CS 3346 Assignment 3

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Q1 a) $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$

b) $P(Cavity = cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$

$P(Cavity = \neg cavity) = 0.016 + 0.064 + 0.144 + 0.576 = 0.8$

$\therefore P(Cavity) = \langle 0.2, 0.8 \rangle$

c) $P(Toothache \mid cavity) = \alpha P(Toothache, cavity)$

$= \alpha [P(Toothache, cavity, catch) + P(Toothache, cavity, \neg catch)]$

$= \alpha [\langle 0.108, 0.072 \rangle + \langle 0.012, 0.008 \rangle]$

$= \alpha \langle 0.12, 0.08 \rangle$

$= \langle 0.6, 0.4 \rangle$

d) $P(Cavity \mid toothache \vee catch)$

$= \alpha P(Cavity, toothache \vee catch)$

$= \alpha [P(Cavity, toothache, catch) + P(Cavity, toothache, \neg catch) + P(Cavity, \neg toothache, catch)]$

$= \alpha [\langle 0.108, 0.016 \rangle + \langle 0.012, 0.064 \rangle + \langle 0.072, 0.144 \rangle]$

$= \alpha \langle 0.192, 0.224 \rangle$

$= \langle 0.462, 0.538 \rangle$

Q2 Let V be the event that a person is carrying the virus
Let NV be the event that a person is not carrying the virus
Let + be the event that the recognize test returns positive
Let − be the event that the recognize test returns negative

$\therefore P(V) = 0.01 \qquad P(NV) = 1 - 0.01 = 0.99$

A: $P(+ \mid V) = 0.95 \qquad P(+ \mid NV) = 0.1$

$P(+) = P(+ \cap V) + P(+ \cap NV) = P(+ \mid V)P(V) + P(+ \mid NV)P(NV)$

$$P(V \mid +) = \frac{P(+ \mid V)P(V)}{P(+)} = \frac{P(+ \mid V)P(V)}{P(+ \mid V)P(V) + P(+ \mid NV)P(NV)}$$

$$= \frac{0.95 \times 0.01}{0.95 \times 0.01 + 0.1 \times 0.99} = 0.0876$$

B: $P(+|V)=0.9 \quad P(+|NV)=0.05$

$P(+) = P(+\cap V)+P(+\cap NV) = P(+|V)P(V)+P(+|NV)P(NV)$

$$P(V|+) = \frac{P(+|V)P(V)}{P(+)}$$

$$= \frac{P(+|V)P(V)}{P(+|V)P(V)+P(+|NV)P(NV)}$$

$$= \frac{0.9\times0.01}{0.9\times0.01+0.05\times0.99}$$

$$=0.1538$$

$\because P_A(V|+) < P_B(V|+)$

$\therefore$ test B returning positive is more indicative of someone really carrying the virus.

Q3. a) Let D be the event that a person has a disease
Let ND be the event that a person does not have a disease
Let + be the event that the test returns positive
Let − be the event that the test returns negative

$P(+|ND)=0.05 \quad P(-|D)=0.02 \quad P(D)=0.0001$

$$P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D)+P(+|ND)P(ND)}$$

$$= \frac{(1-0.02)\times0.0001}{(1-0.02)\times0.0001+0.05\times(1-0.0001)}$$

$$\approx 0.0020$$

$\therefore$ The chance that you have the disease is 0.002

b) $P(+) = P(+\cap D)+P(+\cap ND) = P(+|D)P(D)+P(+|ND)P(ND)$

$= (1-0.02)\times0.0001+0.05\times(1-0.0001)$

$= 0.050093$

$P(++) = P(++|D)P(D)+P(++|ND)P(ND)$

$= (1-0.02)\times(1-0.02)\times0.0001+0.05\times0.05\times(1-0.0001)$

$= 0.00009604+0.00249975$

$= 0.00259579$

$$P(D|++) = \frac{P(++|D)P(D)}{P(++)}$$

$$= \frac{(1-0.02)\times(1-0.02)\times 0.0001}{0.00259579}$$

$$= \frac{0.00009604}{0.00259579}$$

$$\approx 0.0370$$

$\therefore$ 0.0370 is now your chance of having the disease

c) $P(+^k) = P(+^k|D)P(D) + P(+^k|ND)P(ND)$

$$= (1-0.02)^k \times 0.0001 + 0.05^k \times (1-0.0001)$$

$$P(D|+^k) = \frac{P(+^k|D)P(D)}{P(+^k)}$$

$$= \frac{(1-0.02)^k \times 0.0001}{(1-0.02)^k \times 0.0001 + 0.05^k \times (1-0.0001)}$$

