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ORIGINAL RESEARCH

New models of supply chain network design by different decision criteria under hybrid uncertainties

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Abstract
Supply chain network design is to determine factories and distribution centers to produce and distribute products, in order to satisfy the demand of customers to some extent. Supply chain network design plays an important strategic role in supply chain management and faces kinds of uncertainties, such as uncertain demand and cost. In order to deal with the impact of uncertain factors, this paper considers the supply chain network design under hybrid uncertainties, namely, the objective randomness of retailers' demand and the subjective uncertainties of operating costs. We construct three models to handle managers' different needs. They are the expected cost minimization model that minimizes the expected total cost, the β-cost minimization model that minimizes the β-cost which means that the chance measure of the actual cost not exceeding the cost is not less than the confidence level β, and the chance measure maximization model that maximizes the chance measure of the actual cost not exceeding the given cost. This paper then transforms them into deterministic classes by uncertainty theory, and several numerical examples are presented to verify the validity of the models and algorithm.

Keywords Supply chain network design · Uncertainty theory · Expected cost model · β-Cost model · Chance maximization model

1 Introduction

Supply chain network design problem is to determine the optimal configuration structure, so that the whole network has low operating costs in the case of satisfying customers' demand to some extent. And it plays an important role in supply chain management, so many researches studied it widely. Tang et al. (2016) reviewed the literature in the field of supply chain network design according to the objectives, product and the demand. Wang et al. (2011) investigated a multi-objective optimization model for supply chain network design in a green supply chain network. Baud-Lavigne et al. (2014) discussed how to conduct the supply chain network design with the goal of optimal carbon emissions under the environmental constraints. Torabi et al. (2016) put forward that minimizing the cost and maximizing the reliability are the dual goals of the supply chain network design. By taking into account tree types of customers, Coskun et al. (2016) proposed a goal programming model for supply chain network design. Jeihoonian et al. (2016) considered the legislative recovery objective and the various quality levels of returns, designed a closed-loop supply chain network with used durable products with generic modular structures, and proposed a MIP model to optimize the network.

However, due to the existence of complex information interaction, supply chain network design is subjected to uncertainties. In the face of uncertain demand, transporting cost, etc., if we regard them as deterministic factors simply, the design of supply chain network is often difficult to reflect the actual situation and may mislead the decision maker's judgments. Meanwhile, by summarizing and reviewing the literature on supply chain network design, Eskandarpour et al. (2015) considered that supply chain network design was often carried out under deterministic environment, but it was better to fully consider uncertainties in the process of network design. Therefore, there were classes of literature in the domain of supply chain network design considering the randomness on the demand side. In order to solve large-scale

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supply chain network design problems, Santoso et al. (2005) proposed a stochastic programming model and solution algorithm that integrated the sampling strategy, sample average approximation scheme and accelerated Benders decomposition algorithm. Lin and Wang (2011) investigated the randomness of supply and demand in the supply chain network design using the embedded terminal mitigation strategy. Based on the robust optimal policy, Pishvaee et al. (2011) proposed a robust optimal model for the supply chain network design to deal with random parameters. Amin and Zhang (2013) constructed a total cost minimization mixed integer linear programming model that can simultaneously handle demand and return uncertainty in a closed-loop supply chain network. Tsao et al. (2016) performed that the introduction of radio frequency identification in the supply chain could indirectly affect the supply chain network design by influencing the inventory inaccuracy and the randomness of demand. Soleimani et al. (2016) proposed a MILP structure for a multi-period, multi-product closed-loop supply chain network with stochastic demand and price, and obtained the optimal solution by three comparison criteria of mean, standard deviation and coefficient of variation. Golpra et al. (2017) presented a new multi-objective, multi-echelon green supply chain network design problem, and used the Conditional Value at Risk method and the uncertainty set approach to deal with product demand uncertainty and environmental uncertainty respectively. To characterize two types of uncertainties simultaneously, Keyvanshokooh et al. (2016) considered the randomness of transportation cost and gave the polyhedral uncertainty set of uncertain demand and return, and developed a hybrid robust-stochastic programming approach. Golpra (2017) constructed an uncertain supply chain network design problem model, which employed risk averseness of retailers to obtain more realistic model regarding demand and avoided any assumptions about the distribution.

In addition to considering the objective randomness of the demand side, there are also uncertainties existing in the process of product providing. It is well known that the probability distribution is obtained from historical data. Since we can get the corresponding information of customers' demand according to the market survey, and the distribution of demand can be acquired through the statistical analysis of historical data, so that it is reasonable to consider the demands as random variables. Nevertheless, various operating costs in the supply chain, such as founding costs, transporting costs, ordering costs and holding costs, are uncertain in different kinds of circumstances (e.g., employee strike, weather condition, artificial destruction). Due to the lack of historical data, various operating costs can be only estimated relying on subjective judgments of managers or experts. For example, "the establishment of distribution center costs about 2 million yuan", "the ordering cost is between 2000 and 3000 yuan", "what is the possibility that transporting cost is 2 million yuan?".

A lot of literature has tried to analyze the supply chain network design from the perspective of subjective uncertainties through fuzzy set theory. Pishvaee and Torabi (2010) treated demand, delivery time, cost and capacity as fuzzy variables when designing the supply chain network design in a closed-loop network. Based on different criteria, Qin and Ji (2010) proposed three types of fuzzy programming models to deal with the problem of product recycling network design with uncertainty, and designed a hybrid intelligent algorithm combining fuzzy simulation and genetic algorithm to solve it. Pishvaee and Razmi (2012) put forward a multi-objective fuzzy mathematical programming model to solve the endogenous uncertainty of the input data for the supply chain network design. Mohammadi et al. (2014) solved sustainable hub location problem by fuzzy programming in the absence of historical data.

