

# STATE SPACES & BLIND SEARCH

M. Anthony Kapolka III  
Wilkes University  
CS 340 - Fall 2019

# Issues in Search

- Knowledge Representation
- Topology (of State Space)
- Algorithms
- Forward or Backward



# The Farmer, Fox, Goose and Grain...

A farmer wants to move himself, a silver fox, a fat goose, and some tasty grain across a river from the west side to the east side. Unfortunately, his boat is so small he can only take one of his possessions across on any trip. Worse yet, an unattended fox will eat a goose, and an unattended goose will eat grain, so the farmer must not leave the fox and goose alone, or the goose and grain alone.

*What should he do to get everything across?*

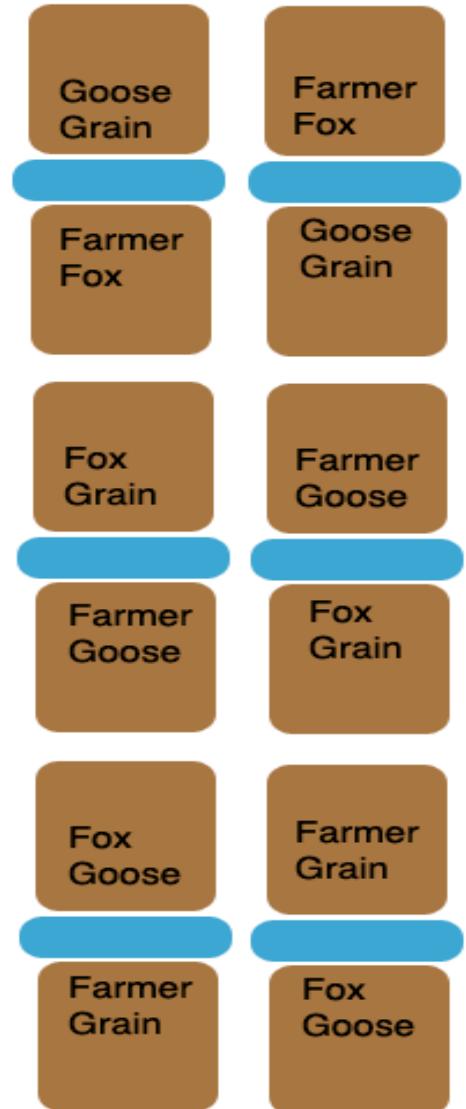
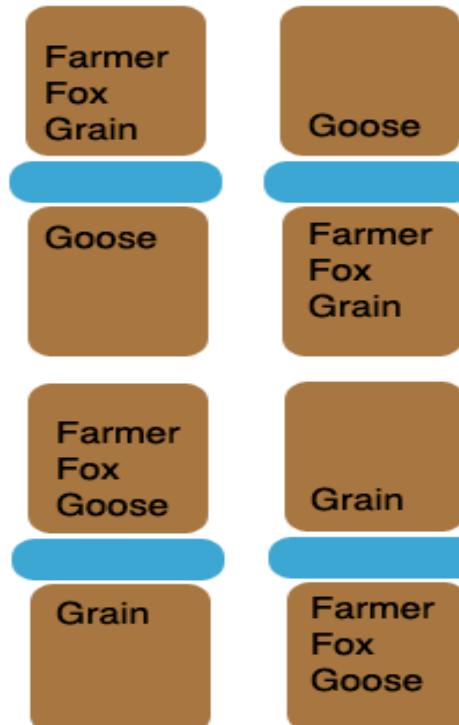
The diagram illustrates the initial state of a classic logic puzzle. It features four distinct colored rectangles arranged vertically. The top rectangle is brown and contains the text "Farmer", "Fox", "Goose", and "Grain". Below it is a blue horizontal bar. The third element is a brown rectangle with no internal text. The bottom element is another brown rectangle with no internal text.

Farmer  
Fox  
Goose  
Grain

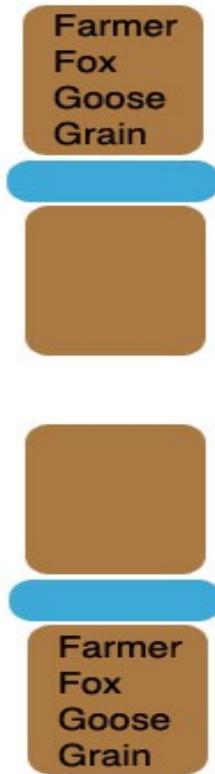
Start State



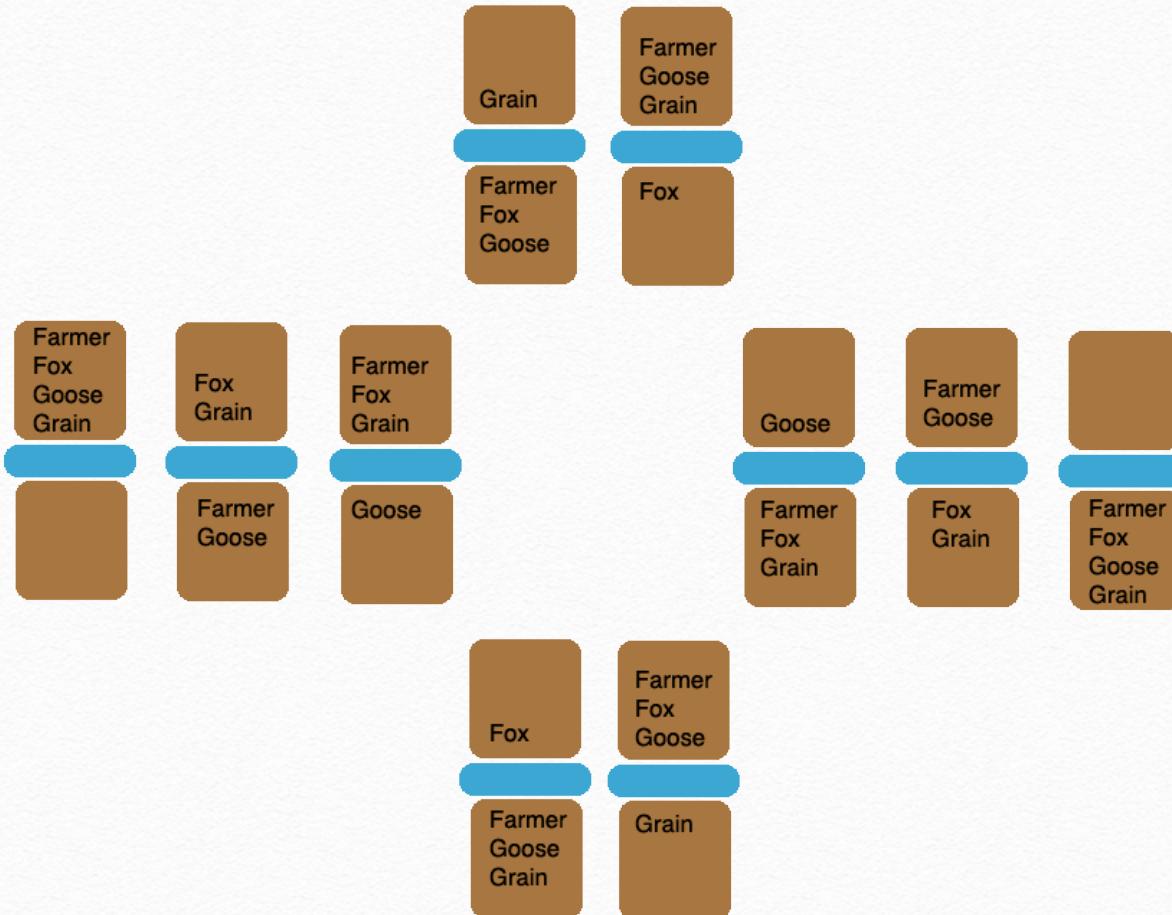
Goal State



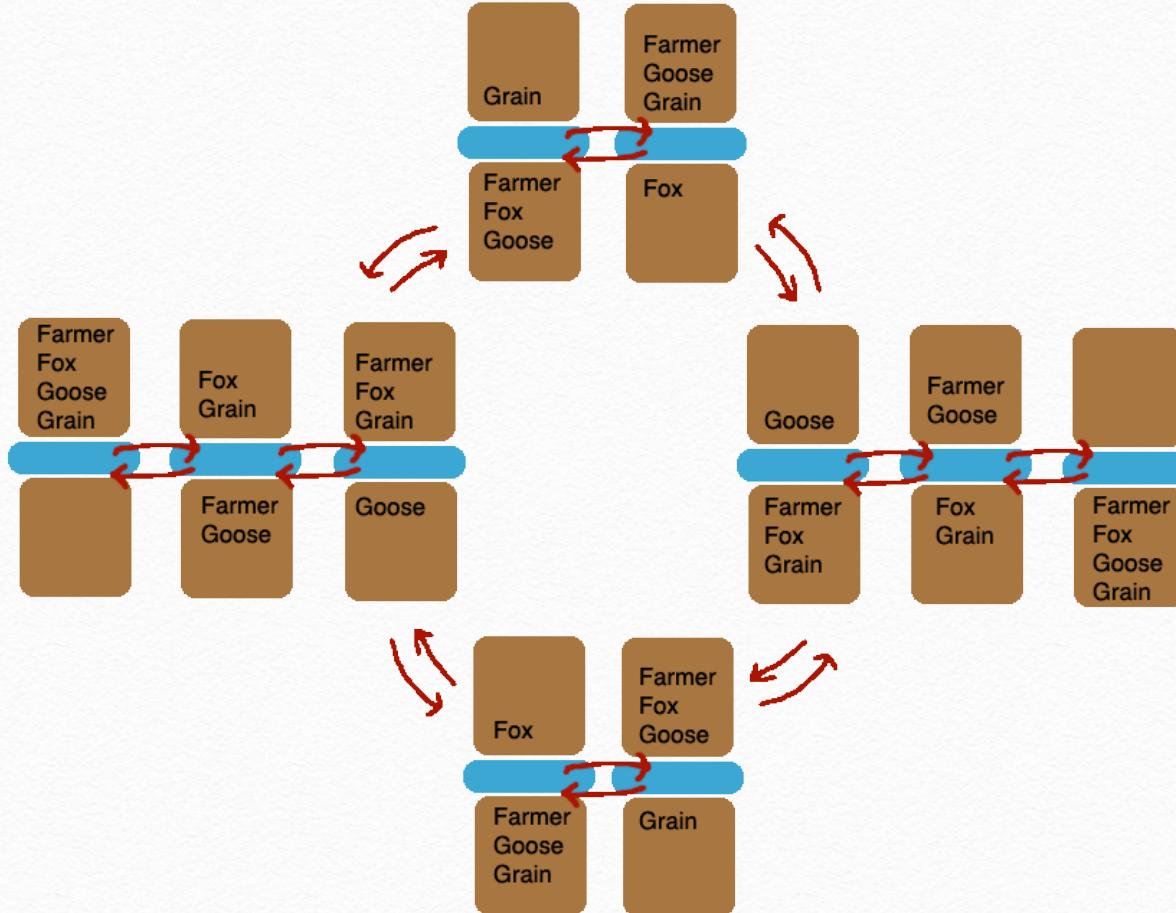
Exhaustively generate all possible states



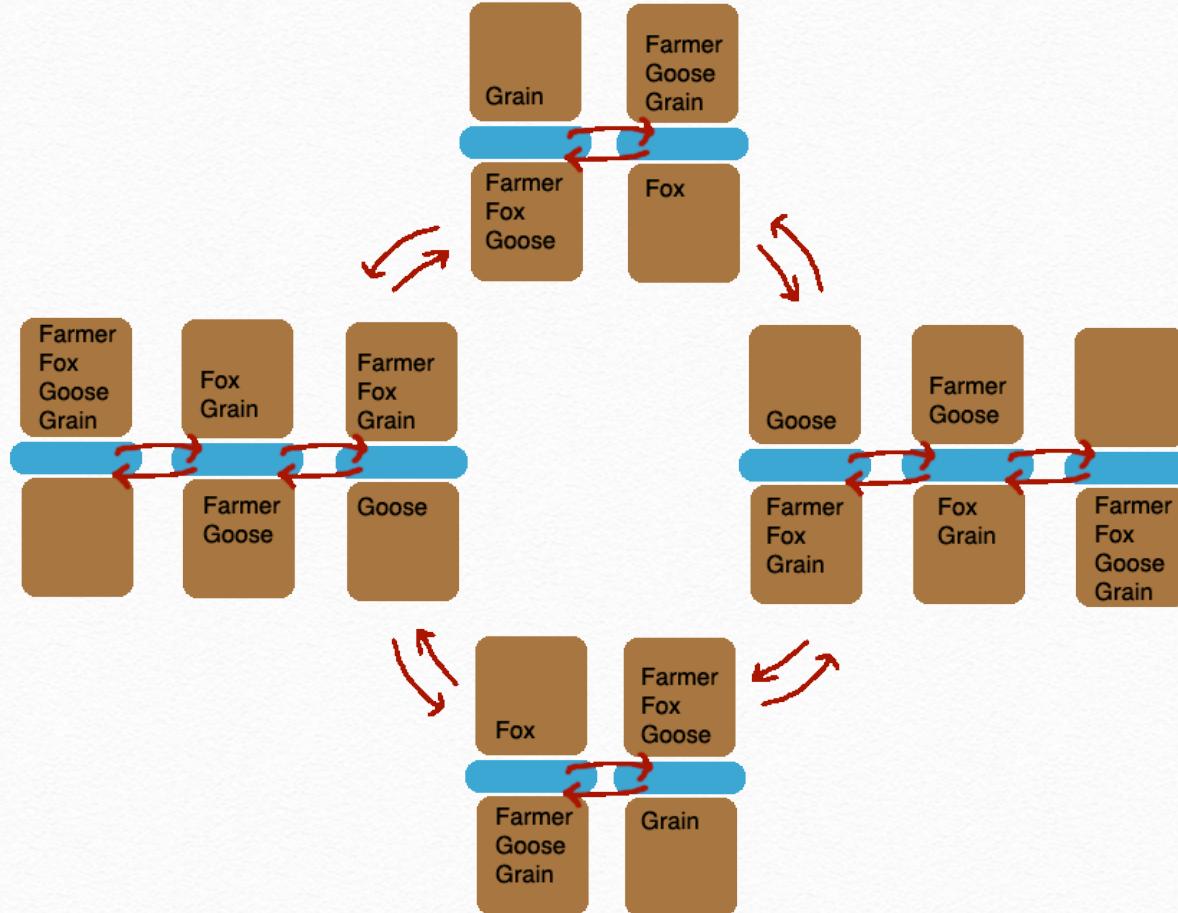
Delete prohibited states



Reorganize states



Solutions are obvious.



What is the topology?

# Problem Space Topology

Problem Space (Newell & Simon, 1972)

Research in human cognition

We search for a solution  
among a set of possible solutions

# A Brief Discourse on Context and History





# Missionaries and Cannibals

Three missionaries and three cannibals come to a river and find a boat that holds two. If the cannibals ever outnumber the missionaries on either bank, the missionaries will be eaten.

*How shall they cross?*

# Problem Space Topology

Topology

Organization of problem space

Standard data structures,  
including graphs and trees

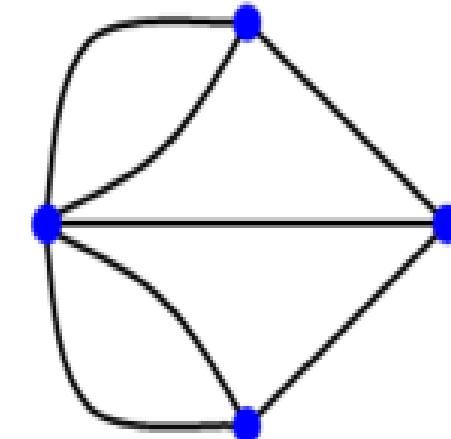
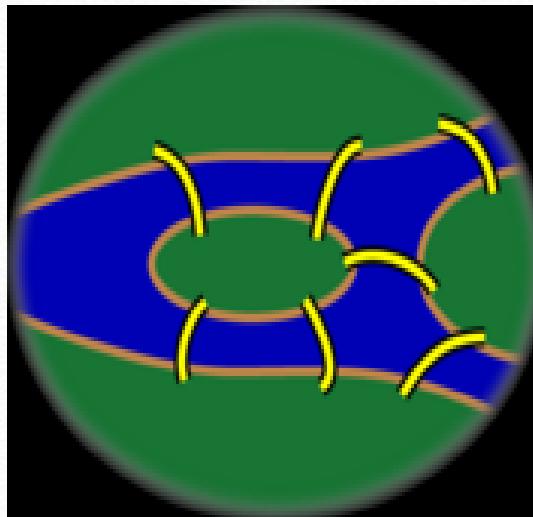
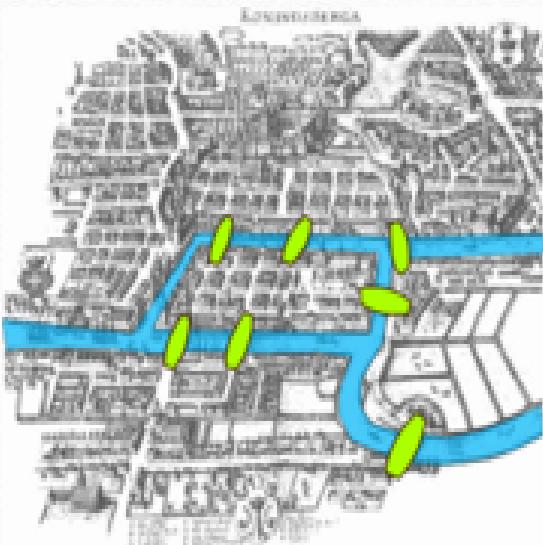
# Seven Bridges of Königsberg



Task: Walk around, crossing over all bridges once and only once.

