Beyond the Boost: Measuring the intrinsic dipole of the CMB using the spectral distortions of the monopole and quadrupole

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We present a general framework for accurate spectral modeling of the low multipoles of the cosmic microwave background (CMB) as observed in a boosted frame. In particular, we demonstrate how spectral measurements of the low multipoles can be used to separate the motion-induced dipole of the CMB from a possible intrinsic dipole component. In a moving frame, the leakage of an intrinsic dipole moment into the CMB monopole and quadrupole induces spectral distortions with distinct frequency functions that respectively peak at 337 GHz and 276 GHz. The leakage into the quadrupole moment also induces a geometrical distortion to the spatial morphology of this mode. The combination of these effects can be used to lift the degeneracy between the motion-induced dipole and any intrinsic dipole that the CMB might possess. Assuming the current peculiar velocity measurements, the leakage of an intrinsic dipole with an amplitude of $\Delta T = 30\mu$K into the monopole and quadrupole moments will be detectable by a *PIXIE*–like experiment at ~ 40 nK (2.5σ) and ~ 130 nK (11σ) level at their respective peak frequencies.

Introduction. The measurements of the COBE/FIRAS instrument show that the intensity of the Cosmic Microwave Background (CMB) has an almost perfect blackbody spectrum [1]. Even though in a frame moving with respect to the CMB the observed intensity is effectively a blackbody in every direction, the intensity harmonic multipoles in this frame generally contain frequency spectral distortions. These distortions are a result of the leakage of the nearby multipoles into each other due to the aberration and Doppler effects [2–6]. The most prominent motion-induced leakage component is that of the monopole into the dipole (i.e. kinematic dipole). The kinematic dipole has a frequency dependence identical to a differential blackbody spectrum which makes it degenerate with any intrinsic (or non-kinematic) dipole that the CMB might possess. Current modeling of the CMB dipole only includes the leakage of the monopole, but ignores any intrinsic dipole component as well as other kinematic corrections to this mode (e.g. the leakage of the quadrupole). Here we present an accurate description of the frequency spectrum of the low multipoles of CMB and show how the kinematic (motion-induced) corrections to these modes can be used by the next generation of CMB surveys to lift the dipole degeneracy.

A kinematic dipole is not the only observational consequence of our motion with respect to the CMB. The motion-induced leakage of the intensity multipoles into each other causes a *boost coupling* between the nearby multipoles. Measuring this boost coupling in a wide range of harmonic modes can actually lead to an independent measure of the peculiar velocity of an observer with respect to the CMB [7–10]. In the CMB rest frame, all motion-induced effects (including the kinematic dipole and the boost coupling) vanish; however, there is no compelling reason for us to believe that the intrinsic dipole moment of the CMB in this frame is precisely zero.

It has been shown that in a flat ΛCDM universe with adiabatic initial perturbations, the intrinsic dipole of the CMB is strongly suppressed [11, 12]. For this reason, the intrinsic dipole of the CMB is usually either ignored or set to zero, and the observed dipole of the CMB is interpreted entirely as a kinematic effect. This results in a peculiar velocity of $\beta \equiv v/c = 0.00123$ in the direction $\hat{\boldsymbol{\beta}} = (264°, 48°)$ in galactic coordinates [13]. If the observed dipole moment only has a kinematic origin, it can be used to define a natural rest frame for CMB (namely, the frame in which the whole dipole vanishes). However, unintended subtraction of an existent non-kinematic dipole in this process will result in obtaining an incorrect CMB rest frame. This can in turn lead to unexpected anomalies, such as the observed power and parity asymmetries in the CMB [14, 15] and the mismatch between the CMB rest frame and the matter rest frame [16–19]. Studying the angular variance of the Hubble parameter over different redshifts (in the CMB dipole-inferred frame) also indicates the presence of a non-kinematic dipole component in the CMB [20, 21]. Furthermore, since isocurvature initial perturbations, and multi-field inflationary scenarios typically invoke a non-negligible intrinsic dipole moment, a detection of this component could have important implications for pre-recombination physics [22–26].

Recently the Planck team has obtained an independent value for the peculiar velocity of the solar system using the boost coupling of the CMB multipoles. Their result $\beta = 0.00128 \pm 0.00026(\text{stat.}) \pm 0.00038(\text{syst.})$ [7] is consis-

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tent with the kinematic interpretation of the dipole and shows that most of the dipole that we observe is induced by our peculiar motion. However, the error bars still allow for a non-kinematic dipole component that remains to be measured.

