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Supply chain network design under the risk of uncertain disruptions

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Facility disruptions in the supply chain often lead to catastrophic consequences, although they occur rarely. The low frequency and non-repeatability of disruptive events also make it impossible to estimate the disruption probability accurately. Therefore, we construct an uncertain programming model to design the three-echelon supply chain network with the disruption risk, in which disruptions are considered as uncertain events. Under the constraint of satisfying customer demands, the model optimises the selection of retailers with uncertain disruptions and the assignment of customers and retailers, in order to minimise the expected total cost of network design. In addition, we simplify the proposed model by analysing its properties and further linearise the simplified model. A Lagrangian relaxation algorithm for the linearised model and a genetic algorithm for the simplified model are developed to solve medium-scale problems and large-scale problems, respectively. Finally, we illustrate the effectiveness of proposed models and algorithms through several numerical examples.

Keywords: supply chain network design; disruption risk; uncertainty theory; Lagrangian relaxation; genetic algorithm

1. Introduction

Supply chain network design (SCND) is a critical and difficult planning activity, which makes decisions including facility location and route assignment to meet customer demands and minimise the total cost. It is the most basic decision of supply chain management, influencing all other works (Simchi-Levi, Kaminsky, and Simchi-Levi. 2004). SCND mainly deals with long-term strategic decisions that are costly and almost impossible to reverse. However, facility disruptions caused by natural disasters, labour strikes, terrorist attacks, etc., can occur at any stage of supply chain network, and inadequate consideration of them in the design stage may lead to serious operational difficulties and economic losses.

For example, during the 2002 West Coast port lockout, the closure of 29 ports resulted in a large number of factories being shut down due to supply disruptions. In the heavily affected automobile industry, Toyota had to close its assembly line which imported parts from the West Coast, and Honda switched some imports of new vehicles to the East Coast, albeit at a higher cost. The estimated economic loss due to this strike is about $15 billion (Hall 2004). According to Carvalho et al. (2016), disruptions caused by the Great East Japan Earthquake spread in supply chain networks, affecting suppliers and customers of disaster-stricken firms. In the year following this disaster, firms that suffered upstream and downstream disruptions experienced 2% and 1.2% decline in performance, respectively. Many other examples of disruptions affecting the supply chain network are analysed in Simchi-Levi, Schmidt, and Wei (2014) and Dolgui, Ivanov, and Sokolov (2018). As can be seen from these references, although disruptions rarely occur, their impact on the supply chain network can be significant and far-reaching. Therefore, it is necessary to take them into account in the design stage.

In the SCND area, Snyder and Daskin (2005) introduce the risk of facility disruptions into the *P*-median problem and the uncapacitated fixed-charge location problem (UFLP), and develop a Lagrangian relaxation (LR) algorithm to solve them. In their paper, disruption probabilities of all facilities are assumed to be the same. Cui, Ouyang, and Shen (2010) relax this assumption and study the UFLP with site-dependent probabilities. They construct a mixed integer programming model which is solved by the LR algorithm and a continuum approximation model. Considering that the impact of random disruptions on the capacity of facilities might be partial, Azad et al. (2013) propose a capacitated two-echelon SCND model and a solution algorithm based on Benders decomposition to make decisions on locations and types of facilities, as well as assignments of customers to facilities. Jabbarzadeh et al. (2016) present a hybrid robust-stochastic optimisation model where the disruption probability is determined by fortification investment and the budget is constrained. They also develop an LR solution method and examine the model using the Monte Carlo simulation.

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All the above models discuss the design problem of two-echelon supply chain network, especially the facility location problem. The multi-echelon SCND problem under the disruption risk has also received much attention. Qi, Shen, and Snyder (2010) consider an integrated SCND problem with one supplier and multiple retailers and customers. Under random disruptions of the supplier and retailers, they determine the retailer and assignments of customers to retailers to minimise the expected total cost. Peng et al. (2011) formulate a scenario-based SCND model to minimise the total cost under normal conditions while reducing the disruption risk using the p-robustness criterion. And a genetic algorithm-based meta-heuristic is proposed. Benyoucef and Tanonkou (2013) establish a two-stage stochastic model that integrates facility location and unreliable supplier selection, and develop an approximation algorithm based on LR and Monte Carlo method. And Shishebori, Snyder, and Jabalameli (2014) optimise the combined facility location and network design to minimise the total cost with constraints on the normal investment budget and disruption cost.

More recently, from the perspective of a single manufacturer, Rezapour, Farahani, and Pourakbar (2017) consider the selection of mitigation strategies in the SCND problem to neutralise the negative impact of upstream supplier disruptions on market share. Snoeck, Udenio, and Fransoo (2019) construct a two-stage stochastic program to deal with the strategic design and operational decisions in a chemical supply chain, and propose a solution method based on sample average approximation. For a review of the SCND problem under the risk of random disruptions, the reader is referred to Snyder et al. (2016) and Ivanov et al. (2017).

In the most existing literature, facility disruption is considered as a random event and the disruption probability is assumed to be accurately predictable regardless of what it depends on. However, because disruptive events rarely occur and are often not repeatable, historical data is very scant and lowly informative, which makes it difficult or even impossible to perfectly estimate their probability (Banks 2005; Taleb 2007). In addition, Saghafian and Oyen (2012) observe that the strategy to mitigate disruption risk by investing in backup capacities may be harmful if the information about disruptive events is not perfect. And Lim et al. (2013) consider a facility location problem where disruptive facilities can be protected with additional investments. They find that misestimating the disruption probability is very expensive when the estimate intervals are wide.

For the problem of inability to obtain enough and valid data to estimate the disruption probability, uncertainty theory found by Liu (2007) provides an approach to cope with it. Uncertainty theory is a mathematical branch to deal with human uncertainty based on four axioms of normality, duality, subadditivity and product. In uncertainty theory, uncertain events are characterised by the belief degree based on experts' estimation, thus avoiding the dependence on perfect historical data. It has been applied in many fields such as portfolio optimisation (Qin 2015), contract design (Chen et al. 2017) and oilfield development (Ji, Yan, and Feng 2017). In particular, this theory is also applied to SCND problem without considering disruption risk. For example, Gao and Qin (2016) study p-hub centre location problem with uncertain travel times, Huang and Song (2018) discuss an emergency logistics distribution routing problem with uncertain demands and travel times, and Tan, Ji, and Yan (2019) consider the SCND problem in which retailers' demand is random and operating costs are uncertain.

By considering facility disruptions as uncertain events, this paper contributes to the literature on the SCND problem under the influence of disruption risk. First, an uncertain programming approach is presented to model the three-echelon SCND problem with different belief degrees of retailer disruptions. Unlike most previous work, this approach can be used when enough historical data of disruptions is unable to be obtained. Another contribution is that we propose a strategy of combining multiple backups and cross-echelon transportation to mitigate the disruption risk and ensure the satisfaction of customer demands in our uncertain SCND model. In addition, we find out the relationship between the optimal total cost and the maximum assignment level of customers, as well as the property about the optimal assignment decision of customers and retailers. Finally, besides a custom-designed LR algorithm, we also develop a genetic algorithm that can solve large-scale problems in a shorter time. In this algorithm, we put forward constraint handling techniques to prevent violations and individual improvement to accelerate the convergence.

The rest of this paper is structured as follows. We introduce the relevant definitions in uncertainty theory and the specific problem description in Section 2. In Section 3, the SCND model under the risk of uncertain disruptions is constructed and simplified to an equivalent formulation. After that, we linearise the simplified model by some techniques. In Section 4, we develop the customised LR algorithm and genetic algorithm. Section 5 illustrates the effectiveness of the proposed model and algorithm by numerical examples. In Section 6, we conclude our work and provide some thoughts on future research.

2. Preliminaries

In this section, basic definitions used in this paper are introduced, as well as a description of the supply chain network design problem with uncertain disruptions we consider.

2.1. Uncertainty theory

Definition 2.1 (Liu 2007) Let Γ be a nonempty set, and $\mathcal{L}$ be a σ-algebra over Γ. Every element Λ in $\mathcal{L}$ is called an event. If a set function $\mathcal{M}$: $\mathcal{L} \to \Re$ satisfies the following conditions:

Axiom 1. *(Normality Axiom)* $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.
Axiom 2. *(Duality Axiom)* $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.
Axiom 3. *(Subadditivity Axiom)* For every countable sequence of $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

We call $\mathcal{M}$ an uncertain measure, and $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

In order to study the uncertain measure of product space, the product uncertain measure is defined.

