Probabilistic Reasoning

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Simple Definitions

- *Sample Space - set of all possible outcomes*
- *an Event, E, is any one set of outcomes of interest*
- *The Probability of an event, Pr(E) is the relative frequency of E over an indefinitely large number of trials.*
- *0 <= Pr(E) <= 1*

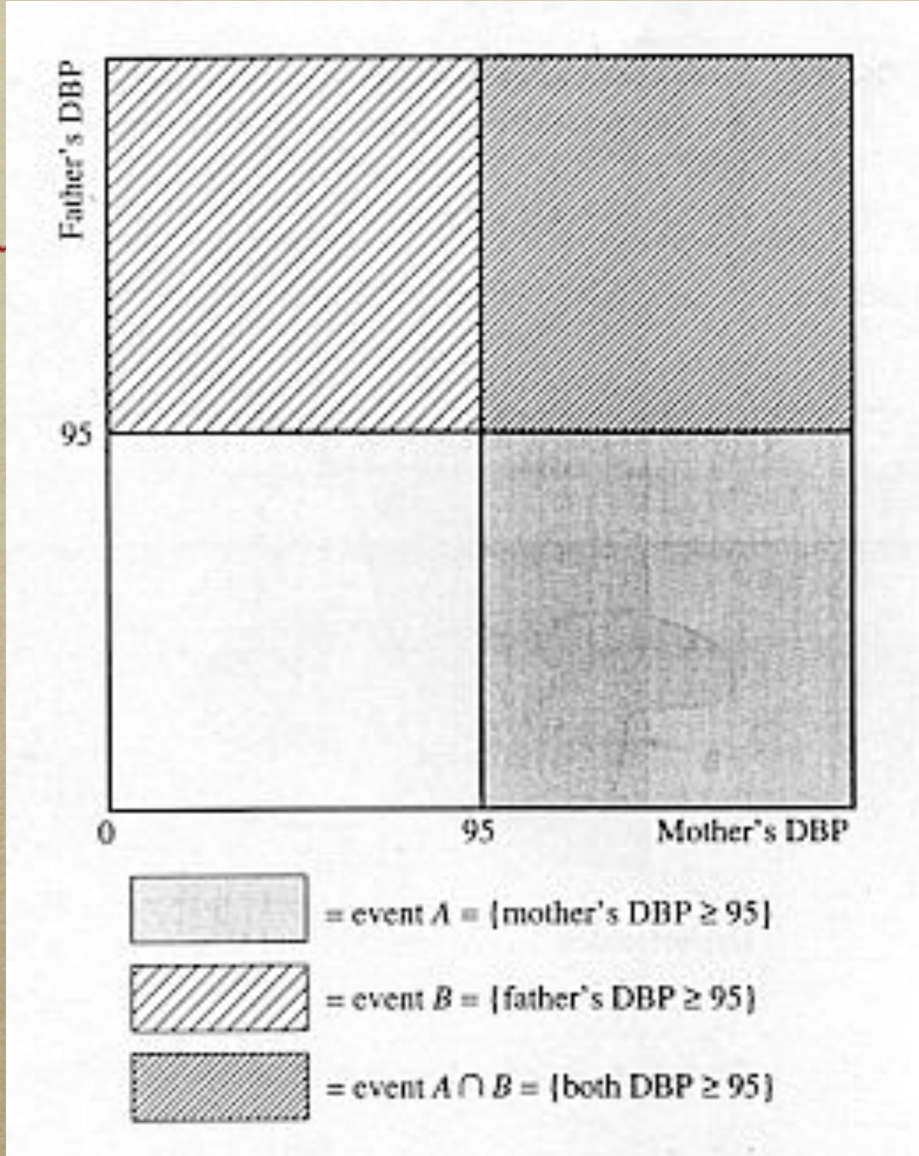
Additive Law of Probability

- *If E & F are mutually exclusive then*
  $Pr(E \text{ or } F) = Pr(E) + Pr(F)$
- *example: coin flip*
  $Pr(H \text{ or } T) = Pr(H) + Pr\,(T)$