With the development of uncertainty theory which is founded by Liu (2007), the subjective uncertainties can be described more appropriately via uncertainty theory. Gao and Qin (2016) treated the travel time as uncertain variables instead of random or fuzzy ones in a p-hub center location problem. Chen et al. (2017) analyzed the supply chain pricing and effort decision of single manufacturer and single retailer, and established one centralized and three decentralized game models based on the expected value criterion that describes consumer demand, manufacturing cost and sales effort cost as uncertain variables. Gao et al. (2017) defined the new uncertain Shapley value and then used the uncertain core to solve the profit allocation problem of the supply chain alliance with transferable payoffs as uncertain variables. Ke and Liu (2017) gave closed-form expressions for equilibriums of the dual-channel supply chain in centralized and decentralized cases, and analyzed the influence of parameter uncertainty distributions on supply chain profits. Cheng et al. (2018) presented the different impacts of store-brand introduction on supply chain players and the entire channel under different power structures. Chen et al. (2018) established a multi-stage model including an original equipment manufacturer and its competitive original design manufacturer (ODM), and analyzed their coopetition decisions after ODM starts to produce its own-brand products, and the impact of decisions on their preferred pricing timing.

However, the problem of supply chain network design is under hybrid uncertainties. In this paper, we consider the objective randomness of consumers' demand and the subjective uncertainty of various operational costs including founding cost, transportation cost, inventory costs, and safety stock cost. According to the different decision-making needs of managers, we construct three models of the supply chain network design problem in the hybrid uncertain environment, which are further transformed into deterministic forms via the uncertainty theory, and give numerical examples to verify their effectiveness.

The rest of the paper is structured as follows. Some basic concepts and properties of uncertainty theory are given in Sect. 2. Section 3 describes the supply chain network design problem in a hybrid uncertain environment. Section 4 presents three different uncertain programming models, and gives the corresponding equivalent classes. This article also considers a case and solves it using genetic algorithm in Sect. 5. The last section gives some important conclusions and management suggestions.

2 Preliminaries

The uncertainty theory was founded by Liu (2007). In the following, the knowledge of uncertainty theory applied in this paper will be introduced.

Let Γ be a nonempty set, Ł be a σ-algebra over Γ, and each element Λ in Ł is called an event. Uncertain measure is defined as a function from Ł to [0,1]. In detail, Liu (2007) gave the concept of uncertain measure as follows.

Definition 1 (Liu 2007) The set function $\mathcal{M}$ is called an uncertain measure if it satisfies:

Axiom 1 $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ;

Axiom 2 $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ;

Axiom 3 For any countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

Besides, in order to provide the operational law, Liu (2009) defined the product uncertain measure on the product σ-algebra $\mathcal{L}$ as follows.

Axiom 4 Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for k=1,2,$\cdots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\},$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for k=1, 2$\cdots$, respectively. Based on the concept of uncertain measure, we can define an uncertain variable.

Definition 2 (Liu 2007) An uncertain variable is a function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{\xi \in B\}$ is an event for any Borel set B of real numbers.

Definition 3 (Liu 2007) The uncertainty distribution of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}, \quad \forall x \in \Re.$$

Definition 4 (Liu 2010) If the inverse function $\Phi^{-1}(\alpha)$ exists and is unique in the interval $\alpha \in (0,1)$ for any uncertain variable ξ, we call $\Phi(x)$ is regular. Then we call the inverse function $\Phi^{-1}(\alpha)$ as the inverse uncertainty distribution of ξ.

An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \Re,$$

denoted by $\mathcal{N}(e,\sigma)$, where e and σ are real numbers with $\sigma > 0$. The inverse uncertainty distribution of normal uncertain variable $\mathcal{N}(e,\sigma)$ is

$$\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}.$$

Theorem 1 (Liu and Ha 2010) *Assume the objective function $f(x,\xi_1,\xi_2,\ldots,\xi_n)$ is strictly increasing with respect to $\xi_1,\xi_2,\ldots,\xi_m$ and strictly decreasing with respect to $\xi_{m+1},\xi_{m+2},\ldots,\xi_n$. If $\xi_1,\xi_2,\ldots,\xi_n$ are dependent uncertain variables with regular uncertainty distributions $\Phi_1,\Phi_,\ldots,\Phi_n$, respectively, then the expected objective function $E[f(x,\xi_1,\xi_2,\ldots,\xi_n)]$ is equal to*

$$\int_0^1 f(x,\Phi_1^{-1}(\alpha),\ldots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\Phi_n^{-1}(1-\alpha))\mathrm{d}\alpha.$$

Theorem 2 (Liu 2007) *Let ξ_1 and ξ_2 be independent normal uncertain variables $\mathcal{N}(e_1.\sigma_1)$ and $\mathcal{N}(e_2.\sigma_2)$, respectively. Then the $\xi_1+\xi_2$ is also a normal uncertain variable $\mathcal{N}(e_1+e_2.\sigma_1+\sigma_2)$, i.e.,*

$$N(e_1,\sigma_1) + N(e_2,\sigma_2) = N(e_1+e_2,\sigma_1+\sigma_2).$$

The product of a normal uncertain variable $\mathcal{N}(e_1.\sigma_1)$ and a scalar number $k \ge 0$ is also a normal uncertain variable $N(ke_1,k\sigma_1)$, i.e.,

$$kN(e_1,\sigma_1) = N(ke_1,k\sigma_1).$$

3 Supply chain network design problem

3.1 Problem introduction

Supply chain network design is to determine which factories and distribution centers we should choose to produce and distribute different kinds of products, in order to satisfy the demand of customers. In a supply chain system, distribution centers order different kinds of products from the manufacturers. We assume that every distribution center follows the (Q, R) ordering policy, the lead time of ordering is fixed, no delayed delivery, and the demand in the lead time is independent. In reality, there may be a large number of candidates for distribution centers. There is the necessity that we should choose some of appropriate distribution centers as candidates at the very beginning by comprehensive evaluation, such as retailers' satisfaction, traffic convenience, etc. According to Fig. 1, this article aims at the following: (1) which distribution centers should be opened in a series of candidates for different kinds of products; (2) each distribution center should receive what kinds of products from which manufacturer and deliver them to which retailer.