Axiom 4. *(Product Axiom)* Let $(\Lambda_k, \mathcal{L}_k, \mathcal{M}_k), k = 1, 2, \ldots$ be uncertainty spaces, the product measure $\mathcal{M}$ is an measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\},$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

Definition 2.2 (Liu 2010) The events $\Lambda_1, \Lambda_2, \ldots, \Lambda_n$ are said to be independent if and only if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n} \Lambda_i^*\right\} = \bigwedge_{i=1}^{n} \mathcal{M}\left\{\Lambda_i^*\right\},$$

where Λ_i^* are arbitrarily chosen from $\{\Lambda_i, \Lambda_i^c, \Gamma\}, i = 1, 2, \ldots, n$, respectively, and Γ is the universal set.

Example 2.1 The impossible set $\varnothing$ and any event Λ are independent because of the following four equations,

$$\begin{aligned}
\mathcal{M}\{\emptyset \cap \Lambda\} &= \mathcal{M}\{\emptyset\} = \mathcal{M}\{\emptyset\} \wedge \mathcal{M}\{\Lambda\}, \\
\mathcal{M}\{\emptyset^c \cap \Lambda\} &= \mathcal{M}\{\Lambda\} = \mathcal{M}\{\emptyset^c\} \wedge \mathcal{M}\{\Lambda\}, \\
\mathcal{M}\{\emptyset \cap \Lambda^c\} &= \mathcal{M}\{\emptyset\} = \mathcal{M}\{\emptyset\} \wedge \mathcal{M}\{\Lambda^c\}, \\
\mathcal{M}\{\emptyset^c \cap \Lambda^c\} &= \mathcal{M}\{\Lambda^c\} = \mathcal{M}\{\emptyset^c\} \wedge \mathcal{M}\{\Lambda^c\},
\end{aligned}$$

which mean that knowing the occurrence of one doesn't change the estimation of the other based on the product axiom.

Definition 2.3 (Liu 2007) An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that

$$\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$$

is an event for any Borel set B of real numbers.

Example 2.2 Take an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to be $\{\gamma_1, \gamma_2\}$ with power set. Then

$$\xi(\gamma) = \begin{cases} 0, & \text{if } \gamma = \gamma_1, \\ 1, & \text{if } \gamma = \gamma_2, \end{cases}$$

is an uncertain variable because $\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$ is an event for any Borel set B of real numbers. Furthermore, we have $\mathcal{M}\{\xi = 0\} = \mathcal{M}\{\gamma_1\}$ and $\mathcal{M}\{\xi = 1\} = \mathcal{M}\{\gamma_2\}$.

Definition 2.4 (Liu 2007) Let ξ be an uncertain variable. The expected value of ξ is

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\}\,\mathrm{d}\,r - \int_{-\infty}^{0} \mathcal{M}\{\xi \leq r\}\,\mathrm{d}\,r,$$

if at least one of the two integrals is finite.

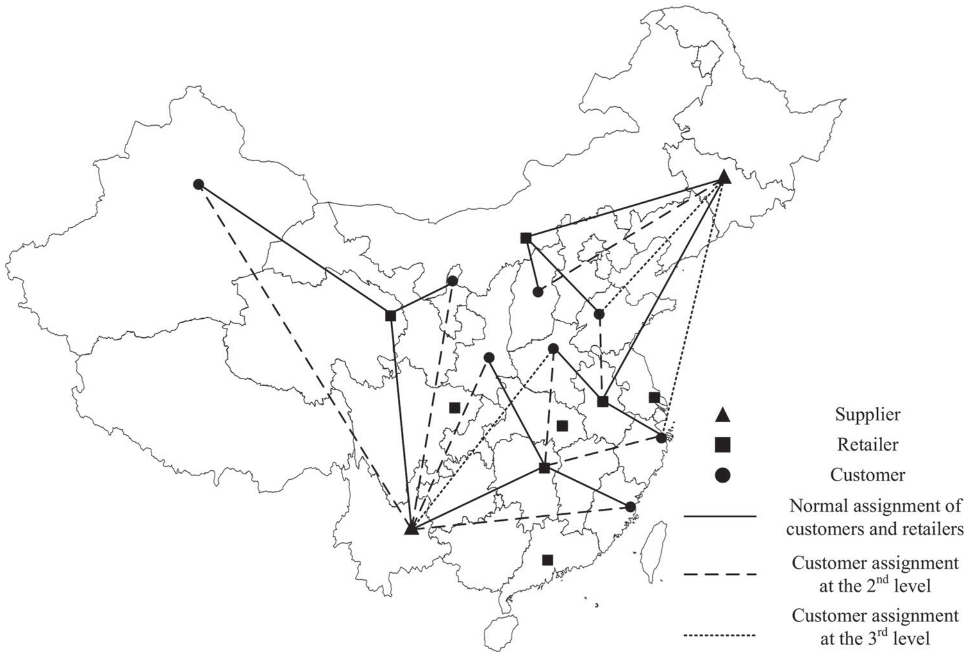

Figure 1. Supply chain network design with the risk of retailers' uncertain disruptions.

2.2. Problem description

This paper considers a three-echelon supply chain network consisting of multiple suppliers, retailers and customers. The company's own suppliers are established and responsible for delivering products to retailers or directly to consumers. Managers need to select external retailers to transport products acquired from suppliers to customers. Candidate retailers are not controlled directly by the company and are at the risk of uncertain disruptions. The fixed cost and disruption belief degree of retailers are different according to their geographical location, operation and other factors. The unit cost of retailer acquiring products from supplier (ordering, transportation costs, etc.) also varies with different route choices. In addition, we assume that the capacity of both suppliers and retailers only results in variable costs, so their capacity constraints are not considered.

The company is make-to-order and acquires customers' demand information through their orders placed in advance. Since retailers may be disruptive, in order to ensure that the demand is met, each customer is assigned to multiple retailers. These retailers serve the customer according to their service assignment levels. When the retailer at the previous level is disruptive, the service is provided by the one at the next level. And when all retailers assigned to a customer are disruptive, this customer will be assigned to one supplier for satisfying the demand. Therefore, a customer has multiple assignment levels and is assigned to a retailer or supplier at each level. (For brevity, we refer to the retailers and suppliers assigned to customers as service providers hereafter.)

In short, we need to select retailers, assign these selected retailers to suppliers, and assign customers to service providers at multiple assignment levels, to meet customer demands and respond to possible disruptions. Our objective is to minimise the total cost of SCND, including the fixed cost of selected retailers, the expected cost of retailers acquiring products from suppliers, and the expected cost of customers acquiring products from providers. A supply chain network with the risk of retailers' uncertain disruptions studied in this paper is shown in Figure 1.

3. Models

In this section, the SCND problem is first formulated as an uncertain mixed integer nonlinear program. Then we analyse the properties of this model and simplify it equivalently. Finally, the simplified model is linearised into a mixed integer linear model.

3.1. Problem formulation

Let S be the set of suppliers, J the set of candidate retailers and I the set of customers. The number of retailers, candidate retailers and customers is $|S|$, $|J|$ and $|I|$, respectively. For customer $i \in I$, the demand is λ_i. And the unit cost of customer i

Table 1. Summary of model notation.

