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Portfolio selection model of oil projects under uncertain environment

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Abstract This paper discusses the oil project optimal portfolio selection under uncertain environment where cash flows of the projects are mostly determined by experts' estimations due to the lack of historical investment data. The oil project investment is usually distinguished by its high input, high risk and highly fluctuating ROI sensitive to the economic, political and technology uncertainties. Besides, in most of the cases, it is quite difficult to find reliable referential historical data for a specific project. All these peculiarities make actual oil project investment decision under high uncertainties. In this paper, we use normal uncertain variables to describe the cash flows and estimate the uncertainty distribution of the cash flows by experts' experimental data. Then, under the constraint of controlling for bankruptcy, we give uncertain programming models to construct portfolios that maximize the expected returns and minimize the sine cross-entropy of the actual return from a prior return. Finally, we provide some numerical examples that fit different risk preference assumptions to further illustrate the feasibility and effectiveness of the models.

Keywords Uncertain measure · Bankruptcy risk · Project portfolio · Uncertain programming

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1 Introduction

Portfolio selection refers to finding a reasonable investment plan that would give a good performance in the trade-off between return and risk. Quantitative researches concerning this topic mainly discuss the investment decision given the requirements for return and risk in the next periods. Markowitz (1952) proposed a mean-variance portfolio model in which the return and risk were measured by expected value and variance, respectively. This paper assumed that investors would choose the portfolio with the lowest risk under the income constraints or the portfolio with the highest return under the risk constraints and got to the highly practical conclusion that investors should choose fully diversified portfolios. After that, many scholars have improved the use of risk indicators in this model, and many of the results have been widely accepted and used. Konno and Yamazaki (1991) proposed a mean-absolute deviation model, which greatly reduced the computational burden and at the same time preserved the good properties of the mean-variance model. Speranza (1993) proposed the use of semi-absolute deviations to measure risk since most investors are more concerned about the tail losses in the asymmetric return distributions. This article did this by simply calculating risk measures for the lower-than-expected returns. Roy (1952) found an optimal solution to the selection under the safety-first criterion by minimizing the probability for the portfolio return to drop below a certain level. Kapur and Kesavan (1992) established a minimization cross-entropy model for minimizing the deviations of portfolio yield and prior yield.

The above models have been proved to be effective in financial portfolio investments. Another kind of important investment in the modern economy is the investment in physical assets such as equipment and plants, which take a great role in the profiting of enterprises. Besides, the highly developed financial investment service has also greatly promoted project investments. In the case of valuing single investment projects, due to the varying age and investment amount of these projects, the existing studies are mostly based on the discounted cash flow (DCF) method to calculate, evaluate and compare return indicators such as net present value, internal rate of return and present value index, which are related to future cash flows and discount rates. However, Zhang (1998) pointed out that these indicators are inconsistent when evaluating for multiple projects. This paper then constructed a new indicator based on the present value index. As for the project portfolio investment, quantitative researches concerning this topic are about the prior decision making that mainly considers project portfolio return and a variety of resource requirements and constraints. Ghasemzadeh et al. (1999) proposed a 0–1 integer linear programming model to select and arrange the optimal project portfolio based on organizational goals and constraints such as resource constraints and interdependencies between projects. Sefair and Medaglia (2005) got the maximization of net present value, the minimization of variance and the optimal investment time of selected projects through a mixed integer programming model. Belenky (2012) considered the reinvestment strategy and got the best project portfolio under constrained budgets. Paquin et al. (2016) studied the conditions for adding candidate projects to the firm's portfolio when considering operational risk. This paper concluded that when the correlation coefficient between a candidate project and a firm's economic activity took a negative value above a threshold, adding it to the portfolio would reduce the operational risk.

In project investments, it is quite different from things in financial market investments because people would have no historical data to estimate cash flows in the future. Besides, oil project investment is of great importance to the survival of enterprises and its belief degree of annual cash flow can only rely on experts' experience. In this case, fuzzy theory and credibility theory are used to deal with this problem. Christer et al. (2007) modeled fuzzy integer programming for portfolio selection. Solak et al. (2010) used a multistage stochastic integer programming model to solve portfolio optimization problems. Huang (2008) studied the mean-variance model and its optimization problem based on credibility measure. Qin et al. (2009) discussed Kapur cross-entropy minimization model. This paper proposed three mathematical models by defining risk as variance, semi-variance, and chance of bad outcome, respectively. But fuzzy theory does not have self-duality, which leads to the existence of a scheme in which the probability of success or failure is 100%. As for the uncertainty problem, Liu (2007) created the mathematical branch of uncertainty theory based on the four axioms of normality, duality, subadditivity and product measure. At present, uncertainty theory has a good development in uncertain programming (Liu 2009), uncertain risk analysis (Liu 2010a), uncertain statistics (Liu 2010b), uncertain process (Liu 2008, 2014) and uncertain differential equation (Liu 2008; Yao and Chen 2013). In the application of uncertain programming, many models are built to solve practical problems. Gao et al. (2015) discussed the characteristics of the diameter of the uncertain graph and proposed an algorithm to determine its distribution function. Ke et al. (2015) built an uncertain random project scheduling model and designed a hybrid intelligent algorithm, which integrated uncertain random simulation and genetic algorithm. Gao and Qin (2016a) deduced the theoretical formula to calculate the uncertain measure that an uncertain graph is k-edge-connected and proposed an algorithm for numerically calculating it. Gao and Qin (2016b) considered travel times as uncertain variables in the p-hub center problem, formulated a chance constrained programming model and a hybrid intelligent algorithm is designed.