3.2 Symbols introduction

This paper considers the supply chain design problem under hybrid uncertainties, namely the consumer demand's objective randomness and operating costs' subjective uncertainties. We give the following symbols to model the problem.

Indexes:

m: Manufacturers, $m = 1, 2, \dots, M$;
n: Different kinds of products produced by manufacturers, $n = 1, 2, \dots, N$;

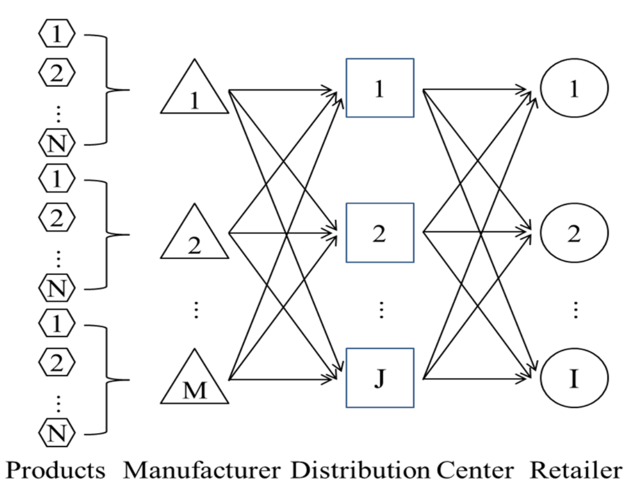

Fig. 1 Sketch map of supply chain network design

i: Retailers, $i = 1, 2, \dots, I$;
zj: Candidates for distribution centers, $j = 1, 2, \dots, J$.
Stochastic variables:
θ_{in}: Daily demand of retailer i for product n, which can be obtained through customers' historical data;
μ_{in}: Mean of the daily demand of retailer i for product n;
σ_{in}: Deviation of the daily demand of retailer i for product n.
Uncertain variables:
ξ_j: The founding cost of each distribution center j;
η_{jn}: The inventory cost of the distribution center j for product n;
τ_{jn}: The ordering cost of the distribution center j for product n;
ζ_{mjn}: The transporting cost of the distribution center j for product n from manufacturer m;
δ_{ijn}: The transporting cost of the distribution center j for product n to retailer i.
Parameters:
l_j: The lead time of distribution center j;
R_j: The volume capacity of the distribution center j;
β: The confidence level that the decision maker sets for the entire supply chain's total cost;
γ: The service level for the distribution center, which means the probability of no shortage of order in the leading period;
d: days for a year;
t_γ: The key value that satisfying $Pr(t \leq t_\gamma) = \gamma$.
Decision variables:
X_j: 0 represents for not choosing to set up a distribution center j, 1 indicates for opening;
Y_{mjn}: 0 represents for manufacturer m delivering products n to distribution center j, 1 indicates for the opposite.
Z_{ijn}: 0 represents for distribution center j delivering products n to retailer i, 1 indicates for the opposite.

3.3 Cost structure and capacity constraint

The programming target of the supply chain network design is no longer a single enterprise but the entire network. In order to be concise, we let $X = \{X_j\}, Y = \{Y_{mjn}\}, Z = \{Z_{ijn}\}$. Then the total cost $C(X, Y, Z)$ of the network consists of four parts consisting of founding cost, transporting cost, inventory cost and safety stock cost. Since we consider operating costs as uncertain variables, the total cost $C(X, Y, Z)$ is also an uncertain variable. It is shown as follows,

$$C(X, Y, Z) = C_{founding} + C_{transporting} + C_{inventory} + C_{safety}.$$

Founding cost $C_{founding}$ refers to the total construction and maintenance costs when establishing distribution center j. To simplify the problem, we assume the cost is linear, that is the founding cost increases as the number of distribution centers increase. We can conclude,

$$C_{founding} = \sum_{j\in J} \xi_j X_j. \tag{3.1}$$

Transporting cost $C_{transporting}$ refers to the cost transporting products from manufacturer m through distribution center j to reach retailer i. This paper supposes that the two stages of transportation costs are linear, which means the transporting cost increases as the transportation volume increases.

$$C_{transporting} = \sum_{i\in I}\sum_{j\in J}\sum_{m\in M}\sum_{n\in N}(\zeta_{mjn}Y_{mjn} + \delta_{ijn}Z_{ijn})E[\theta_{in}]. \tag{3.2}$$

Inventory cost $C_{inventory}$ refers to the total ordering and holding cost according to the (Q, R) ordering strategy of distribution center j for product n. Combined with the classic EOQ model, the economic order quantity and the inventory cost, respectively, are as follows,

$$Q^*_{jn} = \sum_{i\in I}\sum_{j\in J}\sum_{n\in N}\sqrt{2dE[\tau_{jn}]E[\theta_{in}]Z_{ijn}/E[\eta_{jn}]}, \tag{3.3}$$

$$C_{inventory} = \sum_{i\in I}\sum_{j\in J}\sum_{n\in N}\sqrt{2E[\eta_{jn}\tau_{jn}]E[\theta_{in}]Z_{ijn}}. \tag{3.4}$$