	Notation	Representation
Sets and indices	S	Set of suppliers, indexed by $s = 1, 2, \ldots, \lvert S\rvert$
	J	Set of candidate retailers, indexed by $j = 1, 2, \ldots, \lvert J\rvert$
	I	Set of customers, indexed by $i = 1, 2, \ldots, \lvert I\rvert$
	$J \cup S$	Set of service provider, indexed by n where $n = 1, \ldots, \lvert J\rvert$ represent retailers and $n = \lvert J\rvert + 1, \ldots, \lvert J\rvert + \lvert S\rvert$ represent suppliers
Parameters	f_j	Fixed cost of retailer j
	e_{sj}	Unit cost of retailer j acquiring products from supplier s
	d_{in}	Unit cost of customer i acquiring products from service provider n
	α_n	Belief degree of the service provider's uncertain disruption ($\alpha_n \in (0, 1)$ for $n = 1, \ldots, \lvert J\rvert$ and $\alpha_n = 0$ for $n = \lvert J\rvert + 1, \ldots, \lvert J\rvert + \lvert S\rvert$)
Decision variables	X_j	1 if retailer j is selected, 0 otherwise
	Y_{sj}	1 if supplier s is assigned to retailer j, 0 otherwise
	Z_{inr}	1 if provider n is assigned to customer i at level r, 0 otherwise
	M_{inr}	Belief degree that customer i's service providers assigned at levels $1, \ldots, r-1$ are disruptive and the one assigned at level r is normal

acquiring products from service provider $n \in J \cup S$ is d_{in}, where $n = 1, \ldots, |J|$ represent retailers and $n = |J| + 1, \ldots, |J| + |S|$ represent suppliers. Since the scheduling is more difficult and the distance is farther when suppliers service customers directly, the unit cost of customer acquiring products from them is generally much larger than that from retailers. The fixed cost of retailer $j \in J$ is f_j, and the unit cost of acquiring products from supplier $s \in S$ is e_{sj}. The belief degree of the retailer's uncertain disruption is given based on experts' estimates, denoted as $\alpha_n \in (0, 1)$, $n = 1, \ldots, |J|$. And the disruptive events of retailers are independent. The supplier will not be disruptive, so $\alpha_n = 0, n = |J| + 1, \ldots, |J| + |S|$.

There are three types of decision variables corresponding to the network design, namely, the selection variable of retailers X_j, the assignment variable of retailers to suppliers Y_{sj}, and the assignment variable of customers to service providers Z_{inr}:

$$X_j = \begin{cases} 1, & \text{if retailer } j \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_{sj} = \begin{cases} 1, & \text{if supplier } s \text{ is assigned to retailer } j, \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_{inr} = \begin{cases} 1, & \text{if provider } n \text{ is assigned to customer} i \text{ at level } r, \\ 0, & \text{otherwise.} \end{cases}$$

The customer assignment at level r represents the service provider to which customer i is assigned when the assigned service providers at levels $1, \ldots, r-1$ are all disruptive. Unless the customer is assigned to a non-disruptive supplier before level R $(R = 2, \ldots, |J| + 1)$, each customer will have exactly R assignments. And if a customer is assigned to retailers at levels $1, \ldots, R-1$, she must be assigned to a supplier at level R to ensure that the demand can be met when all the assigned retailers at the previous $R-1$ levels are disruptive.

In addition, decision variable M_{inr} is introduced to represent the belief degree that customer i's service providers assigned at levels $1, \ldots, r-1$ are disruptive and the one assigned at level r is normal. All indices, sets, parameters and decision variables used in our model are summarised in Table 1.

The supply chain network design problem under the risk of uncertain disruptions is formulated as follows:

$$\text{(UDM)} \quad \min \sum_{j=1}^{|J|} f_j X_j + \sum_{s=1}^{|S|} \sum_{j=1}^{|J|} \sum_{i=1}^{|I|} \sum_{r=1}^{R} e_{sj} Y_{sj} \lambda_i M_{ijr} Z_{ijr} + \sum_{i=1}^{|I|} \sum_{n=1}^{|J|+|S|} \sum_{r=1}^{R} \lambda_i d_{in} M_{inr} Z_{inr} \tag{1a}$$

subject to:

$$\sum_{s=1}^{|S|} Y_{sj} = X_j, \forall j \in J \tag{1b}$$

$$\sum_{n=1}^{|J|} Z_{inr} + \sum_{p=1}^{r} \sum_{n=|J|+1}^{|J|+|S|} Z_{inp} = 1, \forall i \in I, r = 1, \ldots, R \tag{1c}$$

$$\sum_{r=1}^{R} \sum_{n=|J|+1}^{|J|+|S|} Z_{inr} = 1, \forall i \in I \tag{1d}$$

$$\sum_{r=1}^{R-1} Z_{ijr} \leq X_j, \forall i \in I, j \in J \tag{1e}$$

$$M_{in1} = 1 - \alpha_n, \forall i \in I, n \in J \cup S \tag{1f}$$

$$M_{inr} = \left(\bigwedge_{l=1}^{r-1} \sum_{k=1}^{|J|} \alpha_k Z_{ikl} \right) \wedge (1 - \alpha_n), \forall i \in I, n \in J \cup S, r = 2, \ldots, R \tag{1g}$$

$$X_j, Y_{sj}, Z_{inr} \in \{0, 1\}, \forall s \in S, i \in I, j \in J, n \in J \cup S, r = 1, \ldots, R. \tag{1h}$$

The objective function (1a) is the expected total cost of the entire supply chain network. The first term is the fixed cost of selected retailers, the second term is the expected cost of retailers acquiring products from suppliers and the third term is the expected cost of customers acquiring products from service providers. Constraint (1b) requires that each selected retailer must be supplied by one supplier. Constraint (1c) enforces that for the assignment of each customer at each level, if the customer is assigned to a supplier at a certain level or before, the retailer will no longer be assigned at that level, otherwise there should be a retailer providing services at that level. Constraint (1d) requires that each customer must be assigned to a supplier at one level in order to ensure that the demand is met. Constraint (1e) restricts that customers cannot be assigned to unselected retailers. Constraint (1f) is the belief degree that customer i can be served normally when assigned to service provider n at the first level. According to Definition 2.2, constraint (1g) calculates the belief degree that customer i can be served normally when assigned to service provider n for $r = 2, \ldots, R$, that is, the belief degree that service provider n is normal while providers assigned at previous levels are disruptive. Note that $\sum_{k=1}^{|J|} \alpha_k Z_{ikl} = \sum_{k=1}^{|J|+|S|} \alpha_k Z_{ikl}$ due to $\alpha_k = 0$ for $k = |J| + 1, \ldots, |J| + |S|$, so it can represent the disruption belief degree of provider to which customer i is assigned at level l.

3.2. Model analysis and simplification

About the relationship between the maximum assignment level R and the optimal objective value of (UDM), we have the following proposition.

PROPOSITION 3.1 *As the maximum assignment level R increases, the optimal objective value of (UDM) is nonincreasing.*

Proof The optimal solution of (UDM) is denoted by X_j^*, Y_{sj}^*, Z_{inr}^* and M_{inr}^*, $\forall s \in S, i \in I, j \in J, n \in J \cup S, r = 1, \ldots, \overline{R}$ and the corresponding objective value is denoted by Φ^* when the maximum assignment level is equal to $\overline{R}$. We can construct a feasible solution using the following rule when $R = \overline{R} + 1$. For all $s \in S$, $i \in I$, $j \in J$, $n \in J \cup S$, $r = 1, \ldots, \overline{R}$, let $X_j' = X_j^*$, $Y_{sj}' = Y_{sj}^*$, $Z_{inr}' = Z_{inr}^*$ and $M_{inr}' = M_{inr}^*$. And for $r = \overline{R} + 1$, let $Z_{inr}' = 0$ and $M_{inr}' = 0$. Obviously, when the maximum assignment level is $\overline{R} + 1$, X_j', Y_{sj}', Z_{inr}' and M_{inr}' satisfy constraints (1a)–(1h) and make the objective value remain the same, so its optimal value is less than or equal to Φ^*. ■

Intuitively, since more feasible network structures can be chosen as R increases, the optimal objective value will not become larger at least. However, when the maximum assignment level is set too large, on the one hand, due to the high unit cost of supplier's direct supply, customers may be assigned to distant retailers at some levels. So it is difficult to meet customer demands in time when many retailers are disruptive, which leads to lower customer satisfaction and loyalty. On the other hand, a larger R will also increase the number of decision variables and constraints in the model, making the solution process more complex. Therefore, although the maximum assignment level can be equal to $|J| + 1$ at most, we set this parameter in the model according to the actual needs of the decision-maker, as in Snyder and Daskin (2005) and Cui, Ouyang, and Shen (2010).

