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Multiobjective multistage robust integer optimization model and algorithm for oilfield development planning☆

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ABSTRACT

This paper discusses the long-term planning problem of oilfield development. The planning aims to deploy the workload of stimulation treatments with uncertain indicators. And the uncertainties of initial stimulation effect, annual effect error and new recoverable reserves are considered. With their uncertainty sets and the adaptability of decisions, a multiobjective multistage robust integer optimization model is constructed. In this model, the total development cost is minimized and the total new recoverable reserves is maximized. And the model makes decisions on the workload of each stimulation treatment in each year under constraints of annual oil production and workload balance. In addition, an algorithm for solving multiobjective multistage robust integer optimization model is proposed, which can obtain the finitely adaptive robust efficient solution set. Finally, a numerical example of long-term oilfield development planning is presented. The planning model with given uncertainty sets is solved, to verify the validity of the proposed model and algorithm.

1. Introduction

The life cycle of an oilfield can be divided into five processes: exploration, evaluation, development, production and abandonment. The oilfield development planning guides the overall deployment in the development phase. It determines the workload during the planning period according to actual needs and potential. And the plan has a direct impact on the development cost, oil production, and sustainable development in the future. Therefore, the planning is of great strategic significance. And decision-makers need to find out the best plan, which can meet development requirements under practical resources and capabilities, and achieve optimality in the investment and sustainability.

Regarding the oilfield development planning, Lee and Aronofsky (1958) first solve the production scheduling problem to maximize the profit by linear programming. After that, various mathematical programming methods have been widely used in oilfield development planning. McFarland, Lasdon, and Loose (1984) apply the generalized reduced gradient nonlinear programming method to construct an optimal control model for the development planning. Xiao, Liu, Jiang, and Shi (1998) propose a multiobjective linear programming model for the injection oilfield recovery system, which seeks to maximize production and minimize investment with the constraints of natural resources, equipment, and manpower. Yu, Zhang, Agbemabiese, and Zhang (2017) formulate the two-stage weighted goal-programming models to optimize oil production in the middle and later periods of oilfield development. And Navabi, Khaninezhad, and Jafarpour (2017) present a long-term planning model that can optimize the number, location, type, controls and schedule of the wells simultaneously. These papers all study the planning problem in the deterministic scenario.

For the uncertainties in oilfield development, Jonsbråten (1998) divides them into exogenous and endogenous uncertainties. The former is independent of decisions such as oil price and demand, while the latter is related to decisions such as reservoir size and quality. As exogenous uncertainties are difficult to predict and control, researchers focus more on handling endogenous ones in the development. Tarhan, Grossmann, and Goel (2009) propose a multi-stage stochastic programming model, in which the uncertain initial maximum oil and gas flowrate, recoverable reserves and water breakthrough time are represented by discrete distributions. Gupta and Grossmann (2014) consider endogenous uncertainties including field size, oil deliverability, water- and gas-oil ratio to build a multi-stage stochastic model for offshore oil and gas field infrastructure investment and operation planning. And by treating the

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uncertainty in drilling activities, Jahandideh and Jafarpour (2019) present a multi-stage stochastic optimization to make decisions on well controls and locations. Under uncertainties in oil prices and productivity indicators, Awasthi, Marmier, and Grossmann (2019) formulate a biobjective two-stage stochastic model to maximize the expected value of NPV and total oil production.

In practice, oilfield development involves a wide range of space, and development indicators are influenced by many factors from geology, technology, and economy (Jahandideh & Jafarpour, 2019). Therefore, the uncertainty of development indicators is very high. When dealing with the planning problem in the uncertain environment, most studies regard uncertain indicators as random variables. However, a lot of development work is one-off, so it is impossible to obtain enough sample data to estimate the accurate distribution of these indicators. And the technical and geological factors between different blocks differ significantly, which makes it hard to apply the rules derived from historical data to the future project. In stochastic programming, if the given probability distribution is very different from the actual one, then the optimal solution will behave poorly in practice (Bertsimas & Thiele, 2006).

Recently, other methods have also been applied to the handling of uncertainties in oil development planning. Ji, Yan, and Feng (2017) regard the stimulation effect as uncertain variables, and establish an uncertain multiobjective model which minimizes the expected cost and maximizes the expected new recoverable reserves. Instead of requiring sample data to obtain the probability distribution, this approach needs the experts' detailed subjective estimates. Therefore, it is difficult to use this method when there are many uncertain parameters involved in practical work. Chang, Bouzarkouna, and Devegowda (2015) apply robust multiobjective optimization to maximize the mean and minimize the variance of NPV to provide robust well placement solutions over the geological uncertainty. And going a step further, this paper uses multiobjective multistage robust integer optimization to deal with the uncertain oilfield development problem. In addition to extending the problem to multiple stages, this paper considers the uncertainty related to development activities, which is less considered than the geological uncertainty before but also very important (Jahandideh & Jafarpour, 2019).

Robust optimization, compared with stochastic programming, doesn't require the explicit distribution to characterize uncertainties. Instead, it uses the uncertainty set consisting of all possible realizations of uncertain parameters, to optimize the "worst-case". So the solution is still feasible with any parameter perturbations. Robust optimization can date back to a "worst-case" linear model proposed by Soyster (1973), which considers the uncertainty in the column of the coefficient matrix. Ben-Tal and Nemirovski (1998, 1999, 2000),l Ghaoui and Lebret (1997) and El Ghaoui, Oustry, and Lebret (1998) independently propose the computationally tractable robust counterpart. And the constraint with different types of uncertainty sets can be transformed into a corresponding tractable robust counterpart (Ben-Tal & Nemirovski, 2000; Bertsimas & Sim, 2004).

About robust multiobjective optimization, Kuroiwa and Lee (2012) define the robust efficient solution for the first time, and Fliege and Werner (2014) present the relationship between this definition and efficient solution of deterministic multiobjective model. Ehrgott, Ide, and Schöbel (2014) further discuss the concept of robust efficiency, including weakly robust efficiency, robust efficiency, and strictly robust efficiency. Bokrantz and Fredriksson (2017) introduce the concept of convex hull robust efficiency, and give necessary and sufficient conditions for robust efficiency and convex hull efficiency. Fakhar, Mahyarinia, and Zafarani (2018) deduce nonsmooth sufficient optimality conditions for robust (weakly) efficient solutions. Apart from the planning problem of oilfield development, robust multiobjective optimization has also been applied to supply chain network design (Darestani & Hemmati, 2019), product portfolio(Goli, Zare, Tavakkoli-Moghaddam, & Sadeghieh, 2019), task scheduling(Golpîra & Tirkolaee, 2019), etc.

The oilfield development planning mainly makes long-term decisions on the work related to oil wells, so it is usually described by a multistage integer programming model. In the multistage robust optimization, later decisions are determined according to the realized values of previous uncertain parameters. To deal with this, Ben-Tal, Goryashko, Guslitzer, and Nemirovski (2004) propose the affinely adjustable robust optimization, but it requires that the coefficients of adjustable variables are deterministic and the variables cannot be integers. To solve multistage mixed-integer robust optimization, Bertsimas and Georghiou (2015) treat continuous variables and 0-1 variables as piecewise linear functions and piecewise constant functions respectively, but still require the deterministic coefficients of adjustable variables. Another approach is to partition uncertainty sets to obtain finite adaptability, i.e., a scenario-based approach. Bertsimas and Caramanis (2010) use extreme points of the uncertainty set to partition itself into subsets, and each subset has its own corresponding 0-1 decision variable. Vayanos, Kuhn, and Rustem (2011) partition the uncertainty set using hyperrectangles before calculation. Postek and Hertog (2016) and Bertsimas and Dunning (2016) propose the partition-and-bound method, in which the number of partitions increases continuously during the iteration. The difference is that the former partitions each subset obtained in the last iteration into two parts, while the latter gets multiple partitions to achieve the result more quickly.

