and maximize the overall economic benefits of multiple gas fields under the product-sharing contract model.

Since mathematical optimization methods were applied to solve the oilfield development planning problem, the application of the research results has obtained a very good actual effect. And the nondeterminacy in oilfield development were mainly analyzed based on stochastic programming theory and fuzzy theory in the current research. Gupta and Grossmann (2014) considered the influence of endogenous nondeterminacy and financial policies on development planning, established a multi-stage mixed integer nonlinear programming model based on stochastic programming to maximize the net present value. In order to solve the nondeterminacy of indexes in the later stage of oilfield development, Fang et al. (2015) determined the probability distribution of influencing factors by historical production data, and then the probability distribution of indexes was predicted by the quantitative relationship between indexes and influencing factors. As for the application of fuzzy theory, Chen and Ji (2009) established a fuzzy goal programming model which took into account the fuzziness of stimulation measures, and combined TOPSIS and genetic

algorithm to solve the model. Chen et al. (2009) built a multi-objective fuzzy programming model due to the fuzziness of oilfield development planning parameters, then applied the AHP method to determine the weight, and combined the fuzzy simulation and genetic algorithm to solve the model. Hu (2012) considered the fuzziness of parameters or variables in constraints and objective functions to study the oilfield production optimization under the fuzzy environment.

Oilfield development contains uncertain factors in geological, technical and economic aspects that can affect its process, resulting in the nondeterminacy of yield, cost and new recoverable reserves. In the oilfield development planning, these uncertain factors need to be quantified. However, the object of oilfield development is deep underground and the properties of reservoirs are controlled by natural conditions, so we cannot obtain experimental data to estimate the probability distribution of these factors. And the fuzzy theory does not have self-duality, which leads to the existence of a scheme in which the probability of success or failure is 100%. As for the nondeterminacy problem in oilfield development, the application of the mathematical branch of uncertainty theory created by Liu (2007) based on the four axioms of normality, duality, subadditivity and product measure can not only make the model more realistic, but also make the decision result more scientific and credible. At present, uncertainty theory has a good development in uncertain programming (Liu 2009b), uncertain risk analysis (Liu 2010a), uncertain statistics (Liu 2010b), uncertain process (Liu 2008, 2014) and uncertain differential equation (Liu 2008; Yao and Chen 2013). Also, uncertain theory has been applied to many areas such as economics (Yang and Gao 2016, 2017), management (Gao 2013; Gao et al. 2016) and finance (Chen and Gao 2013). In the application of uncertain programming, many models are built to solve practical problems, such as vehicle routing problem (Jiao 2015), machine scheduling problem (Li and Liu 2017) and so on. Liu and Chen (2015) provided uncertain multi-objective programming and explored some mathematical properties. Besides, in this paper uncertain goal programming was also introduced. And Chen et al. (2017) constructed expected value goal programming models and proposed a chance-constrained goal programming model for the bicriteria solid transportation problem. However, the application of uncertainty theory in oilfield development needs to be further analyzed to solve the practical problem.

In this paper, we study mathematical optimization of oilfield development planning based on uncertainty theory. The uncertain factors involved in the oilfield development are analyzed comprehensively, and an uncertain multi-objective optimal model is constructed. In the given numerical example, the multi-attribute decision-making is

used after obtaining the Pareto solution set, which lays the foundation for the development planning.

This paper proceeds as follows. In Sect. 2, some preliminaries of uncertainty theory and the problem description are clarified. A multi-objective uncertain optimal model of oilfield development planning is formulated, and the model is transformed into a deterministic model based on uncertainty theory in Sect. 3. In Sect. 4, a nondominated sorting genetic algorithm with elite strategy is developed to obtain the Pareto solution set, and then the multi-attribute decision-making is used to realize the optimization of multiple plans. A numerical example is given to illustrate the effectiveness of proposed model and solution algorithm in Sect. 5. Section 6 concludes the paper.

## 2 Preliminaries

### 2.1 Uncertainty theory

**Definition 1** (Liu 2007) Let  $\Gamma$  be a nonempty set, and  $\mathcal{L}$  be a  $\sigma$ -algebra over  $\Gamma$ . Every element  $\Lambda$  in  $\mathcal{L}$  is called an event. If a set function  $\mathcal{M}: \mathcal{L} \rightarrow \mathfrak{R}$  satisfies the following conditions:

**Axiom 1** (Normality Axiom)  $\mathcal{M}\{\Gamma\} = 1$  for the universal set  $\Gamma$ .

**Axiom 2** (Duality Axiom)  $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$  for any event  $\Lambda$ .

**Axiom 3** (Subadditivity Axiom) For every countable sequence of  $\Lambda_1, \Lambda_2, \dots$ , we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

We call  $\mathcal{M}$  an uncertain measure, and  $(\Gamma, \mathcal{L}, \mathcal{M})$  is called an uncertainty space.