Safety stock cost C_{safety} refers to the extra holding cost that the distribution center j keeps in order to reduce the impact of forecasting inaccuracies while balancing the demand at the same time. Compared to the expression of safety stock cost in classic EOQ model $t_\gamma\sqrt{V[\theta_i]}$, this paper no longer counts each retailer's variance in ordering lead time but to replaces it with each distribution center's variance in ordering lead time as a result of the implementation of risk-sharing strategy for distribution centers. The safety quantity and the safety stock cost are as follows,

$$S_{jn} = \sum_{i\in I}\sum_{j\in J}\sum_{n\in N} t_\gamma\sqrt{l_j V[\theta_{in}]Z_{ijn}}. \tag{3.5}$$

$$C_{safety} = \sum_{i\in I}\sum_{j\in J}\sum_{n\in N} \eta_{jn} t_\gamma\sqrt{l_j V[\theta_{in}]Z_{ijn}}. \tag{3.6}$$

The distribution center obeys (Q, R) ordering policy, and the expected demand in the lead time $K_{jn} = \sum_{i\in I}\sum_{j\in J}\sum_{n\in N} l_j E[\theta_{in}]Z_{ijn}$. The capacity constraint is as follows,

$$\sum_{n\in N}\left(Q^*_{jn} + S_{jn} + K_{jn}\right) \le R_j. \tag{3.7}$$

4 New models for supply chain network design

4.1 Expected cost minimization model

In fact, many decision makers are concerned about long-term interests. Therefore, this paper intends to conduct an expected cost minimization model which minimizes the entire expected cost of the supply chain network with capacity constraints.

$$\begin{cases} \min E[C(X,Y,Z)] \\ \text{subject to:} \\ \qquad \sum_{j\in J}\sum_{n\in N} Y_{mjn} \ge 1 & (A1) \\ \qquad \sum_{m\in M} Y_{mjn} \le X_j & (A2) \\ \qquad Z_{ijn} \le X_j & (A3) \\ \qquad \sum_{m\in M} Y_{mjn} \ge Z_{ijn} & (A4) \\ \qquad \sum_{j\in J} Z_{ijn} \ge 1 & (A5) \\ \qquad \sum_{n\in N}(Q^*_{jn} + S_{jn} + K_{jn}) \le R_j & (A6) \\ \qquad X_j, Y_{mjn}, Z_{ijn} = 0, 1. & (A7) \end{cases} \tag{4.1}$$

In the model (4.1), the objective function is to minimize the expected total cost of the supply chain, which is the sum of the founding cost, the transporting cost, the inventory cost and the safety stock cost. The constraint (A-1) represents that each manufacturer must produce at least one kind product. The constraint (A-2) represents each manufacturer delivers productes to the distribution center only when the distribution center is established. The constraint (A-3) indicates each retailer could receive products delivering products from the distribution center only when the distribution center is established. The constraint (A-4) indicates the oueput of a distribution center is no less than its input. The constraint (A-5) means that each retailer has at least one distribution center to deliver each product. The constraint (A-6) shows that the distribution center meets the capacity limits. And the constraints (A-7) means the value for the decision variable can only be 0 or 1.

Theorem 3 *Let $\xi_j, \eta_{jn}, \tau_{jn}, \zeta_{mjn}, \delta_{ijn}$ be independent uncertain variables with regular uncertainty distributions for all i, j and n. Then model (4.1) is equivalent to the following deterministic programming problem,*

$$\begin{cases} \min E[C(X,Y,Z)]^* \\ \text{subject to: Constraints (A1)-(A6)} \end{cases} \tag{4.2}$$

where

$$E[C(X,Y,Z)]^* = \int_0^1 (\Phi_1^{-1}(\alpha)X_j + \sum_{i\in I}(\Phi_4^{-1}(\alpha)Y_{mjn} + \Phi_5^{-1}(\alpha)Z_{ijn})E[\theta_{in}]Y_{ij} + \sqrt{2E[\eta_{jn}\tau_{jn}]E[\theta_{in}]Z_{ijn}} + \Phi_3^{-1}(\alpha)t_\gamma\sqrt{l_j V[\theta_{in}]Z_{ijn}})\mathrm{d}\alpha.$$

Proof 1 Because the total cost is strictly increasing with respect to various operating costs, namely $C(X, Y, Z)$ is strictly increasing with respect to $\xi_j, \eta_{jn}, \tau_{jn}, \zeta_{mjn}, \delta_{ijn}$. According to Theorem 1, we can obtain that $E[C(X,Y,Z)]^*$ is the objective function. The proof is completed. □

4.2 β-cost minimization model

In reality, policymakers (especially the risk-averse decision makers) tend to set a target cost $\bar{C}$, hoping that the actual cost will not be higher than the target cost for a given confidence level β.

Definition 5 The β-cost is defined as

$$\{\bar{C}|\mathcal{M}\{C(X,Y,Z) \le \bar{C}\} \ge \beta\},$$

where β is a predetermined confidence level and $\beta \in [0,1]$.

When other parameters are determined, the decision maker can obtain different designs for distribution center allocations under a given confidence level value β. It is often more practical to minimize the β-cost, which is because the distribution center is not designed for average or extreme situations, such as the train station will not be built based on the average traffic (excessive low capacity) and the airport is not established according to the peak flow in the history (excessive high capacity). Based on the analysis of the actual management situation above, we can thus build a β-cost minimization model with capacity constraints.

$$\begin{cases} \min \bar{C} \\ \text{subject to:} \\ \quad \mathcal{M}\{C(X,Y,Z) \le \bar{C}\} \ge \beta & (B1) \\ \quad \sum_{j\in J}\sum_{n\in N} Y_{mjn} \ge 1 & (B2) \\ \quad \sum_{m\in M} Y_{mjn} \le X_j & (B3) \\ \quad Z_{ijn} \le X_j & (B4) \\ \quad \sum_{m\in M} Y_{mjn} \ge Z_{ijn} & (B5) \\ \quad \sum_{j\in J} Z_{ijn} \ge 1 & (B6) \\ \quad \sum_{n\in N}(Q^*_{jn} + S_{jn} + K_{jn}) \le R_j & (B7) \\ \quad X_j, Y_{mjn}, Z_{ijn} = 0, 1. & (B8) \end{cases} \tag{4.3}$$

For the model (4.3), all other constraints have the same meaning as the model (4.1) does except for constraint (B-1), which means the actual cost will not be higher than the target cost for a given confidence level β.