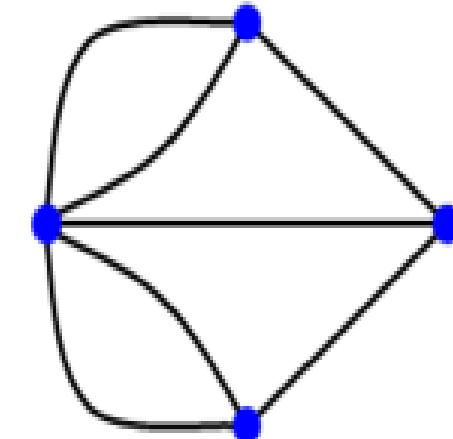
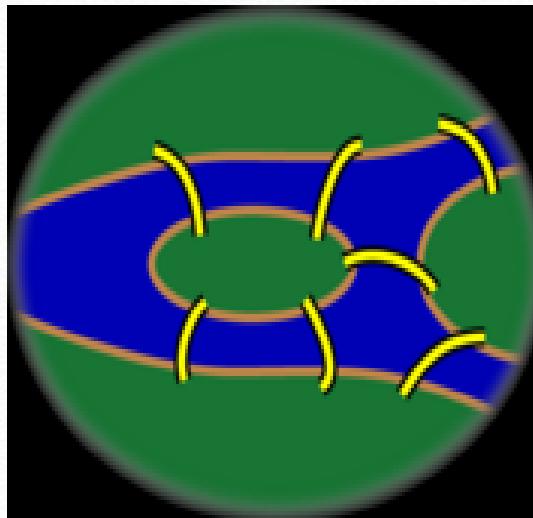
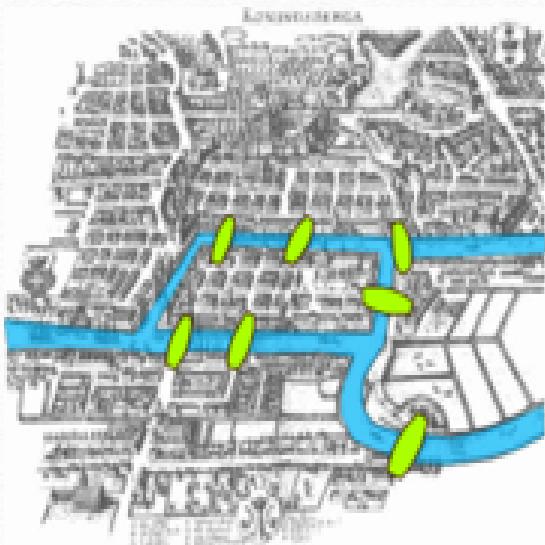
# Seven Bridges of Königsberg

solved by Leonhard Euler, 1736.



# Seven Bridges of Königsberg

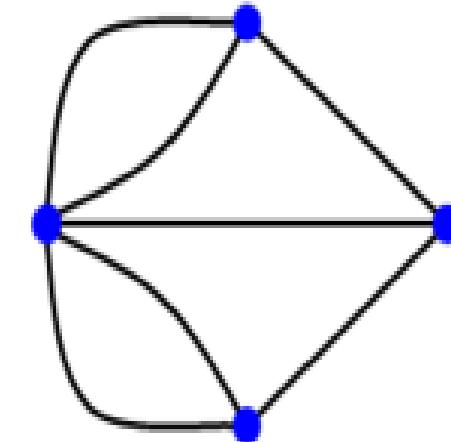
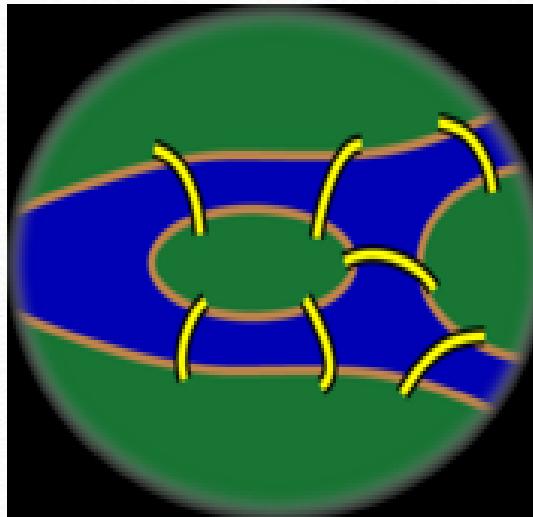
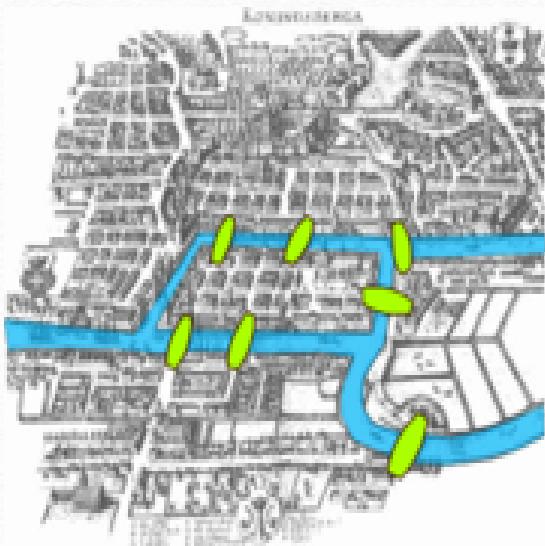
solved by Leonhard Euler, 1736.



Eulerian circuit possible?

# Seven Bridges of Königsberg

solved by Leonhard Euler, 1736.

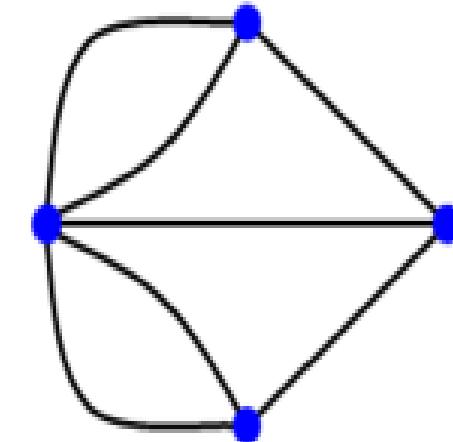
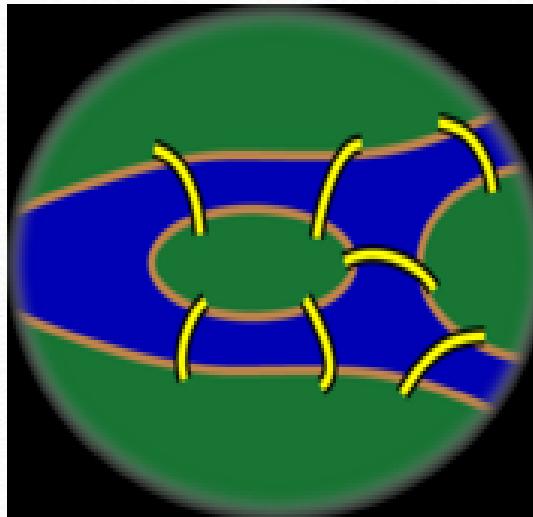
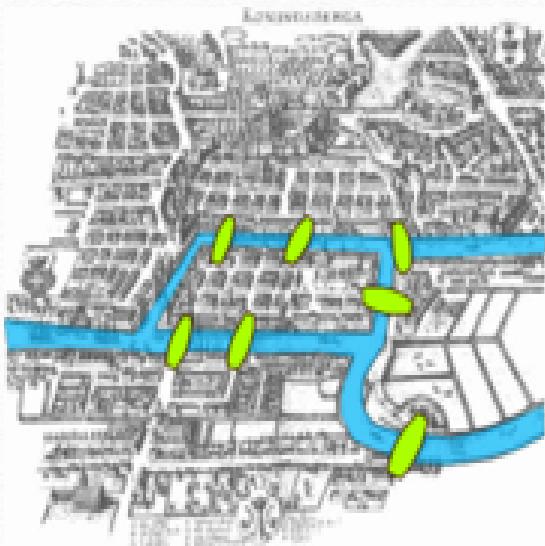


Eulerian circuit possible?

Not if any nodes have odd degree!

# Seven Bridges of Königsberg

solved by Leonhard Euler, 1736.

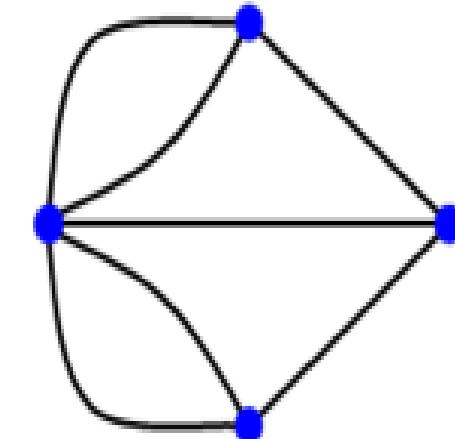
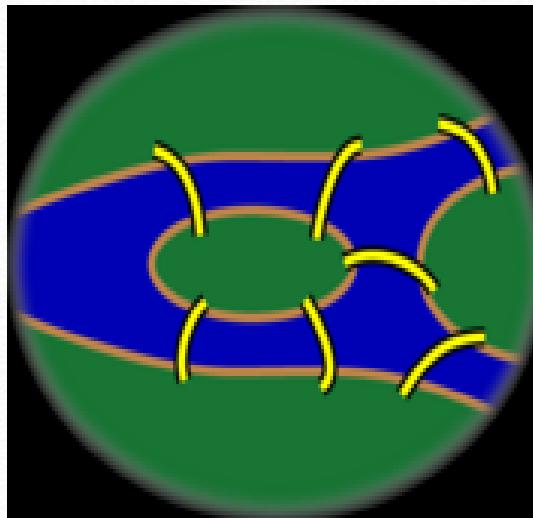
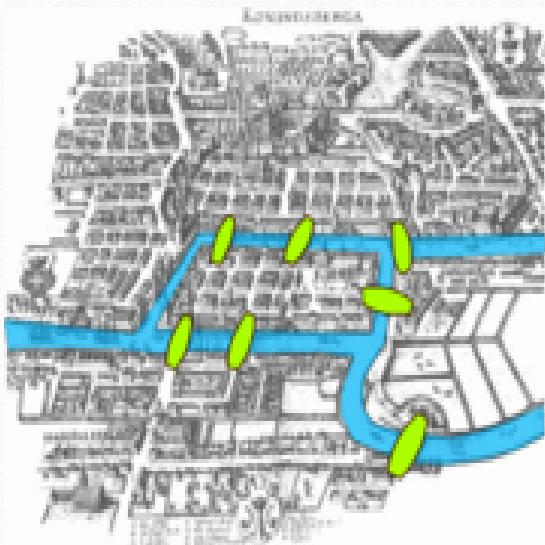


Eulerian walk possible?

Illustrations from Wikipedia, the free encyclopedia.

# Seven Bridges of Königsberg

solved by Leonhard Euler, 1736.



Eulerian walk possible?

Only if 0 or 2 nodes have odd degree!

# Trees vs. Graphs

Main difference: trees are directed, acyclic

Lots of tree traversal & graph algorithms

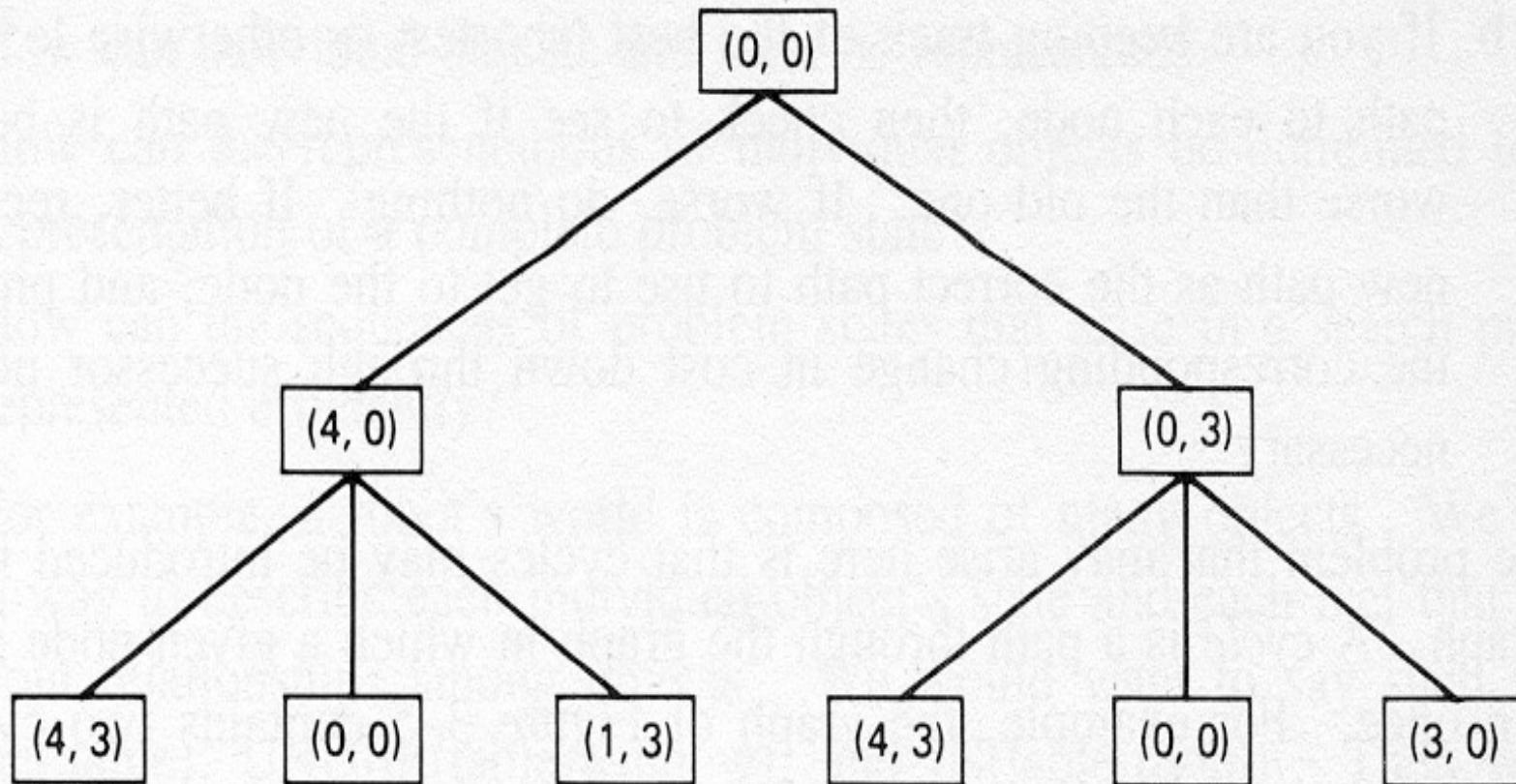
Nodes in trees may be repeatedly generated