As can be seen, multiple objectives and multiple stages are the significant characteristics in most researches on oilfield development planning. And many practical problems are solved by the heuristic algorithm, which is also common in other fields (Gupta, Mogale, & Tiwari, 2019; Mogale, Ghadge, Kumar, & Tiwari, 2020; Mogale, Kumar, & Tiwari, 2020). However, as best as we know, multiobjective multistage robust optimization has not been discussed, nor has its solution algorithm. Therefore, this paper considers using multiobjective multistage robust integer optimization to deal with the uncertain planning problem of oilfield development, and proposes the solution algorithm.

The main contributions in this paper are listed below:

Robust optimization is introduced into the oilfield development planning which avoids the requirement of specific distributions. And the long-term uncertain indicators are analyzed to describe the uncertainty in the planning.

Under the constraints of annual production and workload, this paper constructs a multiobjective multistage robust integer optimization model that aims to minimize total cost and maximize total new recoverable reserves.

A multiobjective partition-and-bound method is proposed. In the iterative process, the uncertainty set is partitioned to obtain the partitioned version of the model, and then the multiobjective genetic algorithm NSGA-II is used to obtain the robust efficient solution set.

Finally, a long-term oilfield development planning model under the box uncertainty set is established. Using the proposed algorithm, a robust effective solution set with finite adaptability is obtained.

The rest of this paper is structured as follows. The planning problem of oilfield development is described and a multiobjective multistage robust integer optimization model is constructed in Section 2. In Section 3, the multiobjective partition-and-bound method is proposed. And Section 4 illustrates the effectiveness of the proposed model and algorithm using a numerical example. In Section 5, some conclusions and thoughts on future research are proposed.

2. Oilfield development planning model

2.1. Problem description and notation

Based on the information of exploration and evaluation, oilfield development planning provides a reasonable arrangement for the development work. The most of work in the development phase is about the oil wells. The drilling of new wells is the main way to increase production and new recoverable reserves. And other stimulation treatments for old wells such as fracturing, triple replacement and perforation adding are also used to boost yield. These types of treatments also have different costs and effects. Therefore, in a large number of candidate plans, the decision-maker is required to determine the number of wells in each year, namely, the annual workload, of each treatment.

In the planning, a serious difficulty is that development indicators have high uncertainty as the reservoir is deeply buried in the ground. These indicators affect the evaluation of development plans, so their uncertainty has to be considered. The essential one is the stimulation effect, that is, the production increment after using treatments. And the treatment continues to be effective in the planning period after implementation. So the uncertainty exists in both the initial effect and the subsequent years. In addition, the new recoverable reserves that new wells bring are related to sustainable development. It refers to the potential increment of recoverable reserves at the current technological level, and it is also susceptible to detection techniques and geological factors. So its uncertainty is also included in the model. But unlike the stimulation effect, it is only effective in the year of implementation.

For the oilfield, production is very important, even a political requirement. Oilfields often want to guarantee a certain production level when planning, which constitutes a production constraint. In addition, the production guarantee needs corresponding workload, and there are many types of treatments. For the workload of each type of treatment, on the one hand, it will be limited by manpower and material resources. On the other hand, considering the balance of work, its number must have the minimum requirements.

And the planning should not only consider the short-term economic indicator, but also consider the future development. In order to pursue the economic benefit, the cost should be as low as possible. Moreover, sustainable development is also considered as an important aspect to obtain a good production succession (Zhou et al., 2020). Better sustainability means longer and better development in the future. So the minimization of cost and the maximization of total new recoverable reserves are the two objectives in the model.

Based on the above characteristics, all sets, parameters and decision variables used to establish the model are presented in Table 1.

In this paper, the decision is the annual workload of each treatment in the planning period. And due to the after-effect on production, the implemented year and observed year may be different. Let I and T be the number of treatment types and years in the planning period, where the treatment is indexed by i and the year is indexed by l and t. Specifically, when it comes to the after-effect, this paper uses l to represent the implemented year of the treatment and t to index the observed year in the planning period; otherwise, the observed year of treatment is also its implemented year, indexed by t.

In practice, the annual stimulation effect of the same treatment presents a regular pattern with respect to time after implementation. So this paper assumes that the annual effect per well of each treatment is the sum of a certain ratio of the initial annual effect per well and an error term, where the initial annual effect is the stimulation effect in the year of implementation. And the initial effect per well and the error are uncertain. For treatment i implemented in year l, its initial annual stimulation effect per well is denoted by $\widetilde{r}_{il}$, the ratio of its annual effect per well in the observed year t (the $(t-l+1)$th year after implementing) to the initial effect is denoted by $k_{i(t-l+1)}$, and the error of annual effect per well in year t is denoted by $\widetilde{e}_{ilt}$. In addition, denote U_r as the uncertainty set to which the initial annual stimulation effect per well $\widetilde{\boldsymbol{r}} = (\widetilde{\boldsymbol{r}}_1, \widetilde{\boldsymbol{r}}_2, \cdots, \widetilde{\boldsymbol{r}}_T)$ belongs, where $\widetilde{\boldsymbol{r}}_l = (\widetilde{r}_{1l}, \widetilde{r}_{2l}, \cdots, \widetilde{r}_{Il})$; denote U_e as the uncertainty set to which the error of annual effect per well $\widetilde{\boldsymbol{e}} = (\widetilde{\boldsymbol{e}}_1, \widetilde{\boldsymbol{e}}_2, \cdots, \widetilde{\boldsymbol{e}}_T)$ belongs, where the error in the tth year $\widetilde{\boldsymbol{e}}_t = (\widetilde{e}_{11t}, \cdots, \widetilde{e}_{I1t}, \cdots, \widetilde{e}_{I(t-1)t}, 0, \cdots, 0)$. Moreover, denote $\widetilde{s}_t$ as the new recoverable reserves per well in the tth year, and denote U_s as the uncertainty set to which the new recoverable reserves per well $\widetilde{\boldsymbol{s}} = (\widetilde{s}_1, \widetilde{s}_2, \cdots, \widetilde{s}_T)$ belongs.

For these uncertainty sets, their specific form can be directly given by the experience of relevant experts in the field. In addition, sometimes experimental data can be collected before planning in practical development. In this condition, the uncertainty set may be obtained by piecewise linear kernel-based support vector clustering (Shang, Huang, & You, 2017), or different uncertainty sets with its own geometric shape and computational properties by pairing assumptions and hypothesis tests (Bertsimas, Gupta, & Kallus, 2018).

In the multistage oilfield development planning, some uncertain parameters have been realized before the implementation of later decisions. Therefore, decision variables should be adaptive with respect to previous uncertain parameters. The decision variable corresponding to the workload of treatment i in year l is denoted by x_{il}, which is adaptive with the initial annual stimulation effect per well, the effect error per well and the new recoverable reserves per well before year l. The development of new wells is taken as the first kind of treatment, and the decision of the first year x_{i1} isn't adaptive and doesn't depend on any uncertain parameters.

Table 1
Summary of model notation.