In order to study the uncertainty measure of product space, Liu (2009a) defined the product uncertain measure.

**Axiom 4** (Product Axiom) Let  $(\Lambda_k, \mathcal{L}_k, \mathcal{M}_k), k = 1, 2, \dots$  be uncertainty spaces, the product measure  $\mathcal{M}$  is an measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where  $\Lambda_k$  are arbitrarily chosen events from  $\mathcal{L}_k$  for  $k = 1, 2, \dots$ , respectively.

**Definition 2** (Liu 2007) An uncertain variable is a function  $\xi$  from an uncertainty space  $(\Gamma, \mathcal{L}, \mathcal{M})$  to the set of real numbers such that

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$$

is an event for any Borel set  $B$  of real numbers.

**Definition 3** (Liu 2007) The uncertainty distribution of an uncertain variable  $\xi$  is defined by

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}, \forall x \in \mathfrak{R}.$$

For example, let  $\xi$  be an uncertain variable, if  $\xi$  has an uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ (x-a)/(b-a), & \text{if } a \leq x \leq b \\ 1, & \text{if } x \geq b \end{cases}$$

we call  $\xi$  a linear uncertain variable, denoted by  $\mathcal{L}(a, b)$ , where  $a$  and  $b$  are real numbers with  $a < b$ .

**Definition 4** (Liu 2010b) If the inverse function  $\Phi^{-1}(\alpha)$  exists and is unique in the interval  $\alpha \in (0, 1)$  for any uncertain variable  $\xi$ , we call  $\Phi(x)$  is regular. Then the inverse  $\Phi^{-1}(\alpha)$  is called the inverse uncertainty distribution of  $\xi$ .

**Theorem 1** (Liu 2010b) Let  $\xi_1, \xi_2, \dots, \xi_n$  be independent uncertain variables with regular uncertainty distributions  $\Phi_1, \Phi_2, \dots, \Phi_n$ . If  $f$  is strictly increasing with respect to  $\xi_1, \xi_2, \dots, \xi_n$ , then  $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$  is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1} = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \dots, \Phi_n^{-1}(\alpha)).$$

**Definition 5** (Liu 2007) Let  $\xi$  be an uncertain variable. If at least one of the following two integrals is finite, then the expected value of  $\xi$  is

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\} dr - \int_{-\infty}^0 \mathcal{M}\{\xi \leq r\} dr.$$

**Theorem 2** (Liu 2010b) Let  $\xi$  be an uncertain variable with regular uncertainty distribution  $\Phi$ . Then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha.$$

### 2.2 Problem description

Oilfield development requires a lot of expertise and involves many steps, with a variety of salient features. First of all, oilfield development is a long-time project. The process

can last for decades or even a hundred years. Therefore, in order to maintain oil production, people need to take effective means to improve oil recovery. Secondly, oilfield development is dynamic. The status changes constantly with time. In the process, it is necessary to plan rationally according to the actual situation and requirements. Thirdly, oilfield development is uncertain. On the one hand, the oil reservoir has the inherent nondeterminacy, such as the relationship between oil reserves, reservoirs and fluids. On the other hand, the process also includes the exogenous nondeterminacy, such as the limitation of oilfield development technology and human subjective cognition, and the change of state policies of oil enterprises (Li et al. 2003). Fourthly, oilfield development has a variety of purposes. Oil production protection is one of the foci. At the same time, oil companies also need to consider development cost and revenue. At last, there are a lot of technical problems to be solved in the specific implementation process like the real-time drilling data communication and management (Almadani 2016).

As the number of years of oilfield development increases, oil production will decrease if we only rely on natural production. In order to ensure oil production to stabilize at a certain level in the planning period, people need to take a variety of stimulation measures, such as fracturing, acidizing and mining new wells to improve production. Through the optimization of various measures, oil production goal can be completed successfully on the basis of saving resources. Therefore, the oilfield development planning needs to consider how many wells should be developed and how many old wells to be taken what measures to achieve the yield stability and the best economic benefit under the workload constraint.