Theorem 4 *Let $\xi_j, \eta_{jn}, \tau_{jn}, \zeta_{mjn}, \delta_{ijn}$ be independent uncertain variables with normal uncertain functions $N(e_{\xi_j}, \sigma_{\xi_j}), N(e_{\eta_{jn}}, \sigma_{\eta_{jn}}), N(e_{\tau_{jn}}, \sigma_{\tau_{jn}}), N(e_{\zeta_{mjn}}, \sigma_{\zeta_{mjn}})$ and $N(e_{\delta_{ijn}}, \sigma_{\delta_{ijn}})$ for all i, j and n. Then model (4.3) is equivalent to the following deterministic programming problem,*

$$\begin{cases} \min e^* + \frac{\sigma^*\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} \\ \text{subject to: Constraints (B2)-(B8)} \end{cases} \tag{4.4}$$

where

$$e^* = \sum_{j\in J}\left(e_{\xi_j}X_j + \sum_{i\in I}\sum_{n\in N}\sum_{m\in M}\left(e_{\zeta_{mjn}}Y_{mjn} + e_{\delta_{ijn}}Z_{ijn}\right)E[\theta_{in}] + \sqrt{2E[\eta_{jn}\tau_{jn}]\sum_{i\in I}E[\theta_{in}]Z_{ijn}} + e_{\eta_{jn}}t_\gamma\sqrt{l_j\sum_{i\in I}V[\theta_{in}]Z_{ijn}}\right),$$

$$\sigma^* = \sum_{j\in J}\left(\sigma_{\xi_j}X_j + \sum_{i\in I}\left(\sigma_{\zeta_j} + \sigma_{\delta_{ij}}\right)E[\theta_{in}]Z_{ijn} + \sigma_{\eta_{jn}}t_\gamma\sqrt{l_j\sum_{i\in I}V[\theta_{in}]Z_{ijn}}\right).$$

***Proof* 2** Since $\xi_j, \eta_{jn}, \tau_{jn}, \zeta_{mjn}, \delta_{ijn}$ are normal uncertain variables, using Theorem 1, we obtain that the total cost $C(X, Y, Z)$ is a normal uncertain variable with normal uncertainty distribution $N(e^*, \sigma^*)$. Thus, the objective of model is equivalent to minimizing $e^* + \frac{\sigma^*\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha}$. □

4.3 Chance measure maximization model

In some cases, the managers will set a budget C_0 and probably hope the chance of satisfying the event that the total cost $C(X, Y, Z)$ is less than a budget could be as large as possible. Therefore, the distribution center may be set up with the greatest chance of letting the total cost be no more than a given cost.

Definition 6 The chance measure is defined as

$$\mathcal{M}\{C(X,Y,Z) \le C_0\},$$

where C_0 is a predetermined total cost.

This paper defines this situation as chance measure maximization, and the corresponding model is conducted as follows,

$$\begin{cases} \max \mathcal{M}\{C(X,Y,Z) \le C_0\} \\ \text{subject to:} \\ \qquad \sum_{j\in J}\sum_{n\in N} Y_{mjn} \ge 1 & (C1) \\ \qquad \sum_{m\in M} Y_{mjn} \le X_j & (C2) \\ \qquad Z_{ijn} \le X_j & (C3) \\ \qquad \sum_{m\in M} Y_{mjn} \ge Z_{ijn} & (C4) \\ \qquad \sum_{j\in J} Z_{ijn} \ge 1 & (C5) \\ \qquad \sum_{n\in N}\left(Q^*_{jn} + S_{jn} + K_{jn}\right) \le R_j & (C6) \\ \qquad X_j, Y_{mjn}, Z_{ijn} = 0, 1. & (C7) \end{cases} \tag{4.5}$$

For the model (4.5), all other constraints have the same meaning as the model (4.1).

Theorem 5 *Let $\xi_j, \eta_{jn}, \tau_{jn}, \zeta_{mjn}, \delta_{ijn}$ be independent uncertain variables with normal uncertain functions $N(e_{\xi_j}, \sigma_{\xi_j})$, $N(e_{\eta_{jn}}, \sigma_{\eta_{jn}})$, $N(e_{\tau_{jn}}, \sigma_{\tau_{jn}})$, $N(e_{\zeta_{mjn}}, \sigma_{\zeta_{mjn}})$ and $N(e_{\delta_{ijn}}, \sigma_{\delta_{ijn}})$ for all i and j. Then model (4.5) is equivalent to the following deterministic programming problem,*

$$\begin{cases} \max \left(1 + \exp\left(\frac{\pi(e^* - C_0)}{\sqrt{6}\sigma^*}\right)\right)^{-1} \\ \text{subject to: Constraints (C1)-(C7)} \end{cases} \tag{4.6}$$

***Proof* 3** Since $\xi_j, \eta_{jn}, \tau_{jn}, \zeta_{mjn}, \delta_{ijn}$ are normal uncertain variables, using Theorem 2, we obtain that the total cost $C(X, Y, Z)$ is a normal uncertain variable with normal uncertainty distribution $N(e^*, \sigma^*)$. Thus, the objective of model is equivalent to minimizing $\left(1 + \exp\left(\frac{\pi(e^* - C_0)}{\sqrt{6}\sigma^*}\right)\right)^{-1}$.

