Part. 1, Coding (50%):

Q2:

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[1, 0.0001]
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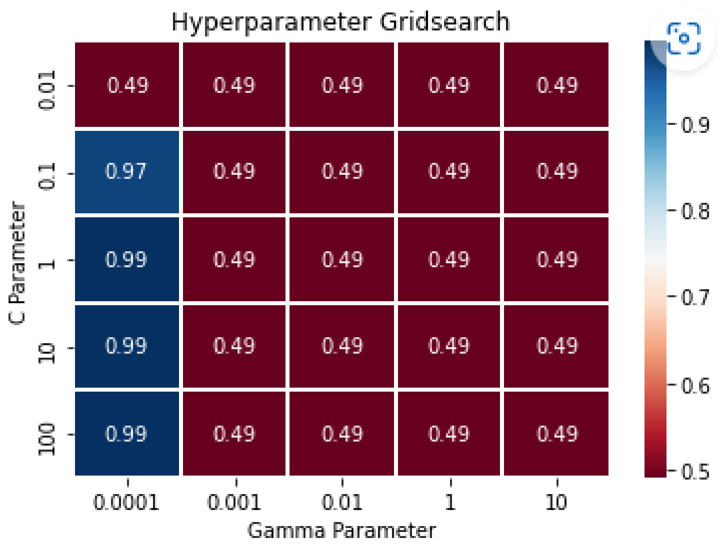
Q3:



Part. 2, Questions (50%):

Q1:

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1. Since K is symmetric, $K = V \Lambda V^T$

V is orthonormal V_t and the diagonal matrix Λ contains the eigenvalues λ_t of K

if K is positive semidefinite, its eigenvalues are non-negative

consider $\phi : x_i \mapsto (\sqrt{\lambda_t} V_{ti})_{t=1}^{n} \in \mathbb{R}^n$

$$\Rightarrow \phi(x_i)^T \phi(x_j) = \sum_{t=1}^{n} \lambda_t V_{ti} V_{tj} = (V \Lambda V^T)_{ij} = K_{ij} = k(x_i, x_j) \ \#$$

Q2:

2. $k(x, x') = \exp(k_1(x, x')) = \exp(k)$

$$= \exp(0) + \exp(0) k + \frac{\exp(0)}{2!} k^2 + \frac{\exp(0)}{3!} k^3 + \cdots$$

$$\exp(k) = 1 + k + \frac{1}{2} k^2 + \frac{1}{6} k^3 + \cdots = k(x, x')$$

$\Rightarrow \exp(k)$ is also a valid kernel $\#$

Q3:

3\. a. $k(x, x') = k_1(x, x') + 1$

let $k_2(x, x') = 1 \Rightarrow k_2$ is valid

$\Rightarrow k_1(x, x') + k_2(x, x')$ is valid $\Rightarrow k(x, x')$ is valid

b. $k(x, x') = k_1(x, x') - 1$

$\Rightarrow$ is not a valid kernel

consider $k_1(x, x') = 0 \Rightarrow k(x, x') = -1$

so its eigenvalues are not non-negative

$\Rightarrow$ not a valid kernel

c. $k(x, x') = k_1(x, x')^2 + \exp(\|x\|)^2 \cdot \exp(\|x'\|^2)$

Let $k_2(x, x') = k_1(x, x') k_1(x, x') \Rightarrow k_2$ is valid

$k_3 = \exp(\|x\|)^2 \cdot \exp(\|x'\|)^2 \Rightarrow f(x) \cdot f(x')$

$\Sigma_{i=1}^n \Sigma_{j=1}^n k(x_i, x_j) c_i c_j = \Sigma_{i=1}^n \Sigma_{j=1}^n f(x_i) f(x_j) c_i c_j$

$= (\Sigma_{i=1}^n f(x_i) c_i)^2 \geq 0$

$\Rightarrow k_3$ is valid

$\Rightarrow k(x, x') = k_2(x, x') + k_3(x, x')$ is a valid kernel

d. $k(x, x') = k_1(x, x')^2 + \exp(k_1(x, x')) - 1$

Let $k_2(x, x') = k_1(x, x') k_1(x, x') \Rightarrow k_2$ is valid

$\exp(k_1(x, x')) - 1 = \exp(0) + \frac{\exp(0) k}{1!} + \frac{\exp(0) k^2}{2!} + \cdots - 1$

$= \frac{\exp(0) k}{1!} + \frac{\exp(0) k^2}{2!} + \cdots \Rightarrow$ is positive (valid)

$\Rightarrow k(x, x') = k_2(x, x') + \frac{\exp(0) k}{1!} + \frac{\exp k^2}{2!} + \cdots$ is a valid kernel

Q4:

4\. $L = x^2 + 4x + 4 + \lambda(x^2 - 2x - 3 - 3)$

$\frac{dL}{dx} = 2x + 4 + \lambda(2x - 2) = 0$

$(2 + 2\lambda)x + (4 - 2\lambda) = 0 \quad x = \frac{4 - 2\lambda}{2 + 2\lambda} = \frac{2 - \lambda}{1 + \lambda}$

$g(\lambda) = (\frac{2-\lambda}{1+\lambda})^2 + 4(\frac{2-\lambda}{1+\lambda}) + 4 + \lambda((\frac{2-\lambda}{1+\lambda})^2 - 2(\frac{2-\lambda}{1+\lambda}) - 6)$ #

dual problem : maximize $(\frac{2-\lambda}{1+\lambda})^2 + 4(\frac{2-\lambda}{1+\lambda}) + 4 + \lambda((\frac{2-\lambda}{1+\lambda})^2 - 2(\frac{2-\lambda}{1+\lambda}) - 6)$

subject to $\lambda \leq 0$