More importantly, Proposition 3.1 can be used to improve the solution process with the result of smaller maximum assignment level. When we solve the equivalent linearised model proposed later by branch and bound algorithm in general solvers, the optimal objective value $\Phi^*(R-1)$ when the maximum assignment level is equal to $R-1$ can be employed as an initial upper bound to accelerate the search process before termination.

The following proposition on the optimal assignment of retailers and customers to suppliers allows us to simplify (UDM).

PROPOSITION 3.2 *In any optimal solution of (UDM),*

(i) *for any retailer $j \in J$, if $e_{s_1j} < e_{s_2j}$ where $s_1, s_2 \in S$, then $Y_{s_2j} = 0$;*
(ii) *for any customer $i \in I$, if $d_{in_1} < d_{in_2}$ where $|J|+1 \le n_1 \le |J|+|S|$ and $|J|+1 \le n_2 \le |J|+|S|$, then $Z_{in_2r} = 0$ for all r.*

Proof (i) Suppose, for a contradiction, that X_j^*, Y_{sj}^*, Z_{inr}^* and M_{inr}^*, $\forall s \in S, i \in I, j \in J, n \in J \cup S, r = 1, \ldots, R$ are optimal for (UDM) where $e_{s_1\bar{j}} < e_{s_2\bar{j}}$ for retailer $\bar{j} \in J$ and $Y^*_{s_2\bar{j}} = 1$. And denote Φ^* as the corresponding objective function value. We can obtain $X^*_{\bar{j}} = 1$ and $Y^*_{s_1\bar{j}} = 0$ from constraints (1b) and (1h). If we let $Y^{*'}_{s_1\bar{j}} = 1$, $Y^{*'}_{s_2\bar{j}} = 0$ and other decision variables in this optimal solution remain the same, then constraints (1b) and (1h) are still satisfied and other constraints are not affected. For the objective function value, denote $\Phi^{*'}$ as its value after adjustment, it follows that

$$\Phi^{*'} - \Phi^* = (e_{s_1\bar{j}} - e_{s_2\bar{j}}) \sum_{i=1}^{|I|} \sum_{r=1}^{R} \lambda_i M_{i\bar{j}r} Z_{i\bar{j}r} < 0.$$

This implies a contradiction to that X_j^*, Y_{sj}^*, Z_{inr}^* and M_{inr}^*, $\forall s \in S, i \in I, j \in J, n \in J \cup S, r = 1, \ldots, R$ are optimal.

(ii) Similarly, suppose that X_j^{**}, Y_{sj}^{**}, Z_{inr}^{**} and M_{inr}^{**}, $\forall s \in S, i \in I, j \in J, n \in J \cup S, r = 1, \ldots, R$ are optimal for (UDM) where $d_{\bar{i}n_1} < d_{\bar{i}n_2}$ for customer $\bar{i} \in I$ and $Z^{**}_{\bar{i}n_2\bar{r}} = 1$ at a certain level $\bar{r}$. And Φ^{**} is the optimal objective value. From constraint (1c) or (1d), there must be $Z^{**}_{\bar{i}n_1\bar{r}} = 0$. If we only adjust $Z^{**}_{\bar{i}n_1\bar{r}} = 0$ and $Z^{**}_{\bar{i}n_2\bar{r}} = 1$ to $Z^{**'}_{\bar{i}n_1\bar{r}} = 1$ and $Z^{**'}_{\bar{i}n_2\bar{r}} = 0$, then all constraints are still satisfied. And in fact, we can easily check that $M^{**}_{\bar{i}n\bar{r}}$, $n = |J|+1, \ldots, |J|+|S|$ are the same. So denote $\Phi^{**'}$ as the objective value after adjustment, it follows that,

$$\begin{aligned}\Phi^{**'} - \Phi^{**} &= \lambda_{\bar{i}} d_{\bar{i}n_1} M^{**}_{\bar{i}n_1\bar{r}} Z^{**'}_{\bar{i}n_1\bar{r}} - \lambda_{\bar{i}} d_{\bar{i}n_2} M^{**}_{\bar{i}n_2\bar{r}} Z^{**}_{\bar{i}n_2\bar{r}} \\ &= \lambda_{\bar{i}} M^{**}_{\bar{i}n_1\bar{r}} (d_{\bar{i}n_1} - d_{\bar{i}n_2}) < 0.\end{aligned}$$

The proposition is proved. ■

Proposition 3.2 shows that the selected retailer is always assigned to the supplier with the lowest unit cost of acquiring products, and the customer will also be assigned to such a supplier if she is assigned to the supplier at a certain level. Using this result, we can simplify (UDM) via equivalent transformation. We assume that there exists a dummy supplier indexed by $n = 0$ and $s = 0$, and denote $\alpha_0 = 0$, $d_{i0} = \min_{s \in S} d_{is}$ for all $i \in I$ and $e_{0j} = \min_{s \in S} e_{sj}$ for all $j \in J$. With Proposition 3.2, we have the following equivalent formulation of (UDM),

$$\text{(UDM}') \quad \min \sum_{j=1}^{|J|} f_j X_j + \sum_{i=1}^{|I|} \sum_{j=1}^{|J|} \sum_{r=1}^{R} e_{0j} X_j \lambda_i M_{ijr} Z_{ijr} + \sum_{i=1}^{|I|} \sum_{n=0}^{|J|} \sum_{r=1}^{R} \lambda_i d_{in} M_{inr} Z_{inr} \tag{2a}$$

subject to:

$$\sum_{n=1}^{|J|} Z_{inr} + \sum_{p=1}^{r} Z_{i0p} = 1, \forall i \in I, r = 1, \ldots, R \tag{2b}$$

$$\sum_{r=1}^{R} Z_{i0r} = 1, \forall i \in I \tag{2c}$$

$$\sum_{r=1}^{R-1} Z_{ijr} \le X_j, \forall i \in I, j \in J \tag{2d}$$

$$M_{in1} = 1 - \alpha_n, \forall i \in I, n \in N \tag{2e}$$

$$M_{inr} = \left(\bigwedge_{l=1}^{r-1}\sum_{k=1}^{|J|}\alpha_k Z_{ikl}\right) \wedge (1-\alpha_n), \forall i \in I, n \in N, r = 2, \ldots, R \tag{2f}$$

$$X_j, Z_{inr} \in \{0,1\}, \forall i \in I, j \in J, n \in N, r = 1, \ldots, R, \tag{2g}$$

where $N = \{0\} \cup J$. The way to obtain the optimal solution of (UDM) from (UDM′)'s is as follows. For all $j \in J$, $Y_{\arg\min_{s\in S} e_{sj},j} = X_j^*$ and $Y_{sj} = 0$ for $s \in S \setminus \{\arg\min_{s\in S} e_{sj}\}$; for all $i \in I$ and $r = 1, \ldots, R$, $Z_{i,\arg\min_{s\in S} d_{is},r} = Z_{i0r}^*$, $Z_{inr} = 0$ for $n \in \{|J|+1, \ldots, |J|+|S|\} \setminus \{\arg\min_{s\in S} d_{is}\}$ and $M_{inr} = M_{i0r}^*$ for $n \in \{|J|+1, \ldots, |J|+|S|\}$, where X_j^*, Z_{i0r}^* and M_{i0r}^* are the corresponding variable values in the optimal solution of (UDM′). And other variables in the two optimal solutions are correspondingly equal.

3.3. Linearised model

In the simplified model (UDM′), there is still a nonlinear constraint (2f) and nonlinear terms $X_j M_{ijr} Z_{ijr}$ and $M_{inr} Z_{inr}$, $\forall i \in I$, $j \in J$, $n \in N$, $r = 1, \ldots, R$ in the objective function (2a). Through the techniques below, we can linearise them to obtain an equivalent mixed integer linear model.