	Notation	Representation
Sets	$\{I\}$	Set of types of treatments, indexed by $i = 1, 2, \cdots, I$
	$\{T\}$	Set of years, indexed by l and $t, l \leqslant t, t = 1, 2, \cdots, T$
	U_r	Uncertainty set of initial effect per well, element $\widetilde{\boldsymbol{r}}$
	U_e	Uncertainty set of annual effect error per well, element $\widetilde{\boldsymbol{e}}$
	U_s	Uncertainty set of new recoverable reserves per well, element $\widetilde{\boldsymbol{s}}$
Parameters	$\widetilde{r}_{il}$	Parameter (nonnegative) to denote the initial effect per well for treatment i implemented in year l
	$k_{i(t-l+1)}$	Parameter (nonnegative) to denote the ratio of annual effect per well in observed year t to the initial effect for treatment i
	$\widetilde{e}_{ilt}$	Parameter (nonnegative) to denote the error of annual effect per well observed in year t for treatment i implemented in year l
	$\widetilde{s}_t$	Parameter (nonnegative) to denote the new recoverable reserves per well implemented in year t
	c^o_{ilt}	Parameter (nonnegative) to denote the unit oil-related cost in year t of treatment i implemented in year l
	c^w_{it}	Parameter (nonnegative) to denote the unit well-related cost of treatment i implemented in year t
Variable	x_{il}	Variable (nonnegative integer) to define the workload of treatment i in year l

2.2. Multiobjective multistage robust integer optimization model

The annual oil production can be considered as the sum of stimulation effect and the natural production from the part developed before the planning period in this year. Since the annual natural production during the planning period is independent of decisions, this paper only pays attention to the annual total stimulation effect in each year. The total stimulation effect in the tth year $\widetilde{Q}_t$ is expressed as,

$$\widetilde{Q}_t = \sum_{i=1}^{I} \sum_{l=1}^{t} (\widetilde{r}_{il} k_{i(t-l+1)} + \widetilde{e}_{ilt}) x_{il}. \tag{1}$$

Note that the annual stimulation effects of all treatments implemented in year t and before should be summed up, due to the after-effect.

The production constraint requires that the actual oil production in each year is not lower than the given target. However, the total annual stimulation effect is uncertain with an infinite number of possible values. In the robust counterpart, constraints are required to be satisfied under all possible realizations of uncertain parameters. The target of total stimulation effect in year t is denoted by Q^0_t, and then the oil production constraint in this year is,

$$\sum_{i=1}^{I}\sum_{l=1}^{t}(\widetilde{r}_{il}k_{i(t-l+1)}+\widetilde{e}_{ilt})x_{il}\geqslant Q_t^0,\forall\widetilde{\boldsymbol{r}}\in U_r,\widetilde{\boldsymbol{e}}\in U_e. \tag{2}$$

For the workload of each treatment in each year, it is necessary to give an upper bound according to the potential of resources and capabilities. And there must be a lower bound to ensure the balance of development work. Therefore, the workload constraint is expressed as,

$$\underline{x}_{it}\leqslant x_{it}\leqslant\overline{x}_{it},x_{it}\in\mathbb{N}^+,\forall i\in\left\{I\right\},t\in\left\{T\right\}, \tag{3}$$

where $\underline{x}_{it}$ and $\overline{x}_{it}$ are the lower and upper workload bounds of treatment i in year t, respectively, and $\{I\}=\{1,2,\cdots,I\},\{T\}=\{1,2,\cdots,T\}$.

The development cost can be divided into the oil-related costs such as material cost, power cost and maintenance fee, and the well-related costs such as downhole operating cost and logging test fee. Since stimulation treatments are effective during the planning period, the treatment implemented before should be considered when calculating the annual oil-related cost. But the annual well-related cost is only associated with the workload in that year. Let c_{ilt}^o denote the unit oil-related cost in year t of treatment i implemented in year l, and c_{it}^w denote the unit well-related cost of treatment i implemented in year t. Then the annual total cost in year t is,

$$\widetilde{C}_t=\sum_{i=1}^{I}\left(\sum_{l=1}^{t}c_{ilt}^o\left(\widetilde{r}_{il}k_{i(t-l+1)}+\widetilde{e}_{ilt}\right)x_{il}+c_{it}^wx_{it}\right). \tag{4}$$

The total cost of the whole planning period is composed of the annual cost in each year, so the time value of money should be considered. Assuming that the annual cost is invested at the beginning of each year and the discount rate is r, the uncertain total cost is,

$$\widetilde{C}=\sum_{t=1}^{T}(1+r)^{-(t-1)}C_t. \tag{5}$$

As for the total new recoverable reserves during the planning period, it is also uncertain because of the uncertainty of new recoverable reserves per well, specifically expressed as,

$$\widetilde{S}=\sum_{t=1}^{T}\widetilde{s}_tx_{1t}. \tag{6}$$

Thus, the multiobjective multistage robust integer optimization model for the long-term oilfield development planning is established as follows,

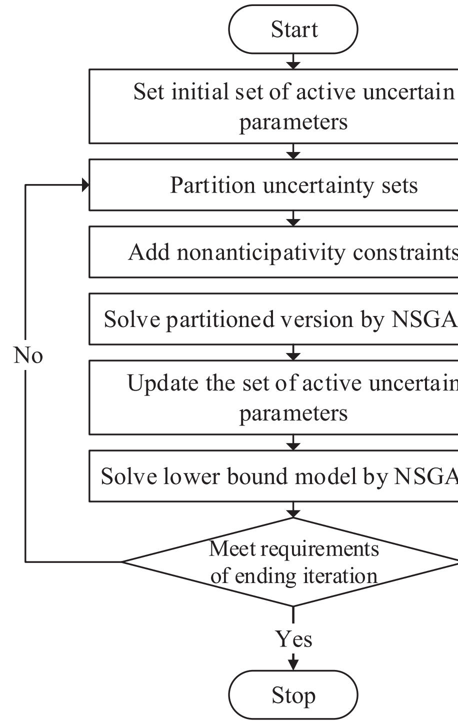


Fig. 1. Flow chart of solution algorithm.

where $\boldsymbol{x}$ is the decision vector composed of the workload of each kind of treatment in each year, and decision variables z_1 and z_2 are introduced to transform two objectives into equivalent constraints. By this transformation, there is no uncertainty in objectives, and the optimal values of z_1 and z_2 are the same as those of the original objective functions.

3. Solution algorithm

Considering the conflict between the minimization of total cost and the maximization of total new recoverable reserves, this paper tries to find out the robust efficient solution set of model (7). For a solution in this set, one objective cannot be improved without weakening the other

$$\begin{aligned}
&\min_{\boldsymbol{x},z_1,z_2} z_1\\
&\max_{\boldsymbol{x},z_1,z_2} z_2\\
&\text{subject to :}\\
&\sum_{t=1}^{T}\left((1+r)^{-(t-1)}\sum_{i=1}^{I}\left(\sum_{l=1}^{t}c_{ilt}^o\left(\widetilde{r}_{il}k_{i(t-l+1)}+\widetilde{e}_{ilt}\right)x_{il}+c_{it}^wx_{it}\right)\right)\leqslant z_1,\forall\widetilde{\boldsymbol{r}}\in U_r,\widetilde{\boldsymbol{e}}\in U_e\\
&\sum_{t=1}^{T}\widetilde{s}_tx_{1t}\geqslant z_2,\forall\widetilde{\boldsymbol{s}}\in U_s\\
&\sum_{i=1}^{I}\sum_{l=1}^{t}(\widetilde{r}_{il}k_{i(t-l+1)}+\widetilde{e}_{ilt})x_{il}\geqslant Q_t^0,\forall\widetilde{\boldsymbol{r}}\in U_r,\widetilde{\boldsymbol{e}}\in U_e,t\in\left\{T\right\}\\
&\underline{x}_{it}\leqslant x_{it}\leqslant\overline{x}_{it},x_{it}\in\mathbb{N}^+,\forall i\in\left\{I\right\},t\in\left\{T\right\},
\end{aligned} \tag{7}$$

one, so it is a good candidate plan for decision-makers.

This paper proposes the multiobjective partition-and-bound method for solving multiobjective multistage robust integer optimization model. By partitioning the uncertainty set, this scenario-based method can deal with the uncertain model and the adjustability of decisions. The partitioned version of model (7) is first established by using a set of active uncertain parameters to partition the uncertainty set and adding nonanticipativity constraints. Then Nondominated Sorting Genetic Algorithm-II (NSGA-II, Deb, Pratap, Agarwal, & Meyarivan, 2002), a popular and reliable multiobjective solution method, is applied to solve the partitioned version to obtain the robust efficient solution set with finite adaptability. The result will update active uncertain parameters. And using the updated set, the lower bound model can be constituted and also solved by NSGA-II. If the termination condition is not satisfied, the new iteration will continue to solve the partitioned version and the lower bound model, and the gap between them will be smaller. The flow chart of the proposed solution algorithm is shown in Fig. 1.

