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1\. (a) the number of spatial increments by which a receptive field is moved

(b) termination of neighborhood to perform convolution

(c) same weights and a single bias are used to generate the convolution values corresponding to all locations in the input image

(d) first:

$$66-(7-1)=60$$

feature map: $66\times66 \to 60\times60$

pooled feature map: $60\times60 \to 30\times30$

second:

$$30-6 \to 24$$

feature map: $30\times30 \to 24\times24$

pooled feature map: 12×12

(e) first:

$$(7\times7)\times\underbrace{1}_{\text{greyscale}}\times6+6=300$$

second

$$(7\times7)\times\underbrace{6}_{\text{first}}\times12+12=3540$$

(f)

$$\delta_{x,y}(l)=\frac{\partial E}{\partial z_{x,y}(l)}=\sum_u\sum_v\frac{\partial E}{\partial z_{u,v}(l+1)}\frac{\partial z_{u,v}(l+1)}{\partial z_{u,v}(l)}$$

$$=\sum_u\sum_v\delta_{u,v}(l+1)\frac{\partial z_{u,v}(l+1)}{\partial z_{u,v}(l)}$$

2. (a) $\begin{cases} f(t) = A, & |t| < \frac{W}{2} \\ f(t) = 0, & |t| \geq \frac{W}{2} \end{cases}$

$$F(u) = \int_{-\infty}^{+\infty} f(t)\, e^{-j2\pi ut}\, dt$$

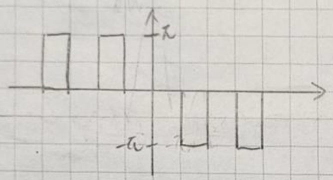
$$F(u) = \int_{-\frac{W}{2}}^{\frac{W}{2}} A e^{-j2\pi ut}\, dt$$

$$= \frac{-A}{j2\pi u}\left[e^{-j2\pi ut}\right]_{-\frac{W}{2}}^{\frac{W}{2}}$$

$$= \frac{-A}{j2\pi u}\left[e^{-j\pi uW} - e^{j\pi uW}\right]$$

$$= \frac{A}{j2\pi u}\left[e^{j\pi uW} - e^{-j\pi uW}\right] = AW\frac{\sin(\pi uW)}{\pi uW} = AW\,\mathrm{sinc}(uW)$$

(b)



3. (a)

perform shifting so that $F(0,0)$ is at the center of $[0, M-1]$ $[0, N-1]$

making it proportional to the average of function

$$F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$$

(b) $(\Rightarrow)$

$f(x,y)$ is real and even

$$F^*(u,v) = R(u,v) - jI(u,v)$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f^*(x,y)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(-\frac{ux}{M}-\frac{vy}{N}\right)}$$
$$= F(-u,-v) \quad \Rightarrow F \text{ is even}$$

$$F(u,v) = R(u,v) + jI(u,v)$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} [\text{even}][\text{even} - j\,\text{odd}][\text{even} - j\,\text{odd}]$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} [\text{even}][\text{even}\cdot\text{even} - 2j\,\text{even}\cdot\text{odd} - \text{odd}\cdot\text{odd}]$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} [\text{even}\cdot\text{even}] - 2j\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}\cdot\text{odd}] - \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\text{even}\cdot\text{even}]$$
$$= \text{real}$$

$(\Leftarrow)$

Same as above, just exchange $F(u,v)$ with $f(x,y)$

(c) $$\mathfrak{F}\left[f(x,y)(-1)^{x+y}\right] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)(-1)^{x+y} e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{j\pi(x+y)}\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{(u-\frac{M}{2})x}{M}+\frac{(v-\frac{N}{2})y}{N}\right)}$$
$$= F\left(u-\frac{M}{2}, v-\frac{N}{2}\right)$$

(d) zero padding: avoid wraparound so that 2 functions has same length P

problem: frequency leakage

4. (a) $P = 2M$, $Q = 2N$

(e) real, symmetric

(f) $G(u,v) = H(u,v)F(u,v)$

(g) $g_p(x,y) = \{ \text{real} [\Im^{-1} [G(u,v)]] \} (-1)^{x+y}$

(h) top, left quadrant

5. (a) A

value	num	sum	CDF
0	1	1	$\frac{7}{25} \times 1 \approx 0$
1	2	3	$\frac{7}{25} \times 3 \approx 1$
2	4	7	$\frac{7}{25} \times 7 \approx 2$
3	5	12	$\frac{7}{25} \times 12 \approx 3$
4	6	18	$\frac{7}{25} \times 18 \approx 5$
5	5	23	$\frac{7}{25} \times 23 \approx 6$
6	1	24	$\frac{7}{25} \times 24 \approx 7$
7	1	25	$\frac{7}{25} \times 25 = 7$

$\Rightarrow$

3	6	5	7	6
6	5	3	5	6
5	5	2	3	7
3	2	2	3	6
0	1	1	2	5

$\downarrow$

(b) B

value	num	sum	CDF
0	2	2	$\frac{7}{25} \times 2 \approx 1$
1	2	4	$\frac{7}{25} \times 4 \approx 1$
2	4	8	$\frac{7}{25} \times 8 \approx 2$
3	2	10	$\frac{7}{25} \times 10 \approx 3$
4	5	15	$\frac{7}{25} \times 15 \approx 4$
5	5	20	$\frac{7}{25} \times 20 \approx 6$
6	3	23	$\frac{7}{25} \times 23 \approx 6$
7	2	25	$\frac{7}{25} \times 25 = 7$

inverse

$\Rightarrow$

3	5	5	7	5
5	5	3	5	5
5	5	2	3	7
3	2	2	3	5
0	0	0	2	5