

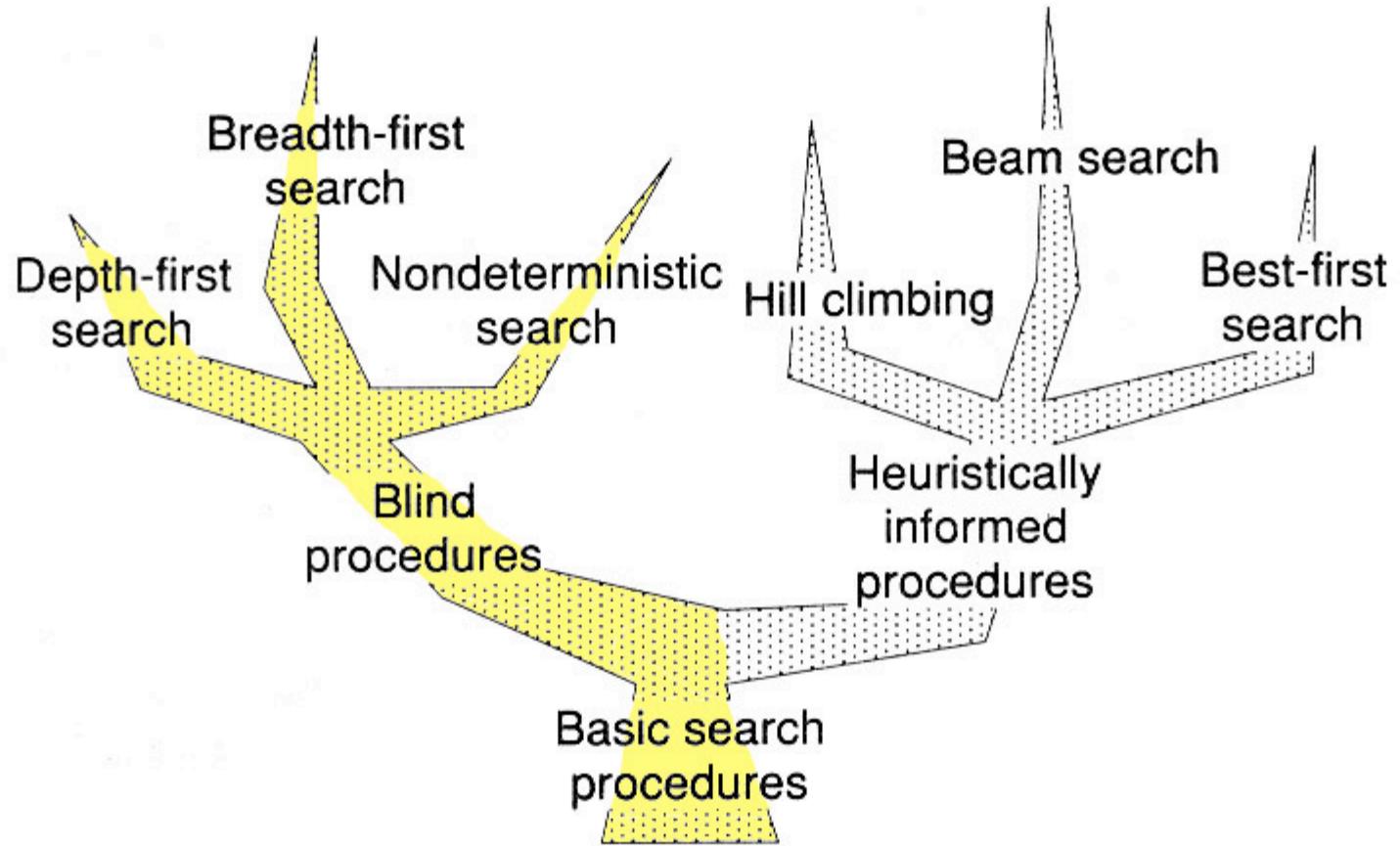
Informed Search

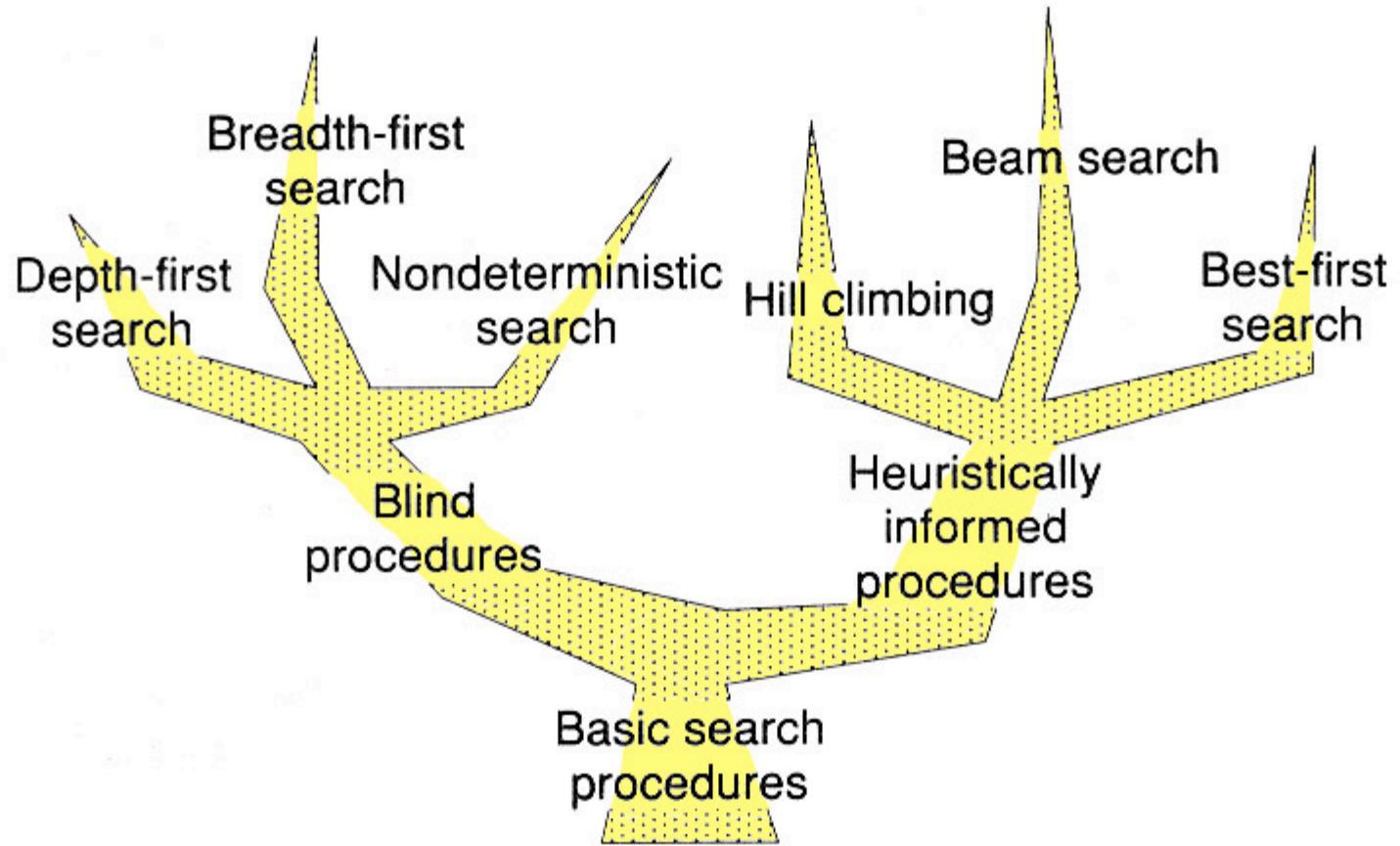
M. Anthony Kapolka III
Wilkes University
CS 340 - Fall 2019

The task that a symbol system is faced with when it is presented with a problem and a problem space, is to use its limited processing resources to generate possible solutions until it finds one that satisfies the problem. If the symbol system could control the order in which potential solutions were generated it would exhibit intelligence if it generated them in an order so actual solutions had a high likelihood of appearing early.

Intelligence for a system with limited processing resources consists in making wise choices of what to do next...

a paraphrase from Newell and Simon's 1976 Turing Award Lecture





\eupiɔKw



Heuristic

from the greek `ευπλοκω meaning to find
a rule of thumb, a guideline
rather than a strict algorithm.