Multiplicative
Law of Probability

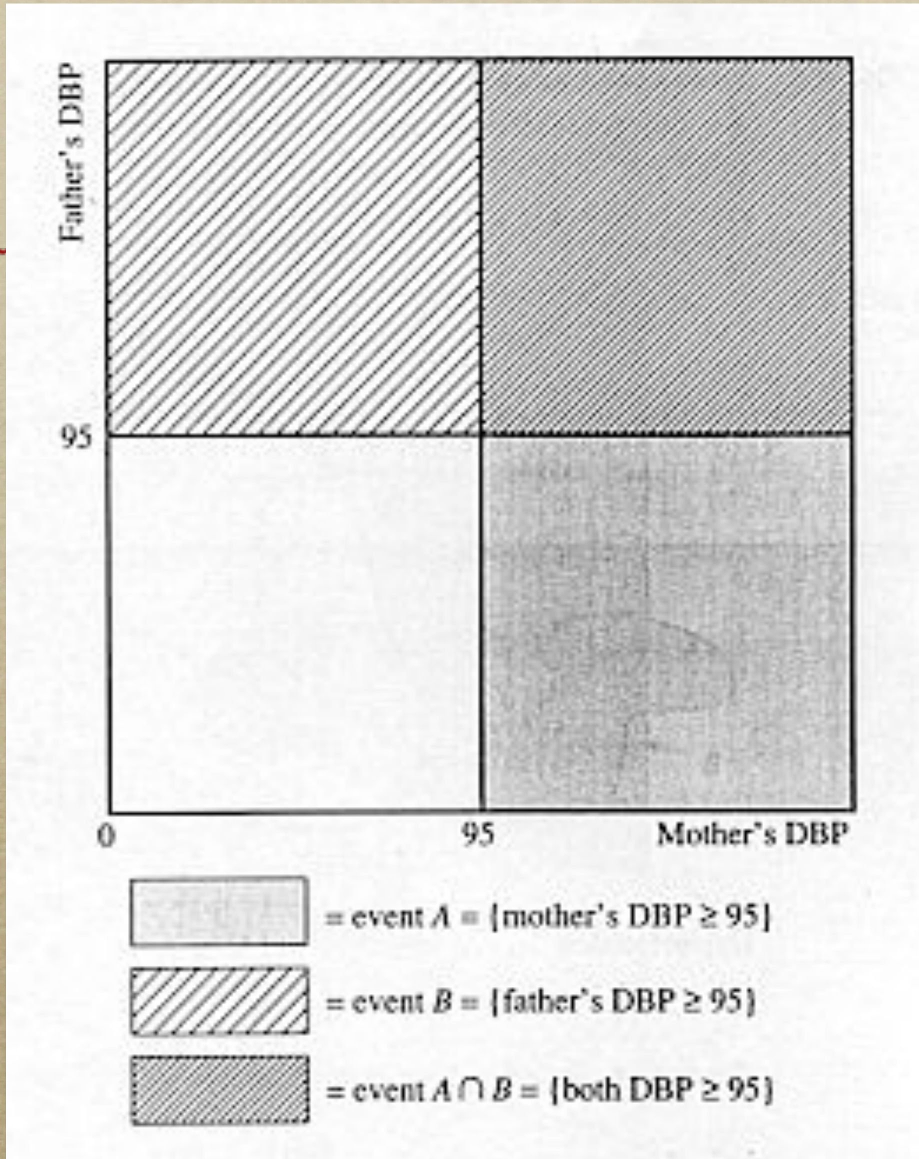
if A & B are independent,

$$Pr(A \cap B) = Pr(A) * Pr(B)$$



$$Pr(A) = 0.1$$
$$Pr(B) = 0.2$$

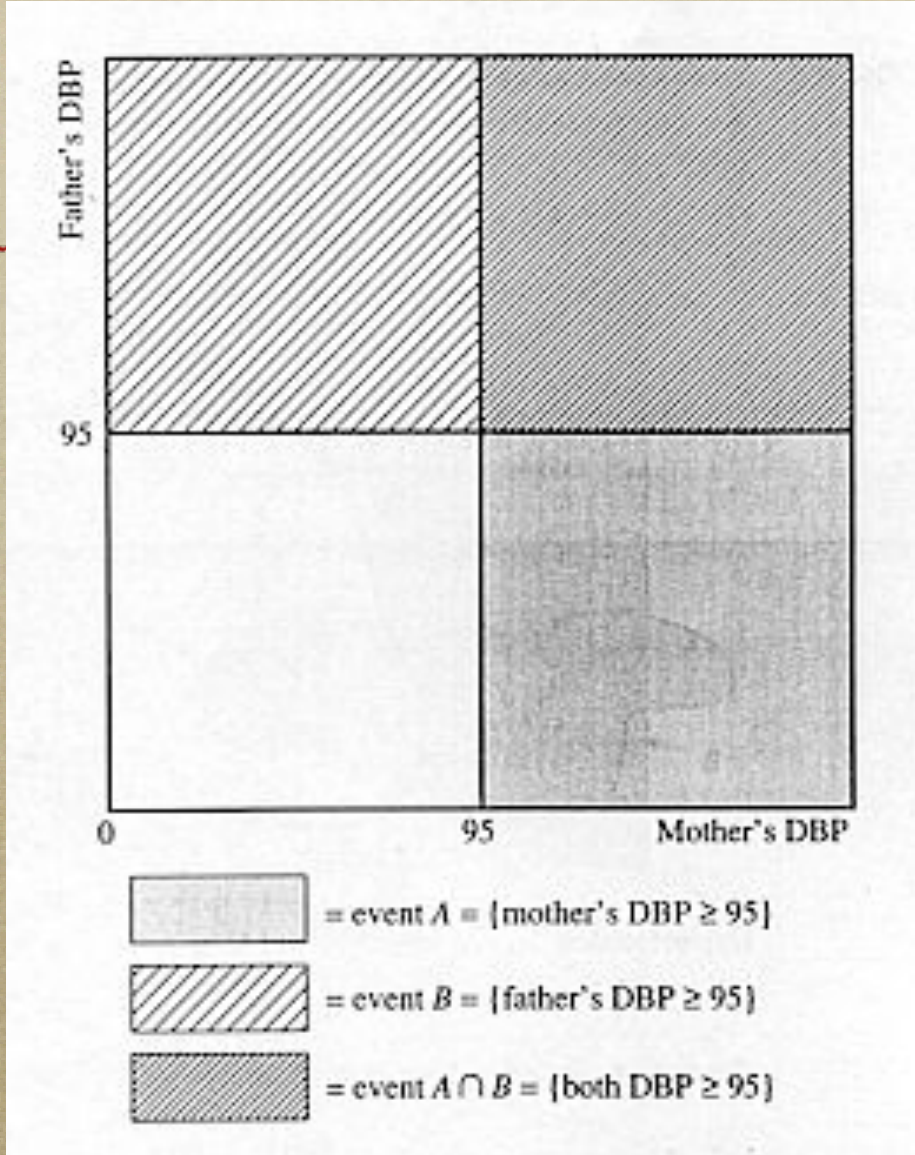
Chance of both parents being hypertensive?