In this letter we show how the kinematic and non-kinematic dipoles can be separated by measuring the motion-induced spectral distortions in the observed low multipoles of the CMB in our local frame. Future microwave surveys, such as *PIXIE* with a sensitivity of 5 Jy/sr, will be able to measure these effects with high precision.

Lorentz boosting the CMB. We define the rest frame of the CMB as the frame in which its kinematic dipole (the leakage of the monopole into the dipole) vanishes.[1] We still allow the CMB to have a non-kinematic dipole in this frame. Then we argue that the full frequency spectrum of the low intensity multipoles in the boosted frame can be exploited to separate the intrinsic dipole from the kinematic part induced by a boost. We assume that the CMB frequency spectrum in its rest frame can be described as a pure blackbody by neglecting any pre-recombination and secondary μ- and y-distortions (see Fig. 12 in [27], also [28]). In this frame, we expand the intensity and the thermodynamic temperature in spherical harmonic multipoles as

$$I_{\nu_{cmb}}(\hat{\boldsymbol{\gamma}}_{cmb}) = \sum_{\ell=0}^{\infty} \sum\nolimits_m^{\ell} a_{\ell m}^{I_{cmb}}(\nu_{cmb})\ Y_{\ell m}(\hat{\boldsymbol{\gamma}}_{cmb}) \tag{1}$$

and

$$T(\hat{\boldsymbol{\gamma}}_{cmb}) = \sum_{\ell=0}^{\infty} \sum\nolimits_m^{\ell} a_{\ell m}^{T_{cmb}}\ Y_{\ell m}(\hat{\boldsymbol{\gamma}}_{cmb}), \tag{2}$$

where the sum notation $\sum_m^{\ell}$ is shorthand for $\sum\limits_{m=-\ell}^{\ell}$. The frequency dependence of the intensity harmonic coefficients for a blackbody—with an average temperature T_0—can be expanded to first order in thermodynamic temperature harmonics as

$$a_{00}^{I_{cmb}}(\nu) = \tilde{B}_\nu(T_0)\ a_{00}^{T_{cmb}}, \tag{3a}$$

$$a_{\ell m}^{I_{cmb}}(\nu) = \tilde{F}_\nu(T_0)\ a_{\ell m}^{T_{cmb}} \quad (\ell > 0), \tag{3b}$$

where $\tilde{B}_\nu(T_0) \equiv T_0^{-1} B_\nu(T_0)$, $B_\nu(T) \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$ is the blackbody spectrum and $\tilde{F}_\nu(T_0) \equiv \tilde{B}_\nu(T_0) f(x)$ is the differential blackbody spectrum with $f(x) \equiv \frac{xe^x}{e^x-1}$ and $x = h\nu/kT_0$.

[1] Indeed, in this frame all the other kinematic effects including the boost coupling and the ones that we are about to discuss will vanish as well.

In order to find the observed multipoles in the boosted frame we use the Lorentz invariance of I_ν/ν^3 to write the observed incoming intensity along the line-of-sight unit vector $\hat{\boldsymbol{\gamma}}$ at frequency ν as

$$I_\nu(\hat{\boldsymbol{\gamma}}) = \Big(\frac{\nu}{\nu_{cmb}}\Big)^3 I_{\nu_{cmb}}(\hat{\boldsymbol{\gamma}}_{cmb}), \tag{4}$$

where

$$\nu_{cmb} = \Big(\frac{1-\beta\mu}{\sqrt{1-\beta^2}}\Big)\nu \tag{5}$$

and

$$\hat{\boldsymbol{\gamma}}_{cmb} = \Big(\frac{(1-\sqrt{1-\beta^2})\mu - \beta}{1-\beta\mu}\Big)\hat{\boldsymbol{\beta}} + \Big(\frac{\sqrt{1-\beta^2}}{1-\beta\mu}\Big)\hat{\boldsymbol{\gamma}} \tag{6}$$

are the frequency and line-of-sight unit vector in the CMB rest frame and $\mu = \hat{\boldsymbol{\gamma}}\cdot\hat{\boldsymbol{\beta}}$. Equations (5) and (6) respectively represent the Doppler and aberration effects. Expanding both sides of Eq. (A.2) in harmonic space allows us to find the observed multipoles in the moving frame as

$$a_{\ell' m'}^{I}(\nu) = \sum_{\ell=0}^{\infty} \sum\nolimits_m^{\ell} \int \Big(\frac{\nu}{\nu_{cmb}}\Big)^3 a_{\ell m}^{I_{cmb}}(\nu_{cmb}) Y_{\ell m}(\hat{\boldsymbol{\gamma}}_{cmb}) Y_{\ell' m'}^*(\hat{\boldsymbol{\gamma}}) \mathrm{d}^2\hat{\boldsymbol{\gamma}}. \tag{7}$$

Substituting Eqs. (5) and (6) into (7) will respectively result in the *Doppler and aberration leakage* of the nearby multipoles into each other. To n-th order in β, the observed multipoles $a_{\ell' m'}^{I}(\nu)$ will have a contribution from $a_{\ell'\pm n, m'}^{I_{cmb}}(\nu)$ of the rest frame. This integral has been computed analytically in Ref.[2]. We do not repeat the calculations here and only use the results hereafter. We also acquire the same notation for the frequency functions.

