Assignment 3

Question 1 **[10 marks]**

a. How many students in a class to guarantee that at least two students received the same score on the final exam. If the exam is graded on a scale from 0 to 100 points. (5 marks)

b. what is the minimum number of students required in a Structure Discrete class so that at least six students will receive the same letter grade (A,B,C,D, or F) (5 marks)

a) $n = ?$

$\frac{n}{k} = 2$

101 total points

$\frac{102}{101} = 2$

$n = 102$ students

b) $n = ?$

$\left\lceil \frac{n}{k} \right\rceil = 6$

k = grades A, B, C, D, or E

$k = 5$

$\left\lceil \frac{n}{5} \right\rceil = 6$

$\left\lceil \frac{26}{5} \right\rceil = 5.2$

$= 6$

∴ minimum no. of students is 26

Question 2 **[25 marks]**

The following table gives information on Mobile phone sold by a certain store:

	Percentage of Customers Purchasing	Of Those Who Purchase, Percentage Who Purchase Extended Warranty
Brand 1	0.70 B_1	0.20 W_1
Brand 2	0.30 B_2	0.40 W_2

A purchaser is randomly selected from among all those bought a mobile phone from the store.

Determine the probability that :

$P(W_1 | B_1) = \frac{P(W_1 \cap B_1)}{P(B_1)}$

a. customer purchased Brand 1. (2 marks)

$P(W_1 | B_1)$

b. customer purchased Brand 2 (2 marks)

c. customer purchase extended warranty given that purchase brand 1. (2 marks)

d. customer who bought brand 1 and purchased extended warranty. (4 marks)

e. customer purchased brand 2 and extended warranty purchased. (5 marks)

f. extended probability purchased. (5 marks)

g. purchased brand 1 item given that also bought extended warranty (5 marks)

$0.20 = \frac{P(W_1) \cdot P(B_1)}{P(B_1)}$

$0.20 = \frac{P(W_1) \cdot 0.7}{0.7}$

$0.14 = P(W_1) \cdot 0.7$

$P(W_1) = 0.2$

$P(B_1)$ - probability of purchasing Brand 1 $= 0.7$

$P(B_2)$ - probability of purchasing Brand 2 $= 0.3$

$P(W_1)$ - probability of purchasing Brand 1 with extended warranty $= 0.2$

$P(W_2)$ - probability of purchasing Brand 2 with extended warranty $= 0.4$

a) $P(B_1) = 0.7$

b) $P(B_2) = 0.3$

c) $P(W_1 | B_1) = \frac{P(W_1 \cap B_1)}{P(B_1)}$

$= \frac{0.2 \times 0.7}{0.7}$

$= 0.2$

d) $P(B_1 \cap W_1) = P(B_1) \cdot P(W_1)$

$= 0.7 \times 0.2$

$= 0.14$

e) $P(B_2 \cap W_2)$

Question 3

Explain the given keyword using your own word and represent your understanding by drawing the graph.

a. Vertices
b. Edges
c. Adjacent Vertices
d. Incident Edge
e. Isolated Vertex
f. Loop
g. Parallel Edges

(7 Marks)

a) vertices :

b) edges : line segments joining vertices

c) adjacent vertices : two vertices that are connected by an edge

d) incident edge : an edge that is associated with 1 or 2 vertices.

e) isolated vertex : a vertex with no edge

f) loop : an edge with only one endpoint.

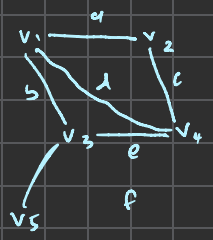
g) parallel edges : two or more distinct edges with the same set of endpoints.

Question 4

Let G = {V, E} be a graph. An undirected graph having V ={v1, v2, v3, v4, v5} and E = {a, b, c, d, e, f}. Where *a* = (*v*1,*v*2), *b* = (*v*1,*v*3), *c* = (*v*2,*v*4), *d* = (*v*1,*v*4), *e* = (*v*3,*v*4) and *f* = (*v*3,*v*5).

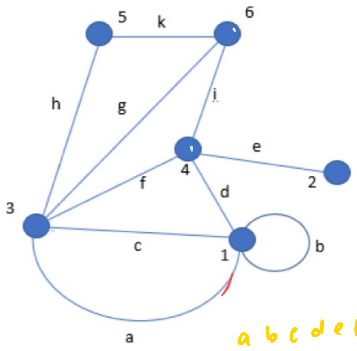
Find the degree of each vertex.