In the oilfield development process, many uncertain factors have an impact on oil production and development cost. Combined with the existing research results and the experts' experience, we take the importance and operability of indicators as the evaluation basis and choose two uncertain factors including the stimulation effect of measures and new recoverable reserves per well. The stimulation effect of measures is the increment of oil production by implementing stimulation measures. Changes in geological conditions, technical conditions and the year of implementation will lead to its nondeterminacy. The new recoverable reserves per well refer to the difference between the recoverable reserves which can be recovered at the current technological level in the two adjacent planning periods, including cumulative proven or unproven resources. And that is related to the sustainable development of oilfield.

Based on the characteristics of the oilfield development, the optimization objectives should take into account the economic benefits and sustainable development. First we minimize the development cost. This is because companies

tend to control it as much as possible to achieve the best economic benefit. In addition, China's oil companies consider achieving the long-term stability of oil production. The new recoverable reserves is a resource index for the sustainable development. The larger the new recoverable reserves, the better the yield. So, the oil companies should maximum the new recoverable reserves. Taking into account the stability goal of production and capacity limitation, we also constrain the oil production and workload in the formulation of development plans.

### 3 Uncertain multi-objective optimal model

As for uncertain factors in oilfield development, we quantify the stimulation effect of measures and new recoverable reserves per well to obtain their uncertainty distributions. The cost optimization goal is converted into the minimization of the expectation of development cost, and the new recoverable reserves optimization goal is converted into the maximization of the expectation of new recoverable reserves. For the oil production requirement, we cannot fully guarantee to achieve the goal because of the existence of nondeterminacy. In this paper, we convert the production constraint into that the uncertainty measure of the oil production goal reaches a certain level. And the workload limitation is constrained directly. Then an uncertain multi-objective annual programming model is constructed to get a reasonable development plan.

#### 3.1 Quantification of uncertain factors

The difficulty of development planning increases significantly because of the uncertain nature of oilfield development. Before building an oilfield development planning model, we need to quantify the two uncertain factors, the stimulation effect of measures and new recoverable reserves per well. According to the experts' experimental data, we obtain the uncertainty distribution of corresponding uncertain variables by Delphi method.

Firstly, we ask the expert to choose a possible value  $x$  that the uncertain variable  $\xi$  represents the stimulation effect of measures or new recoverable reserves per well may take. Then we get the belief degree  $\alpha$  by further asking the expert "How likely is  $\xi$  less than or equal to  $x$ ?". After repeating the above process for multiple times, we can then obtain the following experimental data  $(x_1, \alpha_1), (x_2, \alpha_2), \dots, (x_n, \alpha_n)$ . According to this data, we can use the empirical uncertainty distribution to describe the distribution of uncertain variables. If the data satisfies  $x_1 < x_2 < \dots < x_n$  and  $0 \leq \alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_n \leq 1$ , we can get the empirical uncertainty distribution as follows,

$$\Phi(x) = \begin{cases} 0, & \text{if } x < x_1 \\ \alpha_i + \frac{(\alpha_{i+1} - \alpha_i)(x - x_i)}{x_{i+1} - x_i}, & \text{if } x_i \leq x \leq x_{i+1}, \\ 1, & \text{if } x > x_n. \end{cases} \quad 1 \leq i < n$$

When there are  $m$  experts, we obtain  $m$  different uncertainty distributions  $\Phi_1(x), \Phi_2(x), \dots, \Phi_m(x)$  from their experimental data  $(x_{ij}, \alpha_{ij}), i = 1, 2, \dots, m, j = 1, 2, \dots, n$ . And these distributions are integrated together by weighting to obtain

$$\Phi(x) = \omega_1 \Phi_1(x) + \omega_2 \Phi_2(x) + \dots + \omega_m \Phi_m(x),$$

where  $\omega_1, \omega_2, \dots, \omega_m$  are nonnegative and the sum is 1.

On this basis, if  $|\alpha_{ij} - \Phi(x_{ij})|$  is not less than a given positive number  $\varepsilon$ , the synthesis result is fed back to experts so as to update their opinions according to the feedback information and provide experimental data in the next round until  $|\alpha_{ij} - \Phi(x_{ij})|$  is smaller than  $\varepsilon$ , then the  $\Phi(x)$  is the final uncertainty distribution.