Q4 A: attributes D: defaulted ND: non-defaulted

$$P(A|D) = \frac{2}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{2}{27}$$

$$P(A|ND) = \frac{4}{7}\times\frac{4}{7}\times\frac{2}{7} = \frac{32}{343}$$

$$P(A|D)P(D) = \frac{2}{27}\times\frac{3}{10} = 0.0222$$

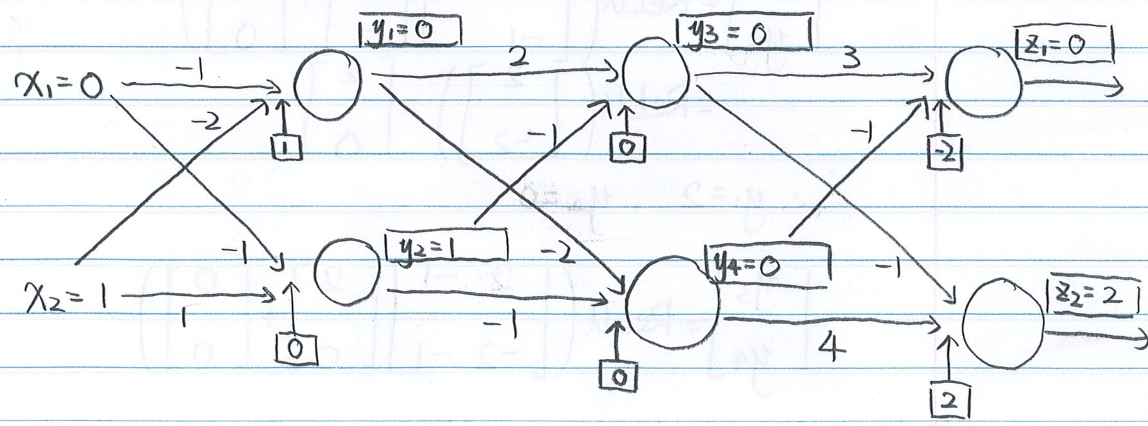
$$P(A|ND)P(ND) = \frac{32}{343}\times\frac{7}{10} = 0.0653$$

$\because P(A|D)P(D) < P(A|ND)P(ND)$

$\therefore$ Bob will not default his loan.

Q5. ReLU function: $f(x) = \max(0, x)$

when $x_1 = 0$, $x_2 = 1$



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathrm{ReLU}\left(\begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$= \mathrm{ReLU}\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore y_1 = 0,\ y_2 = 1$

$$\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} = \mathrm{ReLU}\left(\begin{bmatrix} 2 & -1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right)$$

$$= \mathrm{ReLU}\left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

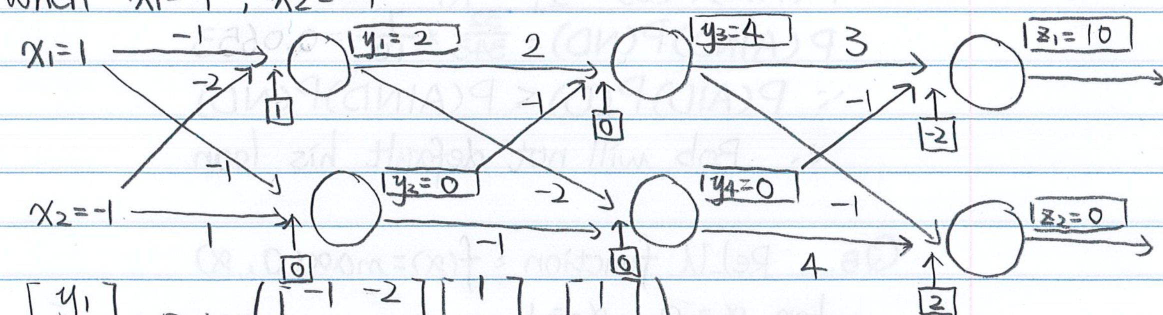
$\therefore y_3 = 0,\ y_4 = 0$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \mathrm{ReLU}\left(\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right)$$

$$= \mathrm{ReLU}\left(\begin{bmatrix} -2 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$\therefore z_1 = 0,\ z_2 = 2$

when $x_1 = 1,\ x_2 = -1$



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathrm{ReLU}\left(\begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$= \mathrm{ReLU}\left(\begin{bmatrix} 2 \\ -2 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\therefore y_1 = 2,\ y_2 = 0$

$$\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} = \mathrm{ReLU}\left(\begin{bmatrix} 2 & -1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right)$$

$$= \text{ReLU}\left(\begin{bmatrix} 4 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$\therefore y_3 = 4, \quad y_4 = 0$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \text{ReLU}\left(\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}\right)$$

$$= \text{ReLU}\left(\begin{bmatrix} 10 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$\therefore z_1 = 10, \quad z_2 = 0$