For constraint (2f), we transform it into a linear form by introducing auxiliary 0-1 decision variable u_q. For each $i \in I$, $n \in N$, $r = 2, \ldots, R$, the following set of new constraints is used to replace this constraint:

$$M_{inr} \le 1 - \alpha_n \tag{3a}$$

$$M_{inr} \le \sum_{k=1}^{|J|} \alpha_k Z_{ikl}, \forall l = 1, \ldots, r-1 \tag{3b}$$

$$M_{inr} + B_r(1 - u_r) \ge 1 - \alpha_n \tag{3c}$$

$$M_{inr} + B_l(1 - u_l) \ge \sum_{k=1}^{|J|} \alpha_k Z_{ikl}, \forall l = 1, \ldots, r-1 \tag{3d}$$

$$\sum_{q=1}^{r} u_q = 1 \tag{3e}$$

$$u_q \in \{0,1\}, \forall q = 1, \ldots, r, \tag{3f}$$

where B_r and B_l, $l = 1, \ldots, r-1$ are some large numbers. Constraints (3a)–(3b) ensure that M_{inr} is not greater than $(\bigwedge_{l=1}^{r-1}\sum_{k=1}^{|J|}\alpha_k Z_{ikl}) \wedge (1-\alpha_n)$. Constraints (3c)–(3f) enforce M_{inr} to be at least greater than or equal to one of values in $\{1-\alpha_n, \sum_{k=1}^{|J|}\alpha_k Z_{ik1}, \ldots, \sum_{k=1}^{|J|}\alpha_k Z_{ikr}\}$, which is equivalent to not less than their minimum. Therefore, constraints (3a)–(3f) are equivalent to constraint (2f).

In addition, for nonlinear terms $X_j M_{ijr} Z_{ijr}$ and $M_{inr} Z_{inr}$, note that X_j and Z_{inr} are 0–1 variables, and M_{inr} are continuous variables on [0,1]. First, we apply the linearisation technique proposed by Sherali and Alameddine (1992) to transform $M_{inr} Z_{inr}$. For each $i \in I$, $n \in N$, $r = 1, \ldots, R$, we use the following set of linear constraints to enforce $V_{inr} = M_{inr} Z_{inr}$:

$$V_{inr} \le M_{inr} \tag{4a}$$

$$V_{inr} \le Z_{inr} \tag{4b}$$

$$V_{inr} \ge 0 \tag{4c}$$

$$V_{inr} \ge M_{inr} + Z_{inr} - 1. \tag{4d}$$

Then, using the same technique, the following linear constraints are added to enforce $W_{ijr} = X_j V_{ijr}$, $\forall i \in I$, $j \in J$, $r = 1, \ldots, R$:

$$W_{ijr} \le X_j \tag{5a}$$

$$W_{ijr} \le V_{ijr} \tag{5b}$$

$$W_{ijr} \ge 0 \tag{5c}$$

$$W_{ijr} \ge X_j + V_{ijr} - 1. \tag{5d}$$

The linearised supply chain network design model under the risk of uncertain disruptions is

$$\text{(LUDM)} \quad \min \sum_{j=1}^{|J|} f_j X_j + \sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\sum_{r=1}^{R} e_{0j}\lambda_i W_{ijr} + \sum_{i=1}^{|I|}\sum_{n=0}^{|J|}\sum_{r=1}^{R} \lambda_i d_{in} V_{inr} \tag{6a}$$

subject to :

$$(2b) \;\; -- \; (2e),\; (2g) \tag{6a}$$

$$(3a) -- (3f), \forall i \in I, n \in N, r = 2, \ldots, R \tag{6b}$$

$$(4a) -- (4d), \forall i \in I, n \in N, r = 1, \ldots, R \tag{6c}$$

$$(5a) -- (5d), \forall i \in I, j \in J, r = 1, \ldots, R. \tag{6d}$$

4. Solution methods

Although the linearised model in Section 3.2 can be solved by general solvers, the solution time is extremely long even for moderately sized problems due to the existence of binary decision variables. Therefore, we develop an LR algorithm and a genetic algorithm. The LR algorithm is used to solve (LUDM), while the genetic algorithm solves (UDM′) to obtain the near-optimal solution.

4.1. Lagrangian relaxation algorithm

In the iterative process of LR algorithm, we first solve the relaxation problem of (LUDM) to obtain the lower bound. Then, a feasible solution is constructed using the lower bound solution to calculate the upper bound. Finally, based on the lower and upper bounds, we update Lagrange multipliers according to subgradient method.

4.1.1. Lower bound

The following Lagrangian problem is yielded by relaxing constraints (2d), (5a) and (5d),

$$\begin{aligned} \text{(UDM} - \text{LR)} \quad \min \; \Phi(\boldsymbol{\mu}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = {} & \sum_{j=1}^{|J|} f_j X_j + \sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\sum_{r=1}^{R} e_{0j}\lambda_i W_{ijr} + \sum_{i=1}^{|I|}\sum_{n=0}^{|J|}\sum_{r=1}^{R} \lambda_i d_{in} V_{inr} \\ & + \sum_{i=1}^{|I|}\sum_{j=1}^{|J|} \mu_{ij}\Big(\sum_{r=1}^{R-1} Z_{ijr} - X_j\Big) + \sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\sum_{r=1}^{R} \eta_{ijr}(W_{ijr} - X_j) \\ & + \sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\sum_{r=1}^{R} \zeta_{ijr}(X_j + V_{ijr} - 1 - W_{ijr}) \end{aligned} \tag{7a}$$

$$\text{subject to : (2b), (2c), (2e), (2g), (5b), (5c), (6c), (6d),} \tag{7a}$$

where $\boldsymbol{\mu}$, $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ are multipliers of corresponding relaxed constraints. The objective function (7a) can be rewritten as follows:

$$\begin{aligned} \Phi(\boldsymbol{\mu}, \boldsymbol{\eta}, \boldsymbol{\zeta}) = {} & \sum_{i=1}^{|I|}\sum_{j=1}^{|J|}\sum_{r=1}^{R} ((e_{0j}\lambda_i + \eta_{ijr} - \zeta_{ijr})W_{ijr} + (\lambda_i d_{in} + \zeta_{ijr})V_{inr} + \mu_{ij} Z_{ijr} - \zeta_{ijr}) \\ & + \sum_{j=1}^{|J|} \Big(f_j - \sum_{i=1}^{|I|} \mu_{ij} - \sum_{i=1}^{|I|}\sum_{r=1}^{R} (\eta_{ijr} - \zeta_{ijr})\Big) X_j. \end{aligned}$$

Because the constraints on decision variable X_j in (UDM-LR) are only binary constraints, for given multipliers $\boldsymbol{\mu}$, $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$, the optimal selection of retailers in the lower bound problem is

$$X_j^* = \begin{cases} 1, & \text{if } f_j - \sum_{i=1}^{|I|} \mu_{ij} - \sum_{i=1}^{|I|} \sum_{r=1}^{R} (\eta_{ijr} - \zeta_{ijr}) < 0 \\ 0, & \text{otherwise.} \end{cases}$$

Thus the remaining problem is only the assignment of customers to service providers (retailers and a dummy supplier) at each level. And in this problem, the assignment of each customer is separable, so we have the subproblem for assignment of customer i in the lower bound problem,

$$(\mathrm{LCAP_i}) \quad \min \sum_{j=1}^{|J|}\sum_{r=1}^{R}((e_{0j}\lambda_i + \eta_{ijr} - \zeta_{ijr})W_{ijr} + (\lambda_i d_{in} + \zeta_{ijr})V_{inr} + \mu_{ij}Z_{ijr} - \zeta_{ijr}) \tag{8a}$$

subject to:

$$(2\mathrm{b})_\mathrm{i},\ (2\mathrm{c})_\mathrm{i},\ (2\mathrm{e})_\mathrm{i},\ (6\mathrm{c})_\mathrm{i},\ (6\mathrm{d})_\mathrm{i} \tag{8a}$$

$$Z_{inr} \in \{0,1\}, \forall n \in N, r = 1,\dots,R, \tag{8c}$$

where constraint (8b) are constraints (2b), (2c), (2e), (6c) and (6d) corresponding to customer i.

4.1.2. *Upper bound*

In each iteration, the following heuristic is used to construct a feasible solution to (LUDM) based on the solution in Section 4.1.1, and then obtain the upper bound.