In fact, model (4.2), (4.4) and (4.6) become non-liner 0–1 integer programming problems in the deterministic environment which can be solved by the classical genetic algorithm. □

5 Numerical examples

5.1 Case study

In this section, we consider a supply chain network containing two manufacturers, three distribution centers and four large retailers (taking Wal-Mart as an example), in which each manufacturer produces as well as delivers two different kinds of products. In the process, each distribution center makes orders according to the (Q, R) strategy. This paper supposes that the lead time for distribution centers is 15 days, the capacity constraint for three distribution centers are 80, 150 and 100 thousand respectively.

Based on the analysis of adequate historical data via the statistical method, this paper supposes that the daily demand for retailers are independent and obey normal distribution, which is shown as follows (Table 1):

Combined with uncertain statistics, we can determine the distribution of uncertain variables using the flowing steps (taking the founding cost of DC_1 for product 1 as an example):

Step 1 Design questionnaires and invite one or more experts to complete the questionnaire. For example, "What do you think is the maximum cost for the founding cost of DC_1", if the answer is 50, then an expert's experimental data (50, 0) is acquired. "What do you think is the minimum cost for the founding cost of DC_1", if the answer is 40, then an expert's experimental data (40, 0) is acquired. "Do you think what is the possibility that the distribution center DC_1's founding cost is less than 43", if the answer is 0.2,

Table 2 The mean and variance of uncertain founding costs (thousand yuan)

Founding cost	ξ_1	ξ_2	ξ_3
$(\mu_{\xi_j}, \sigma_{\xi_j})$	(45, 4)	(110, 10)	(75, 5)

Table 1 The mean and variance of stochastic demand

(μ_{11}, σ_{11})	(μ_{12}, σ_{12})	(μ_{21}, σ_{21})	(μ_{22}, σ_{22})	(μ_{31}, σ_{31})	(μ_{32}, σ_{32})	(μ_{41}, σ_{41})	(μ_{42}, σ_{42})
(200, 120)	(500, 305)	(300, 190)	(1000, 570)	(400, 280)	(1200, 820)	(100, 92)	(300, 235)

Table 3 The mean and variance of uncertain costs (thousand yuan)

Cost	η_{11}	η_{21}	η_{31}	η_{12}	η_{22}	η_{32}
$(\mu_{\eta_{jn}}, \sigma_{\eta_{jn}})$	(0.8, 0.05)	(0.2, 0.02)	(0.5, 0.03)	(0.5, 0.03)	(0.15, 0.02)	(0.3, 0.025)
Cost	τ_{11}	τ_{21}	τ_{31}	τ_{12}	τ_{22}	τ_{32}
$(\mu_{\tau_{jn}}, \sigma_{\tau_{jn}})$	(0.7, 0.07)	(0.6, 0.04)	(0.7, 0.07)	(0.6, 0.05)	(0.5, 0.03)	(0.5, 0.03)
Cost	ζ_{111}	ζ_{121}	ζ_{131}	ζ_{211}	ζ_{221}	ζ_{231}
$(\mu_{\zeta_{mjn}}, \sigma_{\zeta_{mjn}})$	(0.2, 0.02)	(0.25, 0.04)	(0.3, 0.03)	(0.3, 0.03)	(0.3, 0.03)	(0.2, 0.02)
Cost	ζ_{112}	ζ_{122}	ζ_{132}	ζ_{212}	ζ_{222}	ζ_{232}
$(\mu_{\zeta_{mjn}}, \sigma_{\zeta_{mjn}})$	(0.35, 0.04)	(0.3, 0.04)	(0.35, 0.03)	(0.35, 0.03)	(0.3, 0.04)	(0.4, 0.05)
Cost	δ_{111}	δ_{211}	δ_{311}	δ_{411}	δ_{121}	δ_{221}
$(\mu_{\delta_{ijn}}, \sigma_{\delta_{ijn}})$	(0.03, 0.02)	(0.02, 0.015)	(0.025, 0.02)	(0.03, 0.01)	(0.01, 0.005)	(0.015, 0.01)
Cost	δ_{321}	δ_{421}	δ_{131}	δ_{231}	δ_{331}	δ_{431}
$(\mu_{\delta_{ijn}}, \sigma_{\delta_{ijn}})$	(0.04, 0.01)	(0.03, 0.009)	(0.025, 0.01)	(0.015, 0.008)	(0.01, 0.01)	(0.005, 0.007)
Cost (i, j, n)	δ_{112}	δ_{212}	δ_{312}	δ_{412}	δ_{122}	δ_{222}
$(\mu_{\delta_{ijn}}, \sigma_{\delta_{ijn}})$	(0.04, 0.02)	(0.04, 0.02)	(0.03, 0.01)	(0.04, 0.01)	(0.03, 0.01)	(0.02, 0.01)
Cost	δ_{322}	δ_{422}	δ_{132}	δ_{232}	δ_{332}	δ_{432}
$(\mu_{\delta_{ijn}}, \sigma_{\delta_{ijn}})$	(0.04, 0.007)	(0.035, 0.01)	(0.04, 0.015)	(0.02, 0.01)	(0.02, 0.01)	(0.02, 0.005)

then an expert's experimental data (43, 0.2) is acquired. By using the questionnaire survey, the subjective data of experts can be obtained as (40, 0), (43, 0.2), (44, 0.4), (45, 0.6), (47 , 0.85), (48, 0.9), (50, 1).

Step 2 Assume that the distribution center's founding cost obeys the uncertain normal distribution. Using the least square method, based on the above subjective data, we can acquire the mean and variance of uncertain normal distribution for DC_1 are 45 and 4 respectively.