Also, uncertain theory has been applied to many areas such as economics (Yang and Gao 2016, 2017), management (Gao 2013; Gao et al. 2016) and finance (Chen and Gao 2013). As for the application of uncertainty theory in investment, Qin et al. (2009) first introduced uncertainty theory into portfolio selection problem and developed the mean-variance model under uncertain environment. Subsequently, Liu and Qin (2012) defined uncertain semi-absolute deviation and proposed three forms of uncertain mean-semi-absolute deviation model. In project portfolio selection, Zhang et al. (2011) first applied uncertainty theory to solve a multinational project selection problem. After that, Zhang et al. (2015) proposed the profit risk index and cost over-run risk index. This article then developed the domestic project selection model with uncertain average risk index. Huang et al. (2016) discussed optimal project portfolio selection and scheduling using the mean-variance model and the mean-semi-variance model under the condition that the initial outlays and net cash flow of projects are given by expert's estimation. Huang and Zhao (2016) described the cost over-run risk beyond available budgets under uncertain environment and constructed an optimization model that considered both new project selections and existing adjustments.

In fact, compared with other types of investment, oil project investment is of great importance due to its peculiarities such as the requirement for fixed asset investment, a large amount of input, long influence time and poor degree of liquidity. Besides, the cash flows of these projects will come in the strongly uncertain future, and there are no comparable historical data for estimation. In this case, we can use uncertainty theory to deal with the evaluation and selection problems. Since projects cannot continue to operate when the firm goes into bankruptcy due to the fluctuations of future cash flows, investors should avoid bankruptcy and at the same time pursue the highest possible returns. In this paper, based on the uncertainty measure, we consider the impact of uncertainty, bankruptcy risk and the actual situation of investments to obtain the maximum expected returns on the project portfolios. We build two models and give some numerical examples to further demonstrate their effectiveness.

2 Preliminaries

Definition 1 (Liu 2007) Let Γ be a nonempty set, and $\mathcal{L}$ be a σ-algebra over Γ. Every element Λ in $\mathcal{L}$ is called an event. If a set function $\mathcal{M}\colon \mathcal{L} \to \Re$ satisfies the following conditions:

Axiom 1: (Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.
Axiom 2: (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.
Axiom 3: (Subadditivity Axiom) For every countable sequence of $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

we call $\mathcal{M}$ an uncertain measure, and $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

In order to study the uncertainty measure of product space, Liu (2009) defined the product uncertain measure.
Axiom 4: (Product Axiom) Let $(\Lambda_k, \mathcal{L}_k, \mathcal{M}_k), k = 1, 2, \ldots$ be uncertainty spaces, the product measure $\mathcal{M}$ is an measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots,$, respectively.

Definition 2 (Liu 2007) An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$$

is an event for any Borel set B of real numbers.

Definition 3 (Liu 2007) The uncertainty distribution of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}, \quad \forall x \in \Re.$$

For example, let ξ be an uncertain variable, if ξ has an uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \Re,$$

we call ξ a normal uncertain variable, denoted by $\mathcal{N}(e, \sigma)$ where e and σ are real numbers with $\sigma > 0$.

Definition 4 (Liu 2010b) If the inverse function $\Phi^{-1}(\alpha)$ exists and is unique in the interval $\alpha \in (0, 1)$ for any uncertain variable ξ, we call $\Phi(x)$ is regular. Then the inverse $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.

Theorem 1 *(Liu 2010b) Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$. If f is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_n$, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable with inverse uncertainty distribution*

$$\Psi^{-1} = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)).$$

Definition 5 (Liu 2007) Let ξ be an uncertain variable. If at least one of the following two integrals is finite, then the expected value of ξ is

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^{0} \mathcal{M}\{\xi \le r\}\mathrm{d}r.$$

Definition 6 (Liu 2009) Let ξ be an uncertain variable with finite expected value e. Then the variance of ξ is $V[\xi] = E[(\xi - e)^2]$.