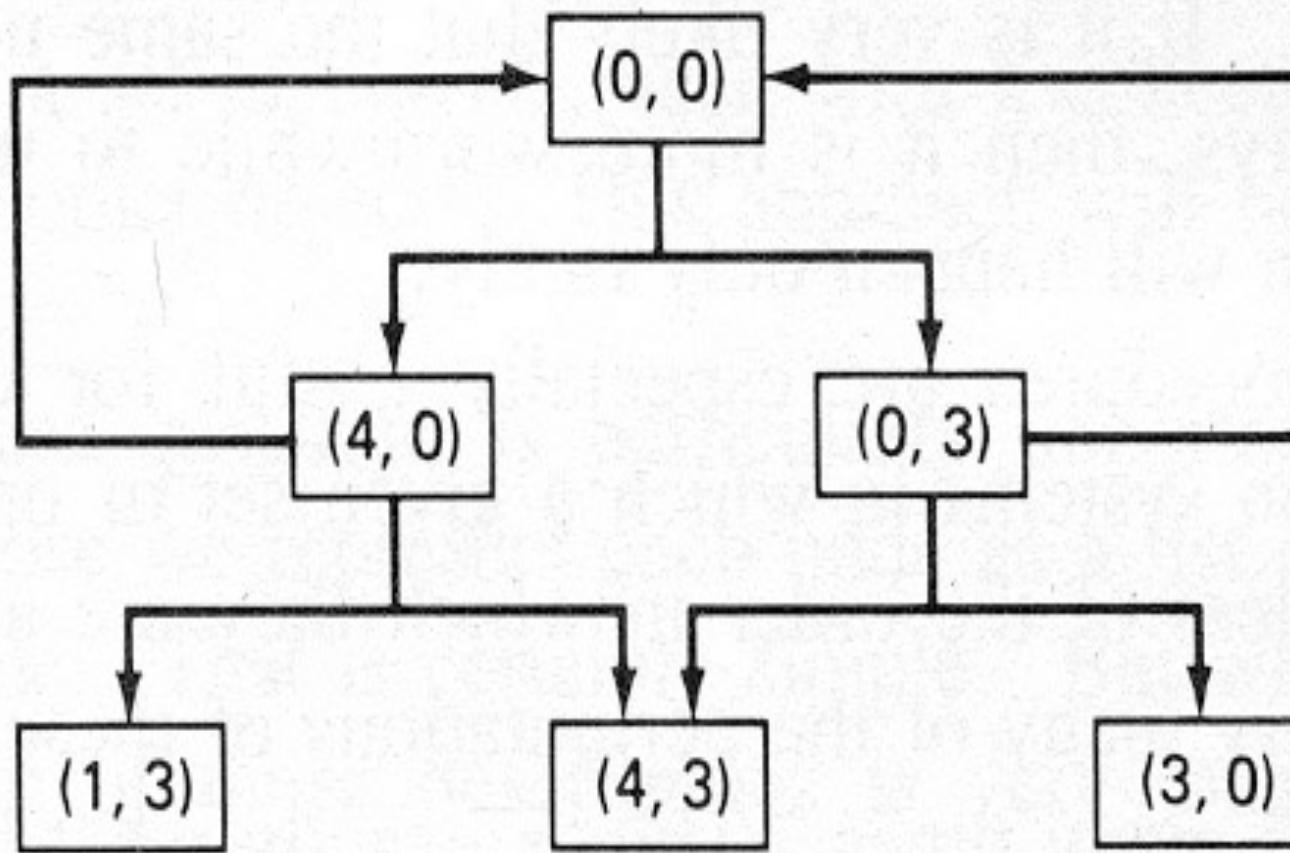
Can convert tree traversals into graph search

# Water Jug Example

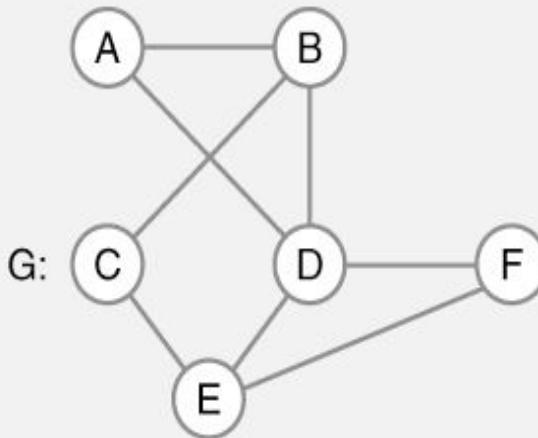
You are given two jugs, a 4-gallon one and a 3-gallon one. Neither has any measurement markings on it. There is a tap to fill them with water.