3.1. Partitioned version of oilfield development planning model

In the partition-and-bound method, the uncertainty set is needed to be partitioned in each iteration. Like Bertsimas and Dunning (2016), this paper adopts the nested partition approach, which uses uncertain constraints to obtain a set of active uncertain parameters, and then achieves the corresponding partition of each parameter by Voronoi diagram scheme.

In the initial iteration, the active uncertain parameter can be any point in uncertainty set U, and its corresponding partition is U, where $U = U_r \times U_e \times U_s = \{\boldsymbol{\zeta} = (\widetilde{\boldsymbol{r}}, \widetilde{\boldsymbol{e}}, \widetilde{\boldsymbol{s}}) | \widetilde{\boldsymbol{r}} \in U_r, \widetilde{\boldsymbol{e}} \in U_e, \widetilde{\boldsymbol{s}} \in U_s\}$. In the subsequent iteration, to obtain active uncertain parameters at iteration $n+1$, a robust efficient solution at iteration n is randomly selected, and denote $(\overline{\boldsymbol{x}}, \overline{z_1}, \overline{z_2})$ as the decision vector of this solution on partition U_p. Then for the production constraint in year t, the set of active uncertain parameters of this partition is,

$$\mathcal{A}_t = \underset{(\widetilde{r,e})}{\operatorname{argmin}} \left\{ \sum_{i=1}^{I} \sum_{l=1}^{t} \left(\widetilde{r}_{il} k_{i(t-l+1)} + \widetilde{e}_{ilt} \right) \overline{x}_{il} - Q_t^0 \right\}. \tag{8}$$

Since $\mathcal{A}_t$ maybe a singleton set or an uncountable set, only one element is taken when selecting the active uncertain parameter. Similarly, active uncertain parameters of constraints corresponding to the two objectives on this partition can also be obtained. Thus, up to $(T+2)$ active uncertain parameters are generated for one partition in each iteration, to further partition itself in the next iteration.

In multistage robust optimization, for two partitions $U(\widehat{\zeta}_p)$ and $U(\widehat{\zeta}_q)$, their decisions x_{itp} and x_{itq} must be the same when uncertain parameters of the first t stages are the same, but not required in the subsequent stages, which is called nonanticipativity. If the uncertainty set is directly partitioned by Voronoi diagram, the model may lose its adaptability when the nonanticipativity is guaranteed.

Denote $Children(\widehat{\zeta})$ as all active uncertain parameters in the next iteration on the partition of active uncertain parameter $\widehat{\zeta}$. And denote $\widehat{\zeta} = Parent(\widehat{\zeta}')$, $Children(\widehat{\zeta}) \backslash \{\widehat{\zeta}'\} = Siblings(\widehat{\zeta}')$ for all $\widehat{\zeta}' \in Children(\widehat{\zeta})$. The partition corresponds to active uncertain parameter $\widehat{\zeta}_p$,

$$\begin{aligned} U(\widehat{\zeta}_p) = & \left\{ \zeta \,|\, \|\widehat{\zeta}_p^{t_{p,q}} - \zeta^{t_{p,q}}\|_2 \leqslant \|\widehat{\zeta}_q^{t_{p,q}} - \zeta^{t_{p,q}}\|_2, \forall \widehat{\zeta}_q \in Siblings\left(\widehat{\zeta}_p\right) \right\} \\ & \cap \left\{ \zeta \,|\, \|Parent\left(\widehat{\zeta}_p\right)^{t'_{p,q}} - \zeta^{t'_{p,q}}\|_2 \leqslant \|\widehat{\zeta}_q^{t'_{p,q}} - \zeta^{t'_{p,q}}\|_2, \forall \widehat{\zeta}_q \in Siblings\left(Parent\left(\widehat{\zeta}_p\right)\right) \right\} \\ & \cap \cdots \cap U, \end{aligned} \tag{9}$$

where, $\zeta^t = (\widetilde{r}_t, \widetilde{e}_t, \widetilde{s}_t)$, $t_{p,q} = \underset{t}{\operatorname{argmin}} \left\{ \widehat{\zeta}_p^t \neq \widehat{\zeta}_q^t \right\}$, i.e., the first time stage where the uncertain parameters begin to differ. Similarly, $t'_{p,q}$ is the first time stage which makes $Parent(\widehat{\zeta}_p)^t \neq \widehat{\zeta}_q^t$ ($\widehat{\zeta}_q \in Siblings(Parent(\widehat{\zeta}_p))$).

Then, the following approximate rule is used to enforce nonanticipativity in finite adaptability.

Step 1: Let $\widehat{\zeta}_A \leftarrow \widehat{\zeta}_p$ and $\widehat{\zeta}_B \leftarrow \widehat{\zeta}_q$.

Step 2: If $\widehat{\zeta}_A \in Siblings(\widehat{\zeta}_B)$, then let $t_{A,B} = \underset{t}{\operatorname{argmin}} \{\widehat{\zeta}_A^t \neq \widehat{\zeta}_B^t\}$. When $t \leqslant t_{A,B}$, the nonanticipativity constraint $x_{itp} = x_{itq}$ is added.

Step 3: Otherwise, if $\widehat{\zeta}_A \notin Siblings(\widehat{\zeta}_B)$, then let $\widehat{\zeta}_A \leftarrow Parent(\widehat{\zeta}_A)$ and $\widehat{\zeta}_B \leftarrow Parent(\widehat{\zeta}_B)$, and return to Step 2.

After partitioning the uncertainty set and adding nonanticipativity constraints, the partitioned version of model (7) with finite adaptability at iteration n is constructed as follows,

$$\begin{aligned} & zo_1^n = \min_{\boldsymbol{x}, z_1, z_2} z_1 \\ & zo_2^n = \max_{\boldsymbol{x}, z_1, z_2} z_2 \\ & \text{subject to:} \\ & \sum_{t=1}^{T} \left((1+r)^{-(t-1)} \sum_{i=1}^{I} \left(\sum_{l=1}^{t} c_{ilt}^{o} \left(\widetilde{r}_{il} k_{i(t-l+1)} + \widetilde{e}_{ilt} \right) x_{ilp} + c_{it}^{w} x_{itp} \right) \right) \leqslant z_1, \forall \boldsymbol{\zeta} \in U\left(\widehat{\zeta}_p\right), \widehat{\zeta}_p \in \mathcal{T}_n \\ & \sum_{t=1}^{T} \widetilde{s}_t x_{1tp} \geqslant z_2, \forall \boldsymbol{\zeta} \in U\left(\widehat{\zeta}_p\right), \widehat{\zeta}_p \in \mathcal{T}_n \\ & \sum_{i=1}^{I} \sum_{l=1}^{t} \left(\widetilde{r}_{il} k_{i(t-l+1)} + \widetilde{e}_{ilt} \right) x_{ilp} \geqslant Q_t^0, \forall t \in \left\{T\right\}, \boldsymbol{\zeta} \in U\left(\widehat{\zeta}_p\right), \widehat{\zeta}_p \in \mathcal{T}_n \\ & x_{itp} = x_{itq}, \forall \widehat{\zeta}_p, \widehat{\zeta}_q \in \mathcal{T}_n, \forall t : \widehat{\zeta}_p^{1,\cdots,t-1} = \widehat{\zeta}_q^{1,\cdots,t-1} \quad \underline{x}_{it} \leqslant x_{itp} \leqslant \overline{x}_{it}, x_{itp} \in \mathbb{N}^+, \forall i \in \left\{I\right\}, t \in \left\{T\right\}, p \in \left\{P_n\right\}, \end{aligned} \tag{10}$$

where $\mathcal{T}_n$ is the set of all active uncertain parameters at this iteration, and P_n is the number of elements in $\mathcal{T}_n$, and $\{P_n\} = \{1, 2, \cdots, P_n\}$.