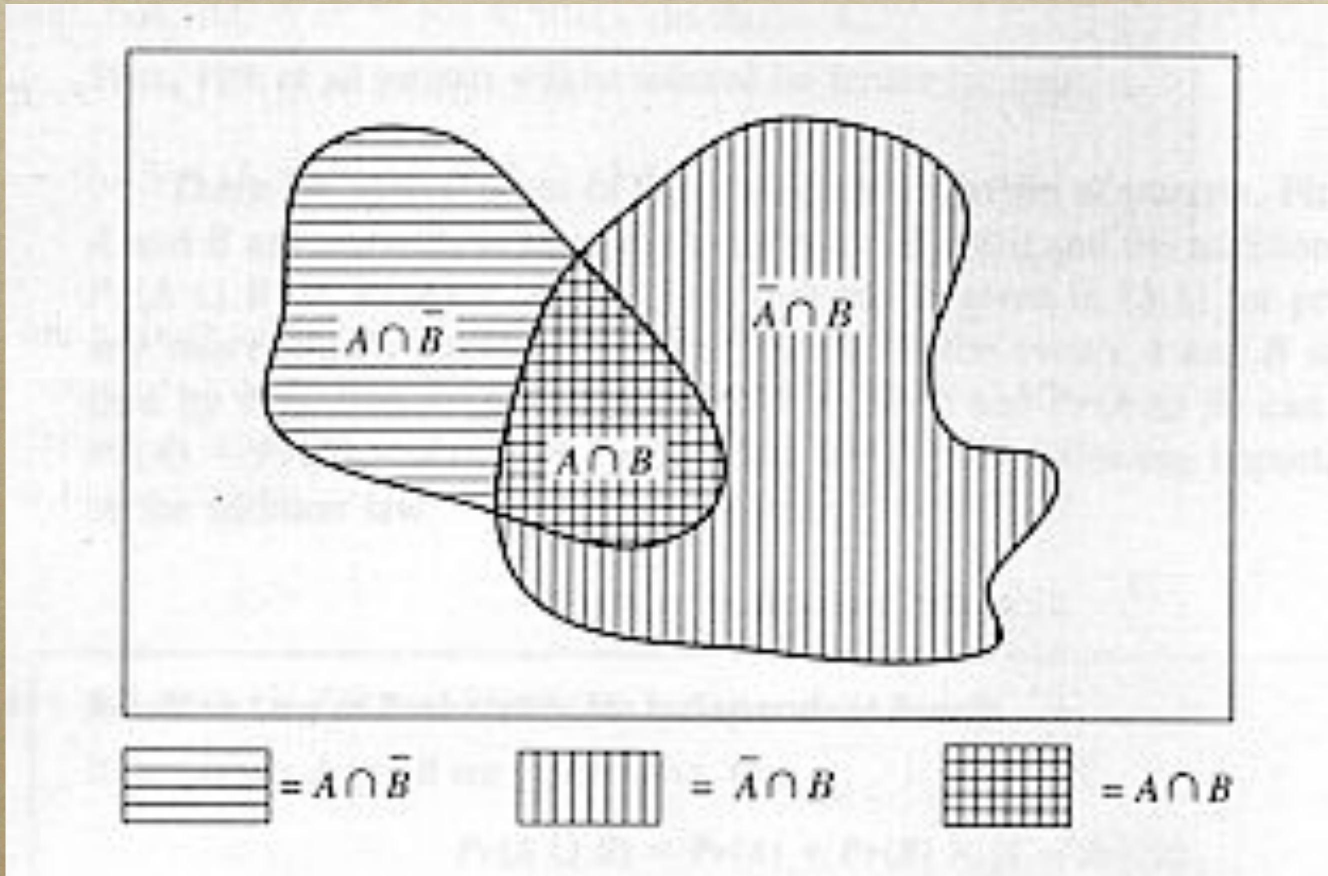
$Pr(A) = 0.1$
$Pr(B) = 0.2$

Chance of both parents being hypertensive?

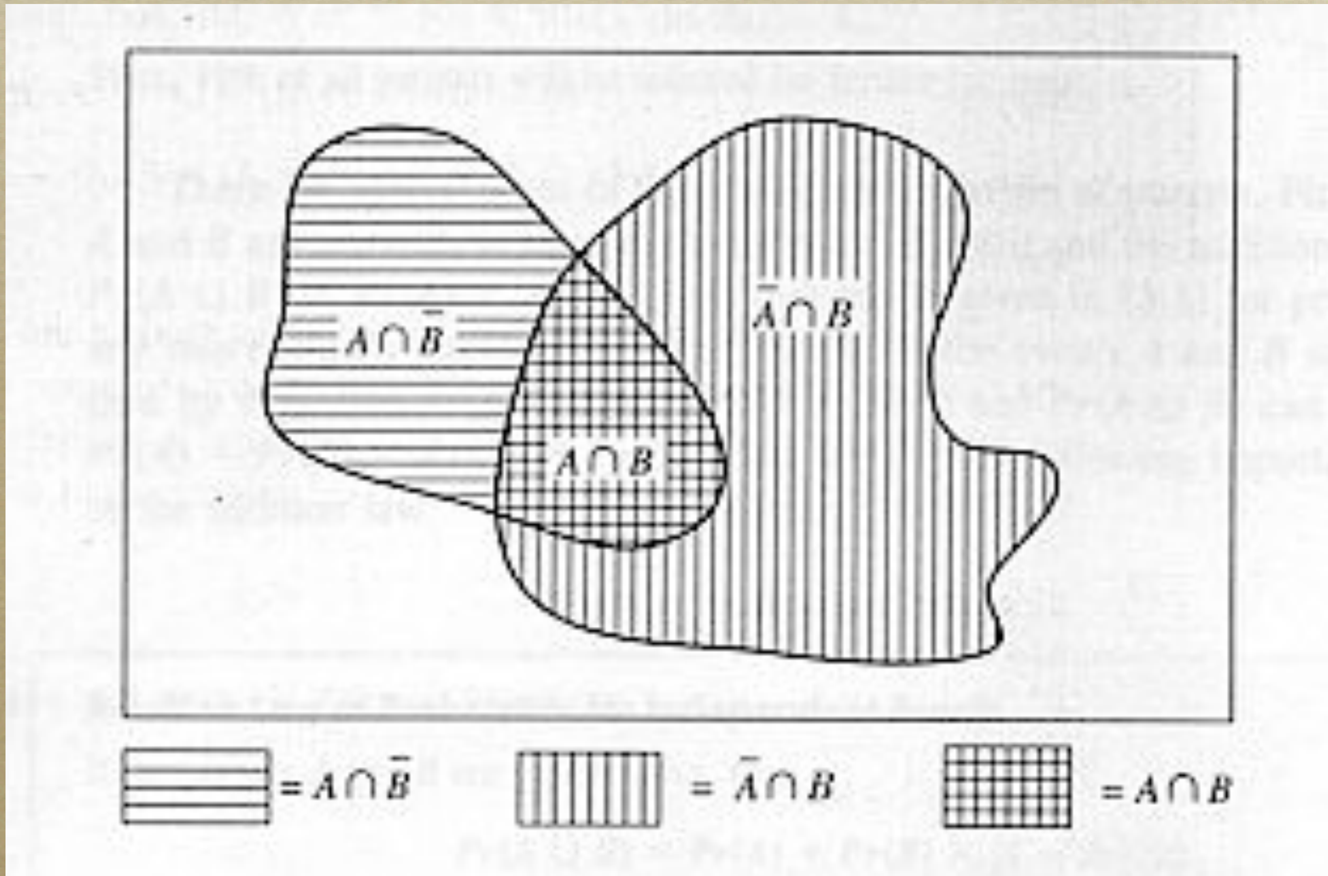
$0.1 * 0.2 = 0.02$



Venn Diagrams



$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$



Special Cases

- *If A & B are mutually exclusive*

  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$

- *If A & B are independent*

  $\Pr(A \cup B) = \Pr(A) + \Pr(B) * [1 - \Pr(A)]$

Conditional Probabilities

- *Multiplicative law works only for independent events*

- *When events are not independent, we need to quantify dependence.*

Let’s Talk About Diseases

Tuberculosis Screening

- *a skin test, SKT, for tuberculosis TB.*
- *A positive skin test {SKT+}*
  should be completely dependent on {TB}
- *(if these events are independent,*
  then the test is useless!)

Tuberculosis Screening

- *Draw a venn diagram for SKT+, TB*
- *Interested in the conditional probability (that is, TB given SKT+)*
- $Pr(SKT+ \cap TB) / Pr(SKT+)$

Generalized

- *conditional probability of B given A*
  $Pr(B|A) = Pr(A \cap B) \,/\, Pr(A)$
- *when A and B are independent then*
  $Pr((B|A) = Pr(B) = Pr(B \mid \text{not } A)$

Screening Tests

- *Predictive Value Positive (PV+)*
  Pr(disease | test+)
- *Predictive Value Positive (PV-)*
  Pr(no disease | test-)
- *Relative Risk (RR) given a positive test*
  Pr(disease | test+) / Pr(disease / test-)

TB example

- *Suppose 1 person in 100 who tests positive actually has TB.*

PV+, Pr(TB | SKT+)

- *Suppose 1 person in 10,000 whose test was negative actually has TB.*

Pr(TB | not SKT+)

Calculate PV- as

Pr(no disease | test-) = 1 - Pr(disease | test-)

1 - 0.0001 = 0.9999

TB example

- *Suppose 1 person in 100 who tests positive actually has TB.*

  PV+, Pr(TB | SKT+)

- *Suppose 1 person in 10,000 whose test was negative actually has TB.*

  Pr(TB | not SKT+)

Calculate RR as Pr(no disease | test+) / Pr(disease | test-)

0.01 / 0.0001 = 100: someone with a positive test is 100 times more likely to have TB

Symptoms, Diseases, & Inference

- *Clinicians often cannot easily measure Pr(disease | symptom)*
- *Can measure how often symptoms appear in people with and without the disease.*

More Definitions

- *Sensitivity of screening test*
  Pr(symptom | condition)

- *Specificity of screening test*
  Pr(no symptom | no condition)

- *Bayes' Rule relates PV+, PV- to these.*

Bayes' Rule

Bayes' Rule

Let A = symptom and B = disease.

