Session 3.2 PCW.

#17.1

1.

linearly separable.
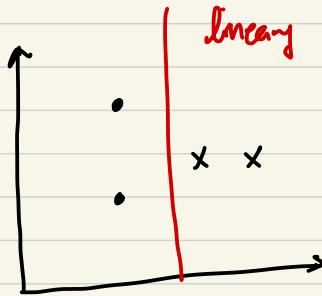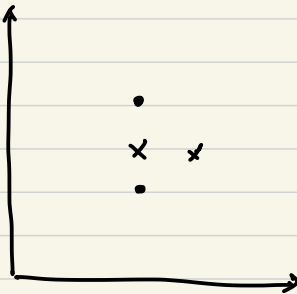
& linearly independent.

2.

linearly independent
but not linearly
separable.

#17.2

(x^n, y^n), $n = (1, \cdots , N)$

$$\sum_{n=1}^{N} (x^n, y^n)/N$$

$y = ax + b$

#17.6

$$\sigma(x) = e^x / (1+e^x)$$

$$\text{lur } \sigma(x) = y$$

$$\sigma^{-1}(x) \Rightarrow \quad x = e^y / (1+e^y)$$

$$x + e^y \cdot x = e^y$$

$$e^y (x-1) = -x$$

$$e^y = \frac{-x}{(x-1)}$$

$$y = -\ln \left(\frac{x}{x-1} \right)$$

$$\therefore \ \sigma^{-1}(x) = - \ln \left(\frac{x}{x-1} \right)$$

#8.3

a)
$$\sigma(a) = \frac{1}{1+e^{-a}}$$

Show:
$$\frac{d\sigma(a)}{da} = \sigma(a)(1-\sigma(a))$$

$$= \frac{1}{1+e^{-a}}\left(1 - \frac{1}{1+e^{-a}}\right)$$

$$= \frac{1}{1+e^{-a}} - \frac{1}{(1+e^{-a})^2}$$

$$= \frac{e^{-a}}{(1+e^{-a})^2}$$

Confirming with Sage:

Type some Sage code below and press Evaluate.

```
1  f(x) = 1/(1+e^(-x))
2
3  f.diff()
```

Evaluate

```
x |--> e^(-x)/(e^(-x) + 1)^2
```