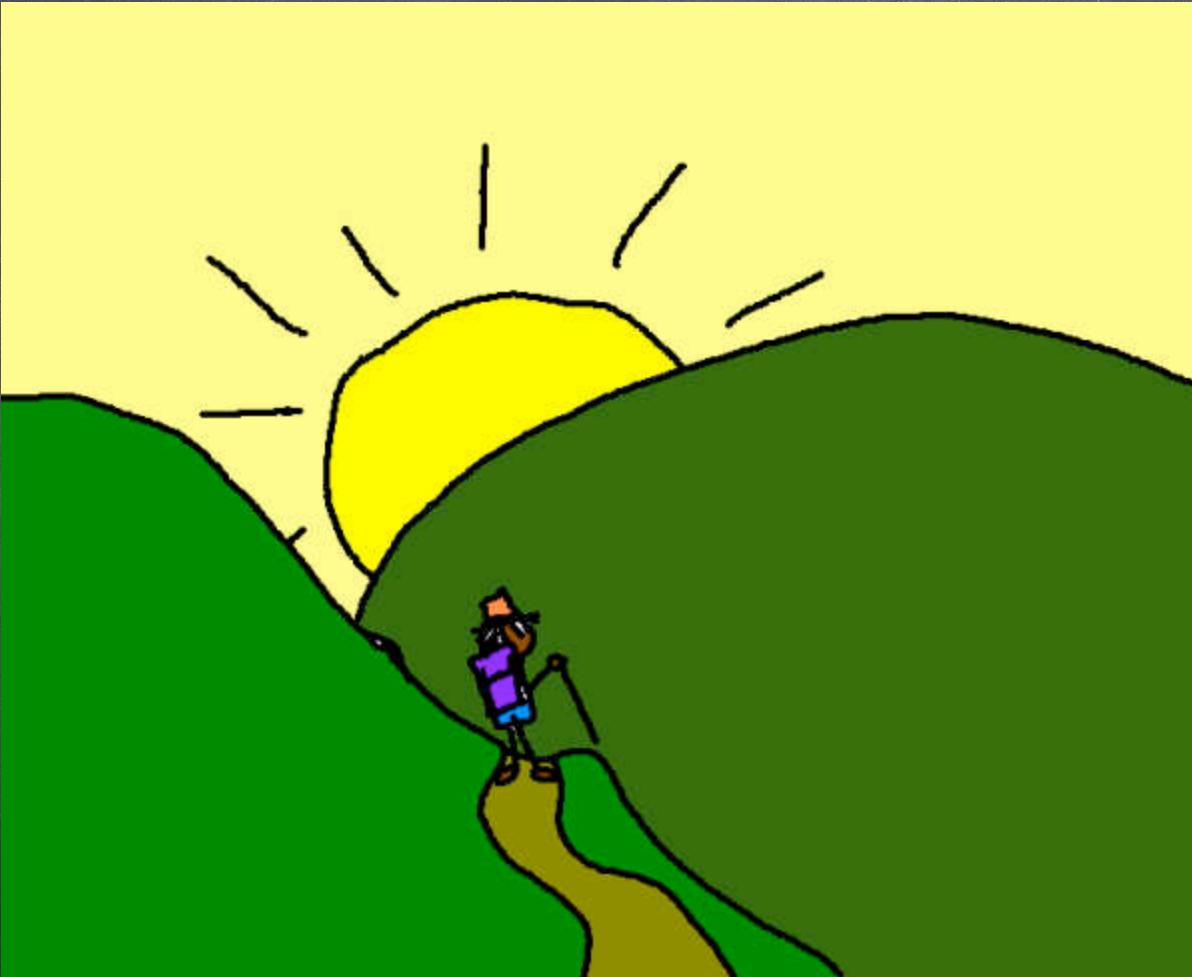
- ⌚ control the center of the chess board.
- ⌚ more expensive beer tastes better.