*How can you get exactly two gallons of water into the four gallon jug?*

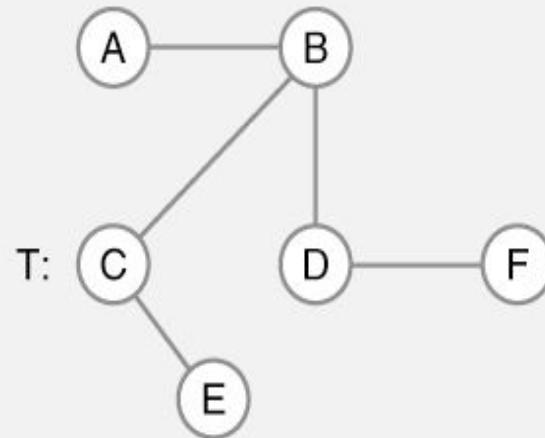




# Spanning Tree



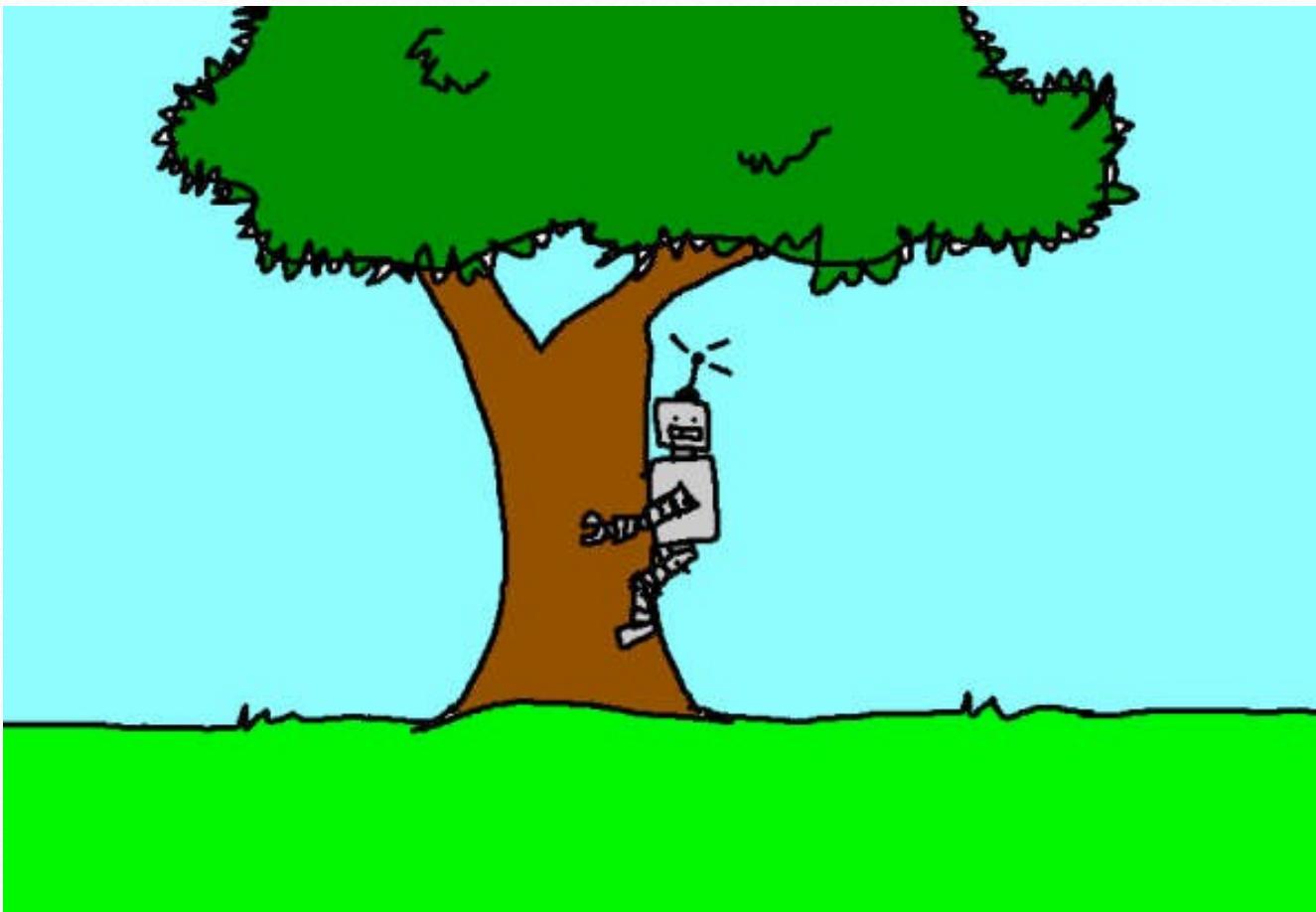
(a) Graph G



(b) Spanning tree T of graph G

All but  $N-1$  edges are dropped to break cycles

# Tree Traversal Algorithms



# The 8-tile Puzzle

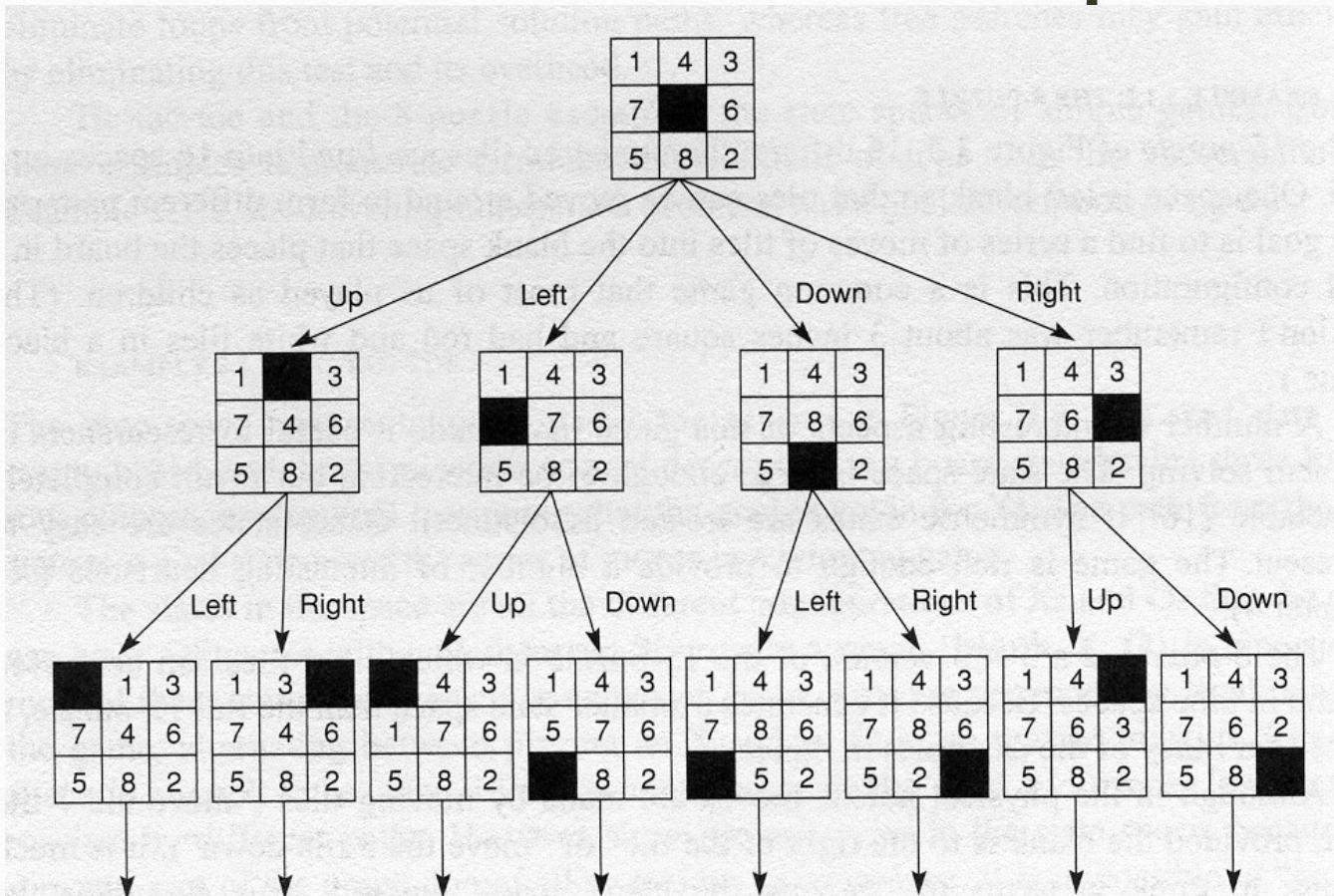


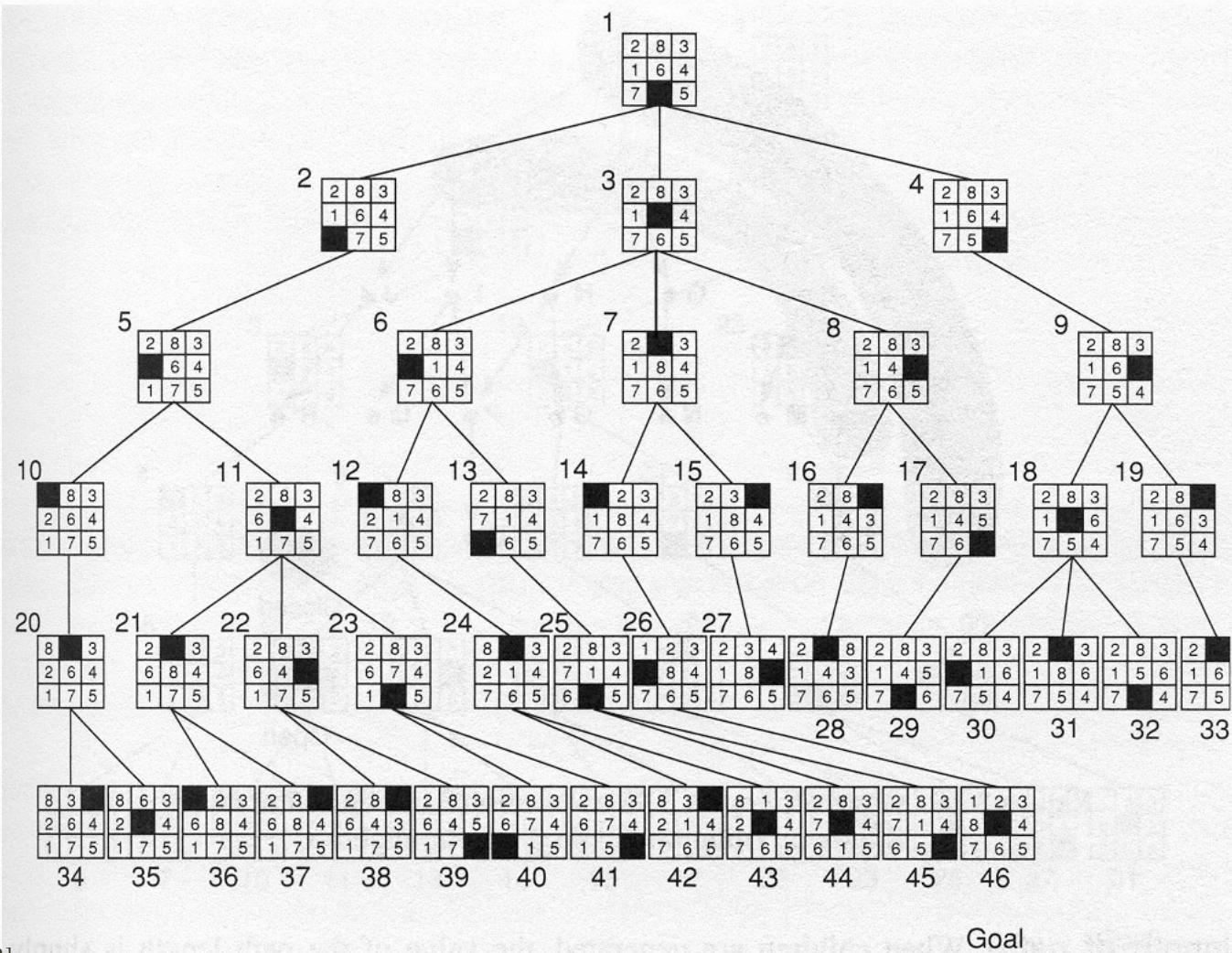
Start State



Goal State

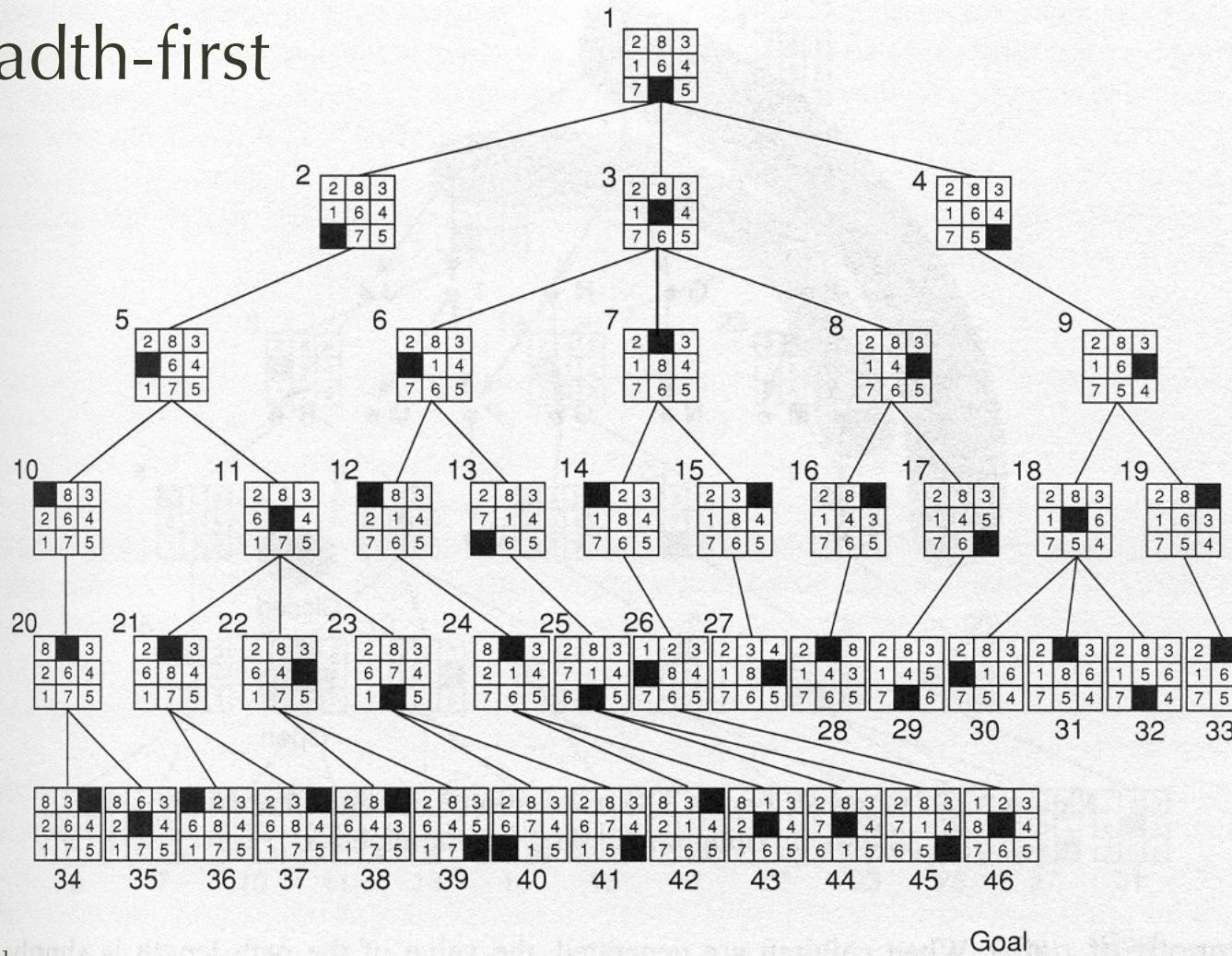
# 8-tile Puzzle State Space





[Lugar 2002]

# Breadth-first



[Lugar 2002]

# Breadth-first search

Explores space in level-by-level fashion

- Always finds shortest path to goal
- When branching is great, takes up much space
- For some problems this makes it unusable.

function **breadth-first-search**

open := [Start]

done := [ ]

while open != [ ] do

X := open.dequeue()

if (X = Goal) return success

else

generate children states of X

discard any children in open, done

for each child C

    open.enqueue(C)

    done.add(X)

return failure

function **breadth-first-search**

open := [Start]

done := [ ]

while open != [ ] do

X := open.dequeue()

if (X = Goal) return success

else

generate children states of X

discard any children in open, done

for each child C

    open.enqueue(C)

    done.add(X)

return failure

Time:  $O(b^{\text{depth}})$

Space:  $O(b \cdot \text{depth})$

## function breadth-first-search

open := [Start]

done := [ ]

while open != [ ] do

X := open.dequeue()

if (X = Goal) return success

else

generate children states of X

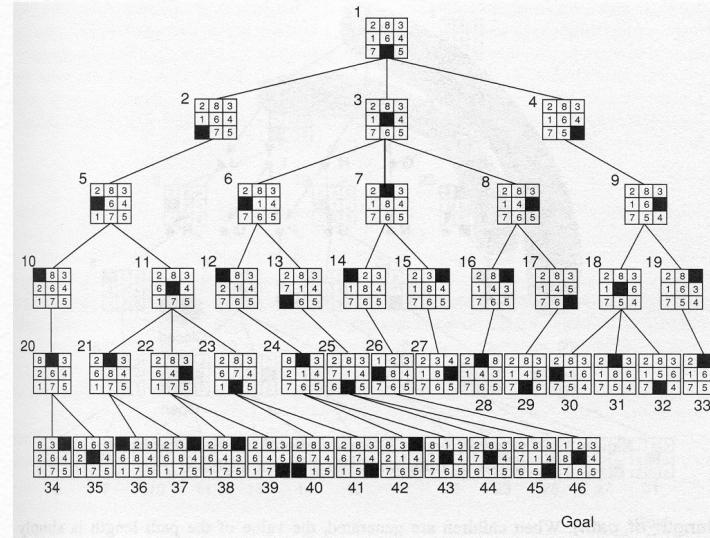
discard any children in open, done

for each child C

    open.enqueue(C)

    done.add(X)

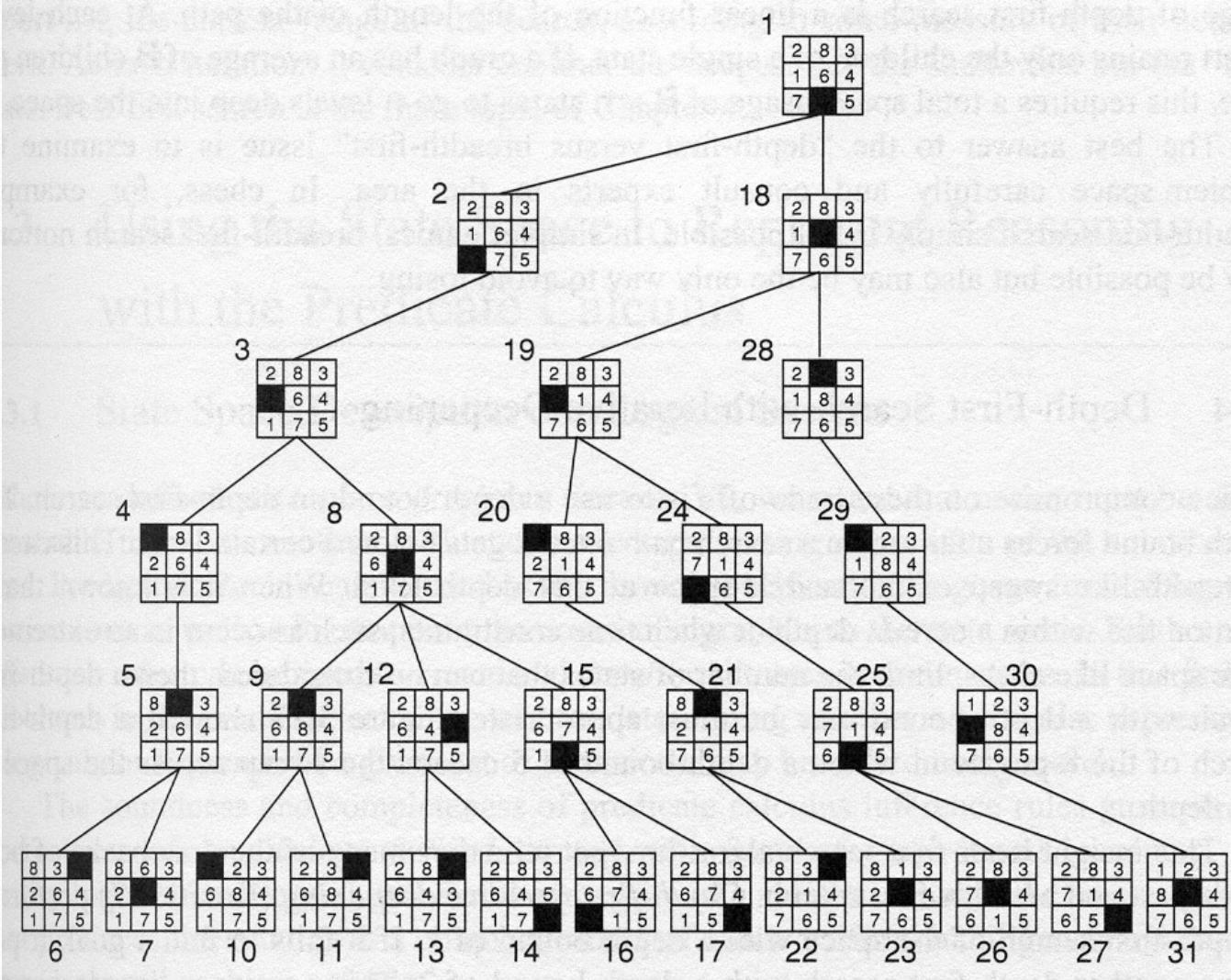
return failure



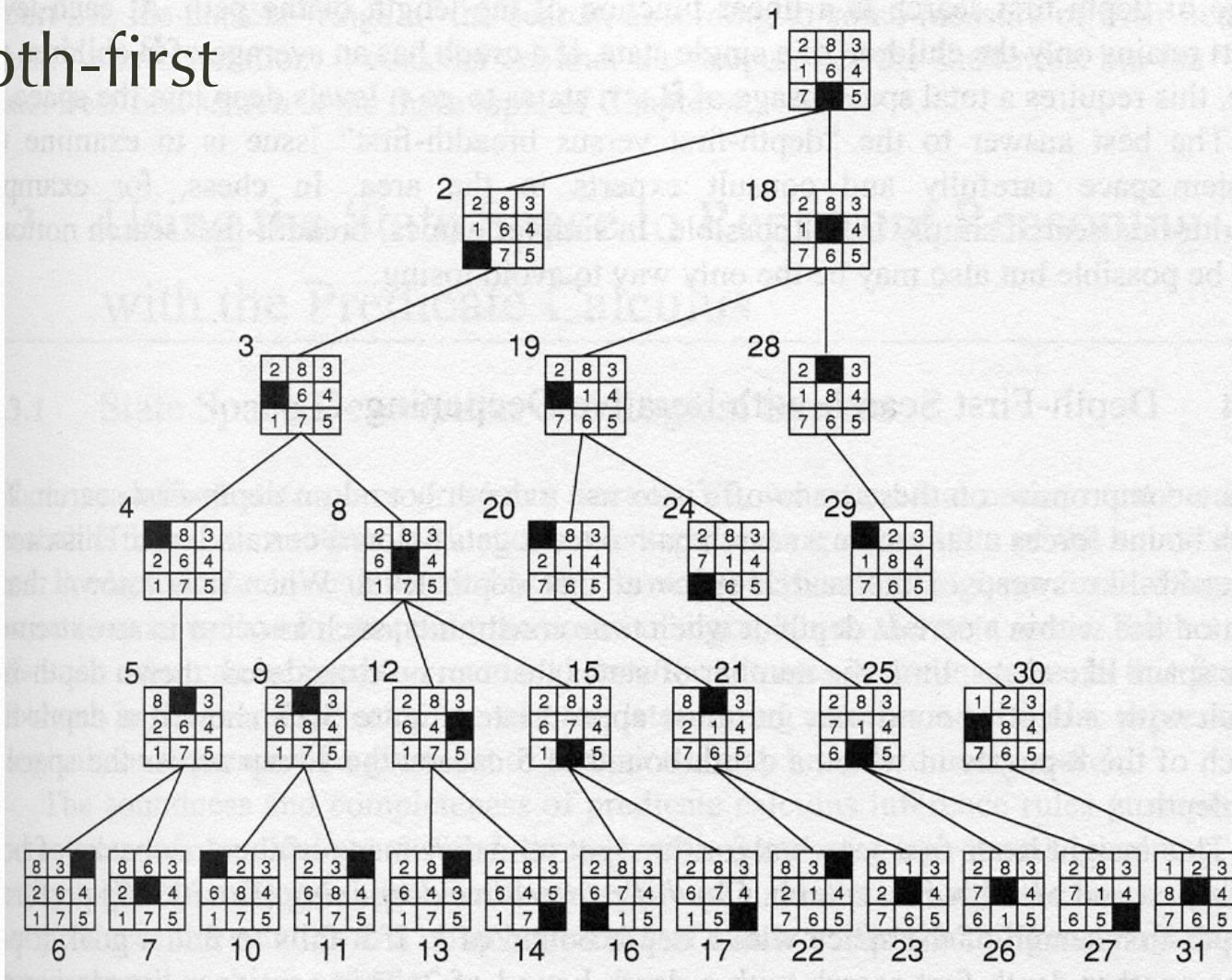
Handwritten notes: "BFS explores all states at distance zero first."

Time:  $O(b^{\text{depth}})$

Space:  $O(b \cdot \text{depth})$



# Depth-first



[Lugar 2002]

Goa

# Depth-first search

- Follows each path to its conclusion
- May not find the shortest state
- Quickly gets deep into problem space
- Good when there are (many) deep solutions
- Space efficient when high branching factor

```
function depth-first-search
  open := [Start]
  done := [ ]
  while open != [ ] do
    X := open.pop()
    if (X = Goal) return success
    else
      generate children states of X
      discard any children in open, done
      for each child C
        open.push(C)
        done.add(X)
  return failure
```

```
function depth-first-search
  open := [Start]
  done := [ ]
  while open != [ ] do
    X := open.pop()
    if (X = Goal) return success
    else
      generate children states of X
      discard any children in open, done
      for each child C
        open.push(C)
        done.add(X)
  return failure
```

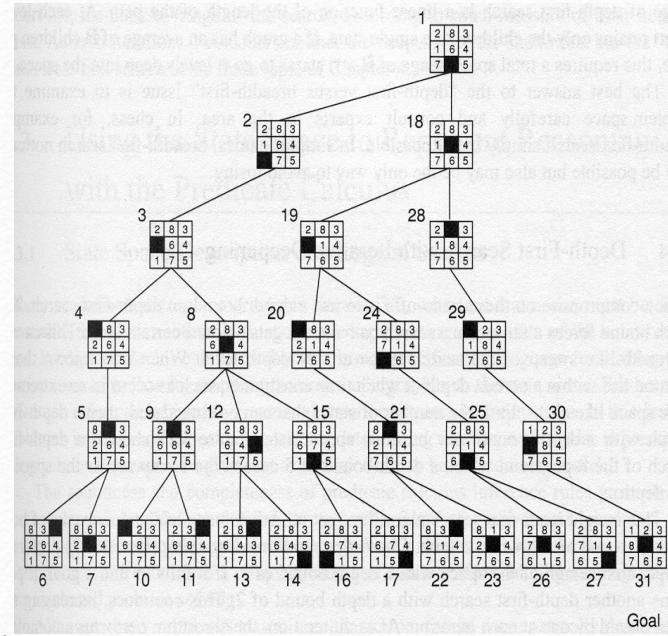
Time:  $O(b^{\text{maxdepth}})$

Space:  $O(b \cdot \text{maxdepth})$

```

function depth-first-search
open := [Start]
done := [ ]
while open != [ ] do
  X := open.pop()
  if (X = Goal) return success
  else
    generate children states of X
    discard any children in open, done
    for each child C
      open.push(C)
      done.add(X)
  return failure

```



Space:  $O(b \cdot \text{maxdepth})$

# Depth-limited search

- Depth first search with depth limit
- Don't add/search nodes deeper than limit
- May not find a solution; but may
  - find shallower solution
  - find solution more quickly

# Depth-limited search

- Depth first search with depth limit
- Don't add/search nodes deeper than limit
- May not find a solution; but may
  - find shallower solution
  - find solution more quickly      Time:  $O(b^{\text{limit}})$   
Space:  $O(b \cdot \text{limit})$