Because of the large number of uncertain variables corresponding to uncertain factors in practical applications, it is possible to consider obtaining the linear empirical uncertainty distribution only using the  $x$  value corresponding to the belief degree 0 and 1 given by the expert, and then get the comprehensive uncertainty distribution. For example, for the stimulation effect of a measure per well, assuming that three experts estimate it, the first expert think that it is not less than 300 tons and not more than 370 tons. Then the uncertainty distribution corresponding to this expert's experimental data is

$$\Phi_1(x) = \begin{cases} 0, & \text{if } x < 300 \\ (x - 300)/70, & \text{if } 300 \leq x \leq 370 \\ 1, & \text{if } x > 370. \end{cases}$$

Meanwhile, the second expert believes that the effect is not less than 290 tons and not more than 360 tons, and the third expert believes that it is not less than 290 tons, not more than 370 tons. In addition, assuming that three experts' opinions are equally important, that is, the weights are all 1/3, then we can get the uncertainty distribution of the variable,

$$\Phi(x) = \begin{cases} 0, & \text{if } x < 290 \\ (x - 290)/112, & \text{if } 290 \leq x < 300 \\ (23x - 6750)/1680, & \text{if } 300 \leq x \leq 360 \\ (x - 258)/112, & \text{if } 360 < x \leq 370 \\ 1, & \text{if } x > 370. \end{cases}$$

Assume the judgment threshold value  $\varepsilon = 0.1$ . For  $x = 350$ , the belief degree given by the first expert is 5/7, while its value calculated using the composite uncertainty

distribution is about 0.773. The difference is about 0.059, less than the threshold 0.1. For any  $x$  value, we calculate the difference. If they are less than 0.1, then the distribution at this time is the uncertainty distribution of the uncertain variable.

### 3.2 Uncertain multi-objective annual optimal model of oilfield development planning

Oilfield development planning can be divided into annual planning and medium-to-long term planning. The annual plan is the implementation plan of short-term development. While a medium-to-long term plan pays attention to the sustainable development and the aftereffect of measures based on the annual plan. Therefore, this paper only establishes the annual optimal model of oilfield development planning. By the annual plan, oil companies can focus on short-term production and economic benefits, while laying the foundation for the medium-to-long term development planning.

The plan formulated by the optimal model is to determine the number of new wells to be mined and the number of old wells to be taken for each stimulation measure, that is, the workload of each measure. We denote the workload of measure  $i$  by  $x_i$  and mining new wells is also classified as a measure. Therefore,  $i = 1$  means mining new wells, while  $i = 2, 3, \dots, I$  means different old well stimulation measures.

Based on the above analysis, the oilfield development planning model needs to consider two optimization goals. And one is the cost and the other is the annual new recoverable reserves.

The oilfield development involves a wide variety of costs during the planning period, and we pool them into two aspects: cost related to oil and cost related to well. For a stimulation measure, the annual total cost related to oil is the product of the cost related to the oil per ton and the amount of oil yield. The annual total cost related to well is the product of the cost related to well per well and the number of wells taken. So the total development cost is the sum of both oil-related cost and well-related cost of each measure. The minimization of cost goal can be considered to be the minimization of the expectation of annual oilfield development cost, i.e.,

$$\min E \left( \sum_{i=1}^I (co_i r_i x_i + cw_i x_i) \right), \quad (1)$$

where  $co_i$  is the cost related to oil of measures  $i$  per ton (yuan/ton),  $cw_i$  is the cost related to well of measures  $i$  per well (yuan),  $r_i$  is the uncertain variable represents the stimulation effect of measure  $i$  per well (ton).

In oilfield development planning, oil companies consider ensuring the sustainable development so that they want to maximum the new recoverable reserves. Besides, in the development, there are new wells every year, and all of their recoverable reserves are the annual new recoverable reserves in that year. Therefore, in the annual planning, the annual new recoverable reserves should be maximized. Its maximization can be considered as the maximization of the expectation of the annual new recoverable reserves, which can be expressed as

$$\max E(s_1 x_1), \quad (2)$$

where  $s_1$  is the uncertain variable represents the annual new recoverable reserves per well (ton).

According to the actual situation of oilfield development, we should also consider the constraints of production and resource potential in the model.