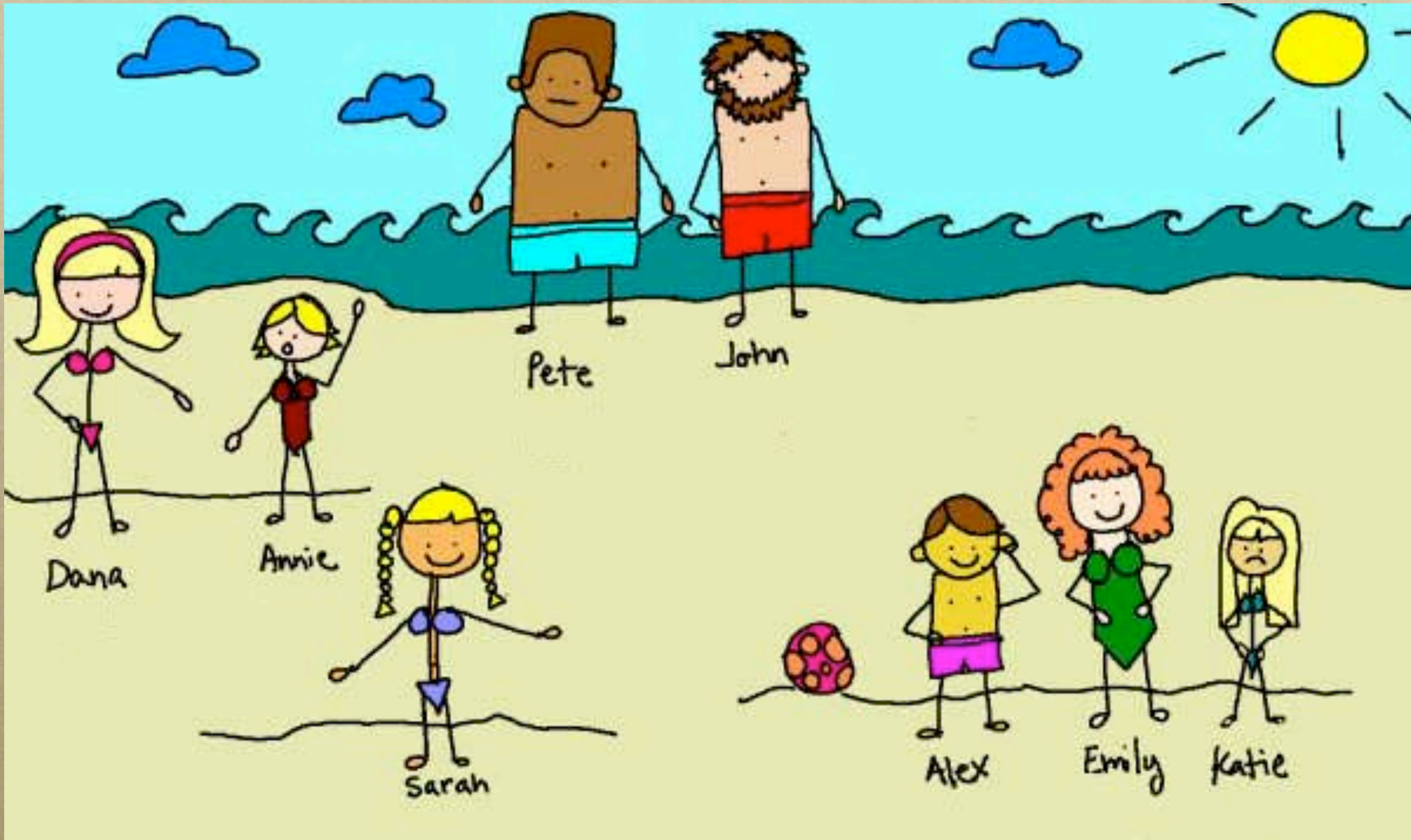
$$PV^+ = Pr(B|A) = \frac{Pr(A|B) \times Pr(B)}{Pr(A|B) \times Pr(B) + Pr(A|\bar{B}) \times Pr(\bar{B})}$$

In words, this can be written as

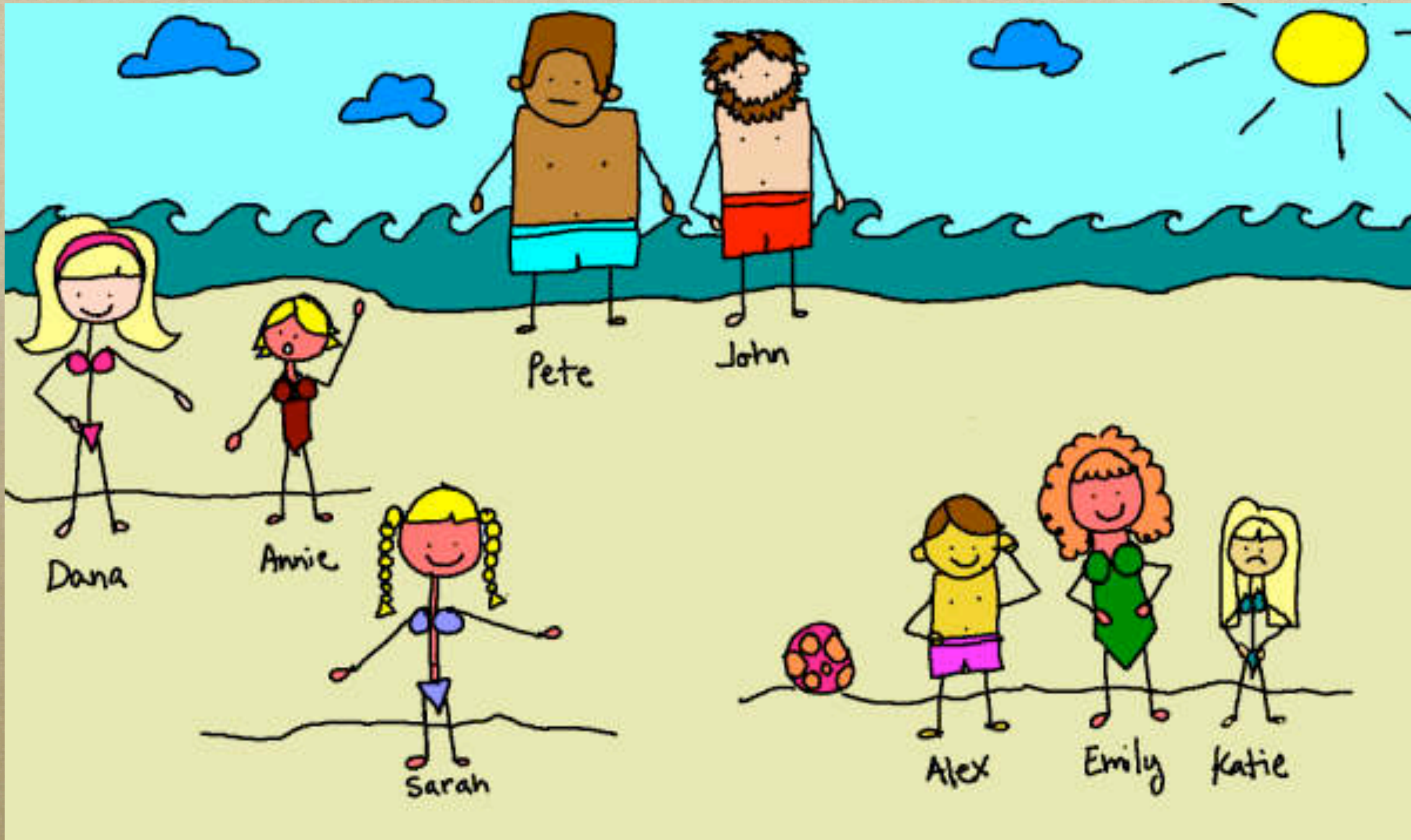
$$PV^+ = \frac{x \times \text{sensitivity}}{x \times \text{sensitivity} + (1 - x) \times (1 - \text{specificity})}$$