Theorem 2 (Liu 2007) *Let ξ_1, ξ_2 be independent normal uncertain variables $\mathcal{N}(e_1, \sigma_1), \mathcal{N}(e_2, \sigma_2)$, respectively. Then the sum $\xi_1 + \xi_2$ is also a normal uncertain variable $\mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2)$, i.e.,*

$$\mathcal{N}(e_1, \sigma_1) + \mathcal{N}(e_2, \sigma_2) = \mathcal{N}(e_1 + e_2, \sigma_1 + \sigma_2).$$

The product of a normal uncertain variable $\mathcal{N}(e, \sigma)$ and a scalar number $k > 0$ is also a normal uncertain variable $\mathcal{N}(ke, k\sigma)$, i.e.,

$$k \cdot \mathcal{N}(e, \sigma) = \mathcal{N}(ke, k\sigma).$$

Definition 7 (Zhang 2015) If ξ and η are uncertain variables with uncertainty distributions $\Phi(x)$ and $\Psi(x)$, then the sine cross-entropy of ξ from η is

$$D[\xi; \eta] = \int_{-\infty}^{\infty} (\Phi(x) - \Psi(x)) \left(\sin\left(\frac{\pi\Phi(x)}{2}\right) - \sin\left(\frac{\pi\Psi(x)}{2}\right)\right) \mathrm{d}x.$$

3 Uncertain models of oil project selection

Oil companies have lots of investment opportunities and differentiated returns, and their projects often have the characteristics of high inputs and high risks. When selecting oil project portfolios, it is necessary to do analysis in terms of political, economic and technology uncertainties. Besides, it is also crucial to make sure that the decisions are in accordance with the company's business development plans and strategic objectives. Under all these constraints, we then select the most suitable portfolio within a reasonable investment scale. For a single oil project, we can first determine whether it is worth investing by judging from its future cash flows. We then can select portfolios within these investable projects to meet different objectives. In order to make the risk controllable after the investment, we will define the bankruptcy risk of the oil project portfolio in the way that investors with different risk preferences can effectively control it. We will then construct two models that would control the bankruptcy risk effectively and at the same time maximize the expected return and minimize the sine cross-entropy of the real returns from prior returns.

3.1 Return and risk assessments of oil project investment based on uncertain measure

Consider an oil project with an n-year investment payback term. The annual cash flow of the project is affected by the technology level, market demand, market supply and other uncertainties. We assume the cash flow in year t as a normal uncertain variable $\xi_t \sim \mathcal{N}(e_t, \sigma_t)$, $t = 1, 2, \ldots, n$, and uncertain variables can further be given by uncertainty statistics (Liu 2010b).

To obtain the uncertainty distribution of uncertain variables from experts' experimental data, we can use linear interpolation method, principle of least squares, method of moments or Delphi method, etc.

We ask the expert to choose a possible value x that the cash flow of a given year named uncertain variable ξ may take. Then we get the belief degree α by further asking the expert "How likely is ξ less than or equal to x?". After repeating the above process for multiple times, we can then obtain the following experimental data,

$$(x_1, \alpha_1), (x_2, \alpha_2), \ldots, (x_n, \alpha_n).$$

If the uncertainty distribution $\Phi(x \mid \theta)$ is determined by an unknown parameter θ, then based on the least squares method, we can use the above data to obtain the optimal solution $\hat{\theta}$ by minimizing the residuals,

$$\min_{\theta} \sum_{i=1}^{n} (\Phi(x \mid \theta) - \alpha_i)^2.$$

Based on the optimal parameter solution, we can then get the least squares uncertainty distribution $\Phi(x \mid \hat{\theta})$.

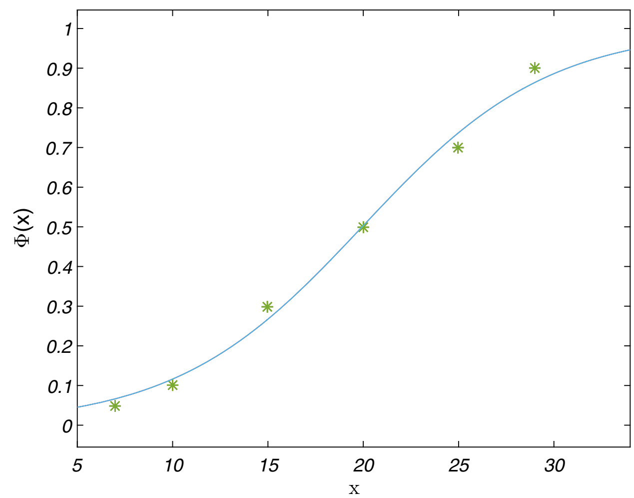

Fig. 1 Normal uncertainty distribution of the cash flow

Example 1 Assume that the cash flow for a project in a given year is a normal uncertain variable with two unknown parameters e and σ, i.e.,

$$\Phi(x \mid e, \sigma) = \left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}.$$