We retain the optimal retailer selection of (UDM-LR). Since decision variable X_j has been determined, X_jV_{ijr} become linear. Therefore, it is no longer necessary to use constraints (5a)–(5d) for linearisation. Similarly, the remaining problem is only about the assignment of customers. The subproblem for the assignment of customer i in the upper bound problem is

$$(\mathrm{UCAP_i}) \quad \min\ \Psi_i(\boldsymbol{X}^*) = \sum_{j=1}^{|J|}\sum_{r=1}^{R} e_{0j}X_j^*\lambda_i V_{ijr} + \sum_{n=0}^{|J|}\sum_{r=1}^{R}\lambda_i d_{in}V_{inr} \tag{9a}$$

subject to:

$$(8\mathrm{b}),\ (8\mathrm{c}) \tag{9a}$$

$$\sum_{r=1}^{R-1} Z_{ijr} \le X_j^*, \forall j \in J. \tag{9c}$$

After obtaining the optimal objective value Ψ_i^* and customer assignments for all subproblems, we remove the retailers that don't serve any customers at any level and get the optimal retailer selection X_j^{**} in the upper bound problem. Then we calculate the upper bound in this iteration $UB = \sum_{j=1}^{|J|} f_j X_j^{**} + \sum_{i=1}^{|I|}\Psi_i^*$.

4.1.3. *Multiplier initialisation and updating*

In our algorithm, we use subgradient method to solve the Lagrangian dual problem, as described in Fisher (1981). In particular, all initial multipliers are set to zero first. And we compute the step-size δ^t at iteration t,

$$\delta^t = \frac{\beta^t(\mathrm{UB}^* - \Phi^*)}{\sum_{i=1}^{|I|}\sum_{j=1}^{|J|}((\sum_{r=1}^{R-1} Z_{ijr} - X_j)^2 + \sum_{r=1}^{R}((W_{ijr} - X_j)^2 + (X_j + V_{ijr} - 1 - W_{ijr})^2))},$$

where β^t is a constant, which is initialised to 40 and halved when 4 consecutive iterations fail to improve the lower bound in this paper; Φ^* is the lower bound in this iteration; UB^* is the best known upper bound; and the value of decision variables in denominator is the optimal solution of the lower bound problem. Then, multipliers are updated according to the rules below,

$$\mu_{ij}^{t+1} \leftarrow \left[\mu_{ij}^t + \delta^t\left(\sum_{r=1}^{R-1} Z_{ijr} - X_j\right)\right]^+,$$

$$\eta_{ijr}^{t+1} \leftarrow [\eta_{ijr}^t + \delta^t(W_{ijr} - X_j)]^+,$$

$$\zeta_{ijr}^{t+1} \leftarrow [\zeta_{ijr}^t + \delta^t(X_j + V_{ijr} - 1 - W_{ijr})]^+.$$

We terminate the Lagrangian process when the gap between upper and lower bounds $(UB^* - \Phi^*)/UB^*$ is less than the given tolerance ϵ or iteration count t reaches $t_{\max}$. If the gap at termination is less than ϵ, then the solution corresponding to the minimum upper bound is taken as the final output; otherwise, we continue to close the gap using branch and bound algorithm.

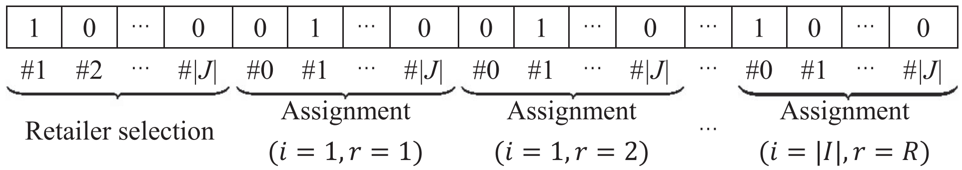

Figure 2. Sample of chromosome.

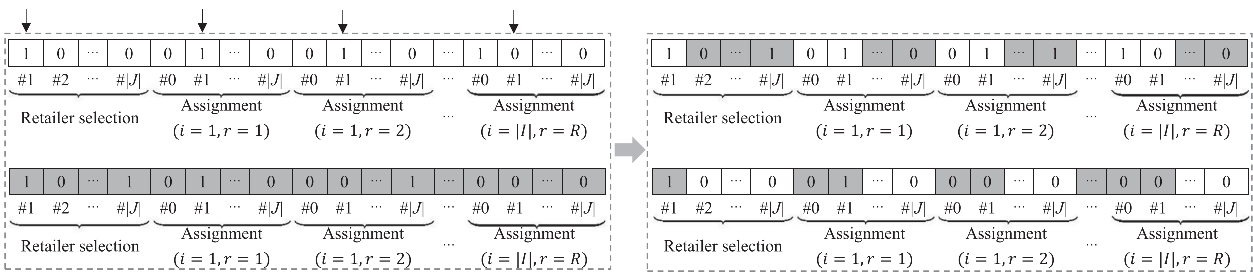

Figure 3. Crossover process.

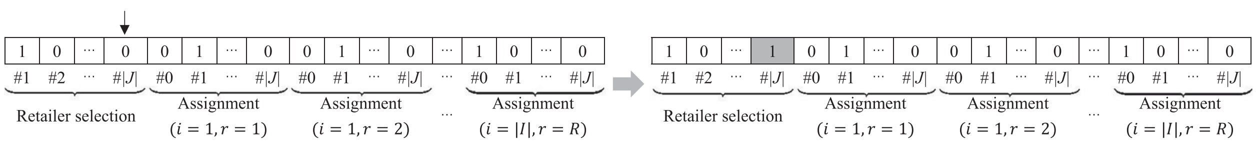

Figure 4. Mutation process.

4.2. Genetic algorithm

In order to solve the problem faster, we develop a genetic algorithm. Because the nonlinearity doesn't determine the efficiency of genetic algorithm and (LUDM) introduces more decision variables and constraints, our proposed algorithm solves the simplified model (UDM′) directly.

4.2.1. Chromosome representation, initialisation and fitness

A chromosome consists of two parts that represent the retailer selection and customer assignments. The value of the first $|J|$ genes represent the decision of retailer selection, and the value of each $(|J|+1) \times R$ genes thereafter corresponds to a customer's assignment decision from level 1 to level R, as shown in Figure 2.

When X_j and Z_{inr} are known, decision variable M_{inr} can be calculated easily using constraints (2e) and (2f). In order to also satisfy constraints (2b)–(2d) in chromosome initialisation, we first make a random selection of retailers. Then, each customer is randomly assigned to service providers level by level based on the retailer selection. After obtaining the randomly generated chromosome, if a customer isn't assigned to the dummy supplier at any level, the assignment at the last level is modified to the dummy supplier. Finally, if the assignment at a certain level is the dummy supplier, no assignments are made at subsequent levels.

For a feasible chromosome, we can obtain the value of objective function (2a) using its corresponding decision variables X_j, Z_{inr} and M_{inr}. Obviously, a chromosome with a smaller objective function value is better. So the reciprocal of objective value is used as fitness, which makes better chromosomes are more likely to be selected into the next generation.

4.2.2. Genetic operator

We use roulette to select chromosomes and elite strategy to preserve the best chromosome in each generation. In the crossover process, we select a retailer serial number randomly. For the part of the chromosome that represents the retailer selection, this serial number is used as crossover point for one-point crossover; and for the part that represents the customer's assignment, we use this serial number as crossover point to crossover each customer's assignment decision at each level. Besides, we adopt simple mutation operator, in which the selection of one retailer is changed to the opposite randomly. Examples of crossover and mutation processes are shown in Figures 3 and 4, respectively.

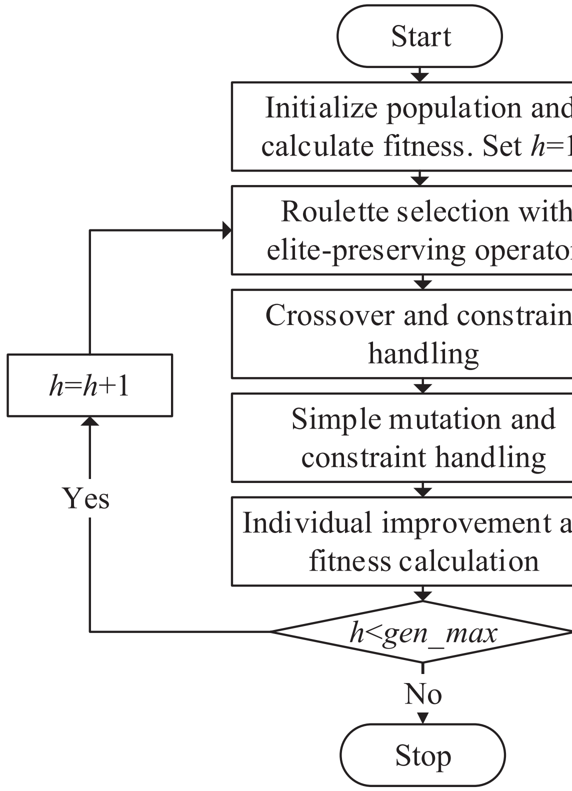

Figure 5. Flow chart of genetic algorithm.