where $x = Pr(B)$ = prevalence of disease in the reference population. Similarly

$$PV^- = \frac{(1 - x) \times \text{specificity}}{(1 - x) \times \text{specificity} + x \times (1 - \text{sensitivity})}$$



At the Beach



Lobster Disease at the Beach

Data Collection

Name	*Hair*	*Height*	*Weight*	*L?*
Sarah	*blond*	*ave*	*light*	*yes*
Dana	*blond*	*tall*	*ave*	*no*
Alex	*brown*	*short*	*ave*	*no*
Annie	*blond*	*short*	*ave*	*yes*
Emily	*red*	*ave*	*heavy*	*yes*
Pete	*brown*	*tall*	*heavy*	*no*
John	*brown*	*ave*	*heavy*	*no*
Katie	*blond*	*short*	*light*	*no*

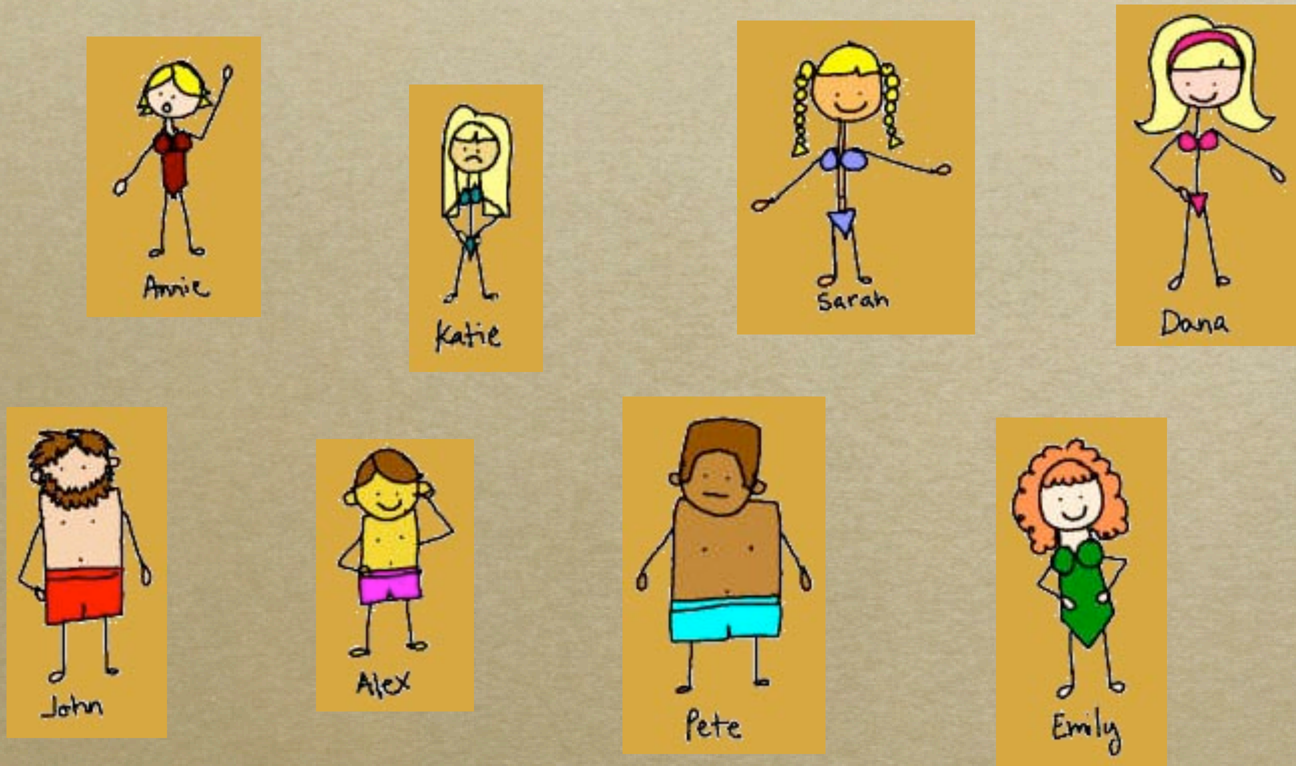
Hair Color as Predictor

		Blond	*Red*	*Dark*
		0.5	*0.125*	*0.375*
Not L	*0.65*	*0.5*	*0*	*1*
L	*0.375*	*0.5*	*1*	*0*

Hair Color as Predictor

		Blond	*Red*	*Dark*
		0.5	*0.125*	*0.375*
Not L	*0.65*	*0.5*	*0*	*1*
L	*0.375*	*0.5*	*1*	*0*

Blond Hair - 0.5



Hair Color as Predictor

		Blond	*Red*	*Dark*
		0.5	*0.125*	*0.375*
Not L	*0.65*	*0.5*	*0*	*1*
L	*0.375*	*0.5*	*1*	*0*

Blond PV+ : Pr(L | blond+)

Half the blonds caught the lobster disease.