Also assume that we have obtained the following expert's experimental data,

$$(7, 0.05), (10, 0.1), (15, 0.3), (20, 0.5), (25, 0.7), (29, 0.9),$$

we then can obtain $e = 20$, $\sigma = 8.9$ by least squares estimation (Fig. 1). This also means the least squares uncertainty distribution function is given by

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(20 - x)}{8.9\sqrt{3}}\right)\right)^{-1}.$$

Assume that the discount rate of the project investment is a constant r and the initial outlay is I, then the present value index of this project is also a normal uncertain variable

$$PI = \frac{1}{I} \sum_{t=1}^{n} \frac{\xi_t}{(1 + r)^{-t}}.$$

According to Theorem 2, the mean and variance of the present value index of the project are given, respectively, by

$$E(PI) = \frac{1}{I} \sum_{t=1}^{n} \frac{e_t}{(1 + r)^{-t}},$$

$$V(PI) = \left(\frac{1}{I} \sum_{t=1}^{n} \frac{\sigma_t}{(1 + r)^{-t}}\right)^2.$$

The mean of the present value index that conforms to the normal uncertainty distribution can reflect the profit level of a unit investment. When the $E(PI)$ is less than or equal to 1, it means the present value of expected cash flows is less than or equal to the initial outlay and the project will not yield profits to investors. Thus, we should reject the project. We can then use the values of $E(PI)$ to determine whether we should incorporate the concerned projects into the portfolio. Besides, the variance $V(PI)$ of the uncertain variable present value index reflects the volatility of the excessive returns. The larger the variance, the greater the risk of investments.

The length of the payback period also plays a significant role in the evaluation of investments. For example, assume we have a 3-year project and a 6-year project. The return of the 6-year project is usually higher than that of the 3-year project. But after the end of the 3-year project, we can further continue to invest in a new similar project to obtain extra returns. Therefore, we should adjust the upper indicators with respect to payback period so that they are still effective when we are evaluating multiple projects with different investment payback periods. In this regard, we can calculate a new normal uncertain variable

$$R = PI \cdot (A/P, r, n),$$

which stands for the net present annuity value of the unit investment, where $(A/P, r, n) = r/[1 - (1+r)^{-n}]$ is the capital recovery factor. And we use its expected value $E(R)$ to evaluate the return on project investment. The indicator is equivalent to the annuity form of net present value for the unit project investment. This means the annual cash flows are regularly discounted to the end of each year. Using this indicator, we can then compare multiple projects with different payback period and scale. The greater the $E(R)$, the higher the profits.

3.2 Oil project portfolio selection model with uncertain bankruptcy risk constraint

Assume there are N feasible projects after the selection of a single investment selection, and the initial outlay of the feasible project i is M_i. Assume that we only carry out one-time investments, and the investment payback period is $T_i, i = 1, 2, \ldots, N$. Further assume that the annual cash flow of each project is independent normal uncertain variable denoted by $\xi_{t,i} \sim \mathcal{N}(e_{t,i}, \sigma_{t,i})$, where i stands for the project i and t stands for year t. The discount rate is denoted by constant r, and I is the total amount of investments. The uncertain present value index of the feasible project I is then given by

$$PI_i = \frac{1}{M_i} \sum_{t=1}^{T_i} \frac{\xi_{t,i}}{(1+r)^{-t}}. \tag{1}$$

The expectation is given by

$$E(PI_i) = \frac{1}{M_i} \sum_{t=1}^{T_i} \frac{e_{t,i}}{(1+r)^{-t}}, \tag{2}$$

which represents the present value of expected return on the unit investment of the project.

The selection for the feasible project i is denoted by $x_i = \begin{cases} 1, & \text{selected project} \\ 0, & \text{abandoned project} \end{cases}$, with the actual investment proportion being given by $\frac{x_i M_i}{\sum_{i=1}^N x_i M_i}$. Let the capital recovery coefficient of the feasible project i be $k_i = (A/P, r, T_i) = r/[1-(1+r)^{-T_i}]$. Then the net present value annuity of the unit investment value is $R_i = PI_i \cdot k_i$, and the expected value $E(R_i)$ can be used to estimate the mean returns.

Here we construct a return indicator that evaluates the project portfolio, i.e., the net present value annuity of the unit total investment

$$R = \sum_{i=1}^{N} \frac{x_i M_i R_i}{\sum_{i=1}^N x_i M_i}. \tag{3}$$

According to Theorem 1, its inverse uncertainty distribution is then given by

$$\Psi^{-1}(\alpha) = \sum_{i=1}^{N} \left(\frac{x_i k_i}{\sum_{i=1}^N (x_i M_i)} \sum_{t=1}^{T_i} \frac{e_{t,i} + \frac{\sigma_{t,i}\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}}{(1+r)^t} \right).$$

Its expectation is given by

$$E(R) = \sum_{i=1}^{N} \left(\frac{x_i k_i}{\sum_{i=1}^N x_i M_i} \sum_{t=1}^{T_i} \frac{e_{t,i}}{(1+r)^t} \right), \tag{4}$$

which represents the mean return of the portfolio.

