Q1: [Probability] From Textbook 3E: Q13.8

13.8 Given the full joint distribution shown in Figure 13.3, calculate the following:

- **a**. **P**(*toothache*).
- **b**. **P**(*Cavity*).
- **c**. **P**(*Toothache* | *cavity*).
- **d**. **P**(*Cavity* | *toothache* ∨ *catch*).

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the Toothache, Cavity, Catch world.

13.8 ***Answer****: The main point of this exercise is to understand the various notations of* ***bold versus non-bold P****, and* ***uppercase versus lowercase variable names****.*

The rest is easy, involving a small matter of addition.

a. This asks for the probability that Toothache is true.

*P(****toothache****) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2*

b. This asks for the vector of probability values for the random variable ***Cavity****. It has two values, which we list in the order (true, false).*

First add up 0.108 + 0.012 + 0.072 + 0.008 = 0.2.

Then we have P(Cavity) = (0.2, 0.8) .

c. This asks for the vector of probability values for ***Toothache****, given that* ***Cavity is true****.*

P(Toothache|cavity) = h(.108 + .012)/0.2,(0.072 + 0.008)/0.2i = (0.6, 0.4)

d. This asks for the vector of probability values for ***Cavity****, given that* ***either Toothache or Catch is true****.*

First compute P(toothache ∨catch) = 0.108+ 0.012+ 0.016+ 0.064+ 0.072 + 0.144 = 0.416.

Then P(Cavity|toothache ∨ catch) = h(0.108 + 0.012 + 0.072)/0.416,(0.016 + 0.064 + 0.144)/0.416i = (0.4615, 0.5384)

Q2 :

13.13 Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

13.13 Let V be the statement that the patient has the virus, and A and B the statements that the medical tests A and B returned positive, respectively.

The problem statement gives:

$P(V) = 0.01$
$P(A|V) = 0.95$ $P(A|\neg V) = 0.10$
$P(B|V) = 0.90$ $P(B|\neg V) = 0.05$
The test whose positive result is more indicative of the virus being present is the one whose posterior probability, $P(V|A)$ or $P(V|B)$ is largest. One can compute these probabilities directly from the information given, finding that $P(V|A) = 0.0876$ and $P(V|B) = 0.1538$, so B is more indicative.

Equivalently, the questions are asking which test has the highest posterior odds ratio $P(V|A)/P(\neg V|A)$. From the odd form of Bayes theorem: $P(V|A)\ P(\neg V|A) = P(A|V)\ P(A|\neg V)\ P(V)\ P(\neg V)$ we see that the ordering is independent of the probability of V and that we just need to compare the likelihood ratios $P(A|V)/P(A|\neg V) = 9.5$ and $P(B|V)/P(V|\neg V) = 18$ to find the answer.

[Probability and Bayes theorem]
After conducting a blood test, the doctor told you that you were tested positive for a fatal disease. Worse, the test is quite accurate: the probability of false-positive (one is tested positive without the disease) is 0.05, and the probability of false-negative (one is tested negative with the disease) is 0.02. Seeing that you are desperate, the doctor told you that overall the disease is rather rare, only 1 in 10,000 people.

(a). What is the chance now that you have the disease? [10 pts]
(b). Naturally, the doctor orders a retest on you. The result of the second independent test is still positive. What is now your chance of having the disease?[15 pts]
(c). If you have done a total of k independent tests, and the results are all positive (quite depressive indeed!), what is your chance of having the disease, expressed in k?
Answer:
One + test:
$p(d|+) = p(+|d)\ p(d)/p(+) = 0.98 * 0.0001 / p(+)$
  $p(+) = p(+|d)\ p(d) + p(+|\sim d)\ p(\sim d) = 0.050093$ (or using normalization; same)
So $p(d|+) = 0.00196$