# Iterative deepening

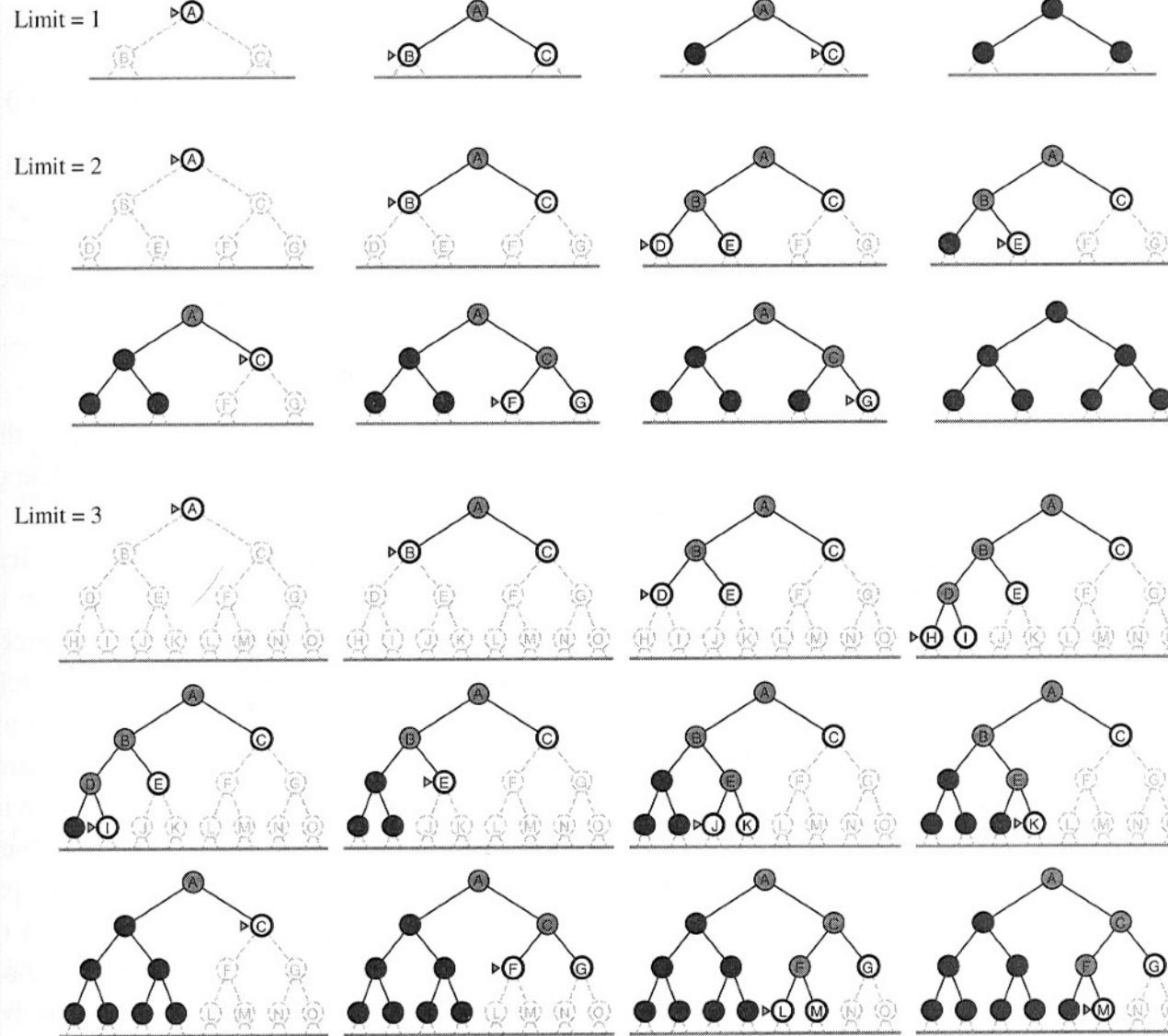
- Depth-first search increasing depth limit
- Combines depth and breadth-first search
- Finds shallowest path
- Preferred method when search space is large and depth of solution is unknown.

# Iterative deepening

- Depth-first search increasing depth limit
- Combines depth and breadth-first search
- Finds shallowest path
- Preferred method when search space is large and depth of solution is unknown.

Time:  $O(b^{\text{limit}})$

Space:  $O(b \cdot \text{limit})$



# Two Assumptions

# Two Assumptions

Working from start state to goal state.  
That is, in a **forward** direction.

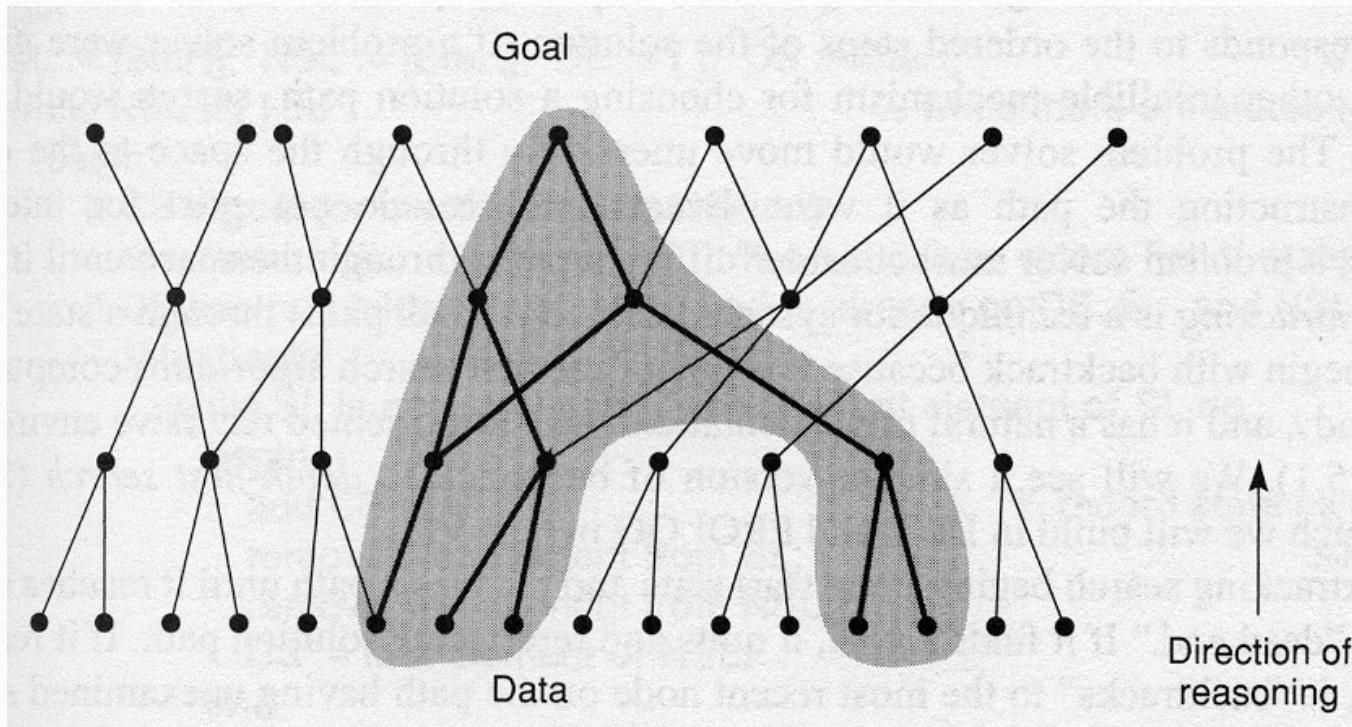
# Two Assumptions

Working from start state to goal state.

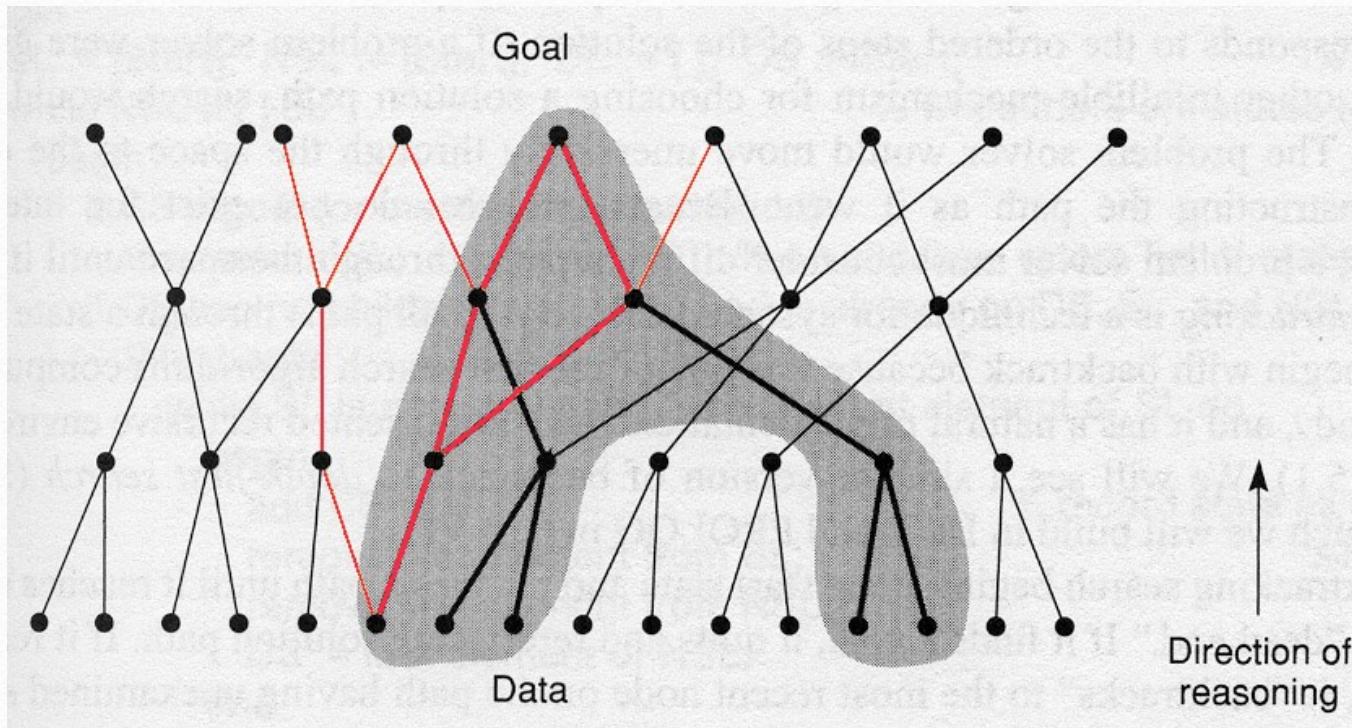
That is, in a **forward** direction.

Searching **blind**, e.g. exhaustively.

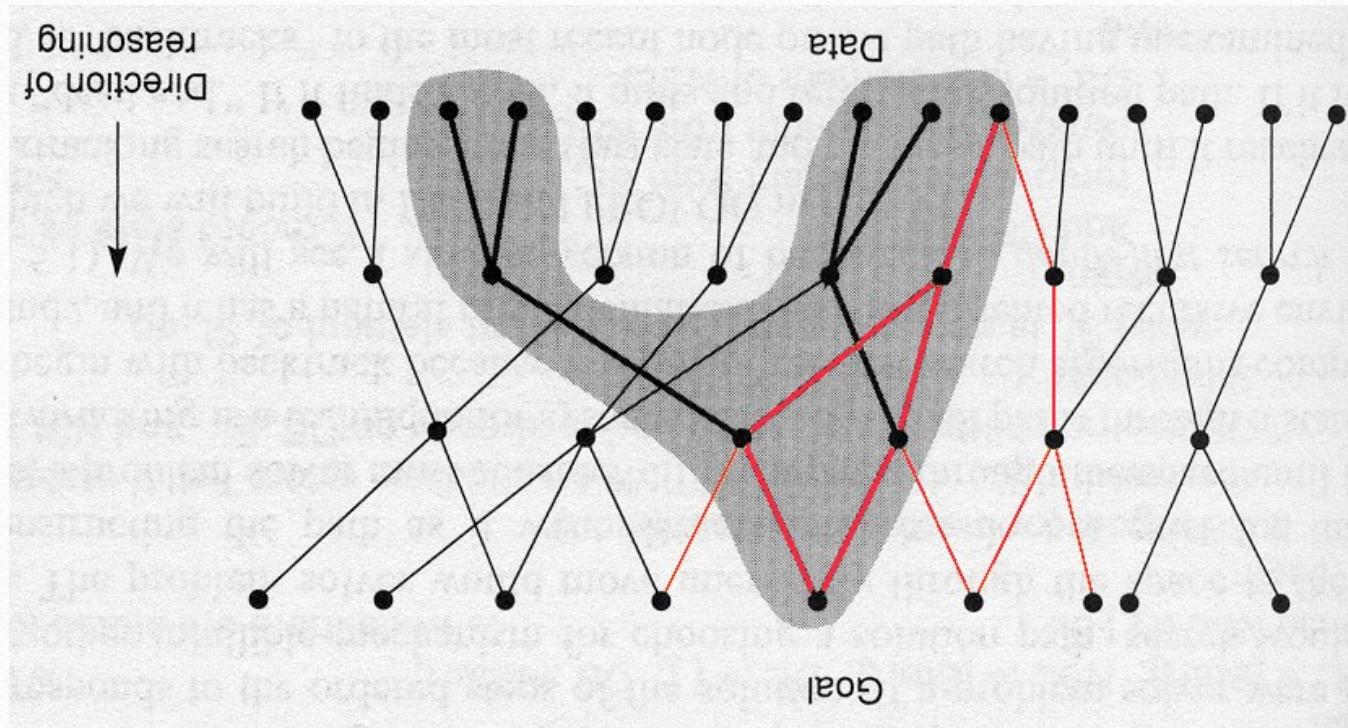
# Forward (data-driven)



# Forward (data-driven)



# Forward (data-driven)

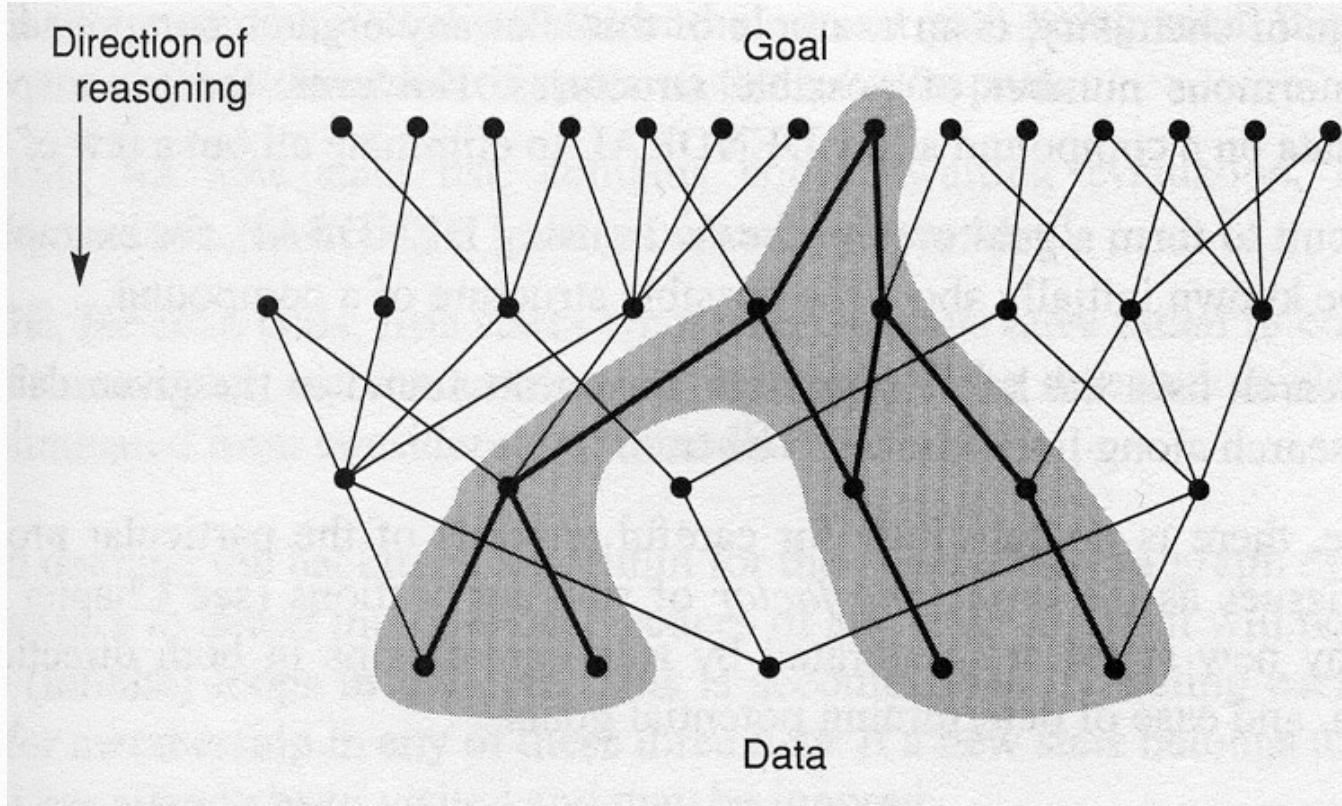


# Forward (data-driven)

Necessary when you can identify the goal but can't state it.