Proposition 1. *For the set of objective function values corresponding to robust efficient solution set S_o^n of the partitioned version (10) at iteration n, i.e., robust efficient frontier Z_o^n, and robust efficient frontier Z corresponding to robust efficient solution set S of model (7), there don't exist elements $(zo_1^n, zo_2^n) \in Z_o^n$ and $(zo_1^{n+1}, zo_2^{n+1}) \in Z_o^{n+1}$ such that $(zo_1^n, -zo_2^n) \prec (zo_1^{n+1},$*

$-zo_2^{n+1})$*, and there don't exist elements* $(z_1, z_2) \in Z$ *and* $(zo_1^n, zo_2^n) \in Z_o^n$ *such that* $(zo_1^n, -zo_2^n) \prec (z_1, -z_2)$.

Proof. The proposition is derived from the nature of nested partition approach. Suppose, for the sake of contradiction, that there are $(zo_1^n, zo_2^n) \in Z_o^n$ and $(zo_1^{n+1}, zo_2^{n+1}) \in Z_o^{n+1}$ such that $(zo_1^n, -zo_2^n) \prec (zo_1^{n+1}, -zo_2^{n+1})$. Denote the element in S_o^n corresponding to (zo_1^n, zo_2^n) by ω_o^n and the element in S_o^{n+1} corresponding to (zo_1^{n+1}, zo_2^{n+1}) by ω_o^{n+1}, respectively. Since model (10) takes into account the "worst-case" of uncertain constraints in each partition, if the value of decision x_{ilp} on partition $U(\widehat{\zeta}_p)$ is taken as the decision value on all the partition corresponding to $Children(\widehat{\zeta}_p)$, then a feasible solution $w_o^{'}$ for the partitioned version at iteration $n+1$ can be obtained while the objective function values remain same. And because $(zo_1^n, -zo_2^n) \prec (zo_1^{n+1}, -zo_2^{n+1}), w_o^{'}$ dominates w_o^{n+1}, resulting in $\omega_o^{n+1} \notin S_o^{n+1}$, which leads to a contradiction. So there don't exist elements $(zo_1^n, zo_2^n) \in Z_o^n$ and $(zo_1^{n+1}, zo_2^{n+1}) \in Z_o^{n+1}$ such that $(zo_1^n, -zo_2^n) \prec (zo_1^{n+1}, -zo_2^{n+1})$.

Because model (7) can be thought of as the partitioned version after infinitely partitioning the uncertainty set, it can be proved similarly that there don't exist elements $(z_1, z_2) \in Z$ and $(zo_1^n, zo_2^n) \in Z_o^n$ such that $(zo_1^n, -zo_2^n) \prec (z_1, -z_2)$. □

3.2. Multiobjective genetic algorithm

NSGA is proposed by Srinivas and Deb (1994) for solving the multiobjective programming. It assigns the nondomination level of individuals according to their domination relationship to obtain the efficient solution set. On this basis, Deb et al. (2002) propose an improved algorithm NSGA-II. This algorithm introduces the fast non-dominated sorting procedure, elitist strategy and fast crowded distance estimation procedure, which reduce the computational complexity effectively and achieve a uniformly spread-out efficient frontier. Its flow chart for solving partitioned version (10) is presented in Fig. 2.

Since the numbers of treated wells are all integers, individuals in the real-coded population are rounded after evolution when using NSGA-II to solve the partitioned version (10). And the bounded SBX crossover and polynomial mutation (Deb & Agrawal, 1995) are used to ensure that the decision doesn't violate the upper and lower bound constraints of the workload. In order to handle nonanticipativity constraints, only one of decision variables with the same value is put into chromosomes in the evolution process. And the complete decision variables are obtained through these constraints when calculating the objective function values and the constraint violation. For other constraints, the constraint-handling approach proposed by Deb et al. (2002) is used. In this approach, the constrained-domination relationship is determined by distinguishing the feasible solution from the infeasible solution and calculating the overall constraint violation of the infeasible solution.

3.3. Multiobjective partition-and-bound method

In practical development, there is only one final realization of uncertain parameters. Therefore, as the number of iterations increases, the solution is closer to the ideal result by partitioning the uncertainty set continuously. But since the uncertainty set can not be partitioned infinitely, this paper proposes the lower bound set of the robust efficient solution in each iteration as the basis for terminating the iteration. When there is the uncertain parameter set $\mathcal{T}_n$, by solving the following model,

$$
\begin{aligned}
& zl_1^n = \min_{\boldsymbol{x}, z_1, z_2} z_1 \\
& zl_2^n = \max_{\boldsymbol{x}, z_1, z_2} z_2 \\
& \text{subject to:} \\
& \quad \sum_{t=1}^{T}\left((1+r)^{-(t-1)}\sum_{i=1}^{I}\left(\sum_{l=1}^{t} c_{ilt}^{o}\left(\widetilde{r}_{il}k_{i(t-l+1)}+\widetilde{e}_{ilt}\right)x_{ilp}+c_{it}^{w}x_{itp}\right)\right)\leqslant z_1, \boldsymbol{\zeta}=\widehat{\boldsymbol{\zeta}}_p, \forall\widehat{\boldsymbol{\zeta}}_p\in\mathcal{T}_n \\
& \quad \sum_{t=1}^{T}\widetilde{s}_t x_{1tp}\geqslant z_2, \boldsymbol{\zeta}=\widehat{\boldsymbol{\zeta}}_p, \forall\widehat{\boldsymbol{\zeta}}_p\in\mathcal{T}_n \\
& \quad \sum_{i=1}^{I}\sum_{l=1}^{t}\left(\widetilde{r}_{il}k_{i(t-l+1)}+\widetilde{e}_{ilt}\right)x_{ilp}\geqslant Q_t^0, \forall t\in\left\{T\right\}, \boldsymbol{\zeta}=\widehat{\boldsymbol{\zeta}}_p, \forall\widehat{\boldsymbol{\zeta}}_p\in\mathcal{T}_n \\
& \quad x_{itp}=x_{itq}, \forall\widehat{\boldsymbol{\zeta}}_p, \widehat{\boldsymbol{\zeta}}_q\in\mathcal{T}_n, \forall t: \widehat{\boldsymbol{\zeta}}_p^{1,\cdots,t-1}=\widehat{\boldsymbol{\zeta}}_q^{1,\cdots,t-1} \\
& \quad \underline{x}_{it}\leqslant x_{itp}\leqslant\overline{x}_{it}, x_{itp}\in\mathbb{N}^+, \forall i\in\left\{I\right\}, \forall t\in\left\{T\right\}, \forall p\in\left\{P_n\right\},
\end{aligned}
\tag{11}
$$

the lower bound set of the robust efficient solution of model (7) S_l^n is obtained. Note that the difference between the lower bound model (11) and the partitioned version (10) is that model (11) only uses the information of active uncertain parameters while model (10) considers each partitions.

Proposition 2. *For robust efficient frontier* Z *corresponding to robust efficient solution set* S *of model* (7) *and robust efficient frontier* Z_l^n *corresponding to* S_l^n*, there don't exist elements* $(z_1, z_2) \in Z$ *and* $(zl_1^n, zl_2^n) \in Z_l^n$ *such that* $(z_1, -z_2) \prec (zl_1^n, -zl_2^n)$*, and there don't exist elements* $(zl_1^n, zl_2^n) \in Z_l^n$ *and* $(zl_1^{n+1}, zl_2^{n+1}) \in Z_l^{n+1}$ *such that* $(zl_1^{n+1}, -zl_2^{n+1}) \prec (zl_1^n, -zl_2^n)$.