Oil production constraint ensures that the belief degree of the implementation of oil production task is greater than the decision makers' requirement. In this model, this constraint is expressed as a chance constraint. That is, the uncertain measure represents that the actual yield is higher than the target yield is not lower than a certain standard. The target yield and standard are based on the production arrangement and decision makers' instructions. In the annual planning, the oil production can be divided into three parts: the natural production of old wells, the measures production of old wells and the production of new wells. And the production of new wells is regarded as a stimulation effect. Therefore, the sum of the natural production and measures production is the total production. Oil production constraint is given by

$$\mathcal{M}\left\{\sum_{i=1}^I r_i x_i + Q_L \geq Q_0\right\} \geq \alpha, \quad (3)$$

where  $\alpha$  is the required confidence level,  $Q_0$  is the annual oil production requirement (ton), and  $Q_L$  is the natural production of old wells in the absence of measures (ton).

Resource potential constraint is the limits of the workload within the planning year. The corresponding workload for each measure during the year is limited, and the workload should be kept relatively balanced and need to be in a reasonable range. So the resource potential constraint is expressed as

$$\underline{x}_i \leq x_i \leq \bar{x}_i,$$

where  $\underline{x}_i$  and  $\bar{x}_i$  express the upper and lower limit on measure  $i$ , respectively.

Thus, we formulate the uncertain multi-objective annual optimal model of oilfield development planning as follows,

$$\begin{cases} \min E(\sum_{i=1}^I (co_i r_i x_i + cw_i x_i)) \\ \max E(s_1 x_1) \\ \text{subject to:} \\ \mathcal{M}\{\sum_{i=1}^I r_i x_i + Q_L \geq Q_0\} \geq \alpha \\ \underline{x}_i \leq x_i \leq \bar{x}_i, \text{ integers.} \end{cases} \quad (4)$$

In the above model, the cost in the objectives and the oil production in the constraints increase monotonically with the increase of stimulation effect of measures, and the new recoverable reserves increase monotonically with the increase of new recoverable reserves of single well. According to Theorems 1 and 2, Eqs. (1) and (2) can be transformed into

$$\min \int_0^1 \left( \sum_{i=1}^I (co_i x_i \Phi_{r_i}^{-1}(\alpha) + cw_i x_i) \right) d\alpha$$

and

$$\max \int_0^1 x_1 \Phi_{s_1}^{-1}(\alpha) d\alpha.$$

In addition, Eq. (3) can be expressed as

$$\sum_{i=1}^I \Phi_{r_i}^{-1}(1 - \alpha) x_i + Q_L \geq Q_0.$$

Therefore, the uncertain multi-objective annual optimal model can be transformed into the following deterministic multi-objective optimal model,

$$\begin{cases} \min \int_0^1 (\sum_{i=1}^I (co_i x_i \Phi_{r_i}^{-1}(\alpha) + cw_i x_i)) d\alpha \\ \max \int_0^1 x_1 \Phi_{s_1}^{-1}(\alpha) d\alpha \\ \text{subject to:} \\ \sum_{i=1}^I \Phi_{r_i}^{-1}(1 - \alpha) x_i + Q_L \geq Q_0 \\ \underline{x}_i \leq x_i \leq \bar{x}_i, \text{ integers.} \end{cases} \quad (5)$$

## 4 Intelligent algorithm

### 4.1 Nondominated sorting genetic algorithm

To solve the multi-objective programming, the nondominated sorting genetic algorithm (NSGA) was put forward in 1993. The basic idea is classifying all individuals according to the level, and sorting them according to the dominance and nondominance relationship before performing the selection operator. Through translating the calculation of multi-objective function into virtual fitness calculation,



NSGA can obtain Pareto optimal solution set. It is better than the direct combination of the genetic algorithm and the multiple attribute decision, which can only get one optimal solution, for example, obtaining feasible solutions using the genetic algorithm and sorting multiple objectives by TOPSIS (Sattarpour et al. 2016). After that, Deb et al. (2000) proposed an improved algorithm NSGAII on the basis of NSGA. NSGAII introduces the fast nondomination sorting algorithm, the elite strategy and the crowding degree comparison operator, reduces the computational

complexity of the algorithm effectively, and achieves the uniform expansion of the optimal solution in the Pareto front. The algorithm is more suitable for solving the multi-objective optimal model, and achieves very good results in the practical application. Aiming at the deterministic multi-objective optimal model of oilfield development planning, the specific algorithm flow chart is shown in Fig. 1.