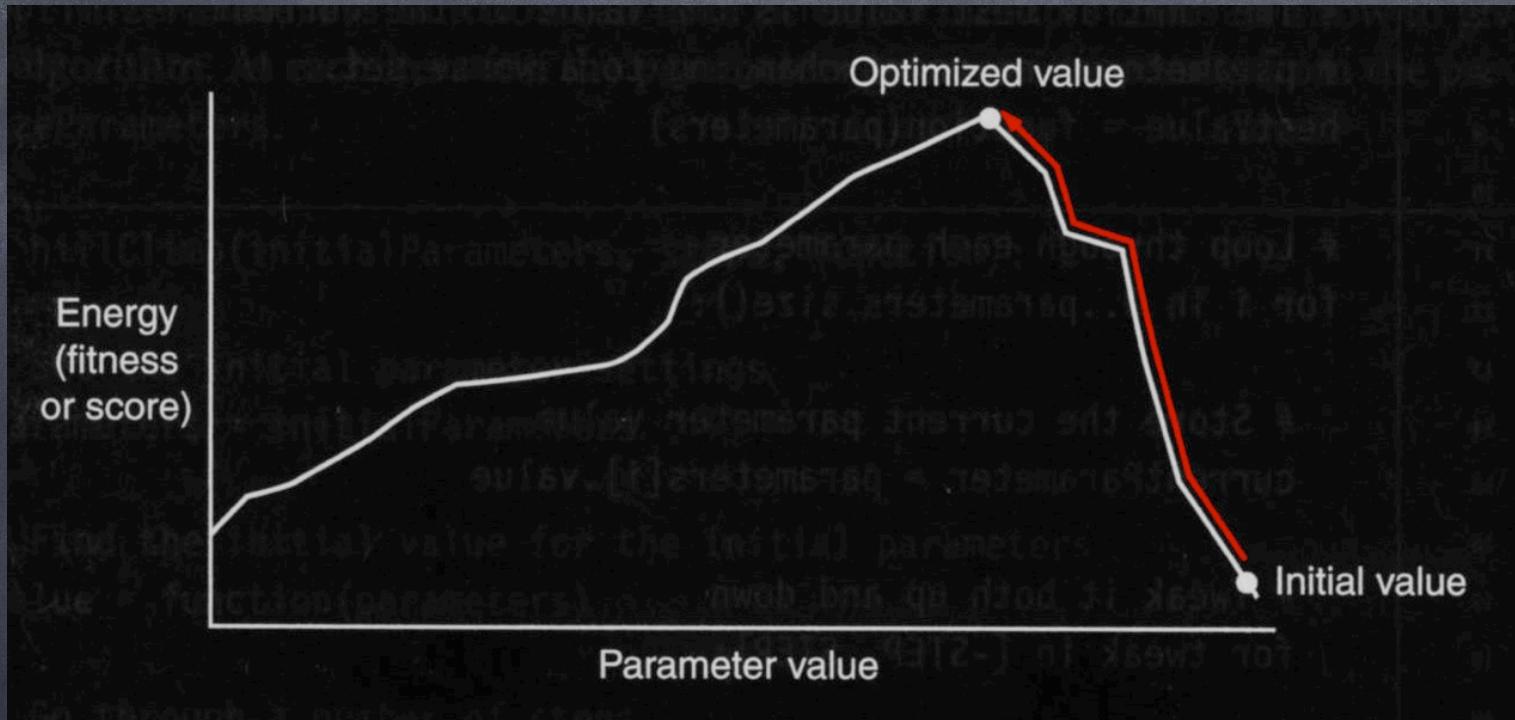
Use heuristics if...

