Homework 6: Q3

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1 Part (a): Proof Idea

I can prove that for any input there is an optimal solution with no idle time by contradiction that there is an input such that there is no optimal solution with no idle time.

I will assume the input to be an list of arrays of size 4 of all athletes, in which in each index of the array is their number, the swim time, their run time, and their bike time.

For such an input to not have an optimal solution with no idle time would mean that given athletes swim right after each other, there is no ordering of the athletes such that no other ordering is faster.

But each ordering has a finish time, and there are n! orderings. I can generate n! orderings and can do a min function on those orderings to find which one finishes the quickest (ties are fine). As such, there is one solution that is faster than the rest that has no idle time, thus is the contradiction.

2 Part (b): Algorithm Idea

input is list of int[] in which each int[] is an athlete, and is always of size 4 where the 0th index is that athletes number, 2^{nd} = swim time, 3^{rd} = bike time, 4^{th} = run time.
Order all athletes by bike+run time in decreasing value

3 Part (b): Algorithm Details

Algorithm: Algorithm1

```
orderAthletes(list<int[]> athletes){
	arraylist<int[]> ret = list.sort(athletes) //by their bike+run time in decreasing order
	return ret;
}
```

4 Part (b): Proof of Correctness Idea

Bi = athlete I's bike+run time
Ai = athlete I's swim time

a)I will prove that given two athletes, the athlete with the bigger b should go first to reduce time.
b)Then I will prove that the optimal solution (defined as ordering that finishes fastest) ,one where the athlete with the biggest b goes first than the rest is in decreasing order based on athlete i's b, by contradiction.

If I assume my solution given by my algorithm isn't an optimal solution, and since I know that given any input there is an optimal solution based on question 3 part a, the optimal solution must be one that isn't just the ordering of all athletes in decreasing value of b, which mean at some point there is an athlete with a b lower than the athlete after it.
I will then prove that there is at least one better solution using (a) and some follow up steps, in which thus contradicts the logical statement that this is the optimal solution.

If I can prove all of this, than my algorithm, that is simply an algorithm that orders the athletes in decreasing b, is correct.

5 Part (b): Proof Details

a)
a1 b1
a2 b2

b1>b2
a1+a2+b2 if a2+b2>b1
a1+b1 else

if I switch b2 and b1, (have the bigger one go second)

a1 b2
a2 b1

b1>b2 still

a1+a2+b1 if a2+b1>b2

a1+b2 else

a1+a2+b1 > a1+a2+b2 since b1>b2
a1+a2+b1 > a1+b1 since it is missing the positive amount of a2
having the larger b second gives a bigger amount

switching b1 and b2 (or having the bigger b go second) always leads to a bigger value.

a2+b1 < b2 is never possible because b1 is already bigger than b2, added with a positive int will just make it further bigger

this is regardless of whether a1 or a2 are the same, or different(one is bigger than the other by definition) since if we assume a1 is the bigger a, and we switch the two, it would be a2+a1+b1, and if we didn't switch it would be a1+a2+b1 which is the same

making the athlete with the bigger b go second will always make the pair end later than if the bigger b goes first regardless if it's a is bigger or smaller

b)There exists a athlete i and athlete i+1 (i is the placement/ordering of athlete in supposed optimal solution) such that
ai bi
ai+1 bi+1

bi<bi+1 given by the situation:(restated below)
since optimal solution isn't mine, which is total ordering of athletes in decreasing order of b, there was a switch, which means that there is no total decreasing order of b, which means that at some point there is an athlete with a b less than the athlete after it)

if I switched the two in the ordering:
ai+1 bi+1
ai bi

the time the two would take to finish after their starting time(we can just say is 0 for simplicity) would change from ai+a(i+1)+b(i+1) to a(i+1)+ai+bi or a(i+1)+b(i+1).
ai+a(i+1)+b(i+1) > a(i+1)+ai+bi since b(i+1) > bi
ai+a(i+1)+b(i+1) > a(i+1)+b(i+1). Since the rhs is the same as lhs but missing a positive value ai the rest is proved in (a).

Thus if we switch the ordering of athlete i and athlete i+1, the pair finish earlier after it's starting time, thus everything that follows start earlier and if finishes earlier. Thus, there is an ordering that is earlier than this supposed optimal solution, thus it isn't an optimal solution.

6 Part (b): Runtime Analysis

O(n) for input
O(nlogn) to sort
O(1) for return
O(n)+O(nlogn)+O(1) = O(nlogn)