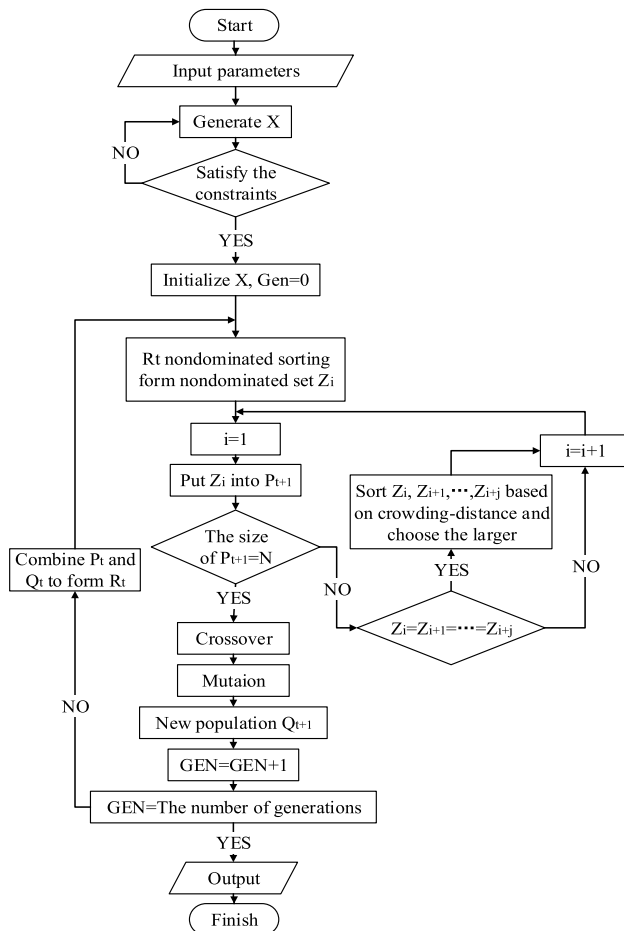
## 4.2 AHP

In order to give the relative weights of multiple indicators, we apply analytic hierarchy process (AHP), a widely used multi-criteria decision-making method developed by Saaty (1980). AHP is regarded as a decision hierarchy, typically incorporating a goal, criteria (objectives), and alternatives of choice. By converting evaluations to numerical values, ratio scale priorities derived from pairwise judgments is given for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. When the optimization model is applied to the oilfield development planning, only a small part in the huge index system can be taken into account usually. This paper focuses on three aspects: production, cost and new recoverable reserves. Outside of these goals, there are many indicators that affect the evaluation of development planning. Therefore, after getting the Pareto solution set of development plans using the optimal model, we can consider these indicators more comprehensively, so as to evaluate the merits of these plans. The steps applying AHP in this paper are described as follows.

*Step 1* In view of the analysis of the practical oilfield development planning, combined with experts' experience, we establish the hierarchy in Fig. 2.

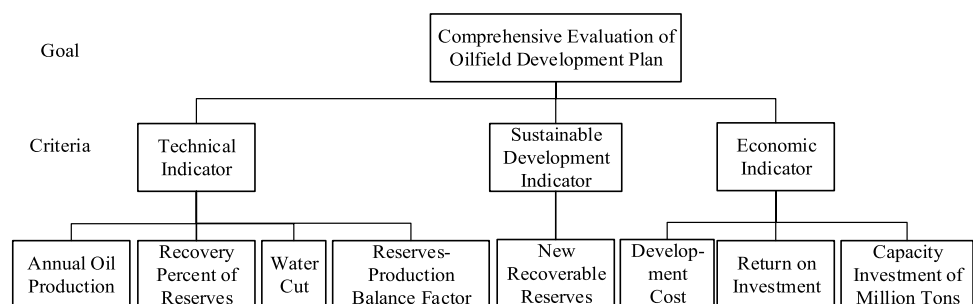
*Step 2* Establish priorities among the elements of the hierarchy by making judgments given by domain experts.

*Step 3* Check the consistency of the pairwise comparison matrix, and the corresponding eigenvector represents the relative weights of the evaluation indicators of the oilfield development plan.



**Fig. 1** NSGAII algorithm flow chart

**Fig. 2** AHP hierarchy of oil-field development



### 4.3 TOPSIS

Technique for order preference by similarity to ideal solution (TOPSIS) (Hwang and Yoon 1981) is a ranking method by computing the similarity to an ideal solution. The ideal solution is the best plan that can be hardly achieved in the real-life problem. Correspondingly, the negative-ideal solution is the assumed worst plan from all alternatives. By calculating the Euclidean distance between plan  $i$  and the ideal solution and negative-ideal solution,  $d_i^+$  and  $d_i^-$ , we can get the similarity between the plan and the ideal solution  $C_i = \frac{d_i^-}{d_i^- + d_i^+}$ . Then the chosen alternative should be most similar to the ideal solution.