The boosted dipole. First, we calculate the observed dipole in the moving frame to illustrate the dipole degeneracy problem. By setting $\ell' = 1$ in Eq. (7) we find (Eq. B.37 in Ref. [2])

$$\begin{aligned} a_{1m'}^{I}(\nu) = {} & \overbrace{\tilde{F}_\nu(T_0) a_{1m'}^{T_{cmb}}}^{\text{Intrinsic dipole}} + \beta\,\overbrace{\frac{2\sqrt{\pi}}{3} Y_{1m'}^*(\hat{\boldsymbol{\beta}}) \tilde{F}_\nu(T_0) a_{00}^{T_{cmb}}}^{\text{Kinematic dipole}} \\ & + \beta \sum_{m,n}^{2,1} {}_0^1\mathcal{G}_{1m'}^{2m}(\hat{\boldsymbol{\beta}}) \tilde{F}_\nu^{(11)}(T_0) a_{2m}^{T_{cmb}} \\ & + \beta \sum_{m,n}^{2,1} {}_1^0\mathcal{G}_{1m'}^{2m}(\hat{\boldsymbol{\beta}}) \tilde{F}_\nu(T_0) a_{2m}^{T_{cmb}} \\ & + O(\beta^2), \end{aligned} \tag{8}$$

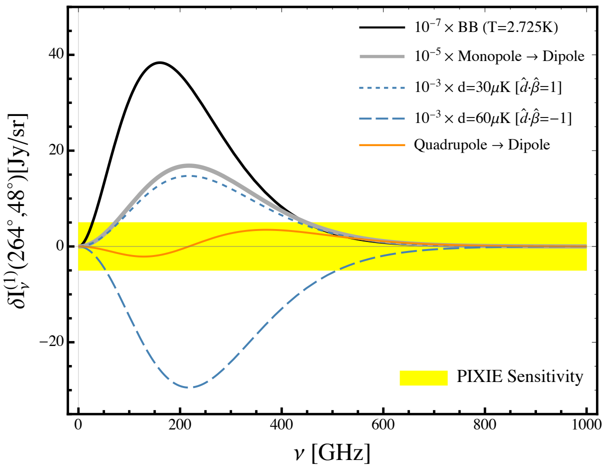

FIG. 1. The CMB dipole constituents observed in a moving frame with $\beta = 0.00128$ and $\hat{\boldsymbol{\beta}} = (264°, 48°)$. The intrinsic dipoles d30 and d60 have identical frequency functions as the kinematic dipole. The average T=2.725 blackbody spectrum (*solid black*) is depicted in all plots for reference.

where $\tilde{F}_\nu^{(11)}(T) = \tilde{F}_\nu(T)(g(x)-1)$ with $g(x) \equiv x\coth(x/2)$, while ${}^{1}_{0}\mathcal{G}^{2m}_{1m'}(\hat{\boldsymbol{\beta}})$ and ${}^{0}_{1}\mathcal{G}^{2m}_{1m'}(\hat{\boldsymbol{\beta}})$ are numerical factors of order ~ 1.

The first term in Eq. (8) is the intrinsic dipole of the CMB with the differential blackbody spectrum $\tilde{F}_\nu(T_0)$. The second term is what is normally identified as the kinematic dipole, which is a result of the Doppler leakage of the monopole into the observed dipole moment. Notice that the frequency dependence of this terms is identical to the intrinsic dipole which makes the two components degenerate. The third and the fourth terms are respectively the Doppler and aberration leakages of the quadrupole into the dipole. These terms have never been considered in the analysis of the CMB dipole.

In order to build some intuition, instead of working with the $a^{T_{cmb}}_{1,m}$ coefficients, we parametrize the three degrees of freedom for the intrinsic dipole in terms of an amplitude and two angles via the definition

$$a^{T_{cmb}}_{1,m} \equiv \frac{4\pi}{3} d Y^*_{1,m}(\theta_d, \phi_d). \tag{9}$$

We define the dipole vector $\vec{\boldsymbol{d}} = d\hat{\boldsymbol{d}}$ where d and $\hat{\boldsymbol{d}} \equiv (\theta_d, \phi_d)$ are the amplitude and direction of the maximum of the dipole on the sky.