Preferred if there are a large number of goals, and few branches

# Backward (goal-directed)

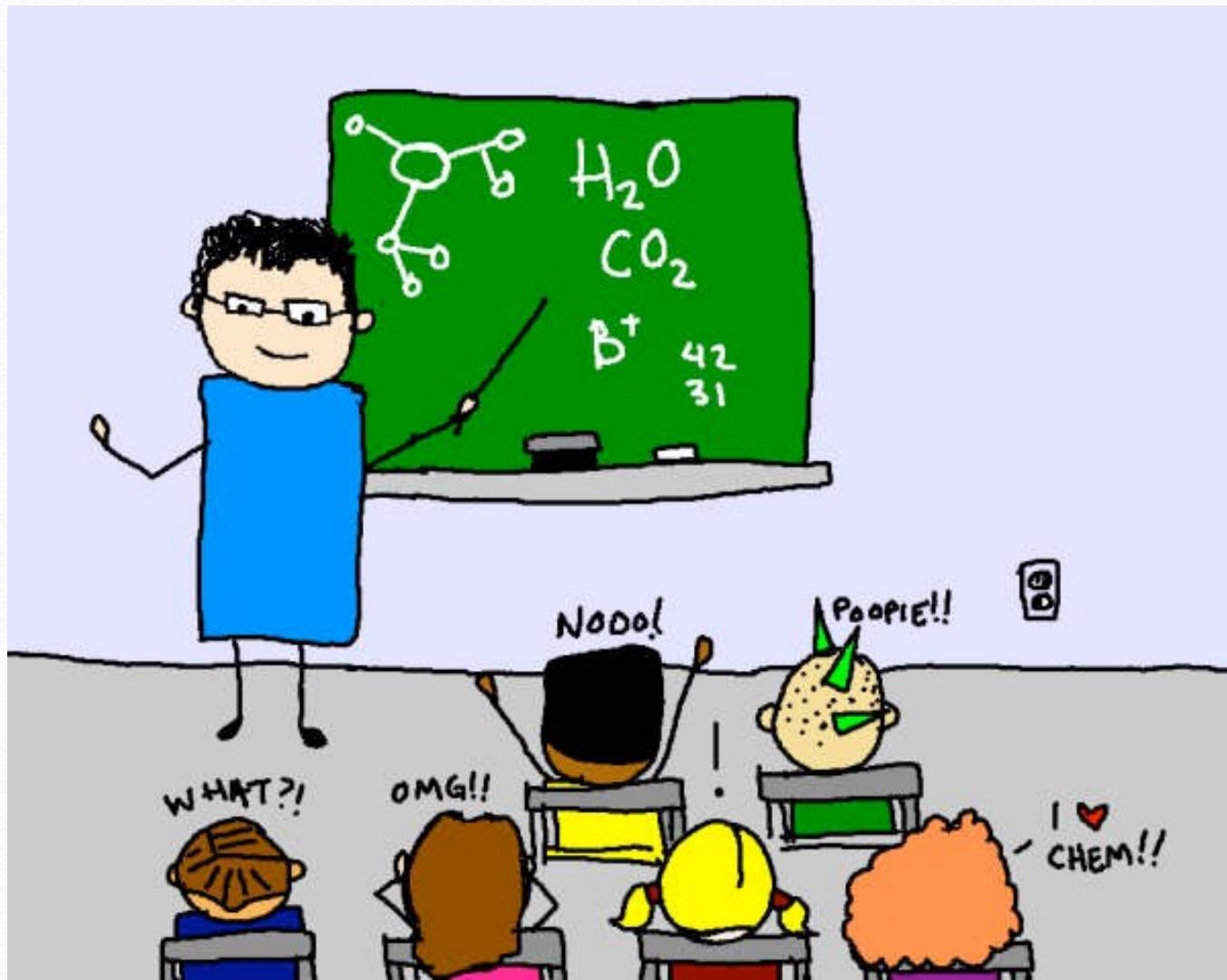


# Backward (goal-directed)

Preferred if there is a clearly identifiable goal,  
and forward search has many branches

Preferred if you have potential for  
gaining more information

e.g. in Medicine, can perform diagnostic  
tests



*Synthia: A Synthetic Design Program*

*M. Anthony Kapolka III*

**CH 500**

*Spring 1987*

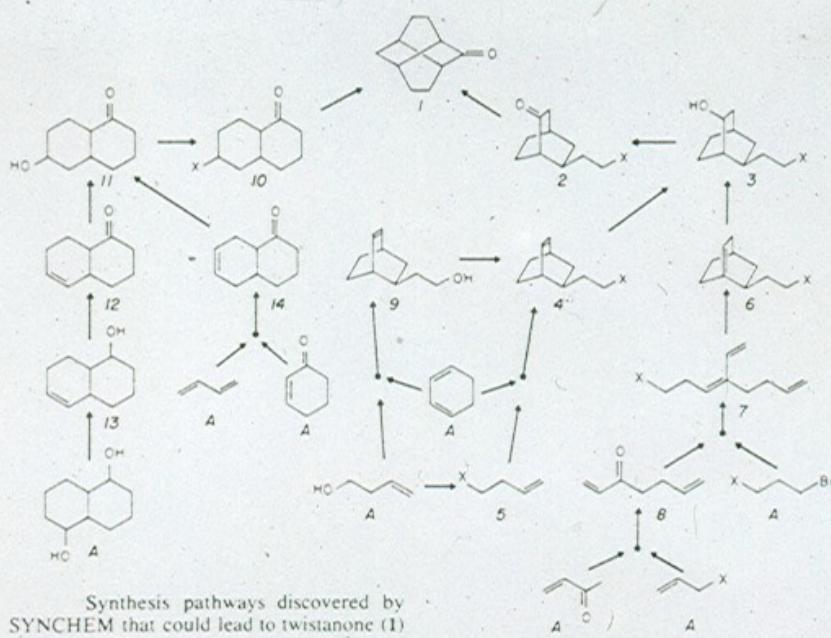


Your instructor as Chemist.

## Computer Synthetic Design

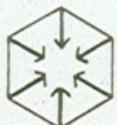
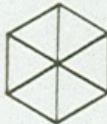
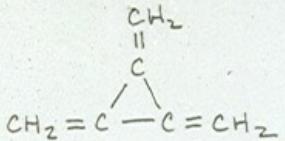
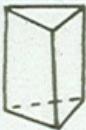
1967-First Work in Synthetic Design Logic

Corey et. al.	SUNY Stonybrook	Hendrickson et. al.
1969	1973	1971
OCSS (1969)	SYNCHEM (1973)	
Fortran	PL/1	
PDP-1	IBM 370/155	
Interactive	Executive	
LHASA (current)		SYNGEN (current)
Fortran		Fortran
VAX-11		PDP 11/23
Interactive		Executive



Synthesis pathways discovered by SYNCHEM that could lead to twistanone (1) from available starting materials. Reaction types used: alkylation alpha to a ketone ( $1 \rightarrow 2, 1 \rightarrow 10, 8 \rightarrow A + A$ ); oxidation of a secondary alcohol ( $2 \rightarrow 3, 12 \rightarrow 13$ ); hydration of an alkene ( $3 \rightarrow 4, 3 \rightarrow 6, 11 \rightarrow 12, 13 \rightarrow A$ ); Diels-Alder reaction ( $4 \rightarrow 5 + A, 6 \rightarrow 7, 9 \rightarrow A + A, 14 \rightarrow A + A$ ); Wittig reaction ( $7 \rightarrow 8 + A$ ); and replacement of an alcohol by a better leaving group ( $10 \rightarrow 11, 4 \rightarrow 9, 5 \rightarrow A$ ). All compounds labeled A were found by SYNCHEM on its list of available compounds.

## Representation of Molecules



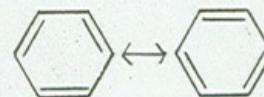
$\phi$



bicarburet  
of Hydrogen



$\text{C}_6\text{H}_6$



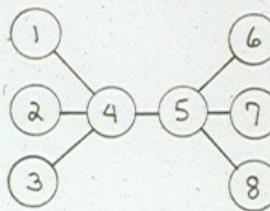
BENZENE

Data Structures

Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	0	0	1	0	0	0	0
2	0	0	0	1	0	0	0	0
3	0	0	0	1	0	0	0	0
4	1	1	1	0	1	0	0	0
5	0	0	0	1	0	1	1	1
6	0	0	0	0	1	0	0	0
7	0	0	0	0	1	0	0	0
8	0	0	0	0	0	1	0	0