According to the above steps, we can get the uncertain normal distributions of the founding cost, the transporting cost, the ordering cost and the holding cost for DC_1, DC_2 and DC_3 respectively, and the mean and variance are as follows (Tables 2, 3):

Because there are different kinds of decision criteria, this paper conducts three numerical examples aiming at different kinds of decision criteria based on three proposed models. Since the service level (the probability of not out of stock) is of great importance of an enterprise short-term performance and has a great influence on the total cost in the long-time run, this paper first gives a fixed service level 0.95 to make decisions for managers, and then conducts the sensitivity analysis with the change of the service level in three circumstances respectively.

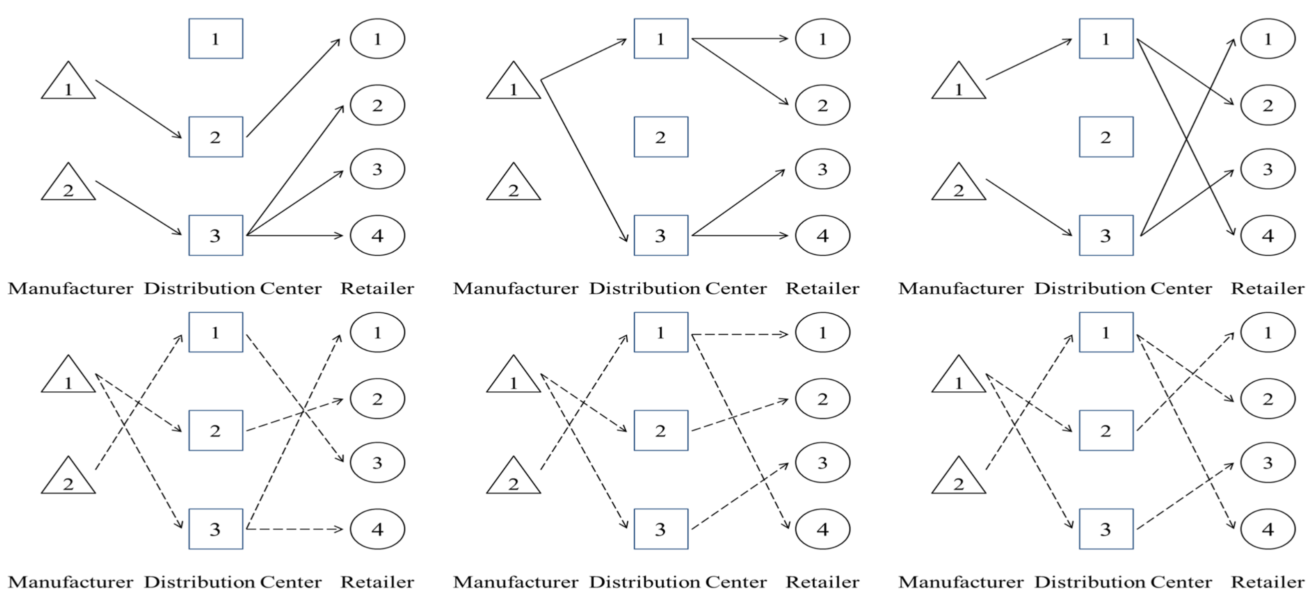

Fig. 2 The supply chain network design under different kinds of decision criteria for differnet products (left: the first model, middle: the second model, right: the third model)

We use genetic algorithm to solve the transformed deterministic models. In the algorithm, we use binary coding so that each individual in the population directly corresponds to a decision scheme, the fitness function is set as the objective function of each model, and constraints are used to limit the search space so that each individual is feasible. In addition, roulette is used to select individuals and we retain the best individual of the previous generation. Crossover and mutation are performed using one-point crossover and simple mutation, respectively. The maximum generation, population size, crossover probability and mutation probability are set to 500, 100, 0.6, and 0.1, respectively. All computations were done in the MATLAB R2017b environment on the laptop.

Figure 2 shows the supply chain network design under different kinds of decision criteria for different products. The above three figures give the supply chain network design for product 1 and the below three figures give the network design for product 2. Above all, we may easily find out that the different types of decision criteria can lead to significantly different supply chain network designs for different products. It does imply that in reality when managers set different business objectives based on the company's strategies, it will have a big influence on the concrete business plans such as the supply chain network designing. Specifically, product 1 has high inventory costs, high ordering costs and low transporting costs (e.g., fresh products), while product 2 has lower inventory costs, lower ordering costs and higher transporting costs (e.g., large furniture). As a result, product 1 tends to be delivered only by two distribution centers while product 2 tends by three distribution centers, which is because distribution centers usually have high inventory costs.

5.2 Extended analysis

This paper conducts extended analysis from two aspects: algorithm and service level for three types of model, aiming to reveal some management insights and supplying guiding for different kinds of managers in real business operations.

Figure 3 shows that new individuals resulting from crossover and mutation operations make the optimal solution continuously better as the generation increases. In the end, the average fitness of the population is close to the optimal solution, showing that the algorithm converges and our models are effective. Moreover, we can see