### 4.4 Hybrid intelligent algorithm

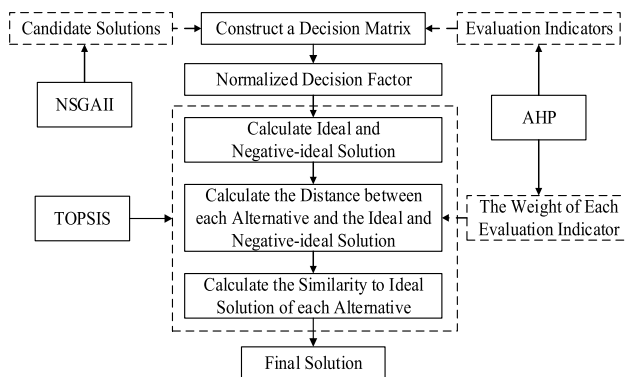
For the uncertain multi-objective optimal model of oilfield development, we design a hybrid intelligent algorithm, integrating nondominated sorting genetic algorithm, AHP and TOPSIS. First, The Pareto solution set is obtained by NSGAI in consideration of the conflict between the optimal cost and the optimal new recoverable reserves. Then, the decision matrix is constructed by using the  $m$  evaluation indicators in AHP and the  $n$  candidate solutions. And the evaluation indicators are divided into the benefit type and the cost type according to their influences. The bigger

benefit-type indicators the better, and the smaller cost-type indicators the better. For cost-type indicators, they can be transformed into benefit-type indicators by adding a negative sign to the corresponding parameter, so that the evaluation trend of each indicator is consistent. In order to solve the problem caused by different dimensions, the data are normalized. Then, for the  $n$  solutions, the best value corresponding to each indicator constitutes an ideal solution, and the worst value constitutes a negative-ideal solution. Combined the weight of each indicator obtained by AHP, the similarity between each alternative and the ideal solution is calculated using TOPSIS to achieve multi-attribute decision-making of oilfield development planning. The flow chart of the hybrid intelligent algorithm is shown in Fig. 3.

## 5 Numerical example

An oil company Z is ready to develop its oilfield D. And Z needs to make the short-term development plan every year. Taking into account the economic benefits and sustainable development, Z minimizes the cost and maximizes the new recoverable reserves in the development of oilfield D. In a given year, the annual oil production target is 20 million tons, the natural production of old wells is 18.8 million tons, and the confidence level of the actual production reaching the target yield is 0.9. The range of workload of each measure is given. And the specific measures of old wells include fracturing, acidizing and perforation adding. Therefore, decision variables of the optimal model,  $x_i$ ,  $i = 1, 2, 3, 4$ , are the number of wells taken mining new well, fracturing, acidizing and perforation adding. Based on the actual situation, assume that uncertain variables  $r_i$  and  $s_1$  all obey the given linear uncertainty distribution. Table 1 shows the specific data required for oilfield development planning.

We formulate this problem as model (5) and solve it by the nondominated sorting genetic algorithm NSGAI. This algorithm is run with parameters as follows. *pop\_size* is 20, the number of generation is 1000, the probability of crossover is 0.8, the probability of mutation is 0.15, the initial random seed is 0.84. The Pareto solution set of the model is



**Fig. 3** Flow chart of the hybrid intelligent algorithm

**Table 1** Basic data for oilfield development planning

Stimulation measure	Lower limit of workload	Upper limit of workload	Cost related to oil (yuan/ton)	Cost related to well (yuan/well)	Stimulation effect of measures per well	New recoverable reserves per well
New wells	900	1500	620	187,700	$\mathcal{L}(550, 700)$	$\mathcal{L}(1800, 2200)$
Fracturing	900	1500	180	174,100	$\mathcal{L}(200, 370)$	/
Acidizing	600	900	270	79,300	$\mathcal{L}(140, 170)$	/
Perforation adding	150	260	170	62,400	$\mathcal{L}(140, 180)$	/



**Table 2** Solutions of oilfield development planning

Candidate solution	New wells	Fracturing	Acidizing	Perforation adding	Cost (10,000 RMB)	New recoverable reserves (ton)
1	1098	1398	862	191	108,080.57	2,196,000
2	1500	900	600	150	115,989.00	3,000,000
3	1490	900	600	150	115,413.80	2,980,000
4	1240	1277	600	150	109,870.68	2,480,000
5	1118	1382	857	150	108,427.99	2,236,000
6	1169	1310	810	150	109,104.43	2,338,000
7	1297	1173	600	150	110,711.56	2,594,000
8	1388	1005	600	150	112,007.96	2,776,000
9	1332	1108	600	150	111,201.16	2,664,000
10	1450	900	600	150	113,113.00	2,900,000

obtained by solving the algorithm with the application of C language. Some results are shown in Table 2.