Its variance is given by

$$V(R) = \left(\sum_{i=1}^{N} \left(\frac{x_i k_i}{\sum_{i=1}^N x_i M_i} \sum_{t=1}^{T_i} \frac{\sigma_{t,i}}{(1+r)^t} \right) \right)^2, \tag{5}$$

which is the fluctuation level of the net present value annuity of the unit total investment and represents the risk of the portfolio.

In oil project investments, it is really crucial that the return of the selected portfolio should reach a certain level to maintain stable cash flows and operations of the firm. Investors should control the uncertain measure for the occurrence of bankruptcy to stay within an acceptable range of $\alpha_0 \in (0, 1)$. Assume that the acceptable level of survival is $d > 0$, namely,

$$\mathcal{M}\{R \le d\} \le \alpha_0,$$

where α_0 and d are determined based on the investor's specific conditions. Investors can make adjustments to the values of α_0 and d depending on their own financial strength, risk preference or affordability of bankruptcy, etc. For example, a high α_0 and a low d will allow investors to pursue high-risk and high-yield projects. While by contrast, a low α_0 and a high d will basically restrict the investors from taking too much risk and let them pursue more stable portfolios.

Assume that an investor wants to control the bankruptcy risk in an acceptable range and restrict the total amount of investment not more than I. In addition, in order to make full use of capital, we assume the actual total investment cannot be less than Q. To get the maximum expected return on this project portfolio, we can therefore construct the following portfolio selection model that considers the previous stated constraints,

$$\begin{cases} \max\ E(R) \\ \text{subject to :} \\ \qquad \mathcal{M}\{R \le d\} \le \alpha_0 \\ \qquad Q \le \sum_{i=1}^{N} x_i M_i \le I \\ \qquad x_i = 0 \text{ or } 1, \quad i = 1, 2, \ldots, N. \end{cases} \tag{6}$$

According to Eq. (4), the objective function is $\sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} x_i M_i} \sum_{t=1}^{T_i} \frac{e_{t,i}}{(1+r)^t}\right)$. And the constraint of bankruptcy can also be converted into the form of inverse uncertainty distribution. Substituting these two terms into model (6), we then get

$$\begin{cases} \max \sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} x_i M_i} \sum_{t=1}^{T_i} \frac{e_{t,i}}{(1+r)^t}\right) \\ \text{subject to :} \\ \qquad \sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} (x_i M_i)} \sum_{t=1}^{T_i} \frac{e_{t,i} + \frac{\sigma_{t,i}\sqrt{3}}{\pi}\ln\frac{\alpha_0}{1-\alpha_0}}{(1+r)^t}\right) \ge d \\ \qquad Q \le \sum_{i=1}^{N} x_i M_i \le I \\ \qquad x_i = 0 \text{ or } 1, \quad i = 1, 2, \ldots, N, \end{cases} \tag{7}$$

which is equivalent to model (6).

We can easily observe from model (7) that after converting the bankruptcy risk constraint into the form of inverse uncertainty distribution, the chance constraint now given by

$$\sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} (x_i M_i)} \sum_{t=1}^{T_i} \frac{e_{t,i}}{(1+r)^t}\right) + \frac{\sqrt{3}}{\pi}\ln\frac{\alpha_0}{1-\alpha_0} \cdot \sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} (x_i M_i)} \sum_{t=1}^{T_i} \frac{\sigma_{t,i}}{(1+r)^t}\right) \ge d.$$

i.e., $E(R) + \frac{\sqrt{3}}{\pi}\ln\frac{\alpha_0}{1-\alpha_0} \cdot \sqrt{V(R)} \ge d$, which is an inequality that is closely related to $E(R)$, $V(R)$, α_0 and d. This shows that the chance constraint in this model considers not only the risk of the portfolio, but constraint for the expected return. Besides, decreasing the value of α_0 or increasing that of d can further reinforce the constraint on the project. It therefore guarantees a better performance than the pure variance constraint.

3.3 Oil project portfolio selection model by minimizing uncertain sine cross-entropy

In oil project investment, investors usually have their own ideal expected returns, which are based on predictions of future market demand and operation conditions. In order to make full use of the funds that come in the future, investors need to make the actual return and the expected prior return as close as possible. The uncertain sine cross-entropy can be used to represent the deviation between the actual and the prior return. Then, investors can achieve this goal by minimizing the sine cross-entropy of the two variables.