Decision Tree

Height as Predictor

		Short	Ave	Tall
		0.375	*0.375*	*0.25*
Not L	*0.65*			
L	*0.375*			

Short People

Height as Predictor

		Short	*Ave*	*Tall*
		0.375	*0.375*	*0.25*
Not L	*0.65*	*0.66*		
L	*0.375*	*0.33*		

Average Height People

Height as Predictor

		Short	Ave	Tall
		0.375	0.375	0.25
Not L	0.65	0.66	0.33	
L	0.375	0.33	0.66	

Tall People

Height as Predictor

		Short	*Ave*	*Tall*
		0.375	*0.375*	*0.25*
Not L	*0.65*	*0.66*	*0.33*	*1*
L	*0.375*	*0.33*	*0.66*	*0*

Height as Predictor

		Short	Ave	Tall
		0.375	0.375	0.25
Not L	0.65	0.66	0.33	1
L	0.375	0.33	0.66	0

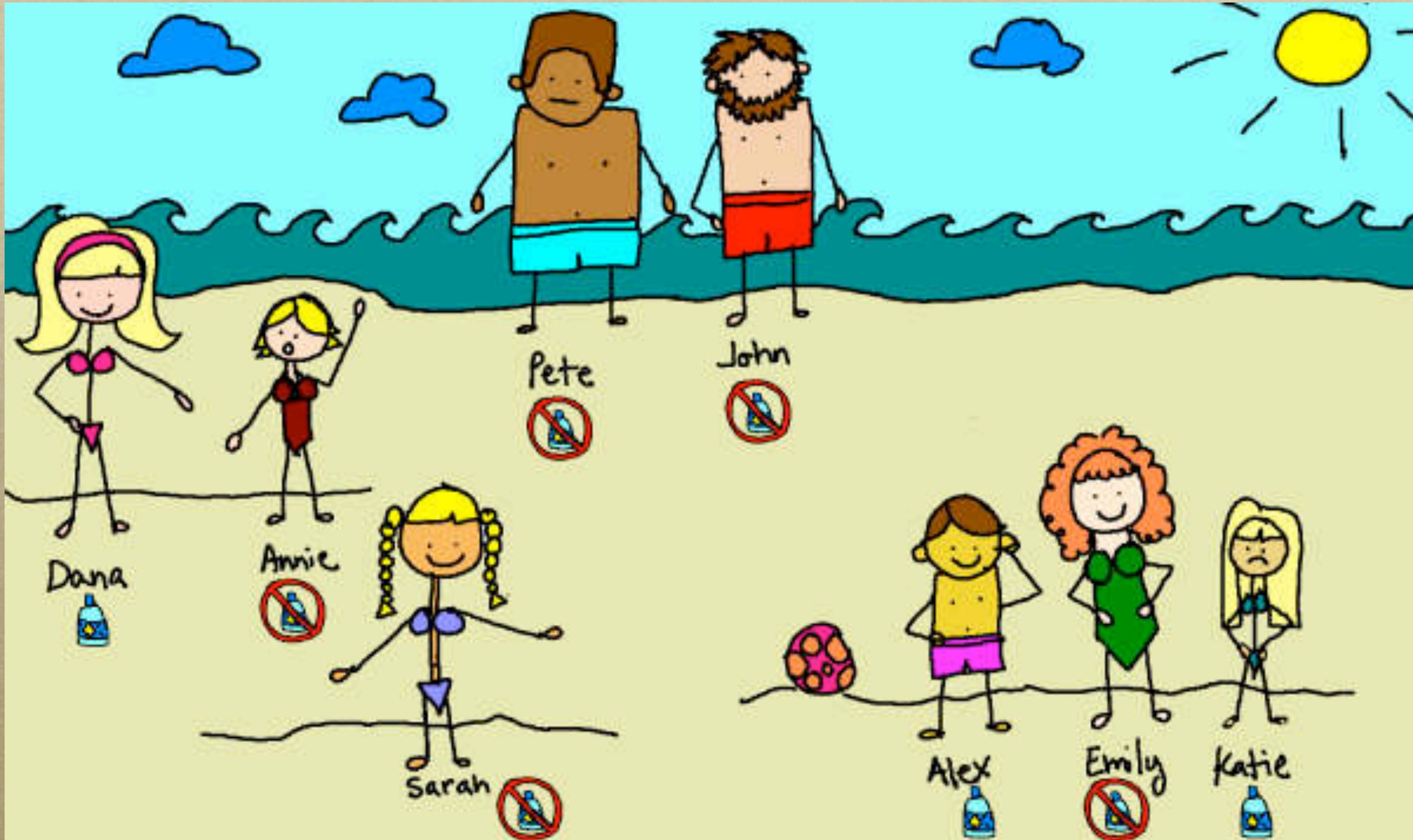
Weight as Predictor

		Light	*Ave*	*Heavy*
		0.25	*0.375*	*0.375*
Not L	*0.65*	*0.5*	*0.66*	*0.66*
L	*0.375*	*0.5*	*0.33*	*0.33*

Problem

No single character trait predicts disease!

investigate more



More History

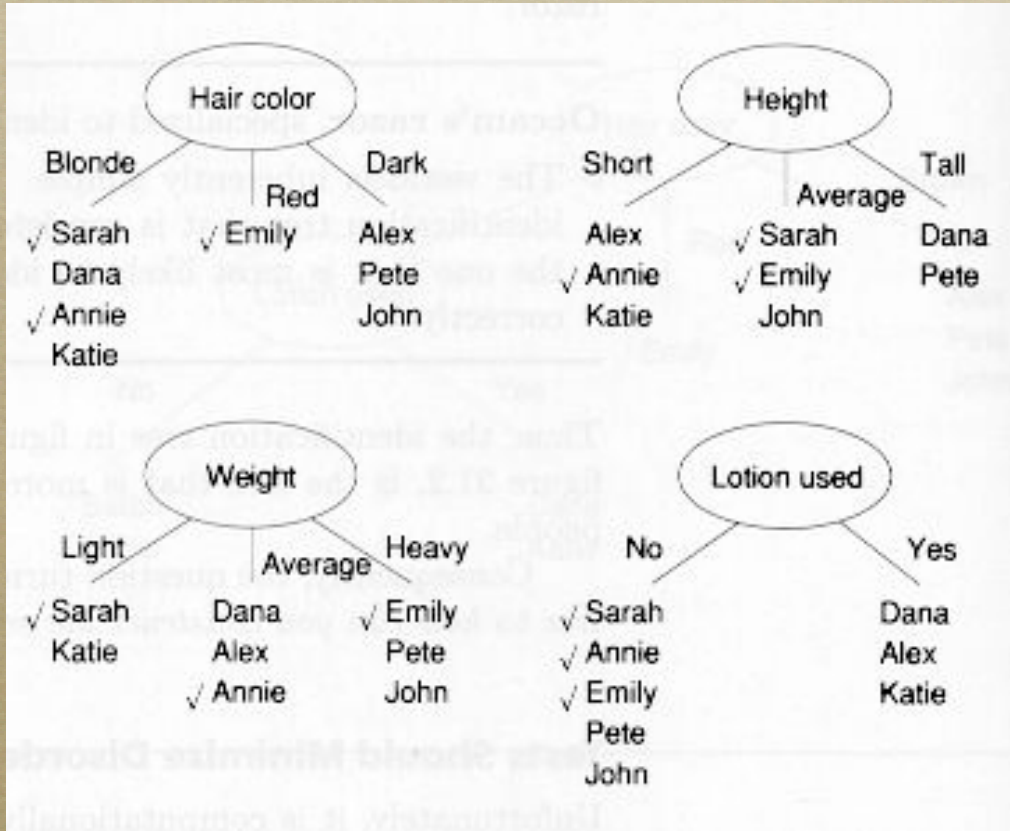
Data Collection

Name	Hair	Height	Weight	Lotion	L?
Sarah	blond	ave	light	no	yes
Dana	blond	tall	ave	yes	no
Alex	brown	short	ave	yes	no
Annie	blond	short	ave	no	yes
Emily	red	ave	heavy	no	yes
Pete	brown	tall	heavy	no	no
John	brown	ave	heavy	no	no
Katie	blond	short	light	yes	no