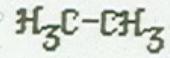
Unidigraph

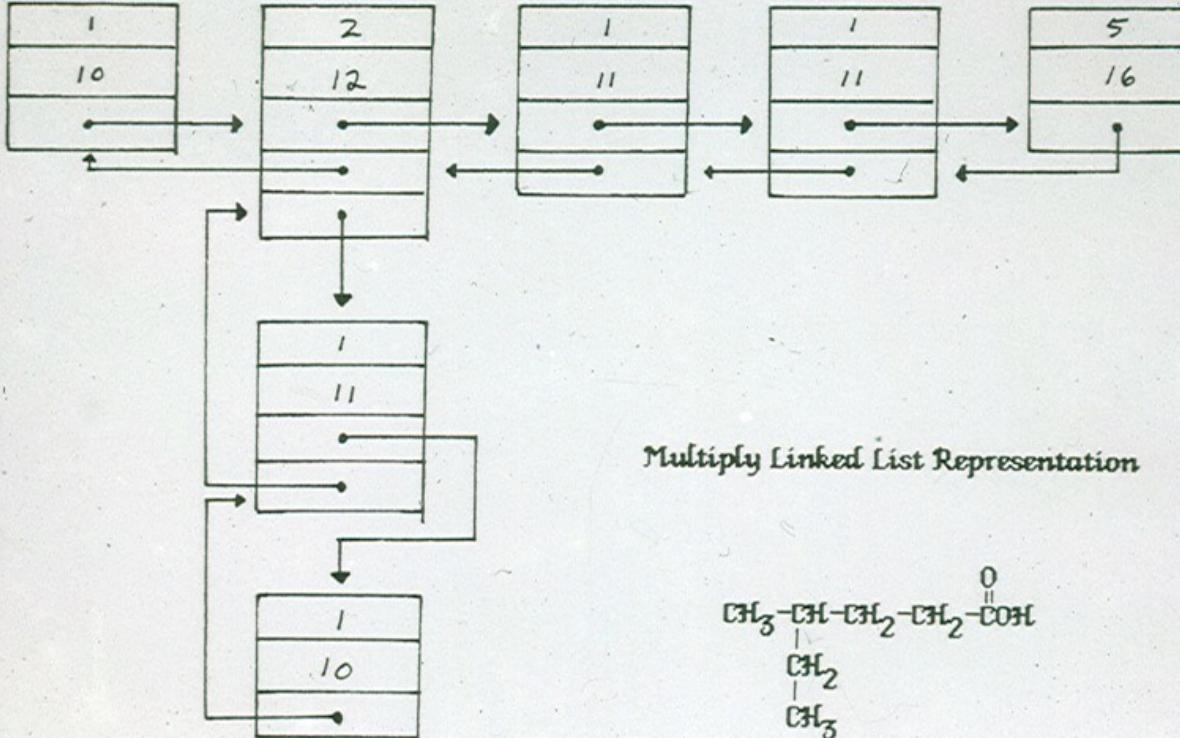


Vertex Code

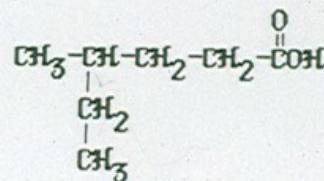
1	H	5	C
2	H	6	H
3	H	7	H
4	C	8	H

Molecule

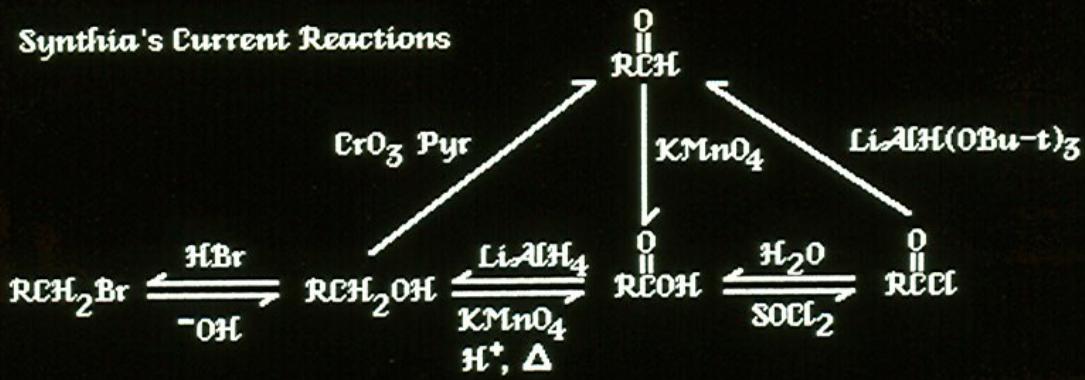


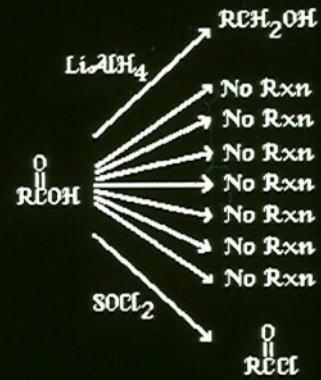


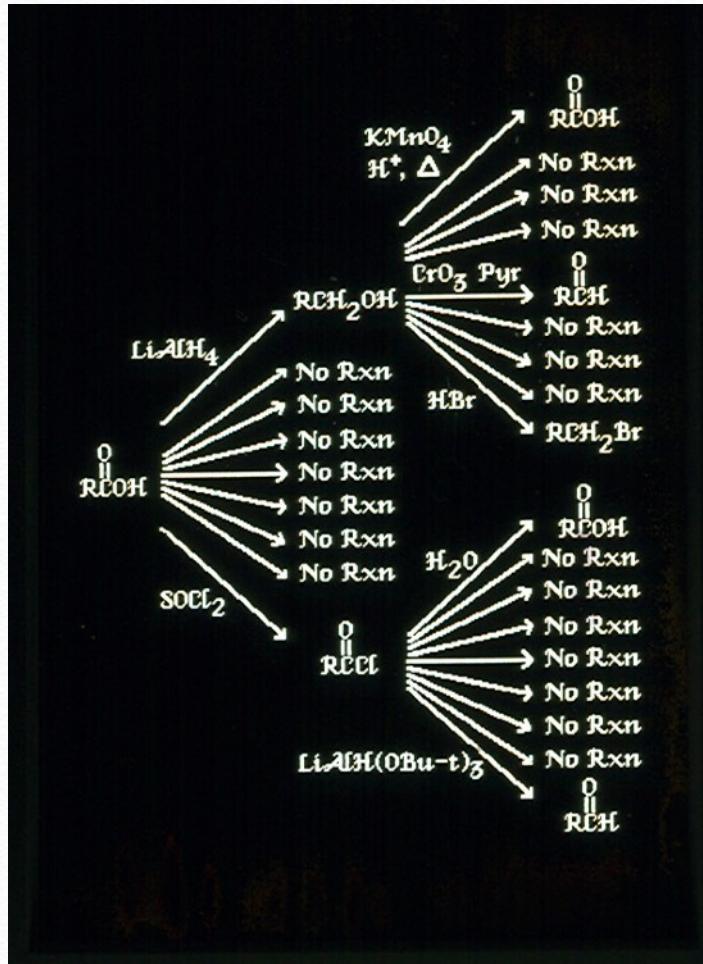
Multiply Linked List Representation

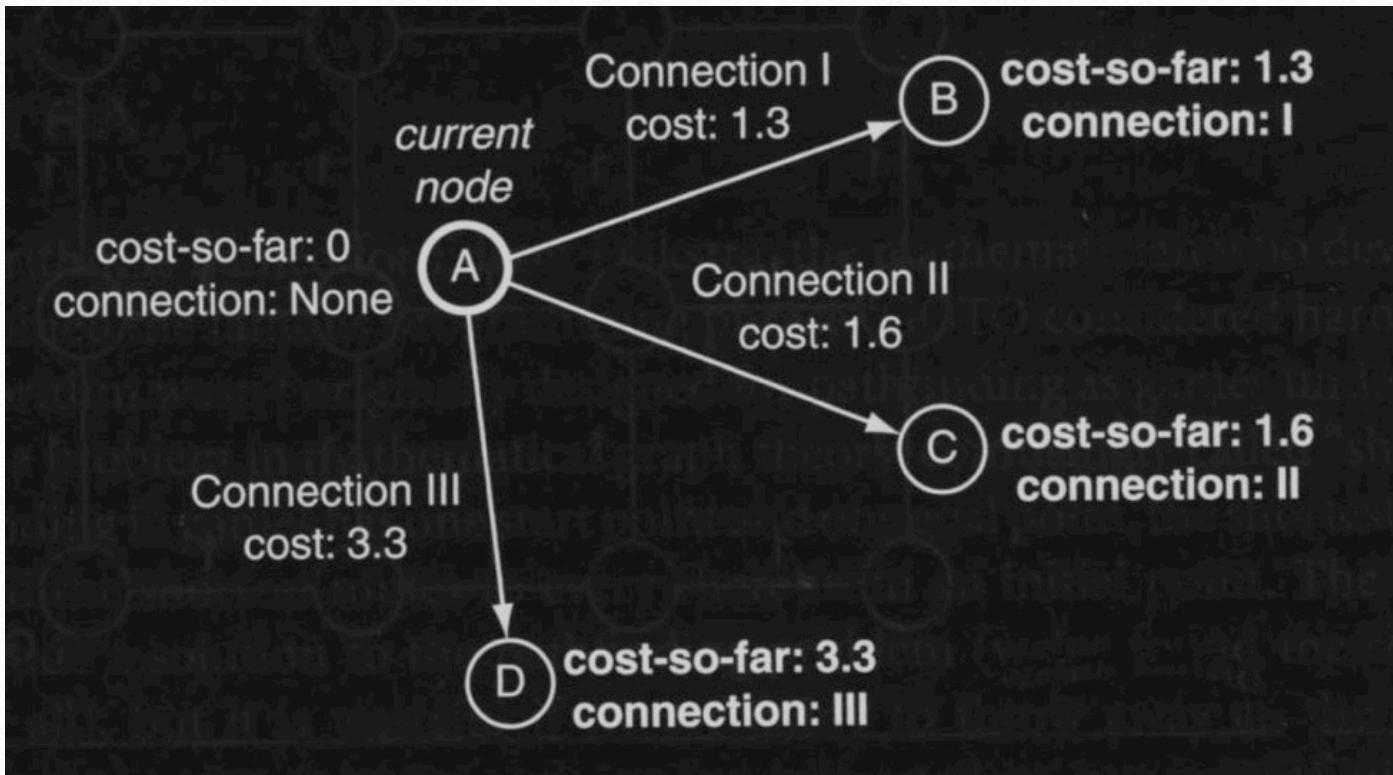


Synthia's Current Reactions

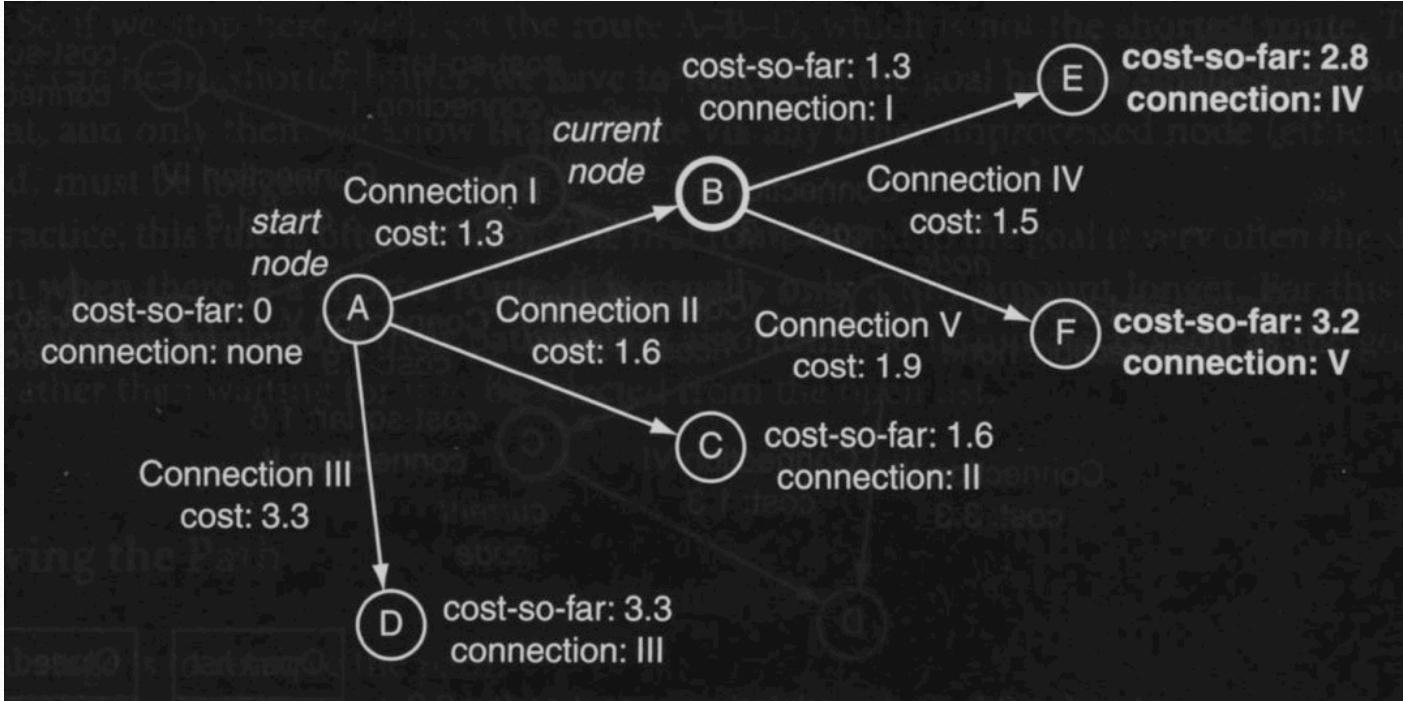




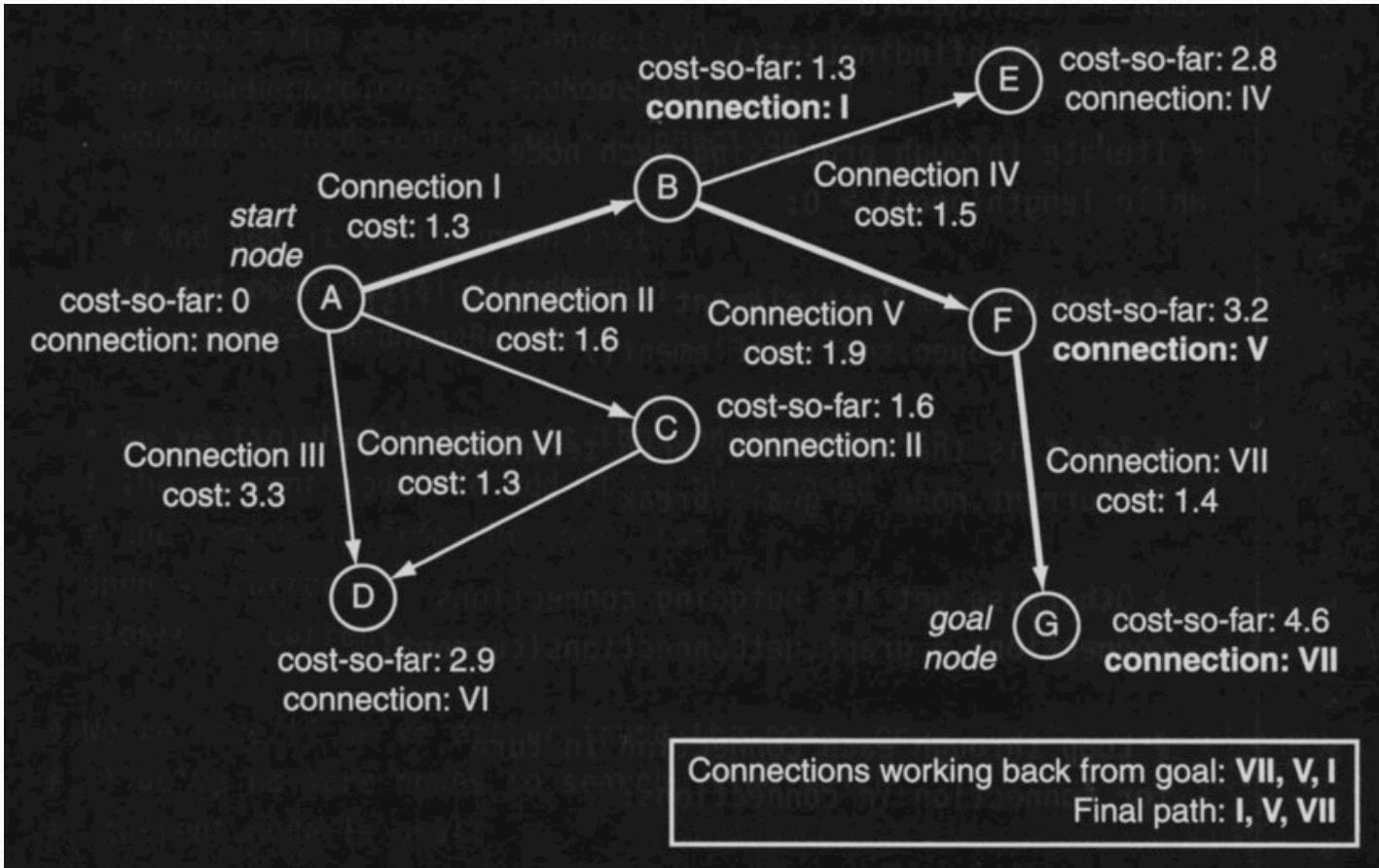




AI for Games, Millington & Funge: Figure 4.8



AI for Games, Millington & Funge: Figure 4.9



AI for Games, Millington & Funge: Figure 4.11

Anthony's Molecular Building Set

$\text{CH}_3$	$\text{CH}_2$	$\text{CH}$	$\text{OH}$	$\text{C}$	$\text{CO}$	$\text{COH}$	$\text{CH}$	$(\text{CH}_3)_3\text{C}$	$(\text{CH}_3)_2\text{CH}$	$\text{Cl}$	$\text{Br}$	$\text{Ph}$
E	B	C	D	E	F	G	H	I	J	K	L	M

Reactant Molecule:

Product Molecule:

Type letter to select, arrow keys to move, or ? for online help

## Anthony's Molecular Building Set

$\text{CH}_3$	$\text{CH}_2$	$\text{CH}$	$\text{OH}$	$\text{O}$	$\text{CO}$	$\text{COH}$	$\text{CH}$	$(\text{CH}_3)_3\text{C}$	$(\text{CH}_3)_2\text{CH}$	$\text{Cl}$	$\text{Br}$	$\text{Fn}$
A	B	C	D	E	F	G	H	I	J	K	L	M

Reactant Molecule

Reactant Complete



I



Product Molecule

Product Complete



Prepare for synthetic design logic!

Type letter to select, arrow keys to move, or ? for online help

## Synthia's Synthetic Path



Reactant Molecule

Reactant Complete



Product Molecule

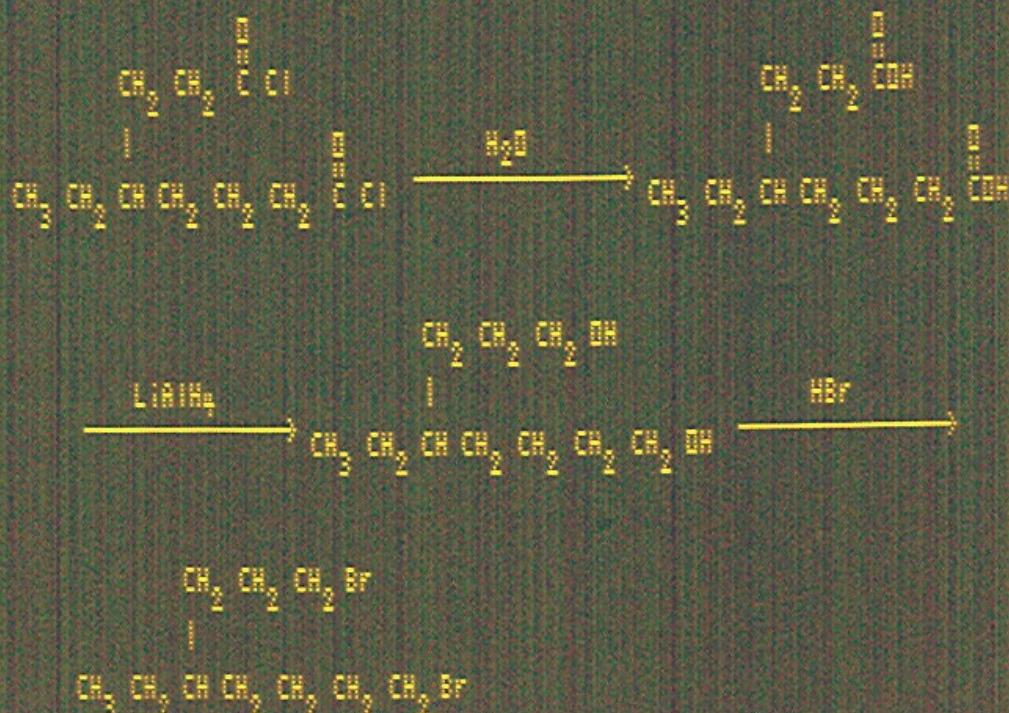
Product Complete

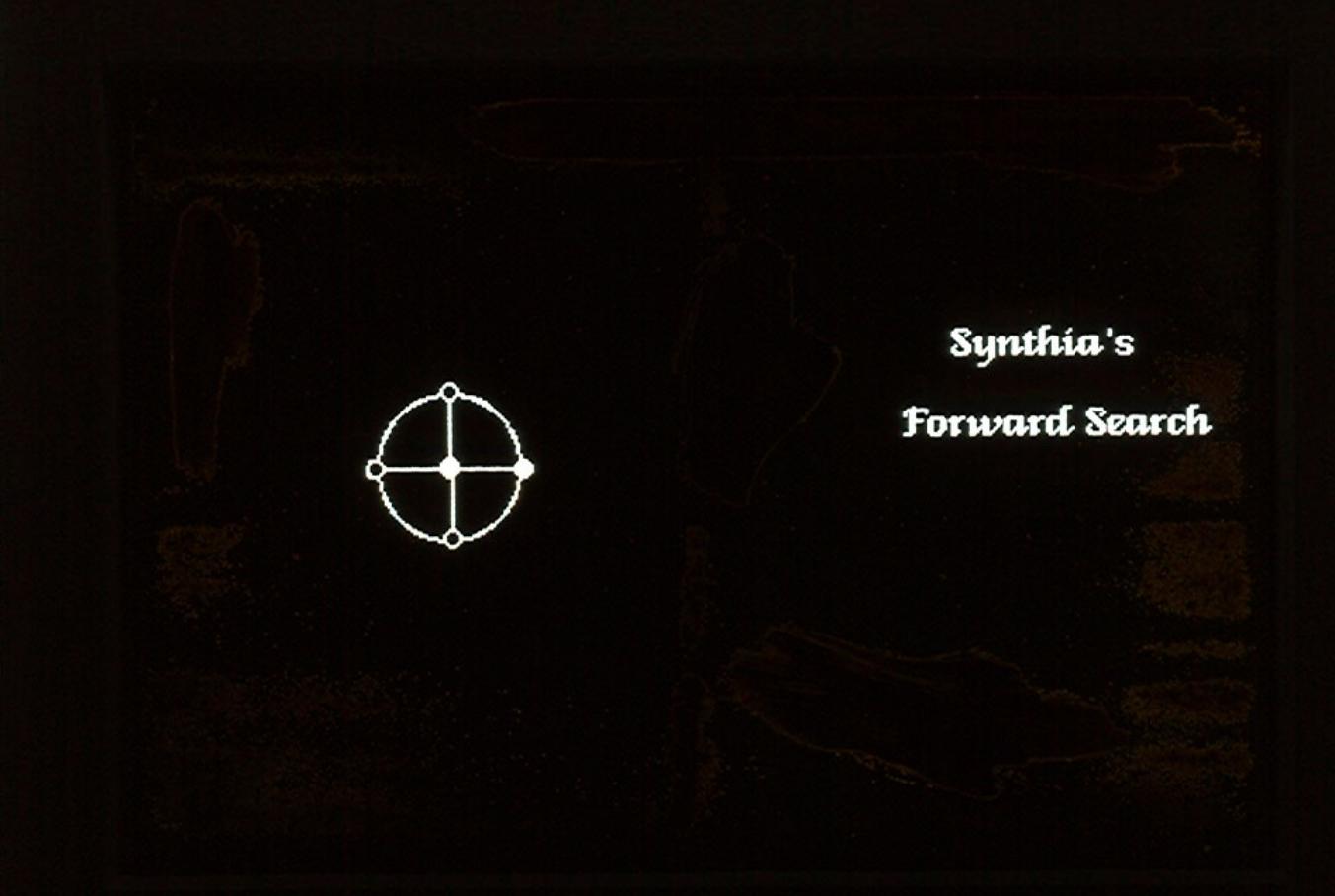


Prepare for synthetic design logic!

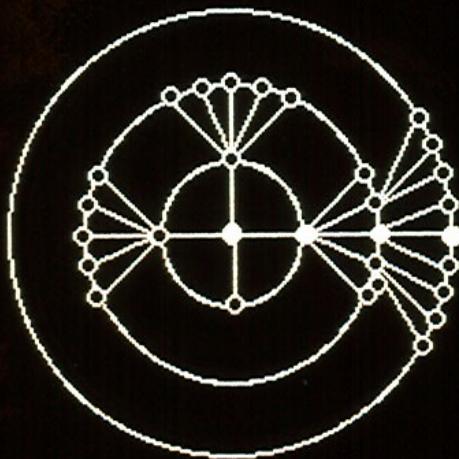
Type letter to select, arrow keys to move, or ?

