DISCRETE STRUCTURE SEM 1 2023/2024

FACULTY: FACULTY OF COMPUTING

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SECTION: 03

ASSIGNMENT: 2

SECI1013-03 - Assignment 2

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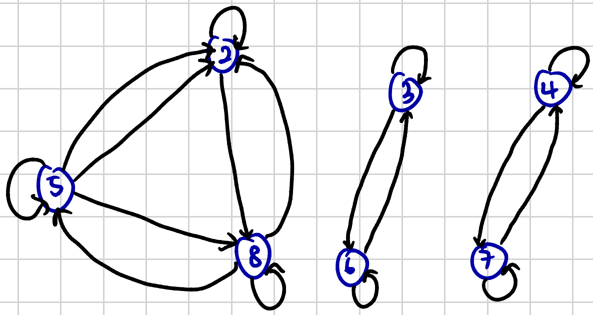
RELATION

1) $A = \{2, 3, 4, 5, 6, 7, 8\}$

xRy if $x - y = 3n$

$R = \{(2,5), (2,8), (3,6), (7,4), (8,5), (5,2), (6,3), (4,7), (5,8), (8,2), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8)\}$



2) $A = \{1, 2, 3\}$

$B = \{9, 8, 7\}$

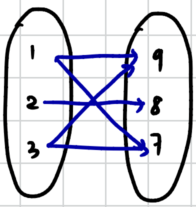
$\forall (a,b) \in A \times B$, $aRb \leftrightarrow a+b$ is an even no.

a) $R = \{(1,7), (1,9), (2,8), (3,9), (3,7)\}$

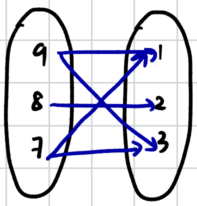
$R^{-1} = \{(7,1), (9,1), (8,2), (9,3), (7,3)\}$

b) arrow diagram

R



R^{-1}



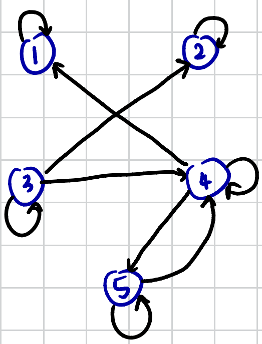
c) R^{-1} is the inverse of R that will still give output according to the condition.

3) $A = \{1, 2, 3, 4, 5\}$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & & & & & \\ 2 & & & & & \\ 3 & & & & & \\ 4 & & & & & \\ 5 & & & & & \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Digraph

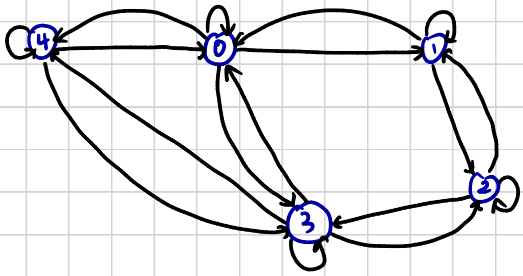


in degrees / out degrees

	1	2	3	4	5
in degrees	2	2	1	3	2
out degrees	1	1	3	3	2

4) $A = \{0, 1, 2, 3, 4\}$

$R = \{(0,0), (0,1), (0,3), (0,4), (1,0), (1,1), (1,2), (2,1), (2,2), (2,3), (3,0), (3,2), (3,3), (3,4), (4,0), (4,3), (4,4)\}$



Every elements in A are associated with itself in R ∴ this relation is reflexive

$$m_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \qquad m_R^T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

since $m_R = m_R^T$ ∴ this relation is symmetric.

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \otimes \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

since $m_R \otimes m_R \neq m_R$
∴ this relation is not transitive.

5) $A = \{1, 2, 3, \ldots 13, 14\}$
$R = \{(x, y) : 3x - y = 0\}$

$R = \{(1,3), (2,6), (3,9), (4,12)\}$

a) R is not reflexive because non of the element in A is related to itself in R.

b) R is not symmetric because for all $(x, y) \in R$, there is no $(y, x) \in R$.

c) R is not transitive (1,3) and (3,9) exist but there is no (1,9).

67)

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

a) $RS =$

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b) $SR =$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

FUNCTION

7) i) The difference between relation and function is a relation might have several outputs for a single input, but a function can only have one input and one output.

ii) Relations can be many-to-one, many-to-many, or one-to-one but functions are specifically one-to-one or many-to-one but not many-to-many.

8) $A = \{2, 3, 4, 5\}$

i) $\{(2,3), (3,4), (4,5), (5,2)\}$

⇒ function. every element in the domain is related with only one element from the codomain.

ii) $\{(2,4), (3,4), (5,4), (4,4)\}$

⇒ function. every element in the domain is related with only one element from the codomain.

iii) $\{(2,3), (2,4), (5,4)\}$

⇒ not a function. because element '2' in the domain is related to more than one element in the codomain which are '3' and '4'.

iv) $\{(2,3), (3,5), (4,5)\}$

⇒ not a function. there is an element in A, which is 5 in the domain that is not associated to any element in the codomain.

(v) $\{(2,2), (2,3), (4,4), (4,5)\}$

⇒ not a function. Both elements in the domain are associated with more than one element in the codomain.