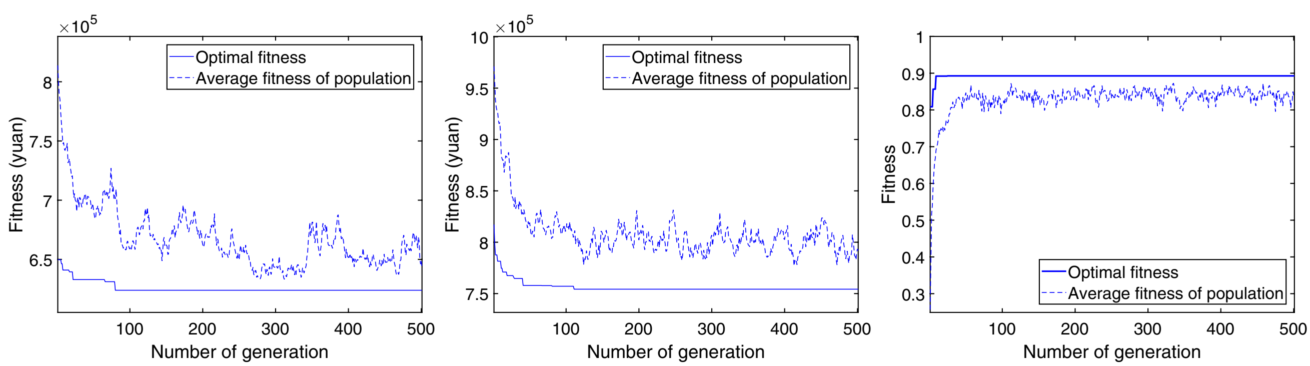

Fig. 3 The extended analysis of algorithm (left: the first model, middle: the second model, right: the third model)

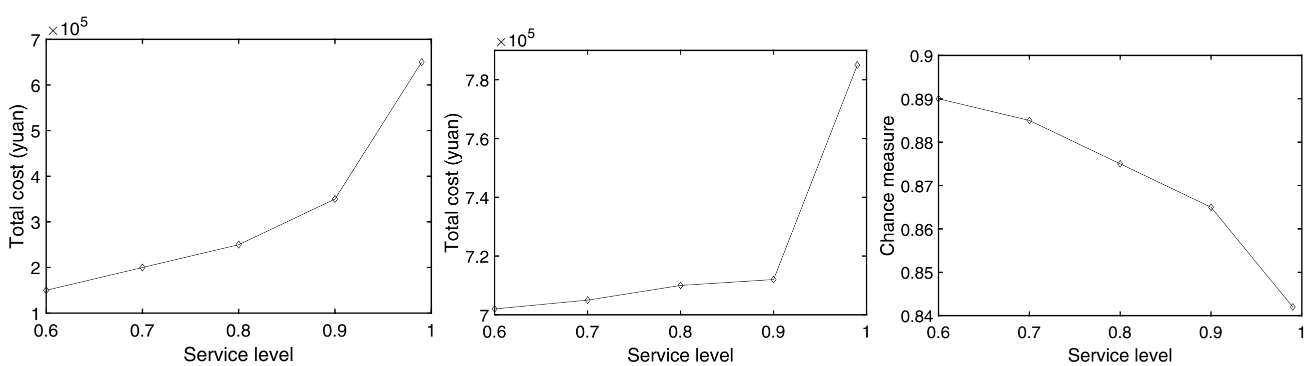

Fig. 4 The sensitive analysis of service level (left: the first model, middle: the second model, right: the third model)

that the first model and the second model reach convergence around the 100th generation, and the third model converges around the 30th generation, indicating that the convergence speed is fast.

Figure 4 shows that as the service level increases, the total cost is increasing or the chance measure is decreasing. Fundamentally, it is because that distribution centers set up more safety stock to deal with the market demand fluctuations in the future. In reality, managers should pursue the balance between the improvement of service level and total cost or chance measure. What's more, when the service level increases to 0.9, the total cost has a sudden increase or the chance measure has a sudden decrease. Essentially, it is because when managers try to provide an extremely high service level, they have to prepare much more products in cope with the demand surge which in turn lead to a rather huge inventory and ordering costs. The reason why β-cost is larger than the expected cost at the same service level, is because managers using β-cost minimization model need to bear fewer risks and can guarantee the actual costs no more than the optimal cost at 0.9 confidence level.

To sum up, managers always should make a trade-off between the total cost and the service level under different types of decision criteria, which essentially is a balance between long-term strategies and short-term plans. What's more, there is a threshold for the optimal service level, which means when the service level exceeds the threshold, the total cost or chance measure will have a sudden increase. Therefore, in reality, decision makers should not simply pursue the highest service level by increasing the total cost or decreasing the chance measure. Instead, they would better provide the service at the threshold level (like 0.9 in the above sensitive analysis) to better balance its service and cost.

6 Conclusions

The supply chain network design has attracted more and more attention with the economic growth. Due to the existence of complex information interaction, the supply chain network design is subjected to hybrid uncertainties. Based on the uncertainty theory, this paper takes into account the objective randomness of customers' demand and the subjective uncertainty of various operating costs. The article also constructs the three mathematical models aimed at different kinds of decision criteria. For those decision makers who focus on long-term interests and pursue the minimum cost of the entire supply chain network, this paper establishes an expected cost minimization model. For those decision makers who are willing to take a certain risk and purse the target cost with an acceptable level of confidence, this paper sets up a β-cost minimization model. For those managers who hope the chance that the total cost is no more than the budget is as large as possible, the paper presents a chance measure maximization model. After that, the three uncertain models are further transformed into deterministic classes and are solved via genetic algorithm.

This paper also gives the corresponding numerical examples and gives the optimal supply chain network designs under different decision criteria. This paper further concludes that: (1) different decision criteria can lead to significantly different supply chain network designs. Specifically, products that have high inventory and ordering costs but low inventory costs tend to be delivered through fewer distribution centers; (2) managers should always make a trade-off between the total cost (or the chance measure) and the service level under different decision criteria, which essentially is a balance between long-term strategies and short-term plans. What's more, managers would better provide services at the threshold, in which the quality of service and the optimal supply chain goals can be better balanced. There are also many interesting and meaningful directions in the future, such as supply chain network designs for special products under hybrid uncertainties.

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Compliance with ethical standards

Conflict of interest Author Yajing Tan declares that she has no conflict of interest. Author Xiaoyu Ji declares that she has no conflict of interest. Author Sen Yan declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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