Lotion as Predictor

		No	*Yes*
		0.625	*0.375*
Not L	*0.65*	*0.4*	*1*
L	*0.375*	*0.6*	*0*

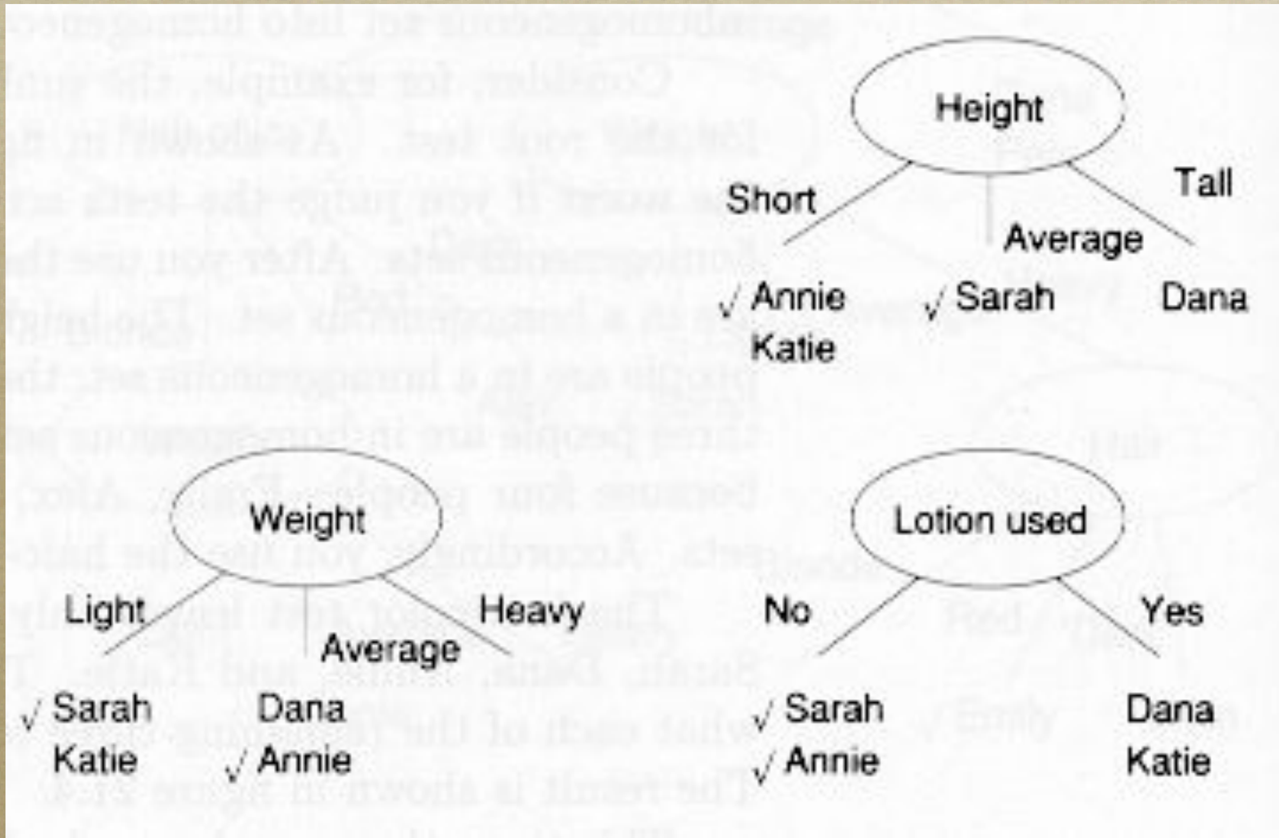
Possible Trees



Diagnostic Rules

- *if your hair is red, you'll burn*
- *if your hair is dark, you won't*
- *if your hair is blond ???*

Now, look at just the blonds



Height as Predictor for Blonds

		Short	Ave	Tall
		0.5	*0.25*	*0.25*
Not L	*0.5*	*0.5*	*0*	*1*
L	*0.5*	*0.5*	*1*	*0*