9) $R = \{(x, y) \mid y = x + 5,\ x \text{ is } \mathbb{Z}^+ < 6\}$

$R = \{(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)\}$

domain $= \{0, 1, 2, 3, 4, 5\}$

range $= \{5, 6, 7, 8, 9, 10\}$

10) v) $f = \mathbb{R} \rightarrow \mathbb{R},\ f(x) = 1 - 2x$

$$f(x_1) = f(x_2)$$
$$1 - 2x_1 = 1 - 2x_2$$
$$-2x_1 = -2x_2$$
$$x_1 = x_2 \quad \therefore \text{one-to-one}$$

$f(x) = 1-2x$

$y = 1-2x$

$2x = 1-y$

$x = \dfrac{1-y}{2}$

there exists $\dfrac{1-y}{2}$ in $\mathbb{R}$;

$f\left(\dfrac{1-y}{2}\right) = 1-\cancel{2}\left(\dfrac{1-y}{\cancel{2}}\right)$

$= 1-(1-y)$

$= y$

$\therefore$ onto

$\therefore$ $f(x) = 1-2x$ is bijective

vi) $f = \mathbb{R} \to \mathbb{R}, f(x) = 5x^2 - 1$

$f(x_1) = f(x_2)$

$5x_1^2 - 1 = 5x_2^2 - 1$

$5x_1^2 = 5x_2^2$

$\sqrt{x_1^2} = \sqrt{x_2^2}$

$x_1 = \pm x_2$

x_1 is not always the same as x_2

$\therefore$ f is not one-to-one .

$y = 5x^2-1$

$\dfrac{y+1}{5} = x^2$

$x = \pm\sqrt{\dfrac{y+1}{5}}$ $\therefore$ not onto

$\therefore$ f is neither one-to-one nor onto.

(vii) $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^4$

$$f(x) = x^4$$
$$x_1^4 = x_2^4$$
$$x_1 = x_2$$

Therefore, it is a one-to-one function

$$f(x) = y$$
$$y = x^4$$
$$x = \sqrt[4]{y}$$

Therefore, it is an onto function.

$\therefore$ It is a bijective function because it includes one-to-one and onto function.

(viii) $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \left(\frac{x-2}{x-3}\right)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$$
$$\cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 2x_1 - 3x_2 + \cancel{6}$$
$$-3x_1 - 2x_2 = -2x_1 - 3x_2$$
$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$
$$-x_1 = -x_2$$
$$x_1 = x_2$$

Therefore, it is a one-to-one function

$$f(x) = y$$
$$y = \frac{x-2}{x-3}$$

$$y(x-3) = x - 2$$
$$xy - 3y = x - 2$$
$$xy - x = 3y - 2$$
$$x(y-1) = 3y - 2$$
$$x = \frac{3y-2}{y-1} \; ; y \neq 1$$

Therefore, it is not onto function

$\therefore$ It is not a bijective function because it is a one-to-one function but not onto function.

11) ix) $f(x) = 3x - 1$

$g(x) = x^2 - 1$

$f[g(x)] = 3(x^2-1) - 1$

$= 3x^2 - 3 - 1$

$= 3x^2 - 4$

$f[g(0)] = 3(0)^2 - 4 = -4$

$f[g(1)] = 3(1)^2 - 4 = -1$

$f[g(2)] = 3(2)^2 - 4 = 8$

$f[g(3)] = 3(3)^2 - 4 = 23$

xi) $f(x) = x - 1$

$g(x) = x^3 + 1$

$f[g(x)] = (x^3+1) - 1$

$= x^3$

$f[g(0)] = 0$

$f[g(1)] = 1$

$f[g(2)] = 8$

$f[g(3)] = 27$

x) $f(x) = x^2$

$g(x) = 5x - 6$

$f[g(x)] = (5x-6)^2$

$= 25x^2 - 60x + 36$

$f[g(0)] = 25(0)^2 - 60(0) + 36 = 36$

$f[g(1)] = 25(1)^2 - 60(1) + 36 = 1$

$f[g(2)] = 25(2)^2 - 60(2) + 36 = 16$

$f[g(3)] = 25(3)^2 - 60(3) + 36 = 81$

RECURRENCE RELATION

12) (xii) $a_n = 6a_{n-1} - 9a_{n-2}$;

$a_0 = 1$, $a_1 = 6$

$a_2 = 6a_1 - 9a_0$

$= 6(6) - 9(1)$

$= 27$

$a_3 = 6a_2 - 9a_1$

$= 6(27) - 9(6)$

$= 108$

$a_4 = 6a_3 - 9a_2$

$= 6(108) - 9(27)$

$= 405$

$\therefore$ 1, 6, 27, 108, 405, ...

xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;

$a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$a_3 = 6a_2 - 11a_1 - 6a_0$

$a_3 = 6(15) - 11(5) - 6(2)$

$= 47$

$a_4 = 6a_3 - 11a_2 - 6a_1$

$a_4 = 6(47) - 11(15) - 6(5)$

$= 87$

$\therefore$ 2, 5, 15, 47, 87, ...

xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$;

$a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3a_2 - 3a_1 - a_0$

$= -3(-1) - 3(-2) + 1$

$= 10$

$a_4 = -3a_3 - 3a_2 + a_1$

$= -3(10) - 3(-1) + (-2)$

$= -29$

$\therefore$ 1, -2, -1, 10, -29, ...

13) $a_{n+1} = 5a_n - 3$; $a_1 = k$, $k \neq 0$

i) value of a_4 in terms of k

$a_2 = 5a_1 - 3$

$= 5k - 3$

$a_3 = 5a_2 - 3$

$= 5(5k - 3) - 3$

$= 25k - 15 - 3$

$= 25k - 18$

$a_4 = 5(25k - 18) - 3$

$= 125k - 90 - 3$

$a_4 = 125k - 93$

ii) $a_4 = 7$

$125k - 93 = 7$

$125k = 100$

$k = \frac{100}{125}$

$k = \frac{4}{5}$