### Synthia's Synthetic Path

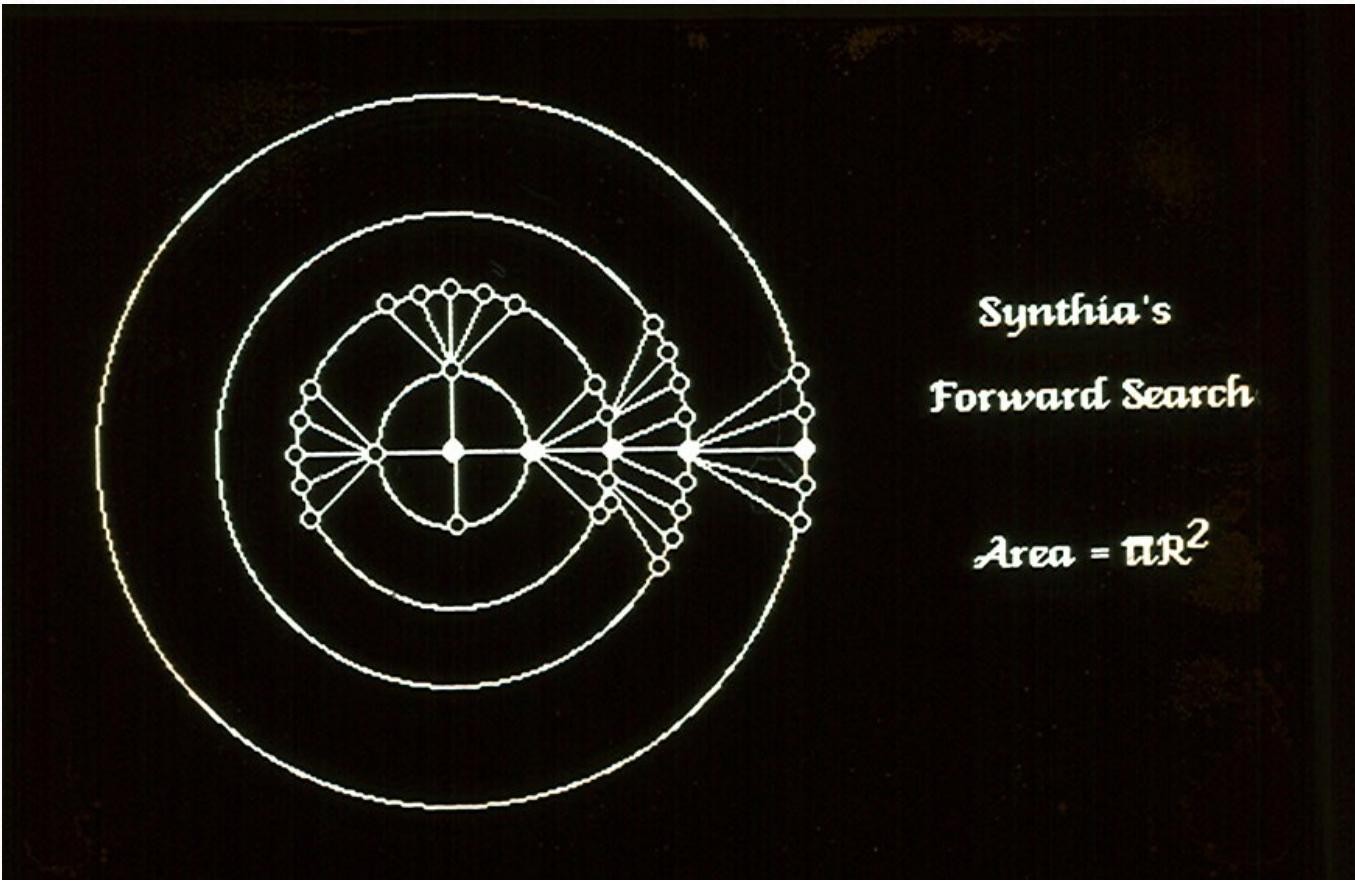




**Synthia's  
Forward Search**



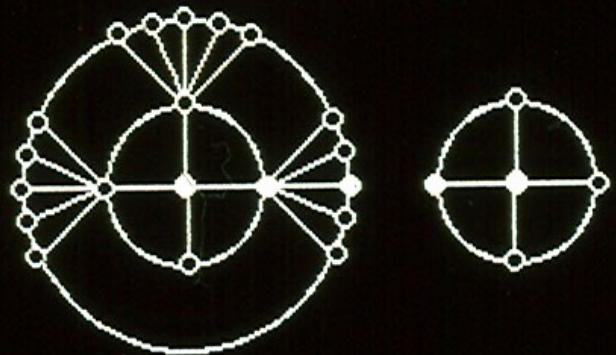
*Synthia's  
Forward Search*



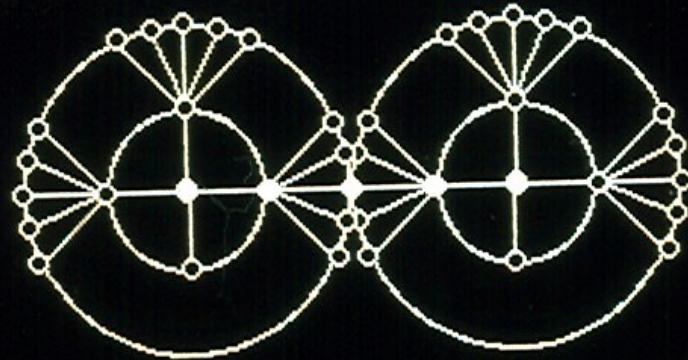
# Dijkstra's Algorithm



## Dijkstra's Algorithm



## Dijkstra's Algorithm

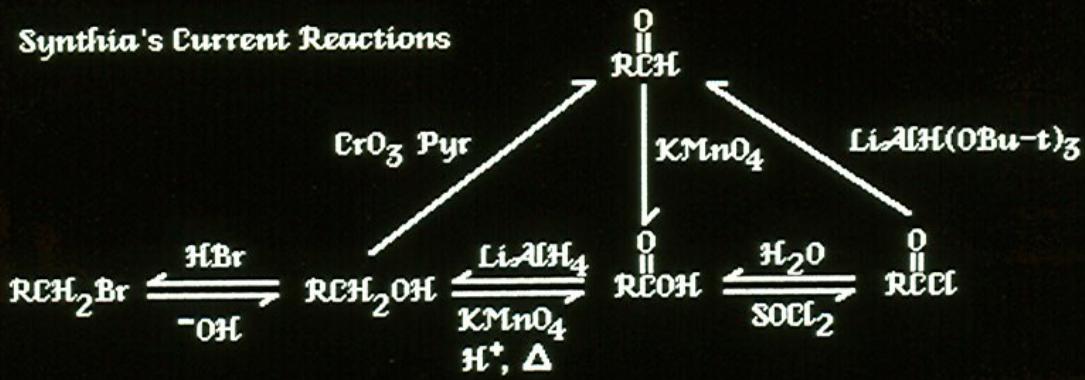


$$\begin{aligned} \text{Area} &= 2\pi(1/2 R)^2 \\ &= 1/2 \pi R^2 \end{aligned}$$

## Synthia's Synthetic Path



Synthia's Current Reactions



## Synthia's Synthetic Path

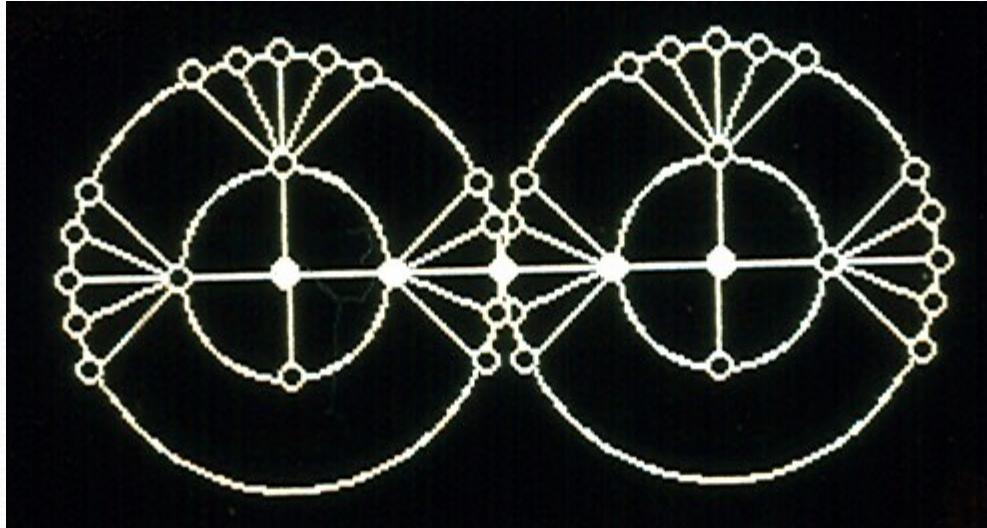


### Synthia's Synthetic Path

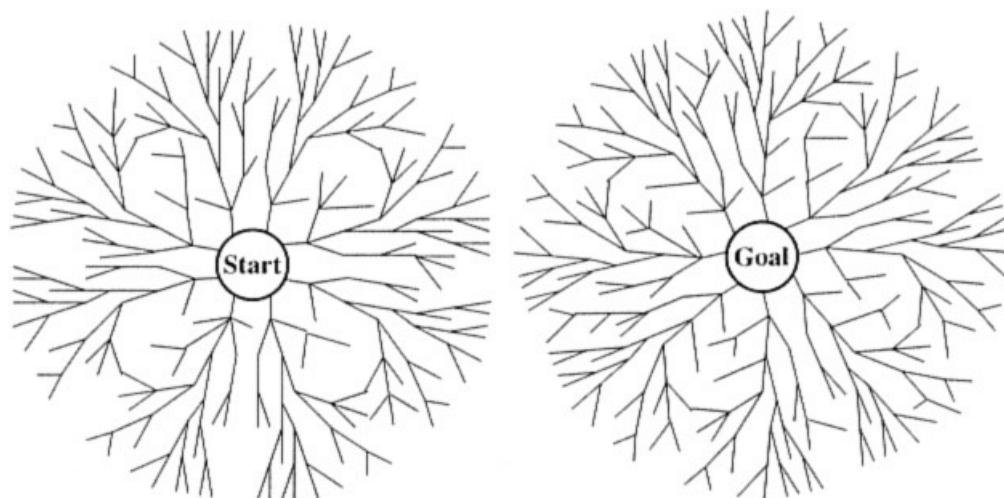


## Synthia's Synthetic Path

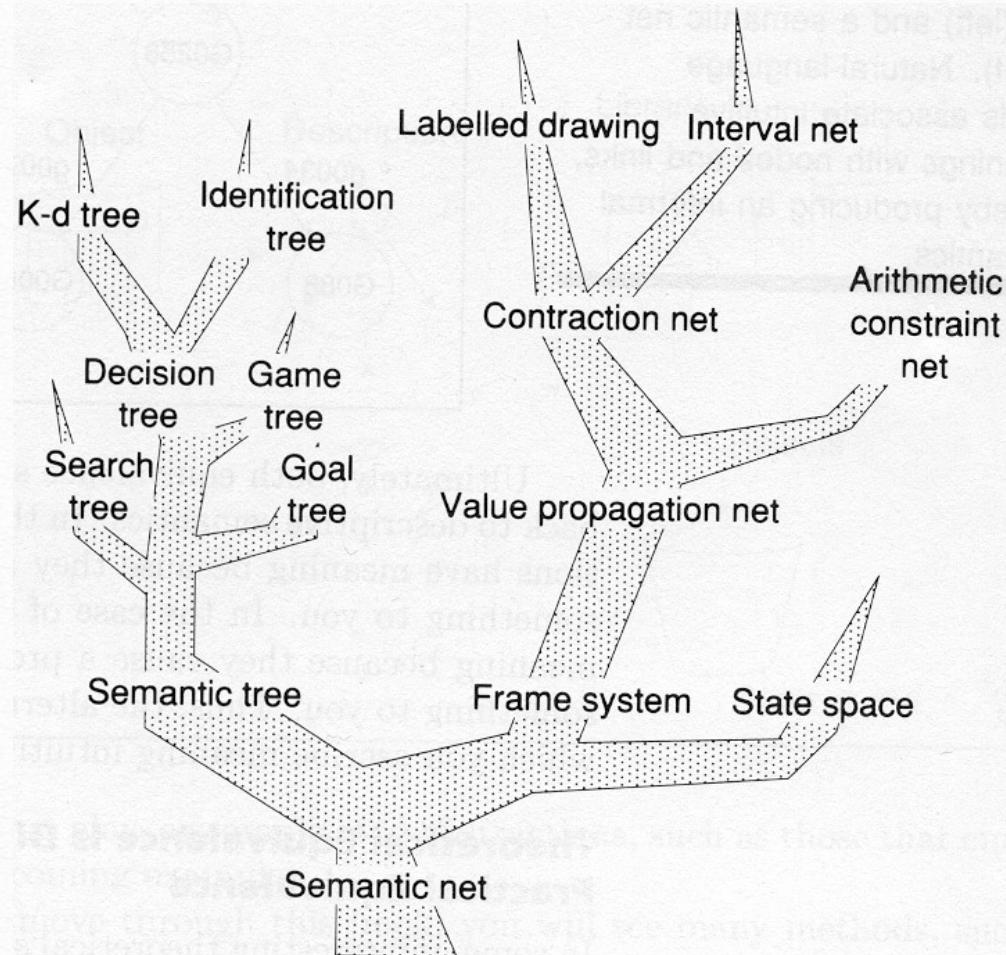


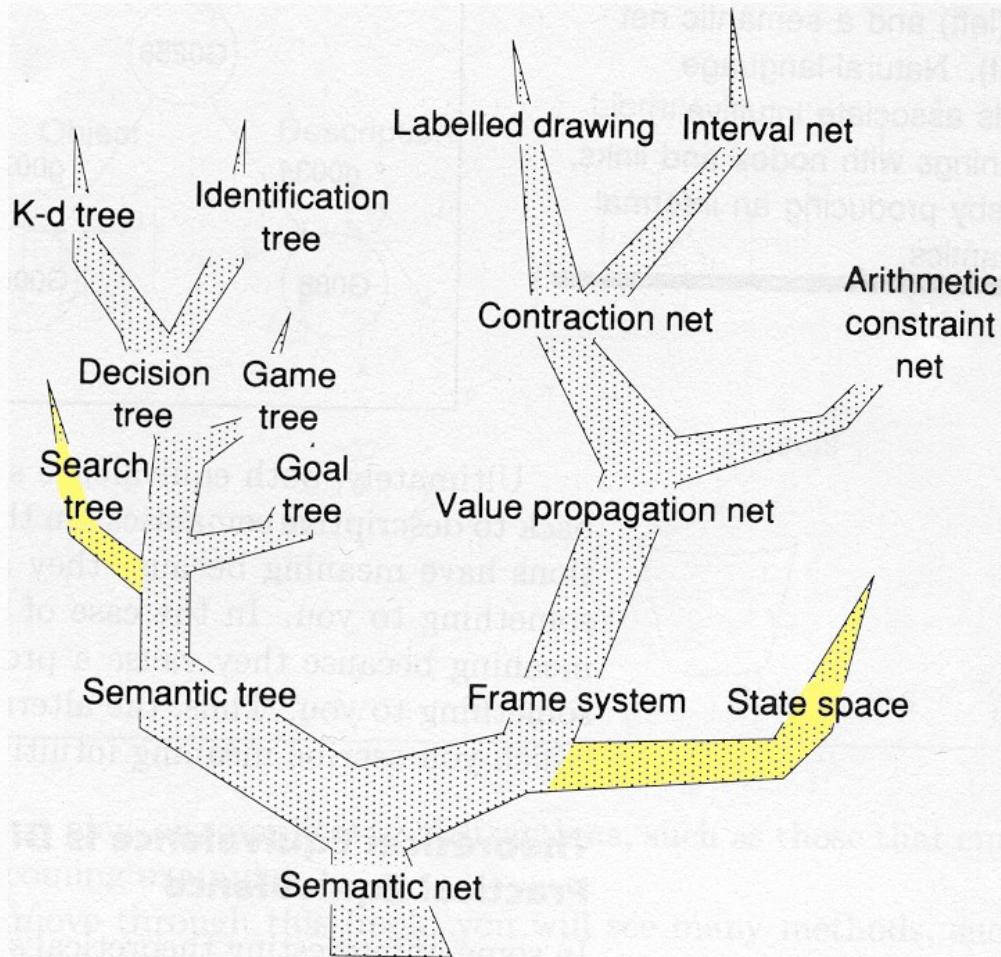


Time:  $O(b^{d/2})$   
Space:  $O(b^{d/2})$



[Russell & Norvig 2003]





ANY  
QUESTIONS  
??

# Study Questions

Solve the Missionaries and Cannibals problem.

What is the major difference between implementations of breadth-first and depth-first search?

Solve the Water Jug problem by completing the graph until the goal is reached.