- no exact solution exists
- finding exact solution too expensive

heuristics are fallible

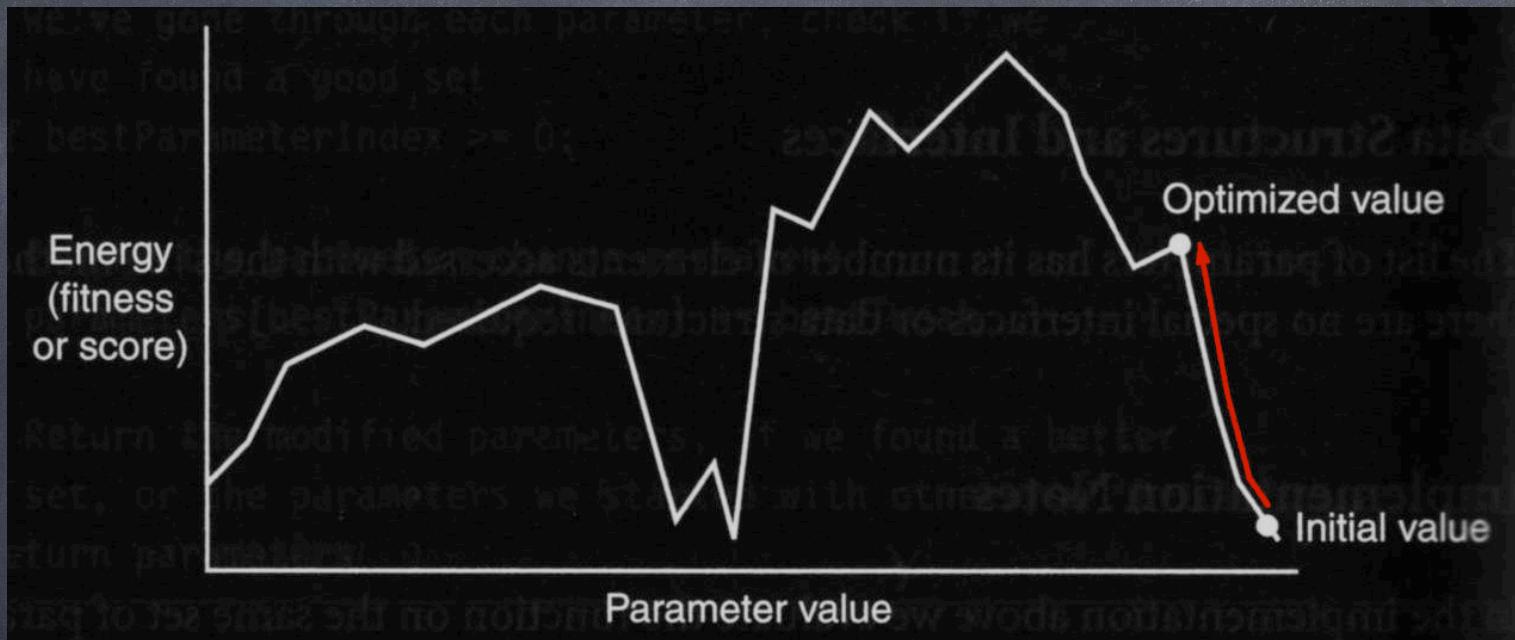
Hill Climbing



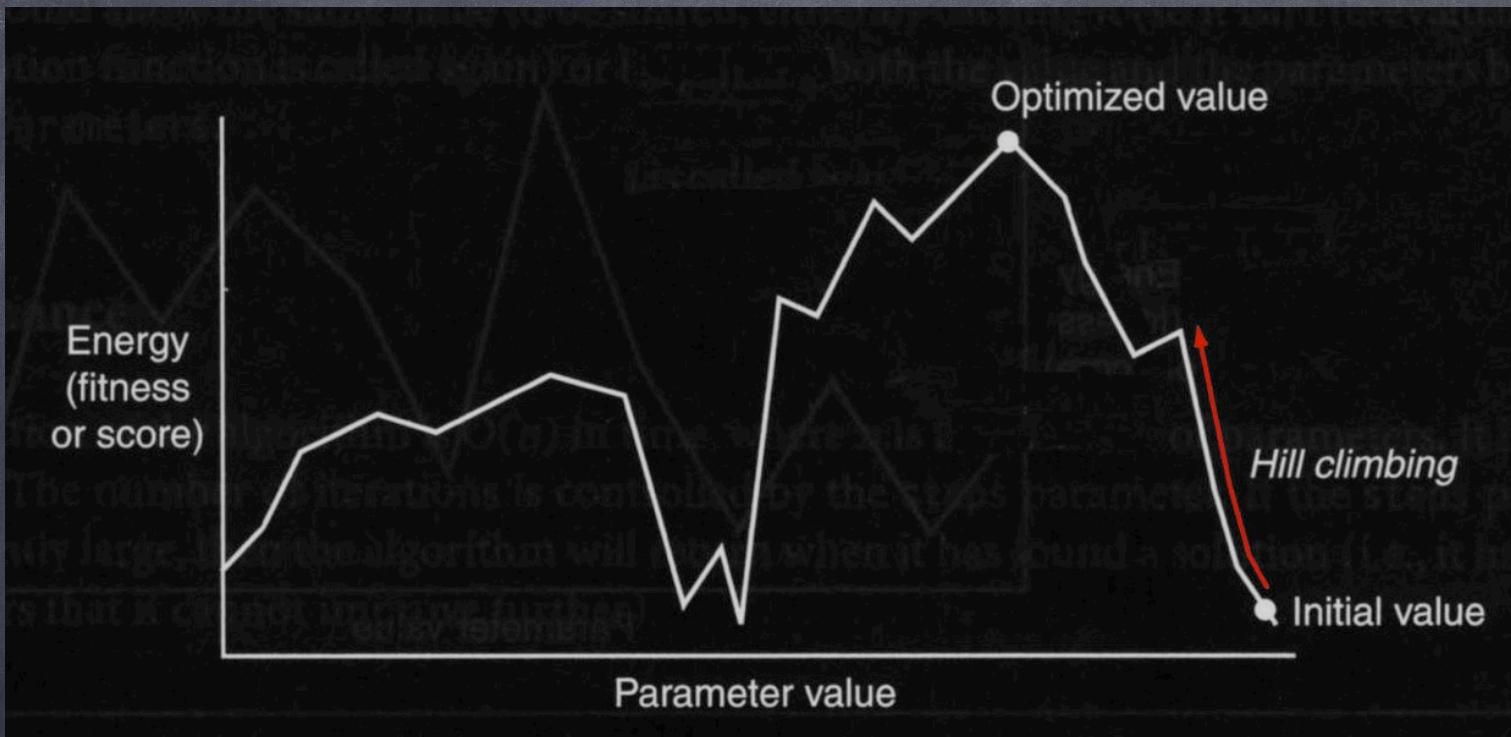