With this new definition, we set out to study the observable effects of an intrinsic dipole of order $\sim 10^{-5}$ on the local dipole, monopole and quadrupole of the CMB. In order to gauge the expected magnitude of the effect we will consider two different dipoles with the amplitudes $d = 30\mu$K and $d = 60\mu$K (motivated by Ref. [11] Eqs. 31-33). We will refer to these dipoles respectively as d30 and d60.

The observed dipole intensity in the direction (θ, ϕ) is defined as $\delta I^{(1)}_\nu(\theta,\phi) \equiv \sum^1_{m'} a^I_{1m'}(\nu) Y_{1m'}(\theta,\phi)$. Fig. 1 shows the contribution of each term in Eq. (8) to $\delta I^{(1)}_\nu(\theta_\beta, \phi_\beta)$ at different frequencies. Unless the intrinsic dipole is much larger than the one we chose, the dominant term in this equation is the leakage of the monopole into the dipole (kinematic dipole) with the thermodynamic temperature $\delta T^{(1)} \equiv \delta I^{(1)}_\nu / \tilde{F}_\nu(T_0) = 3.35$ mK. The next order contribution is due to the intrinsic dipole with the same frequency function as that of the kinematic dipole. The leakage of the quadrupole into the dipole is a motion-induced effect which does not depend on the intrinsic dipole at all. Since this term has a different frequency dependence, technically it could be used as an independent measure of β. However, the peak amplitude of this component—assuming the observed value of the quadrupole as input—is lower than the sensitivity of *PIXIE*, and therefore it is not likely to be useful for lifting the dipole degeneracy. Nevertheless, this extra leakage component should be taken into account for a precise analysis of the observed dipole in the future CMB surveys.

Now we show how the dipole degeneracy can be removed by looking at the motion induced spectral distortions in the dipole's neighbors: the monopole ($\ell' = 0$) and the quadrupole ($\ell' = 2$).

The boosted monopole. Using Eq. (7), it is easy to find the monopole of the CMB in a boosted frame (Eq. B.36 in Ref. [2])

$$\begin{aligned} a^I_{00}(\nu) =& \tilde{B}_\nu(T_0) a^{T_{cmb}}_{00} + \beta^2 \tilde{B}^{(20)}_\nu(T_0) a^{T_{cmb}}_{00} \\ &+\beta \sum\nolimits^1_m \frac{2\sqrt{\pi}}{3} Y_{1m}(\hat{\boldsymbol{\beta}}) \tilde{F}^{(11)}_\nu(T_0) a^{T_{cmb}}_{1m} \\ &-\beta \sum\nolimits^1_m \frac{4\sqrt{\pi}}{3} Y_{1m}(\hat{\boldsymbol{\beta}}) \tilde{F}_\nu(T_0) a^{T_{cmb}}_{1m} + O(\beta^2), \end{aligned} \tag{10}$$

with $\tilde{B}^{(20)}_\nu(T_0) = \frac{1}{6}\tilde{F}_\nu(T)(g(x)-3)$. Here the first term is the well known $T = 2.725$ blackbody spectrum, the second term is the second order Doppler correction to the monopole, and the third and fourth terms are respectively the Doppler and aberration leakages of the dipole into the monopole.

The observed monopole intensity $I^{(0)}_\nu(\theta,\phi) = a^I_{00}(\nu) Y_{00}(\theta,\phi) = a^I_{00}(\nu)/2\sqrt{\pi}$ is plotted in Fig. 2 for different amplitudes and orientations of the intrinsic dipole. Using Eq. (9), we can rewrite Eq. (10) as

$$\delta I^{(0)}_\nu = \tilde{B}_\nu(T_0) T_0 + \beta \tilde{B}^{(20)}_\nu(T_0)[\beta T_0 + 2d(\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{\beta}})]. \tag{11}$$

Since the frequency dependence of the intrinsic monopole T_0 is different from the motion induced terms, it can be fit and measured separately. Since the motion-induced spectral distortions depend the combination of the kinematic dipole (βT_0) and the projection of the intrinsic dipole along the direction of motion ($d(\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{\beta}})$), it might seem like these two components still remain degenerate. However, combining this with the observed dipole in $\hat{\boldsymbol{\beta}}$ direction (with the quadrupole leakage term dropped, as-