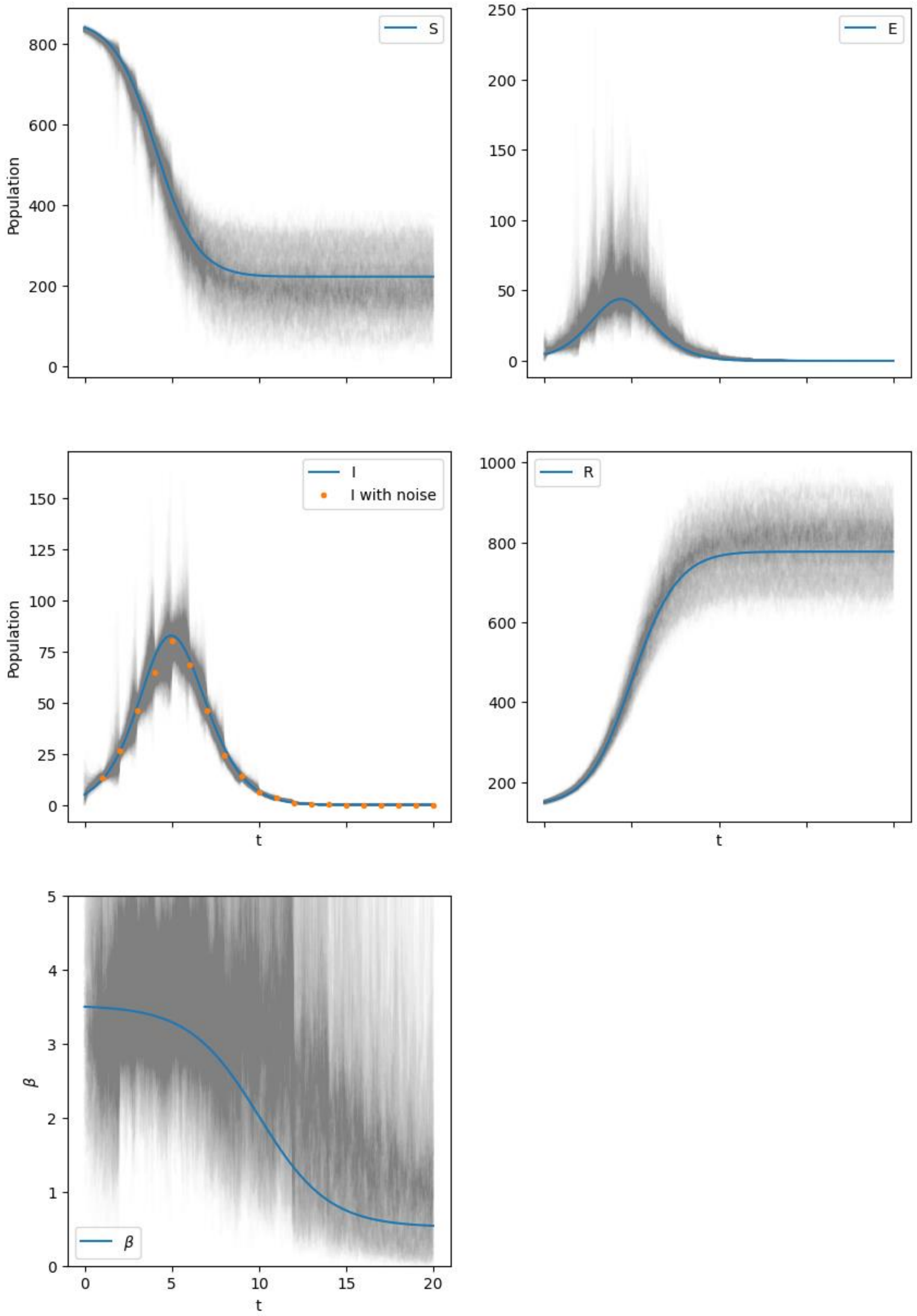

Estimating constant and time-varying parameters in dynamical system

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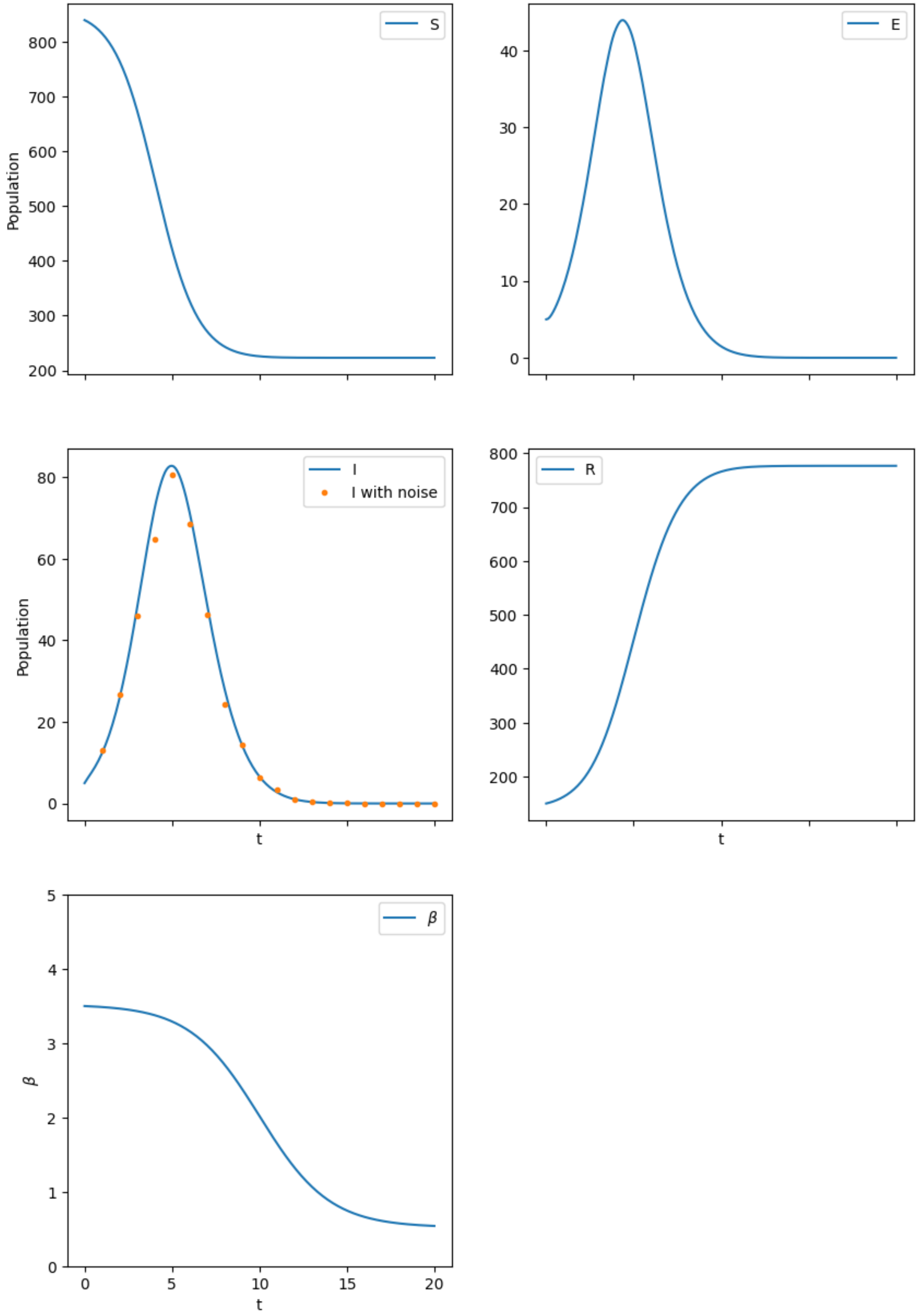
Introduction

- Goal: learn constant and time-varying parameters in the SEIR disease transmission model
- Motivation: study system parameter estimation, especially particle Markov-chain Monte Carlo (PMCMC)
- Focus of project: estimate all the parameters for a simulated SEIR model via a combination of state-parameter augmentation and adaptive PMCMC



Mathematical formulation and algorithm

- SEIR: Susceptible / Exposed / Infected / Removed population modeled by ODE
- Model time-varying β as a Brownian process and augment to the system states
- Assume system process is deterministic and only available data is infected population with log-normal noise
- Estimate all the system parameters $\theta = \{S_0, E_0, I_0, R_0, c_0, \sigma, k, \gamma\}$
- Use adaptive MCMC to sample the parameters

$$P(\theta \mid \mathcal{Y}_T) = \frac{P(\mathcal{Y}_T \mid \theta)P(\theta)}{P(\mathcal{Y}_T)}$$

- For each proposed sample, use particle filter to recursively calculate the likelihood

$$P(\mathcal{Y}_T \mid \theta) = \prod_{k=1}^{T} P(y_k \mid \theta)$$

$$P(\mathcal{Y}_T \mid \theta) \approx \prod_{k=1}^{T} \left[\frac{1}{N}\sum_{i=1}^{N} P(y_k \mid x_k^{(i)}, \theta)\right]$$