Proof. Suppose, for the sake of contradiction, that there are $(z_1, z_2) \in Z$ and $(zl_1^n, zl_2^n) \in Z_l^n$ such that $(z_1, -z_2) \prec (zl_1^n, -zl_2^n)$. Note that the constraints in model (7) are satisfied for any realization of uncertain parameters, while the constraints in model (11) are only satisfied for the uncertain parameter in $\mathcal{T}_n$. For any feasible solution of model (7), let the workload corresponding to each uncertain parameter in the model (11) be the workload in this solution. Then the transformed workload is feasible for model (11) when the value of objective functions remains same. Therefore, element ω in S corresponding to (z_1, z_2) can be transformed into $\omega^{'}$ which dominates element ω_l^n in S_l^n corresponding to (zl_1^n, zl_2^n), so that $\omega_l^n \notin S_l^n$, which leads to a contradiction. So there don't exist elements $(z_1, z_2) \in Z$ and $(zl_1^n, zl_2^n) \in Z_l^n$ such that $(z_1, -z_2) \prec (zl_1^n, -zl_2^n)$.

Since more uncertain parameters are considered in model (11) at iteration $n+1$ than iteration n, for the same active uncertain parameter, the feasible solution of iteration n can be obtained using the decision

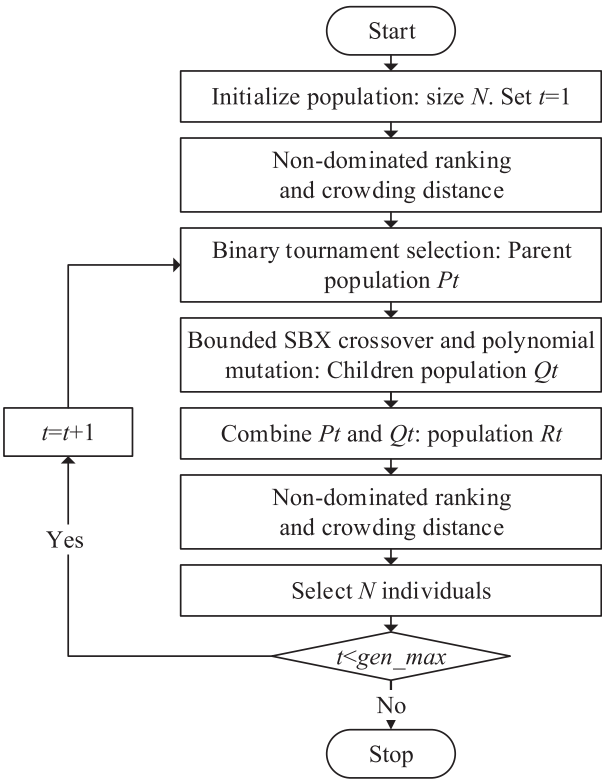


Fig. 2. Flow chart of NSGA-II.

variables in the feasible solution of iteration $n+1$ while the objective function values are the same. Similarly, it can be seen that there don't exist elements $(zl_1^n, zl_2^n) \in Z_l^n$ and $(zl_1^{n+1}, zl_2^{n+1}) \in Z_l^{n+1}$ such that $(zl_1^{n+1}, -zl_2^{n+1}) \prec (zl_1^n, -zl_2^n)$. $\square$

From Proposition 1 and 2, the following corollary can be obtained immediately.

Corollary 1. *There don't exist elements* $(zo_1^n, zo_2^n) \in Z_o^n$ *for any iteration n and* $(zl_1^m, zl_2^m) \in Z_l^m$ *for any iteration m such that* $(zo_1^n, -zo_2^n) \prec (zl_1^m, -zl_2^m)$.

The specific steps for solving model (7) using the multiobjective partition-and-bound method are as follows.

Step 1: Initialize. Select any $\zeta \in U$ randomly to form the initial set of active uncertain parameters. Set iteration counter $n \leftarrow 1$.

Step 2: Solve. Solve the partitioned version (10) with $\mathcal{T}_n$ using the algorithm in Section 3.2, until the first conclusion of Proposition 1 holds.

Step 3: Grow. Set $\mathcal{T}_{n+1} \leftarrow \mathcal{T}_n$, and add all $Children(\hat{\zeta}_p)$ to $\mathcal{T}_{n+1}$ for each $\hat{\zeta}_p \in \mathcal{T}_{n+1}$.

Step 4: Bound. Solve model (11) with $\mathcal{T}_{n+1}$ using the algorithm in Section 3.2, until Corollary 1 holds. Check if the latter conclusion of Proposition 2 is true. If not, then recalculate the previous lower bound sets forward by iteration until it holds.

Step 5: Terminate. Define the shortest distance between the robust efficient frontier Z_o^n and Z_l^{n+1} as $\bar{d}_n = \min_{v,w} \left\| (zo_1^n, zo_2^n)_v - (zl_1^{n+1}, zl_2^{n+1})_w \right\|$, where $(zo_1^n, zo_2^n)_v$ is the vth element in Z_o^n, and $(zl_1^{n+1}, zl_2^{n+1})_w$ is the wth element in Z_l^{n+1}. Terminate if $\bar{d}_n / \bar{d}_1 \leqslant \delta$ is satisfied with the given threshold δ, or iteration count n reaches requirement. Otherwise, set $n \leftarrow n+1$ and return to Step 2.

4. Numerical example

In this section, the long-term oilfield development planning of an oilfield in China is taken as an example. The box uncertainty sets to which uncertain parameters belong are given, as well as the upper and lower bounds of the workload and the target of stimulation effect in each year. A corresponding multiobjective multistage robust integer optimization model is constructed, in which the constraints are transformed under the given uncertainty sets, and then the multiobjective partition-and-bound method is used to solve it. All computations were done in MATLAB R2017b for Windows x64 on the laptop with a 2.50 GHz Intel Core i5-7200U CPU and 8 GB RAM.

4.1. Box uncertainty sets and constraint transformation

Decision-makers need to obtain a there-year development plan of the oilfield, and are prepared to adopt four types of stimulation treatments, namely, new well, fracturing, triple replacement and perforation adding. The box uncertainty sets $U_r = \left\{ \tilde{r} | \underline{r}_{il} \leqslant \tilde{r}_{il} \leqslant \bar{r}_{il}, \forall l \in \{T\}, i \in \{I\} \right\}$ and $U_e = \left\{ \tilde{e} | \underline{e}_{ilt} \leqslant \tilde{e}_{ilt} \leqslant \bar{e}_{ilt}, l \leqslant t, \forall l, t \in \{T\}, i \in \{I\} \right\}$ are given by the experience of relevant experts, where $\underline{r}_{il}$ and $\bar{r}_{il}$ are the lower and upper bounds of $\tilde{r}_{il}$, and $\underline{e}_{ilt}$ and $\bar{e}_{ilt}$ are the lower and upper bounds of $\tilde{e}_{ilt}$. For the new

Table 2
The upper and lower bounds of the initial annual stimulation effect per well, the annual stimulation effect error per well, and the annual new recoverable reserves per well (ton).

Implemented year	Treatment	Initial annual effect per well	Annual effect error per well in each observed year			Annual new recoverable reserves per well
			1	2	3	
1	NW	622 ± 81	0	±8.39	±7.39	2000 ± 205
	FR	340 ± 57	0	±1.01	±0.4	–
	TR	160 ± 26	0	±0.39	±0.12	–
	PA	164 ± 41	0	±0.5	±0.19	–
2	NW	624 ± 88	–	0	±8.39	2050 ± 210
	FR	338 ± 53	–	0	±1.01	–
	TR	159 ± 42	–	0	±0.39	–
	PA	164 ± 56	–	0	±0.5	–
3	NW	624 ± 91	–	–	0	2100 ± 215
	FR	339 ± 67	–	–	0	–
	TR	159 ± 46	–	–	0	–
	PA	164 ± 60	–	–	0	–

* NW: New well; FR: Fracturing; TR: Triple replacement; PA: Perforation adding.

