Homework 1: Q2

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1 Part (a) Proof Idea

If the lowest rated show of A is a higher rated show than the highest rated show of b, if b were to match up those two shows, A would win that match up. For the rest of the shows of A, the shows would have a higher (or equal) rating as the lowest rated show of A. The rest of the shows of B would have a lower (or equal a rating) as the highest rated show of B. I will label the set of shows of A as A(s) (excluding the lowest rated show, labeled as "a") and the set of shows of B as A(s) (excluding the highest rated show, labeled as "b"). Every item in A(s)≥a and b≥B(s). If A(s)≥a≥b≥B(s), all shows in A(s) has a higher rating that all shows in B(s) so no match up will end in a winning time slot for B and thus A will win all match ups.

Ex. A- 10 9 8
B- 5 4 2
None of B's shows with ratings 5,4,2 put in the time slot of A's show of rating 10 will win the match up of that time frame. This is the same for A's 9 and 8 rated shows.

2 Part (b) Proof Idea

Ex.A- 20 18 15
B- 19 16 12
If B switches it's 19 and 16 rated show's time frames, B would win the second time slot (A with show rated 18 and B with show rated 19). Thus, this match-up is instable. In fact, any match up where a company has show ratings that go in between the show ratings of other shows and outside (below and/or above) will cause an instable match up. There are 6 different ways each show could organize their shows in this case (3 shows = 3! Possibilities because n shows = n! possibilities). There are also 3! possible schedules of the other company for each possible schedule of the first company, as in 3!*3! = 36 (n!*n!) = (n!)^2. If one match up of schedules is instable, than all match ups combinations are instable.

3 Part (b) Proof Details

I am going to describe a concept I will name criss-cross. If both companies have shows with ratings that go in between and "out" of the other companies show's rating than there will be instability. That is to say:

If A is the list of company A's shows (as their rating) with "a" being shows in A
and B is the list of company B's shows (as their rating) with "b" being shows in B,

Subscripts will be assigned to denote different b's and a's. They are NOT and indication of any sort of ordering and thus, definitely not any sort of consecutive ordering. If there exists some b (denoted as b_1) that is greater than some a (denoted as a_1) but less than another a (denoted as a_2), and some other b (denoted as b_2) that is less than the a_1 , there is criss cross, and there is instability.

To visualize, if ordered from left to right:
…….a_2 …… b_1 ……. a_1 ……..b_2 ……..
In place of …… are the rest of A, B ordered from left to right from highest to lowest.

Regardless of the rest of A,B, if there just exists a_1 ,a_2 ,b_1 ,b_2 that follow this inequality/ordering than this match up in instable because lets say in one match up, a_2 is matched with b_1 and a_1 with b_2. Than, a_2 would have won their match up and a_1 would have won that match up too. But if company b were to switch up b_2 ,b_1's time, a_2 would still win their match up but a_1 would have lost. Thus, because a company could change their schedule to win more match ups, the combination of match ups are instable by definition.

If one match up combination of company A's shows and schedules and company B's shows and schedule is instable than all $(n!)^2$ possible match ups are instable because the definition of instability involves seeing if given S_a (schedule of shows of company A) and S_b (schedule of shows of company B), is there some different S_b (denoted as S_b`) that will win more match ups with $S_a$ or is there some different $S_a$ (denoted as $S_a$`) that will win more match ups with S_b. Thus, the definition of instability isn't really pertaining to a specific S_a and S_b but the sets A, and B. If B is winning the most match ups possible B can, that means A can rearrange in some way to win some more (given that there is criss cross) and the vice versa.

This is to say, that there exists a pair of schedules that are not stable, which as said previously means that any pair of schedules made by company A and B is also instable, which means there in a set of n shows provided by company A and B, there is a possibility that their provided shows respectively will create only instable pairs of schedules.
Thus, because there exists a match up that has no stable pair of schedules, it can cannot be so that for every set of n movies, their always exists a stable pair of schedules.