$$\frac{dS_t}{dt} = -\beta_t S_t \frac{I_t}{N}$$

$$\frac{dE_t}{dt} = \beta_t S_t \frac{I_t}{N} - kE_t$$

$$\frac{dI_t}{dt} = kE_t - \gamma I_t$$

$$\frac{dR_t}{dt} = \gamma I_t$$

$$\frac{dc_t}{dt} = \sigma dW$$

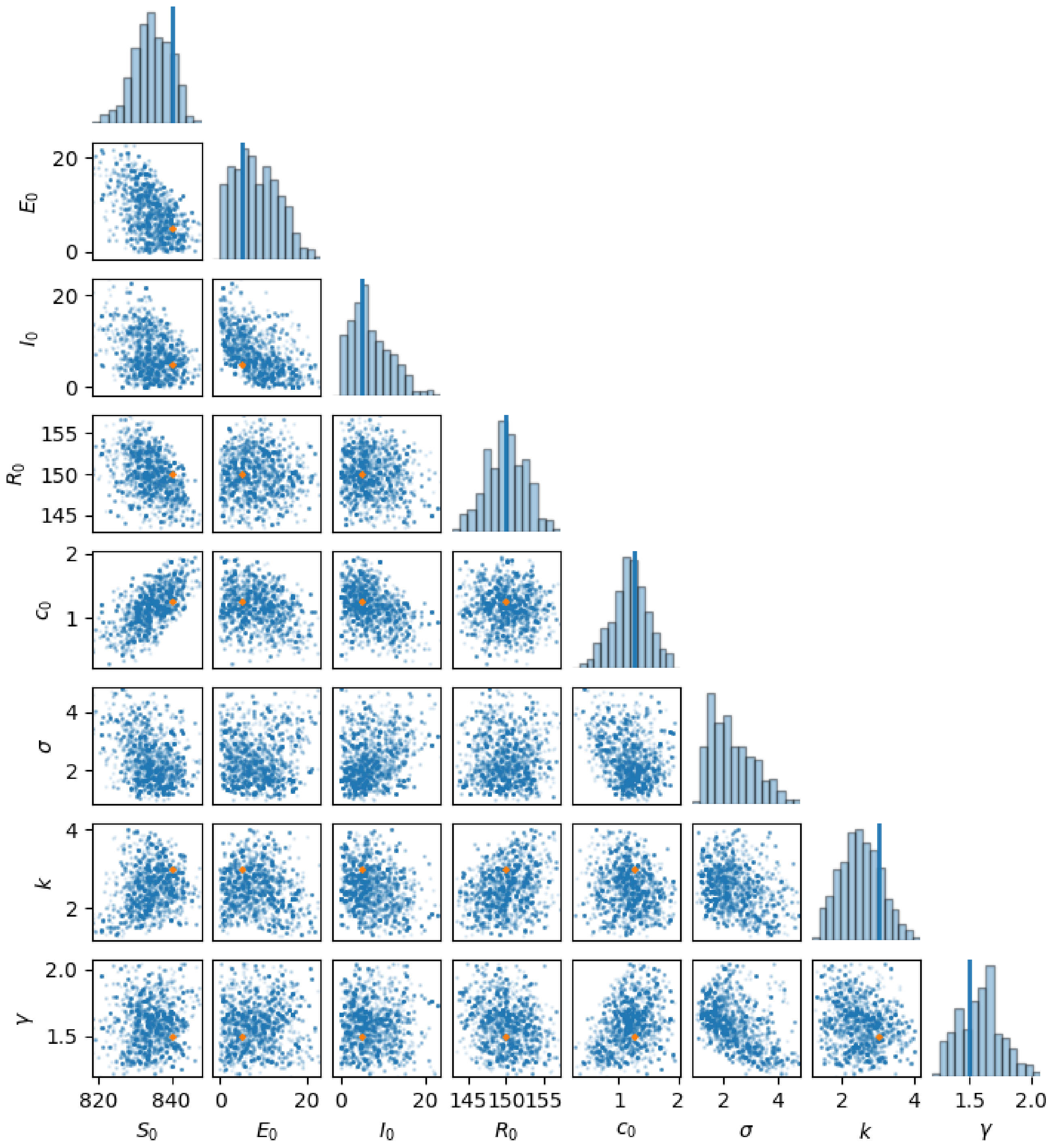
$$\beta = \exp(c_t)$$

Results

Priors:

$$S_0 = 1000 - E_0 - I_0 - R_0$$
$$E_0 = \mathcal{U}(0, 10)$$
$$I_0 = \mathcal{U}(0, 10)$$
$$R_0 = \mathcal{N}(150, 10)$$
$$c_0 = \mathcal{U}(-2, 2)$$
$$\sigma = \mathcal{U}(0, 1)$$
$$k = \mathcal{N}(3, 0.5)$$
$$\gamma = \mathcal{N}(1.5, 0.5)$$

IAC:
{47.13, 52.35, 47.96, 37.18, 42.32, 66.07, 46.53, 56.00}



Conclusions

- Adaptive PMCMC has successfully estimated all the constant system parameters, as well as the trajectory of β
- The trajectory for the system states has also been well recovered.

- Future work:
  - Effect of sample size
  - Incorporate adaptive rejection
  - More analysis on the particle filter in PMCMC (e.g. MSE, marginals of empirical state distributions, etc.)