Denote an estimated prior return by an uncertain variable $\eta \sim \mathcal{N}(e_\text{p}, \sigma_\text{p})$ with a distribution of $\Upsilon(x)$ and the uncertain sine cross-entropy of the actual return and the prior return by $D[R; \eta]$. Make sure that the value of uncertain measure for the occurrence of bankruptcy event $R \le d$ is within the acceptable range $\alpha_0 \in (0, 1)$. We can then construct a project portfolio selection model that minimizes the deviation between actual return and the prior return,

$$\begin{cases} \min\ D[R; \eta] \\ \text{subject to :} \\ \qquad \mathcal{M}\{R \le d\} \le \alpha_0 \\ \qquad Q \le \sum_{i=1}^{N} x_i M_i \le I \\ \qquad x_i = 0 \text{ or } 1, \quad i = 1, 2, \ldots, N. \end{cases} \tag{8}$$

According to Sect. 3.2, the uncertainty distribution of the actual return R in the selected portfolio is given by,

$$\Psi(x) = \left(1+\exp\left(\frac{\pi\left(\sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} x_i M_i}\sum_{t=1}^{T_i}\frac{e_{t,i}}{(1+r)^t}\right)-\ln x\right)}{\sqrt{3}\left(\sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N} x_i M_i}\sum_{t=1}^{T_i}\frac{\sigma_{t,i}}{(1+r)^t}\right)\right)}\right)\right)^{-1},$$

and the uncertainty distribution of the prior return is $\Upsilon(x) = (1+\exp(\pi(e_\mathrm{p}-\ln x)/(\sqrt{3}\sigma_\mathrm{p})))^{-1}$. Substituting these two terms into model (8), we then get the following transformed form,

$$\begin{cases} \min \int_{-\infty}^{\infty}(\Psi(x)-\Upsilon(x))\left(\sin\left(\frac{\pi\Psi(x)}{2}\right)-\sin\left(\frac{\pi\Upsilon(x)}{2}\right)\right)\mathrm{d}x \\ \text{subject to :} \\ \quad \sum_{i=1}^{N}\left(\frac{x_i k_i}{\sum_{i=1}^{N}(x_i M_i)}\sum_{t=1}^{T_i}\frac{e_{t,i}+\frac{\sigma_{t,i}\sqrt{3}}{\pi}\ln\frac{\alpha_0}{1-\alpha_0}}{(1+r)^t}\right)\geq d \\ \quad Q\leq\sum_{i=1}^{N}x_i M_i\leq I \\ \quad x_i=0 \text{ or } 1,\quad i=1,2,\ldots,N. \end{cases} \tag{9}$$

4 Numerical examples

Assume that the investors have evaluated 12 projects A, B, C, D, E, F, G, H, I, J, K and L. And the investors have already estimated the uncertainties such as the length of the payback period, the market demand or the operation status, etc. They thus have got the following series of normal uncertain variables (Table 1), in which the first phase cash flow of the project D is the result in Example 1. In the last row of this table, we may also find the initial outlays of each project.

To use the model that we constructed before, we need to make additional assumptions. Here we assume the discount rate to be 10% and the actual total outlay to lie within the range of 6 million dollars to 10 million dollars. Based on all the assumptions, we can then step on the road to find the optimal solution to the selection models.

Based on the numbers in Table 1, we further calculated the values for the following indicators in Table 2, including the expected net present value of return, their variance and the annuity return of expected net present value of the unit outlay of each project.

We can then initially screen out the unacceptable projects based on the values of $E(PI)$. For those projects in which the values of $E(PI)$ are less than 1, we would drop them from our portfolio since they have a net present value lower than the initial outlay. Thus, our portfolio is now left with the remaining 9 projects: A, B, D, E, G, I, J, K and L. In addition,

Table 1 Uncertainty distribution of cash flows and initial outlays (unit: thousand dollar)