Table 3
The ratio of annual stimulation effect to initial annual effect, the unit oil-related cost (dollar/ton), the unit well-related cost (dollar/well) and workload bounds.

Implemented year	Treatment	Effect ratio in each observed year			Unit oil-related cost in each observed year			Unit well-related cost	Workload bounds	
		1	2	3	1	2	3		Lower	Upper
1	NW	1	1.72	1.39	96	99	103	29466	390	1145
	FR	1	0.61	0.19	57	60	60	27331	510	1245
	TR	1	0.6	0.2	59	60	60	12920	190	925
	PA	1	0.59	0.19	49	50	50	11837	160	925
2	NW	–	1	1.72	–	96	99	31240	125	945
	FR	–	1	0.61	–	58	58	28964	165	965
	TR	–	1	0.6	–	62	62	13061	100	825
	PA	–	1	0.59	–	53	56	11962	65	545
3	NW	–	–	1	–	–	104	35102	390	905
	FR	–	–	1	–	–	60	32559	180	915
	TR	–	–	1	–	–	64	13265	100	690
	PA	–	–	1	–	–	57	12135	65	545

recoverable reserves per well, its box uncertainty set $U_s = \left\{\widetilde{s}\,\middle|\,\underline{s}_t \leqslant \widetilde{s}_t \leqslant \overline{s}_t, \forall t \in \{T\}\right\}$ are also given, where $\underline{s}_t$ and $\overline{s}_t$ are the lower and upper bounds of $\widetilde{s}_t$. These upper and lower bounds of uncertain parameters are shown in the Table 2.

In fact, the box set is a special hypercube set. And the uncertainty set is partitioned by multiple hyperplanes in Eq. (9), so the subsets obtained are still hypercube sets. When the uncertainty set is a hypercube set, the uncertain linear constraint in robust optimization can be transformed. For example, in one iteration, the pth partition is $U_p = \{\boldsymbol{\zeta}|\boldsymbol{D\zeta} + \boldsymbol{q} \geqslant \boldsymbol{0}\}$, where $\boldsymbol{D} \in \mathbb{R}^{m\times\frac{1}{2}\left(3IT+IT^2+2T\right)}$, $\boldsymbol{q} \in \mathbb{R}^m$, and $\boldsymbol{m}$ is the number of constraints for uncertain parameters in this partition. For the constraint transformed from the objective of total new recoverable reserves maximization, the equivalent "worst-case" is first considered as follows,

$$\max_{\zeta\in U_p} - \sum_{t=1}^{T} \widetilde{s}_t x_{1tp} \leqslant -z_2. \tag{12}$$

Then by the strong duality, the maximization problem in Eq. (12) and its dual problem can get the same optimal objective value, so the constraint is equivalent to,

$$\min_w \left\{\boldsymbol{q}^T\boldsymbol{w} : \boldsymbol{D}^T\boldsymbol{w} = \left(x_{11p}, 0, \cdots, 0, x_{12p}, 0, \cdots, 0, \cdots, x_{1Tp}, 0, \cdots, 0\right)^T, \boldsymbol{w} \geqslant 0\right\} \leqslant -z_2. \tag{13}$$

Therefore, when solving the partitioned version (10) by the algorithm in Section 3.2, the decision vector $\boldsymbol{x}_p$ of this partition is used in each chromosome, and the linear programming on the left side of Eq. (13) is solved,

$$-z_2^p = \min_w \boldsymbol{q}^T\boldsymbol{w}$$

$$\text{subject to}: \tag{14a}$$

$$\boldsymbol{D}^T\boldsymbol{w} = \left(x_{11p}, 0, \cdots, 0, x_{12p}, 0, \cdots, 0, \cdots, x_{1Tp}, 0, \cdots, 0\right)^T$$

$$\boldsymbol{w} \geqslant 0, \tag{14b}$$

to obtain the value of decision vector $\boldsymbol{w}$ while ensuring that constraint (14a) and (14b) are satisfied. And z_2^p is actually the total new recoverable reserve for $\boldsymbol{x}_p$ in the "worst-case" of this partition. Therefore, the minimum value of z_2^p on all partitions is assigned to z_2 for this chromosome so that Eq. (12) is also satisfied. The annual oil production constraints and the constraint transformed from total cost minimization are also treated in similar approaches. However, since the right side of production constraints is a given target, no assignment is required, and these constraints may not be satisfied.

4.2. Computational results

The ratio of annual stimulation effect per well to initial annual effect per well in each year, the unit oil-related cost and the unit well-related cost, and the upper and lower workload bounds are given, as shown in Table 3.

In addition, the targets of annual stimulation effect are 800,000 tons, 1,200,000 tons and 2,000,000 tons respectively, and the discount rate is 6%. The maximum generation, population size, crossover probability, mutation probability, distribution index for crossover and distribution index for mutation are set to 100, 300, 0.8, 0.1, 20 and 100, respectively. Set the threshold $\delta = 0.1$ in the termination condition. The algorithm terminates when iteration count is 3, and the number of partitions at each iteration is 1, 3, and 10, respectively.

In the iterative process, the number of partitions is continuously increased, so that more and more uncertain parameter realizations are considered, which affects the optimal decision at each iteration. The robust efficient frontiers of partitioned version (10) and the lower bound at each iteration are illustrated in Fig. 3.

As can be seen from Fig. 3, the robust efficient frontier of partitioned version is getting better, and the gap between it and the corresponding lower bound is getting smaller in the iterative process. Finally, a robust efficient solution set with finite adaptability of model (7) is obtained. Each robust efficient solution in this set is a development plan. And finite adaptations can be made in the latter decisions according to the realization of previous uncertain parameters. So this set provides good decision support for development planning. Decision-makers can conduct a more comprehensive investigation of these plans, and further use multi-attribute decision-making to select the final development plan.

In order to compare with multistage stochastic programming and non-adjustable robust optimization, two models for the problem are constructed. In the multistage stochastic model, the normal distribution is used to describe uncertain parameters, where 1/6 of their range is