For the chromosome after crossover, constraints (2d)–(2g) are satisfied. In order not to violate constraints (2b) and (2c), we first only retain one of the multiple retailers assigned to the same customer at the same level randomly. Then, if a customer isn't assigned to any retailer at a certain level and isn't assigned to the dummy supplier at the previous levels, she will be randomly assigned to a selected retailer or the dummy supplier at that level. Finally, the chromosome is adjusted according to the steps after obtaining the randomly generated chromosome in the initialisation.

In the mutated chromosome, we need to deal with constraint (2d) because the retailer selection is changed. If a customer is assigned to an unselected retailer, she will be reassigned to a selected and unserviced one.

4.2.3. Individual improvement

After crossover, mutation and corresponding constraint handling processes, we apply the following method to make the algorithm converge faster. For each individual in the population, the selection of retailers remains unchanged. And the customer is assigned to selected retailers or the dummy supplier level by level in the order of increasing unit cost of acquiring products, while ensuring that she is assigned to the dummy supplier at least at the last level. After that, we calculate the value of objective function (2a) under this greedy decision. If this value is less than the original objective value, then the individual will be replaced with the chromosome corresponding to the greedy decision.

The flow chart of our proposed genetic algorithm is shown in Figure 5.

5. Numerical examples

This section demonstrates the performance of proposed algorithms on four datasets. And the benefit of considering uncertain disruptions, as well as the impact of belief degree and maximum assignment level on decisions are examined.

5.1. Computational results

We tested proposed algorithms on four datasets. The first two have two and four suppliers, respectively. And their 35 candidate retailers are located in all provincial capitals and municipalities of mainland China, while customers come from

Table 2. Parameter values for Lagrangian relaxation algorithm.

Parameter	Value
Optimality tolerance (ϵ)	0.01
Maximum number of iterations ($t_{\max}$)	100
Initial value of the scalar β	4
Number of nonimproving iterations before halving β	5
Minimum value of the scalar β	10^{-2}
Initial value of Lagrange multipliers $\mu_{ij}, \eta_{ijr}, \zeta_{ijr}$	0

Table 3. Parameter values for genetic algorithm.

Parameter	Value	
	Dataset 1 & 2	Dataset 3 & 4
Population size	200	400
Maximum number of generations	300	300
Probability of crossover	0.3	0.6
Probability of mutation	0.1	0.1

70 medium- and large-sized cities. In the third and fourth datasets, there are 3 and 5 suppliers respectively, 70 candidate retailers in these medium- and large-sized cities, and 338 customers from all prefectures and municipalities in mainland China.

In all four datasets, customer demands λ_i are set to the nominal GDP of the corresponding city in 2017 divided by 10^8. The fixed cost f_j is generated randomly in the range of [17000, 160000] yuan. The belief degree of retailer's uncertain disruptions α_j is estimated based on experience and doesn't exceed 0.2. The unit cost of retailer acquiring products from supplier e_{sj} is their great-circle distance divided by 1000 plus 0.2, the unit cost of customer acquiring products from supplier $d_{in}, n = 1, \ldots, |J|$ is their great-circle distance divided by 50 plus 2, and the unit cost of customer acquiring products from supplier $d_{in}, n = |J| + 1, \ldots, |J| + |S|$ is their great-circle distance divided by 50 plus 10.

We used Gurobi solver, LR algorithm and genetic algorithm to solve 16 problems with the maximum assignment level of 2 to 5 on the four datasets. The parameter values for LR algorithm and genetic algorithm are shown in Tables 2 and 3, respectively. All computations were implemented in MATLAB R2017b for Windows $\times 64$ on the laptop with an Intel i5-7200U CPU and 8 GB RAM. And solutions of (LUDM) and linear models in the iterative process of LR were obtained by invoking Gurobi 8.1.0. Table 4 lists the result of three approaches. The column "Objective" is the objective function value at the termination of the algorithm; "Gap$_G$" and "Gap$_L$" are the gaps between upper and lower bounds of the solver and LR algorithm, respectively; and the column "Time" is the running time.

As can be seen from Table 4, it is difficult for Gurobi to solve moderately sized problems in the given time, although it can obtain the exact solution and solve the small-scale problem fast. When the problem size increases, both Gap$_G$ and running time increase significantly. And the custom-designed LR algorithm can solve the model in a relatively short time. The gap between its result and the exact solution is very small or even zero. However, for large-scale problems, LR algorithm cannot solve them within the given time limit. The genetic algorithm solves all problems effectively and obtains the near-optimal solution. Although the gap between the near-optimal solution and the exact solution may be relatively large, its solution time is the shortest of three approaches except for the four small-scale problems with $R = 2$. Therefore, in practice, we can solve small-scale problems directly by general solvers; the LR algorithm is used for medium-scale problems; and for large-scale problems that require to be solved in a given time, genetic algorithm is recommended.

5.2. Managerial insights

5.2.1. The benefit of considering uncertain disruptions

When the risk of uncertain disruptions is not considered, we don't need to assign customers to multiple service providers to mitigate it. The deterministic model proposed by Daskin (1995) is used to obtain the optimal network design under this normal scenario. But once retailers are disruptive, the demand assigned to them cannot be satisfied, which results in additional penalty costs. The unit penalty cost of unmet demand for each customer is set to the minimum unit cost of acquiring products from suppliers. Besides, to compare with the deterministic model, we calculate the total cost under the normal scenario for the optimal network obtained by our model. The dataset here consists of 2 suppliers, 21 candidate

Table 4. Algorithm results and comparison.

				Gurobi solver			Lagrangian relaxation			Geneticalgorithm	
$\vert S\vert$	$\vert J\vert$	$\vert I\vert$	R	Objective	Gap_G (%)	Time (s)	Objective	Gap_L (%)	Time (s)	Objective	Time (s)
2	31	70	2	**6,755,452**	0	50	6,776,497	0.95	96	6,765,474	211
2	31	70	3	**5,790,894**	0	1281 (924)	5,795,151	0.69	851	5,881,382	305
2	31	70	4	5,772,485	13.97	–	**5,754,928**	0.89	2426	5,756,825	413
2	31	70	5	5,772,225	15.34	–	**5,744,068**	0.49	17,173	5,752,443	528
4	31	70	2	**6,421,872**	0	42	**6,421,872**	0.65	94	6,423,098	206
4	31	70	3	**5,643,545**	0	1593 (937)	**5,643,545**	0.61	591	5,678,933	308
4	31	70	4	5,617,821	12.87	–	**5,565,260**	0.59	2100	5,591,137	414
4	31	70	5	5,651,482	18.38	–	**5,565,260**	0.49	8341	5,589,748	509
3	70	338	2	**9,494,005**	0	764	9,502,265	1.00	4627	9,555,630	4051
3	70	338	3	7,950,745	5.26	–	**7,901,249**	0.95	31,993	7,921,295	5031
3	70	338	4	8,512,974	21.93	–	8,156,984	9.83	–	**7,677,167**	6951
3	70	338	5	14,287,436	53.69	–	13,689,974	48.55	–	**7,599,135**	9164
5	70	338	2	**9,063,155**	0	718	**9,063,155**	0.70	3887	9,110,210	3325
5	70	338	3	7,883,320	4.56	–	**7,727,928**	0.88	34,125	7,740,831	5221
5	70	338	4	8,440,781	19.01	–	7,978,054	9.11	–	**7,529,624**	7024
5	70	338	5	14,166,274	46.54	–	13,389,672	44.97	–	**7,525,883**	9219

Notes. (a) "–" means the algorithm is terminated when the running time reaches 36,000 s. (b) Solution time improved by Proposition 3.1 in parentheses. (c) For each problem, the best result is marked in bold.