(5 Marks)



vertex	degree
V_1	3
V_2	2
V_3	2
V_4	3
V_5	1

Question 5



h g f c a

a b c d e f g h i k

Given the graph shown above, Find:

i. Incidence Matrix (6 Marks)
ii. Adjacency Matrix (6 Marks)

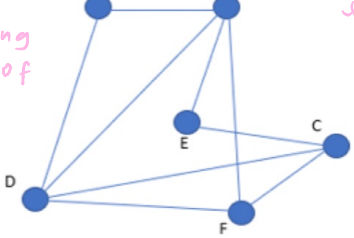
i)

	a	b	c	d	e	f	g	h	i	k
1	1	2	1	1	0	0	0	0	0	0
2	0	0	0	0	1	0	0	0	0	0
3	1	0	1	0	0	1	1	1	0	0
4	0	0	0	1	1	1	0	0	1	0
5	0	0	0	0	0	0	0	1	0	1
6	0	0	0	0	0	0	1	0	1	1

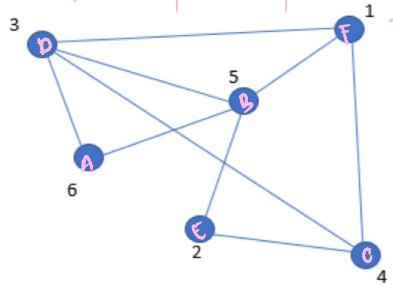
ii)

	1	2	3	4	5	6
1	1	0	2	1	0	0
2	0	0	0	1	0	0
3	2	0	0	1	1	1
4	1	1	1	0	0	1
5	0	0	1	0	0	1
6	0	0	1	1	1	0

Question 6



Graph Y



Graph Z

Determine whether Graph Y and Z above are isomorphic. If it is proven isomorphic, find their adjacency matrix.

(12 Marks)

	Y	Z	degree
vertices	6	6	Y : 2, 3, 4
edges	9	9	Z : 2, 3, 4

$f : Y \to Z$, where $Y = \{A, B, C, D, E, F\}$ and $H = \{1, 2, 3, 4, 5, 6\}$

$f(A) = 6,\ f(B) = 5,\ f(C) = 4,\ f(D) = 3,\ f(E) = 2,\ f(F) = 1$

Proven

adjacency matrix

A_Y

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	0	1	1	1
C	0	0	0	1	1	1
D	1	1	1	0	0	1
E	0	1	1	0	0	0
F	0	1	1	1	0	0

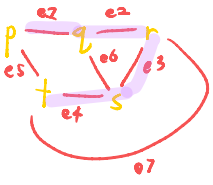
A_Z

	6	5	4	3	2	1
6	0	1	0	1	0	0
5	1	0	0	1	1	1
4	0	0	0	1	1	1
3	1	1	1	0	0	1
2	0	1	1	0	0	0
1	0	1	1	1	0	0

Question 7

Consider an undirected graph with vertices V = {p, q, r, s, t} and edges E = {e1, e2, e3, e4, e5, e6, e7}.
The edges are defined as follows:

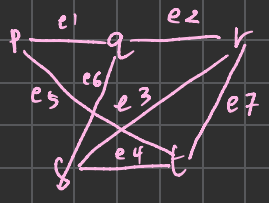
e1 = (p, q); e2 = (q, r); e3 = (r, s); e4 = (s, t); e5 = (t, p); e6 = (q, s); e7 = (r, t)

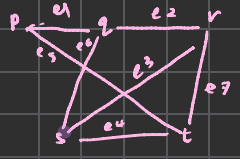


Draw the graph and from the graph:

i. Find all possible paths from vertex p to vertex t. (5 Marks)
ii. Determine all possible trails from vertex p to vertex t. (5 Marks)
iii. Identify the shortest and longest path from vertex p to vertex t. (2.5 marks)
iv. Find the shortest and longest trail from vertex p to vertex t. (2.5 Marks)

(14 Marks)





(p, q, r, t)
(p, t)
(p, q, s, t)

(p, q, s, r, t)
(p, q, r, s, t)

i) all possible path from p to t

(p, q, r, t)
(p, t)
(p, q, s, t)
(p, q, r, t)
(p, q, s, r, t)
(p, q, r, s, t)

p, e1, q, e2, r, e7, t
p, e5, t — shortest
p, e1, q, e6, s, e4, t
p, e1, q, e6, s, e3, r, e7, t
p, e1, q, e2, r, e3, s, e4, t

ii) all possible trails

Path, Trail, Cycle, Circuit, Walk.