Period number	Project A	Project B	Project C	Project D	Project E	Project F
1	$\mathcal{N}(200, 48)$	$\mathcal{N}(170, 70)$	$\mathcal{N}(120, 45)$	$\mathcal{N}(200, 89)$	$\mathcal{N}(270, 83)$	$\mathcal{N}(100, 38)$
2	$\mathcal{N}(260, 98)$	$\mathcal{N}(230, 153)$	$\mathcal{N}(190, 123)$	$\mathcal{N}(290, 137)$	$\mathcal{N}(320, 90)$	$\mathcal{N}(190, 82)$
3	$\mathcal{N}(320, 142)$	$\mathcal{N}(350, 215)$	$\mathcal{N}(240, 139)$	$\mathcal{N}(310, 198)$	$\mathcal{N}(350, 150)$	$\mathcal{N}(140, 104)$
4		$\mathcal{N}(470, 235)$	$\mathcal{N}(300, 195)$	$\mathcal{N}(480, 225)$	$\mathcal{N}(400, 202)$	$\mathcal{N}(310, 154)$
5		$\mathcal{N}(560, 271)$	$\mathcal{N}(370, 245)$	$\mathcal{N}(510, 269)$	$\mathcal{N}(430, 249)$	
6		$\mathcal{N}(720, 358)$		$\mathcal{N}(640, 305)$		
7				$\mathcal{N}(670, 366)$		
8						
Initial outlay	600	950	1000	1150	1250	900
Period number	Project G	Project H	Project I	Project J	Project K	Project L
1	$\mathcal{N}(310, 87)$	$\mathcal{N}(150, 61)$	$\mathcal{N}(370, 86)$	$\mathcal{N}(280, 79)$	$\mathcal{N}(180, 62)$	$\mathcal{N}(370, 73)$
2	$\mathcal{N}(290, 135)$	$\mathcal{N}(160, 110)$	$\mathcal{N}(390, 140)$	$\mathcal{N}(350, 172)$	$\mathcal{N}(200, 104)$	$\mathcal{N}(420, 140)$
3	$\mathcal{N}(430, 177)$	$\mathcal{N}(190, 153)$	$\mathcal{N}(450, 175)$	$\mathcal{N}(400, 229)$	$\mathcal{N}(320, 159)$	$\mathcal{N}(430, 175)$
4	$\mathcal{N}(400, 227)$	$\mathcal{N}(230, 208)$	$\mathcal{N}(510, 250)$	$\mathcal{N}(520, 296)$	$\mathcal{N}(330, 181)$	$\mathcal{N}(470, 214)$
5	$\mathcal{N}(520, 270)$		$\mathcal{N}(640, 289)$	$\mathcal{N}(530, 330)$	$\mathcal{N}(510, 262)$	
6	$\mathcal{N}(500, 307)$		$\mathcal{N}(750, 355)$	$\mathcal{N}(600, 374)$	$\mathcal{N}(500, 282)$	
7			$\mathcal{N}(820, 410)$		$\mathcal{N}(620, 302)$	
8			$\mathcal{N}(900, 470)$		$\mathcal{N}(600, 349)$	
Initial outlay	1300	700	1500	1080	1300	1140

Table 2 Evaluation indicators of each project

	A	B	C	D	E	F	G	H	I	J	K	L
$E(PI)$	1.062	1.771	0.881	1.743	1.050	0.628	1.325	0.812	2.004	1.729	1.532	1.165
$V(PI)$	0.148	0.863	0.283	0.790	0.197	0.100	0.398	0.332	0.757	0.866	0.621	0.163
$E(R)$	0.427	0.407	0.232	0.358	0.277	0.198	0.304	0.256	0.376	0.397	0.287	0.367

Table 3 Selection with maximizing expected return under different risk preferences

Parameters	$\max E(R)$	$\sqrt{V(R)}$	The selected project portfolio
$\alpha_0 = 0.03, d = 0.03$	0.384	0.175	(1, 1, 1, 0, 0, 1, 1, 0, 1)
$\alpha_0 = 0.03, d = 0.05$	0.373	0.168	(1, 1, 0, 0, 1, 1, 1, 0, 1)
$\alpha_0 = 0.03, d = 0.1$	–	–	–
$\alpha_0 = 0.05, d = 0.03$	0.384	0.175	(1, 1, 1, 0, 0, 1, 1, 0, 1)
$\alpha_0 = 0.05, d = 0.05$	0.384	0.175	(1, 1, 1, 0, 0, 1, 1, 0, 1)
$\alpha_0 = 0.05, d = 0.1$	0.368	0.163	(1, 1, 0, 1, 0, 1, 1, 0, 1)

we can see the variance of each project. The variance of the high-yield project often has higher risk level but is related to the length of payback period and other various uncertainties that investors consider.

For these 9 candidates, we can then fit them into model (7) with different combinations of bankruptcy condition d and acceptable level α_0.

When α_0 is fixed, the higher the investor' s perceived minimum level of survival d, the smaller the expected return. When d is high enough such as to be 0.1, for a certain value $\alpha_0 = 0.03$ there is even no eligible portfolio that can fit into these parameter combinations. So in order to get a portfolio that can at least obtain the minimal expected return, the investor should adjust the value for parameters, in other words, take more risks. In contrast, when parameter d is fixed, the larger the value of α_0, the higher the acceptable level of the occurrence of bankruptcy event, which also means higher risk and a higher expected return.

Investors therefore can choose their own sets of parameters to fit for their different risk preferences. In Table 3, we can obtain the optimal project portfolio (A, B, D, I, J, L) with expected return $E(R) = 0.384$. The parameter combinations (α_0, d) for this portfolio are $(0.03, 0.03)$, $(0.05, 0.03)$ or $(0.05, 0.05)$, which just represents different risk preferences.