Table 5. Performance comparison between the deterministic model and the model considering uncertain disruptions.

		C_N	Δ_N	$\%\Delta_N$	C_D	Δ_D	$\%\Delta_D$
Deterministic model		3,649,648	–	–	7,117,298	–	–
Our model	$R=2$	3,941,730	292,082	8.00	5,465,501	−1651797	−23.21
	$R=3$	3,803,441	153,793	4.21	5,341,665	−1775633	−24.95
	$R=4$	3,803,441	153,793	4.21	5,341,406	−1775892	−24.95

Table 6. Comparison of decisions under different belief degrees.

	$R=2$			$R=3$			$R=4$		
Belief degree of disruption risk	C	n_R	ρ_D (%)	C	n_R	ρ_D (%)	C	n_R	ρ_D (%)
0	3,649,648	6	2.51	3,649,648	6	2.51	3,649,648	6	2.51
$0.5\alpha_n$	4,703,615	6	3.54	4,562,155	7	2.93	4,562,025	7	2.93
α_n	5,465,500	6	7.08	5,341,665	7	5.74	5,341,406	7	5.74
$1.5\alpha_n$	6,189,427	5	10.25	6,102,734	7	8.61	6,102,734	7	8.61
$2\alpha_n$	6,893,616	4	13.17	6,813,296	5	12.06	6,813,296	5	12.06

retailers and 40 customers in Section 5.1, and the maximum assignment level is up to 4, ensuring that the exact solution can be obtained. Table 5 summarises the performance comparison of the two models, where C_N is the total cost under the normal scenario, C_D is the total cost under the disruption risk, Δ_N ($\%\Delta_N$) is the cost increase (rate) of our model compared with the deterministic model under the normal scenario, and Δ_D ($\%\Delta_D$) is the cost increase (rate) under the disruption risk.

Table 5 shows that considering uncertain disruptions in the design phase can reduce costs significantly compared to the deterministic model. And even under the normal scenario, it will not increase the cost too much. Particularly, when the maximum assignment level is equal to 4, we can reduce the cost under the disruption risk by 24.95% while the normal cost only increases by 4.21%.

5.2.2. Decisions under different belief degrees

In order to study the influence of the belief degree of uncertain disruptions on the decision result, we set them to 0 (deterministic model), 0.5, 1, 1.5 and 2 times the value given before. For the small-scale dataset in Section 5.2.1, the results of the maximum assignment level equal to 2, 3, and 4 are listed in Table 6. And we denote the minimum total cost, the number of selected retailers and the expected demand coverage of dummy supplier as C, n_R and ρ_D, respectively.

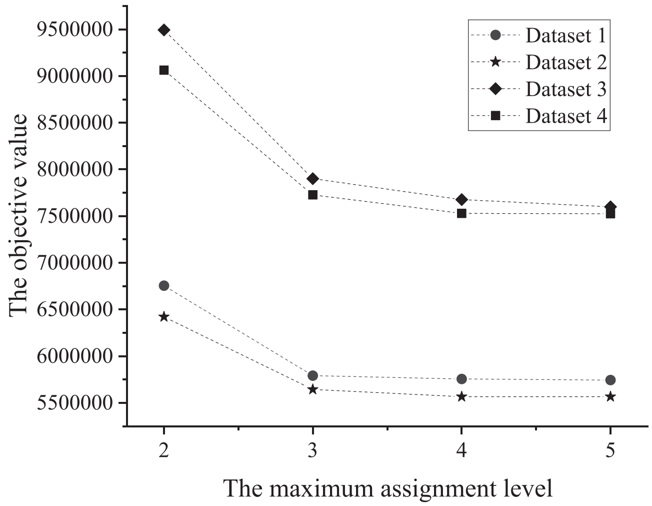

Figure 6. The change of objective value with R on different datasets.

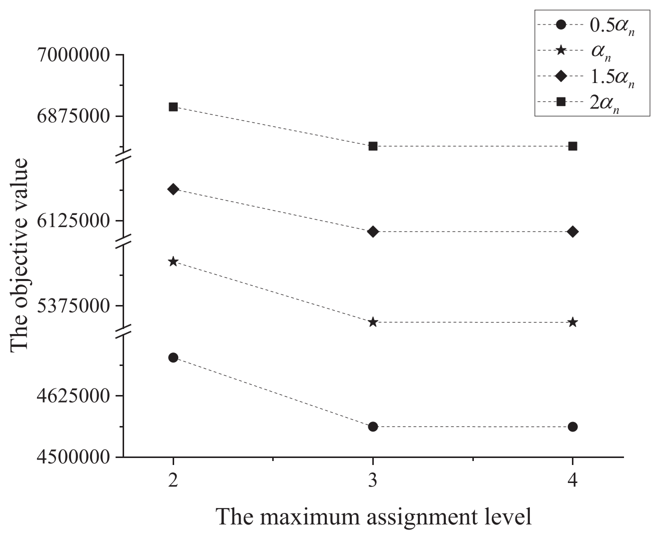

Figure 7. The change of objective value with R at different belief degrees.

As shown in Table 6, for the same R, it always takes more cost to mitigate the risk of uncertain disruptions as the belief degree increases. And the number of selected retailers n_R increases first and then decreases. This is because the increase in belief degrees within a certain range can be mitigated by selecting more retailers compared to the case of no disruption or very low risk. However, when the belief degree is high, more retailers are not able to mitigate it well, but increase the fixed cost. In this case, we should reduce the number of retailers and serve customers more through direct transportation from suppliers. In addition, with the increase of risk, the demand coverage of direct service provided by suppliers also increases significantly.

5.2.3. The setting of maximum assignment level

From Table 4, we also see that the minimum total cost decreases with the increase of maximum assignment level R for all datasets, as stated in Proposition 3.1. And Table 6 shows such results under different belief degrees. We illustrate the relationship between the total cost and R in these numerical examples in Figures 6 and 7.

For these examples, the change of total cost is most obvious when the maximum assignment level changes from 2 to 3, and the subsequent decrease is not significant. This can provide a basis for decision-makers to set a reasonable maximum assignment level, which makes the planning scheme not only cope well with the impact of uncertain disruptions to reduce the cost, but also prevent retailers assigned to customers from failing to meet demands in a timely manner when R is too large.

6. Conclusion

In this paper, we study the design problem of a three-echelon supply chain network consisting of suppliers, customers and candidate retailers with the risk of uncertain disruptions. An uncertain nonlinear mixed integer programming model is formulated. In the model, we consider disruptions of the retailer as uncertain events and obtain the belief degree of disruptions based on experts' estimation, which is suitable for the situation where the historical data of disruptions is not enough and valid. The proposed model selects retailers, assigns retailers to suppliers and assigns customers to retailers and suppliers to meet customer demands and minimise the total cost. In addition, by analysing the property of the model, we put forward two propositions about the relationship between the maximum assignment level and the optimal objective value, and the optimal assignment of retailers and customers to suppliers. These two propositions are used to accelerate the solution process and simplify the model. And the simplified model is then linearised into a mixed integer linear model by some techniques. After that, we develop the LR algorithm for solving the linearised model and the genetic algorithm for solving the simplified model, which are applicable to different scale problems.

One limitation of our model is that the capacity of facilities is not considered. While this is a common assumption in SCND models, it may not be realistic in practice. Therefore, our model is more suitable for the case that capacities don't affect the design significantly, or that the change in capacities mainly affects variable costs, which can be included in e_{sj} and d_{in}. Also, we assume that the demand is deterministic, which requires decision-makers to know it accurately. This is not strange in make-to-order manufacturing industries. But when the demand is highly uncertain or affected by disruptions, our model may not be appropriate.

These limitations also suggest opportunities for future research. First, it is possible to introduce capacity constraints as model parameters or decision variables. And uncertain disruptions may also lead to partial rather than full facility failures. Second, when disruptions are uncertain events, the demand is not deterministic. The uncertainty of demand may arise from exogenous factors or depend on disruptions. In addition to these, our model considers retailer disruptions under a specific supplier setting, so that more supplier disruption scenarios can be taken into account on this basis. Therefore, another possible extension is to consider uncertain disruptions of both suppliers and retailers.

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