Two positive tests...
$p(d|+,+) = p(+|d,+)\ p(d|+)/p(+|+) = p(+|d)\ p(d|+)/p(+|+)$
  $= 0.98 * 0.00196 / p(+|+) = 0.0383$
  $p(+|+) = p(+|+,d)p(d) + p(+|+,\sim d)p(\sim d) = 0.050093$ (or normalization; same)
OR
$p(d|+,+) = p(+,+|d)\ p(d)/p(+,+) = p(+|d)\ p(+|d,+)\ p(d) / p(+,+)$
  $= p(+|d)\ p(+|d)\ p(d) / p(+,+)$
$p(+,+)= p(+,+|d)\ p(d) + p(+,+|\sim d)\ p(\sim d)$
  $= p(+|d)^2\ p(d) + p(+|\sim d)^2\ p(\sim d)$ (or using normalization)

So for k positive tests:
= p(+|d)^k p(d)/p(+,+,...)
p(+,+,...)= p(+|d)^k p(d) + p(+|~d)^k p(~d)

[Naive Bayes Classifier]

Given the training data in the below table, predict if Bob will default his loan.

<u>Bob:</u>
Homeowner: No
Marital status: Married
Job experience: 3
Answer:

Home owner	Marital Status	Job experience (1-5)	Defaulted
Yes	Single	3	No
No	Married	4	No
No	Single	5	No
Yes	Married	4	No
No	Divorced	2	Yes
No	Married	4	No
Yes	Divorced	2	No
No	Married	3	Yes
No	Married	3	No
Yes	Single	2	Yes

- $P(Y = No) = 7/10$
- $P(Home\ owner = No|Y = No) = 4/7$
- $P(Marital\ status = Married|Y = No) = 4/7$
- $P(Job\ experience = 3|Y = No) = 2/7$

$P(Y=Yes) = 3/10$

$P(HO=No|Y=Yes)=\frac{2}{3}$

$P(Married|Y=Yes)=\frac{1}{3}$

$P(Job=3|Y=Yes)=\frac{1}{3}$

$P(Bob\ will\ default)=3/10 * \frac{2}{3} * \frac{1}{3}*\frac{1}{3} =0.022$

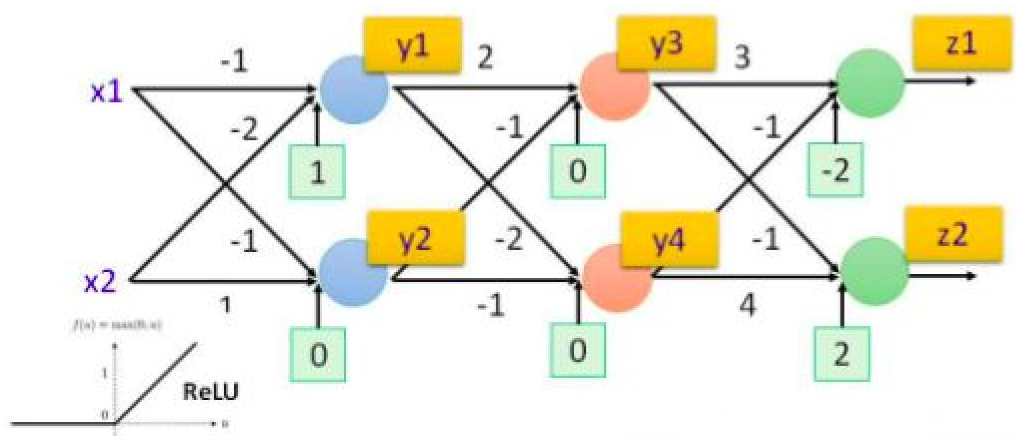
P(Bob will NOT default) =7/10*4/7*4/7*2/7=0.065

Predict: BOB WILL NOT DEFAULT

Another naive Bayes example.

[Deep Neural Network]

A deep neural network with given weights and biases is shown in the top figure. The activation function is the ReLU function (see the insert). When (x1, x2) = (0, 1) and (1, -1), what will the outputs (z1, z2) be, respectively? (Note: to show your work, also write down the hidden layer output (y1, y2), and (y3, y4) in your solutions).



solution:

(x1, x2) = (0, 1), (y1, y2) = (0,1),

(y3, y4)=(0,0), (z1, z2) = (0,2)

(x1, x2) = (1, -1), (y1, y2) = (2,0),

(y3, y4)=(4,0), (z1, z2) = (10,0)