For different project portfolios, we can then plot points $(V(R), E(R))$ in the $E(R) - V(R)$ chart to correspond the risk and expected returns. The chart can demonstrate in a more intuitive way the different impacts that the bankruptcy risk constraint and variance constraint have in the selection process. Figure 2 shows the selection process under the bankruptcy constraint $\alpha_0 = 0.03$, $d = 0.03$ and the variance constraint $\sqrt{V(R)} \leq 0.16$. The points in the figure are the coordinates of the expectation and the variance of the portfolios' returns.

In Fig. 2, the bankruptcy risk constraint requires that the coordinates should lie above the baseline $E(R) = d - \frac{\sqrt{3}}{\pi} \ln \frac{\alpha_0}{1-\alpha_0} \cdot \sqrt{V(R)}$, while the variance constraint requests the points should stay within the left region of $\sqrt{V(R)} = 0.16$. The points in the figure show the correspondence between high risk and high yield. But the variance constraint only requires volatility, which cannot effectively measure the high return gain from high risk. And the bankruptcy risk constraint excludes those portfolios with a lower return at the same risk.

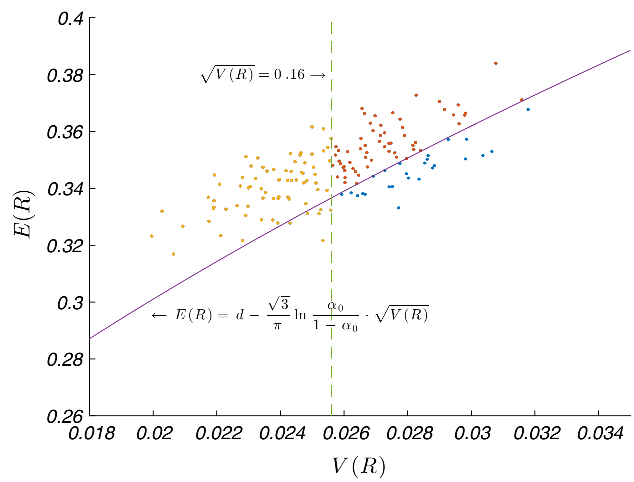

Fig. 2 Comparison of project portfolio selection under bankruptcy risk and variance constraint

Assume that $e_p = 0.3$, $\sigma_p = 0.2$. Fitted with different sets of parameter values, we can also use model (9) to select the portfolio that has minimal deviation between actual returns and expected returns.

We can observe from Table 4 that when $\alpha_0 = 0.05$ and $d = 0.03$ or 0.05, we get the portfolio that has the smallest uncertain sine cross-entropy $D[R; \eta] = 0.0025$ of the actual return from a priori return. When the higher requirements for the risk of bankruptcy, investors are more difficult to get

Table 4 Selection with minimizing sine cross-entropy under different risk appetites

Parameters	$\min D[R;\eta]$	$\sqrt{V(R)}$	The selected project portfolio
$\alpha_0 = 0.03, d = 0.03$	0.0032	0.151	(1, 1, 1, 1, 0, 1, 0, 0, 0)
$\alpha_0 = 0.03, d = 0.05$	0.0043	0.141	(0, 1, 1, 1, 0, 1, 1, 0, 0)
$\alpha_0 = 0.03, d = 0.1$	–	–	–
$\alpha_0 = 0.05, d = 0.03$	0.0025	0.159	(1, 1, 1, 0, 1, 1, 0, 0, 0)
$\alpha_0 = 0.05, d = 0.05$	0.0025	0.159	(1, 1, 1, 0, 1, 1, 0, 0, 0)
$\alpha_0 = 0.05, d = 0.1$	0.0055	0.142	(0, 1, 1, 1, 0, 1, 1, 1, 0)

returns in line with the expectations. When α_0 is fixed, the higher the investors perceived minimal level of survival d, the smaller the deviation; when d is fixed, the higher α_0, the smaller the deviation. The level of the deviation also relates to the distribution of a prior return and investors with different risk preferences can choose different parameters for making decisions. Besides, it can also be obtained that the risk preferences described by parameters α_0 and d are consistent with the actual risk level of the selected portfolios.

5 Conclusions

In the actual oil project portfolio selection, we use uncertainty theory to handle the experts' experimental data which are estimated by integrating a variety of uncertainties on the selection operation. The cash flows of the projects are then estimated by experts' empirical data and described by normal uncertain variables. During the selection process, we also need to control the bankruptcy risk effectively and to meet the different risk preferences of the investors. Usually, investors would seek to maximize their returns and reuse the recycling funds within the projects. Therefore, under the pre-defined constraint of bankruptcy risk, we constructed uncertain programming models to maximize the portfolio expected return and minimize the uncertain sine cross-entropy of the actual return from a prior return. By adjusting different parameters to fit different risk preference assumptions in the model, we therefore can simulate a better use of the existing capital and get a higher return than the simple project investment. Further researches may consider investment decision at different time and reinvestment after the recovery of cash flows and so on.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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