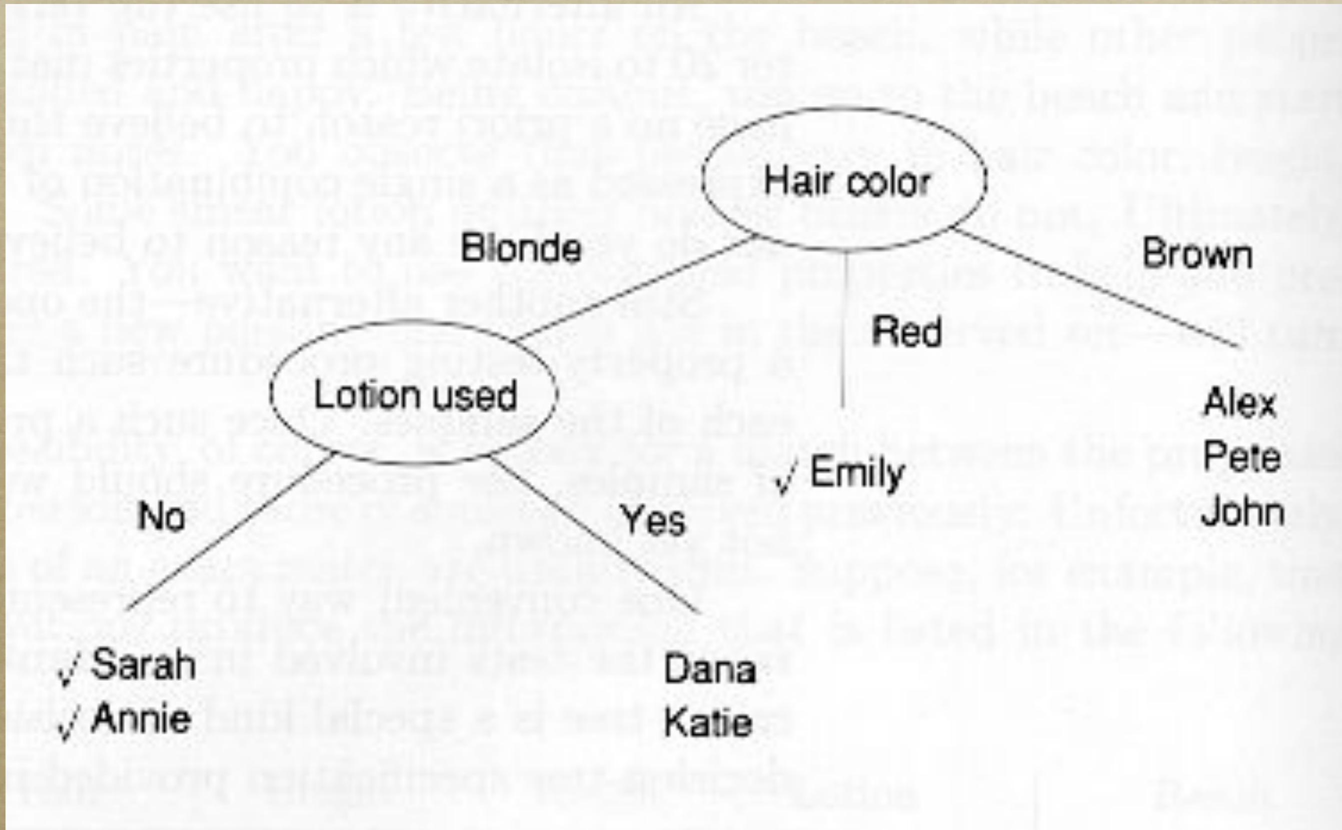
Weight as Predictor for Blonds

		Light	*Ave*	*Heavy*
		0.5	*0.5*	*0*
Not L	*0.5*	*0.5*	*0.5*	*0*
L	*0.5*	*0.5*	*0.5*	*0*

Lotion as Predictor for Blonds

		No	*Yes*
		0.5	*0.5*
Not L	*0.5*	*0*	*1*
L	*0.5*	*1*	*0*

Success Multiple Tests



Predictive algorithm

First ask about hair color

if red, burn

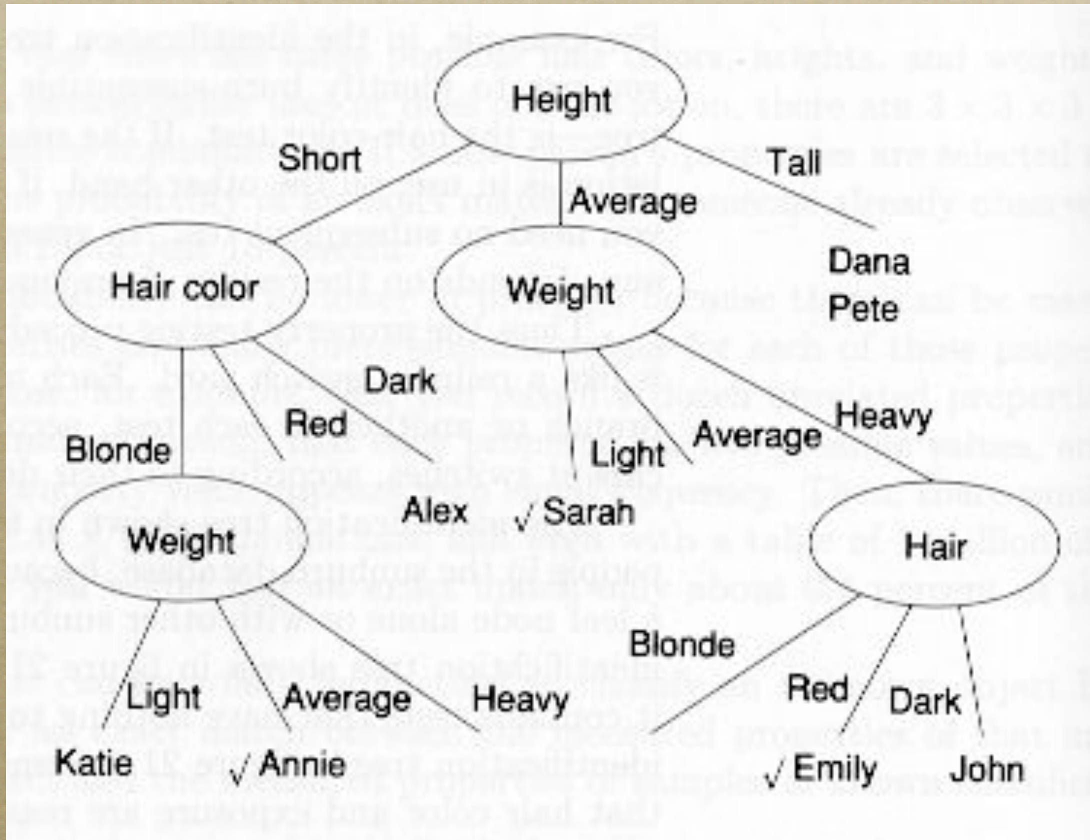
if dark, not burn

Next ask about lotion

if lotion, not burn

otherwise, burn

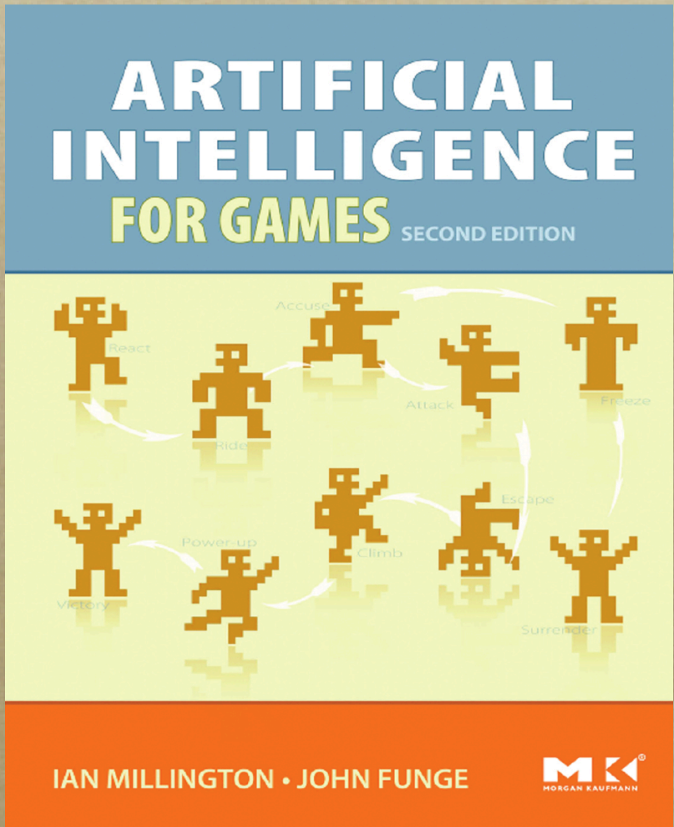
Was that solution unique?



real life considerations

- *For TB, skin test had many false positives*
- *Still chosen because the best test, a chest x-ray, is expensive and potentially harmful*

Reading



AI for Games

Section 7.5,

Naive Bayes Classifiers

Study Question

- *Check the predictive value of gender in the beach example.*

  (Determine sex from the names or pictures.)

- *Does this change the predictive algorithm?*