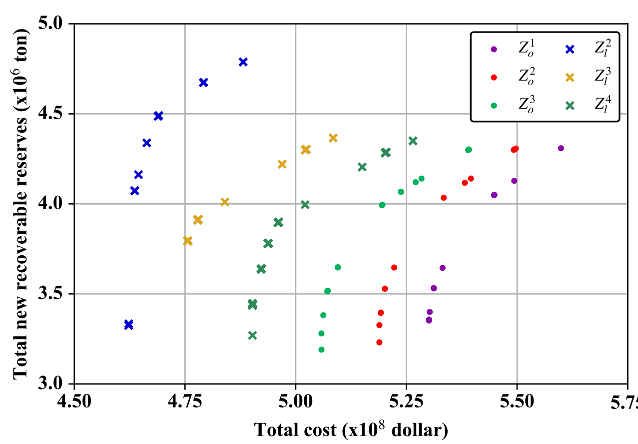
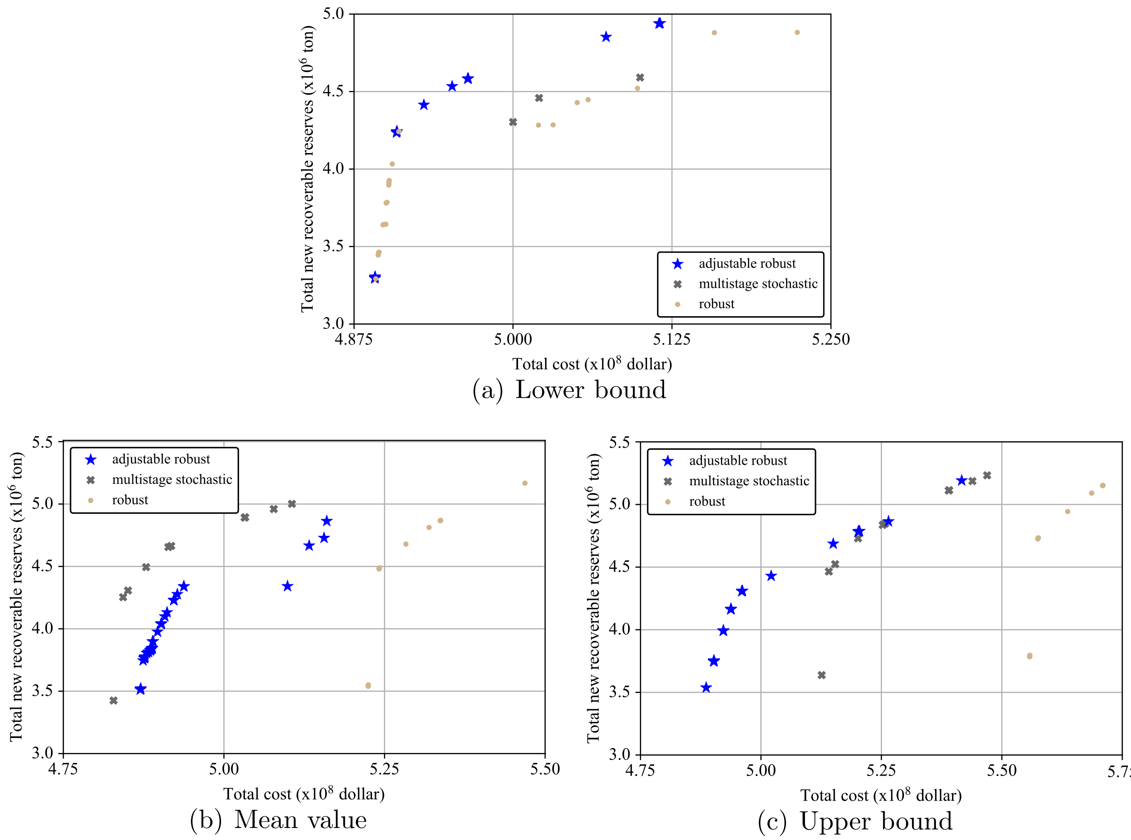


Fig. 3. Robust efficient frontier of model (10) and the lower bound at each iteration.



(a) Lower bound

(b) Mean value

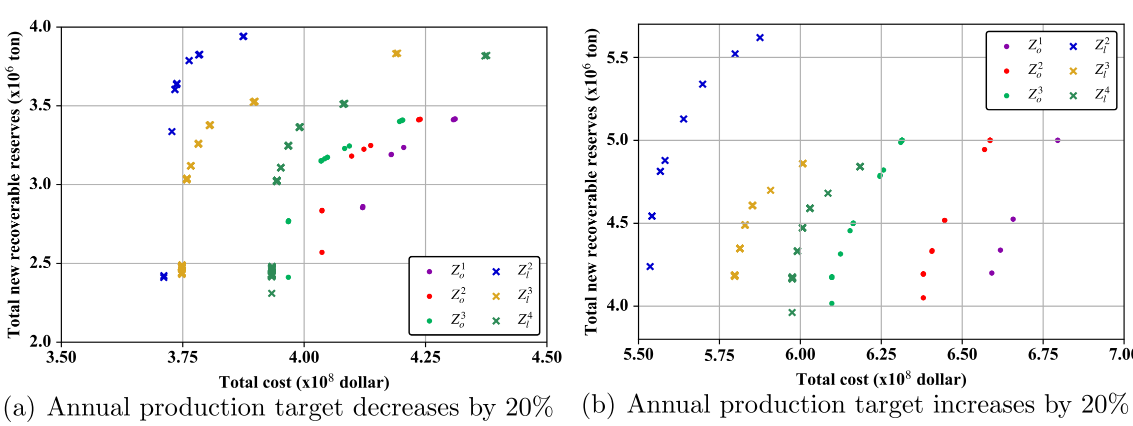
(c) Upper bound

Fig. 4. Comparison of three methods under different realizations of parameters.

taken as the standard deviation and the mean value remains unchanged. Besides, the model optimizes the expectation of cost and new recoverable reserves, and requires that the probability of achieving target is not less than 90% in the annual production constraint. The scenario-based method is also used to deal with this multistage model by Monte Carlo simulation and adding nonanticipativity constraints. In the robust optimization model, the "worst-case" is directly optimized without considering the adjustability caused by multiple stages. And NSGA-II is still used to solve the transformed multistage stochastic and robust model. The running time of the proposed method, multistage stochastic model and non-adjustable robust optimization are 10945s, 16847s and 2334s, respectively. In Fig. 4, the effective frontiers of these three methods are compared under different realizations of uncertain parameters.

According to Fig. 4, the proposed model has the best performance when the realization is the upper or lower bound of parameters, and has more feasible solutions. When the realization is equal to the mean value, its result is slightly worse than the multistage stochastic programming, but far better than the non-adjustable robust model. On the one hand, the solution set is robust to the uncertainty in parameters, so it can keep feasibility under different realizations. On the other hand, the adjustability of multistage decisions is considered, so that the decision can be adjusted after the previous parameters are observed, avoiding the over-conservative problem in the robust optimization, as shown in Figs. 4(b) and (c).

In practice, the setting of production targets will affect the planning result significantly, as well as company performance. This paper does the sensitivity analysis to observe the impact of their values on the workload of different treatments and the objective values. In Fig. 5, the annual production target decreases and increases by 20%, respectively,



(a) Annual production target decreases by 20%

(b) Annual production target increases by 20%

Fig. 5. Robust efficient frontiers under different production targets.

and then the corresponding robust effective frontier is obtained by solving the proposed model.

Compared with Fig. 3, when the production target is lower, the robust effective frontier becomes more "flat", which means that the improvement of sustainable development will cost a lot. In this scenario, the annual workload of different treatment types decreases by 13%, 16%, 26% and 27% on average. When the target is higher, the robust effective frontier becomes more "steep", and the measure workload increases by 32%, 1%, 5% and 11% on average. Because the number of new wells has a direct impact on the new recoverable reserves, the decision-maker can exchange a low investment for better sustainability in this setting.

5. Conclusions

In this paper, the robust optimization approach is used to solve the uncertain oilfield development planning problem. This planning is a long-term strategic decision, and the uncertainty of development indicators is very high. With constraints of oil production and workload balance, this paper constructs a multiobjective multistage robust integer optimization model that optimizes the treatment workload in each year to reduce cost and enhance sustainable development as much as possible. In order to solve the proposed model, a multiobjective partition-and-bound method is designed. In the iterative process, the uncertainty set is first partitioned and nonantipativity constraints are added. Then NSGA-II is used to solve the partitioned version and the lower bound model. And finally the robust efficient solution set with finite adaptability is obtained. Therefore, decision-makers can use these development plans for selection.

This paper also has some limitations. In future research, the cost of oilfield development can be more completely characterized, and other objectives and constraints can be considered according to practical needs. In addition, the proposed multiobjective partition-and-bound method can be extended to other fields that need to solve the multiobjective multistage robust mixed-integer optimization model. Finally, the transformed model does not necessarily remain linear under other types of uncertainty sets. And other solution methods for multiobjective optimization instead of NSGA-II can also be used and may have better performance (Mogale, Kumar, Kumar, & Tiwari, 2018; Mogale, Lahoti, et al., 2018; Mogale, Cheikhrouhou, & Tiwari, 2020). So the efficiency of algorithm will be improved.

CRediT authorship contribution statement

Sen Yan: Conceptualization, Methodology, Writing - Original Draft. **Xiaoyu Ji:** Supervision, Formal analysis, Writing- Reviewing & Editing. **Yanjun Fang:** Visualization, Investigation. **Hongguo Sun:** Resources, Data Curation.

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