We can see that the upper limit of workload can be sufficient to meet the actual workload of each measure. So it is fully achievable that the confidence level of the oil production target reaches 0.9 in the current allocation of resources. What's more, the expected value of cost fluctuates in the range of about 1.08 billion to 1.16 billion yuan, and the cost per ton fluctuates in the range of about 900–940 yuan. The expectation of the maximum new recoverable reserves

fluctuates between 2.19 million and 3 million tons, indicating that the oilfield can continue to develop in the long term.

For using multi-attribute decision-making, we investigate domain experts to complete the relative importance evaluation of indicators and get the results in Table 3.

After calculation, we can get the weight of eight specific indicators: 0.315607, 0.085948, 0.160370, 0.075834, 0.104506, 0.040278, 0.047596, 0.169860. The data for candidate solutions on the corresponding indicators are given in Table 4.

**Table 3** Candidate solution evaluation indicators survey results

Criteria	Technical indicator		Sustainable development indicator		Eco- nomic indicator
Technical indicator	1		5		3
Sustainable development indicator	1/5		1		1/3
Economic indicator	1/3		3		1
Technical indicator	Annual oil production	Recovery percent of reserves	Water cut	Reserves-pro- duction balance factor	
Annual oil production	1	3	2	5	
Recovery percent of reserves	1/3	1	1/2	1	
Water cut	1/2	2	1	2	
Reserves-production balance factor	1/5	1	1/2	1	
Sustainable development indicator				New recoverable reserves	
New recoverable reserves				1	
Economic indicator	Development cost		Return on investment		Capacity invest- ment of million tons
Development cost	1		1		1/5
Return on investment	1		1		1/3
Capacity investment of million tons	5		3		1

**Table 4** Oilfield development planning solution selection data

Solution	Annual oil production	Recovery percent of reserves	Water cut	Reserves-production balance factor	New recoverable reserves	Cost	Return on investment	Capacity investment of million tons
1	20,118,750	50%	22%	0.53	2,196,000	108,080.57	50%	49
2	20,156,000	46%	25%	0.5	3,000,000	115,989.00	52%	47
3	20,149,750	39%	21%	0.48	2,980,000	115,413.80	49%	45
4	20,119,795	40%	28%	0.56	2,480,000	109,870.68	50%	50
5	20,118,555	42%	20%	0.58	2,236,000	108,427.99	52%	48
6	20,119,025	37%	29%	0.46	2,338,000	109,104.43	47%	53
7	20,120,580	30%	23%	0.57	2,594,000	110,711.56	51%	52
8	20,121,175	35%	30%	0.59	2,776,000	112,007.96	56%	46
9	20,120,680	33%	24%	0.48	2,664,000	111,201.16	53%	48
10	20,124,750	44%	28%	0.49	2,900,000	113,113.00	48%	49
Type <sup>1</sup>	0	1	1	0	0	1	0	1

<sup>1</sup>0 Benefit type, 1 cost type

And then using TOPSIS, we get the best plan is solution 3 which closest to the ideal solution, and the remaining plans are ranked in the order of 7, 5, 9, 2, 1, 8, 10, 6, 4.

## 6 Conclusions

Oilfield development planning is an important part of making plans and deploying mining tasks for oil companies. It is necessary to make scientific arrangements and handle the uncertain and multi-objective natures of oilfield development. For oil companies, oilfield development planning is related to the implementation of various operational work and the long-term development. In oilfield development, the impact of nondeterminacy on the final decision is unavoidable. So they should be fully considered in decision making. We mainly consider the influence of two uncertain factors on oilfield development and quantify them. Then, an uncertain multi-objective annual optimal model of oilfield development planning is constructed to determine the optimal workload arrangement. And the multi-attribute decision-making is used to obtain the most suitable development plan under comprehensive evaluation indicators. The numerical example shows that the model constructed in this paper has practical application value and the algorithm is also effective.

In future studies, more uncertain factors and the oil block can be considered. In addition, it can be considered that the medium-to-long term planning is divided into several interrelated stages, and the workload is arranged at each stage to achieve the optimal goal of the whole planning period.

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