AI for Games, Millington & Funge: Figure 7.2

Sub-optimal Results

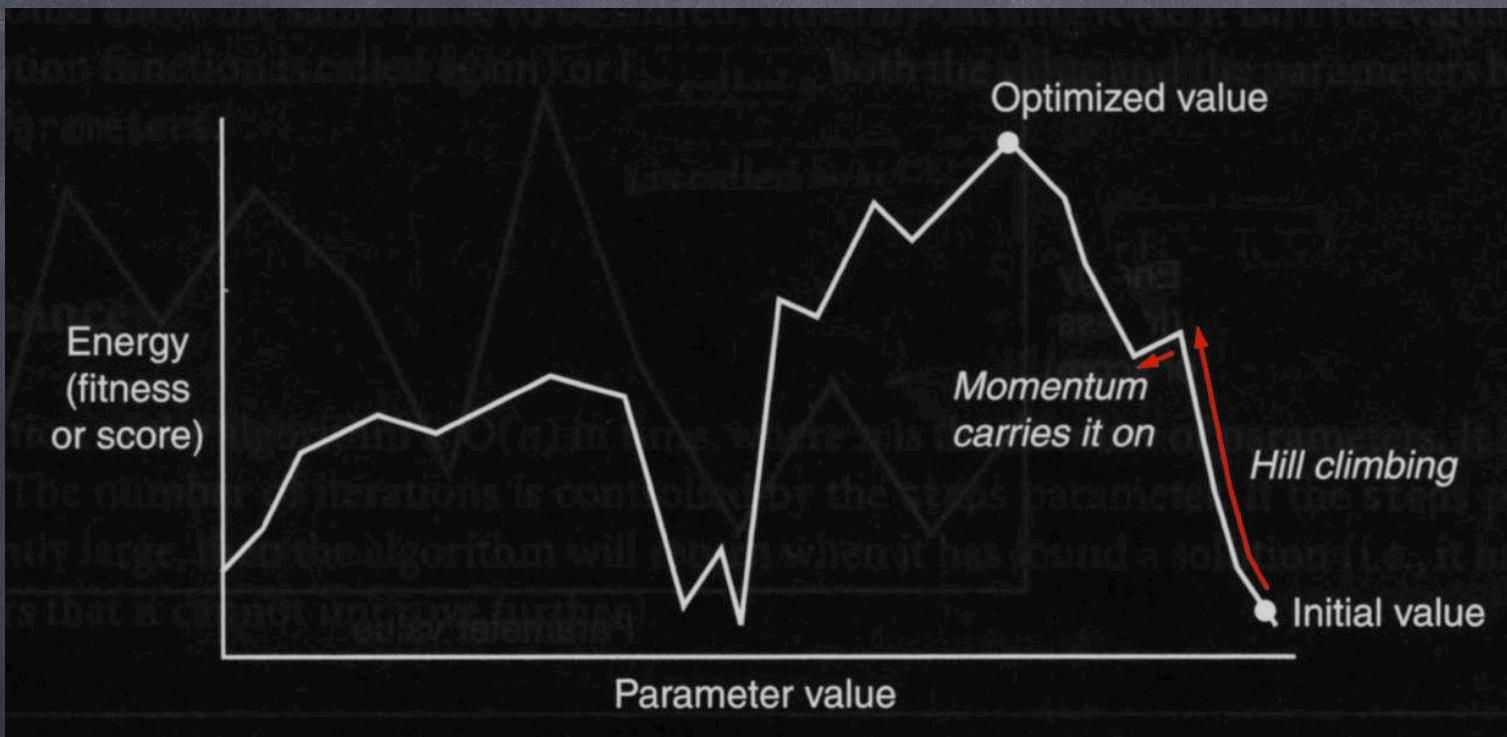


AI for Games, Millington & Funge: Figure 7.3



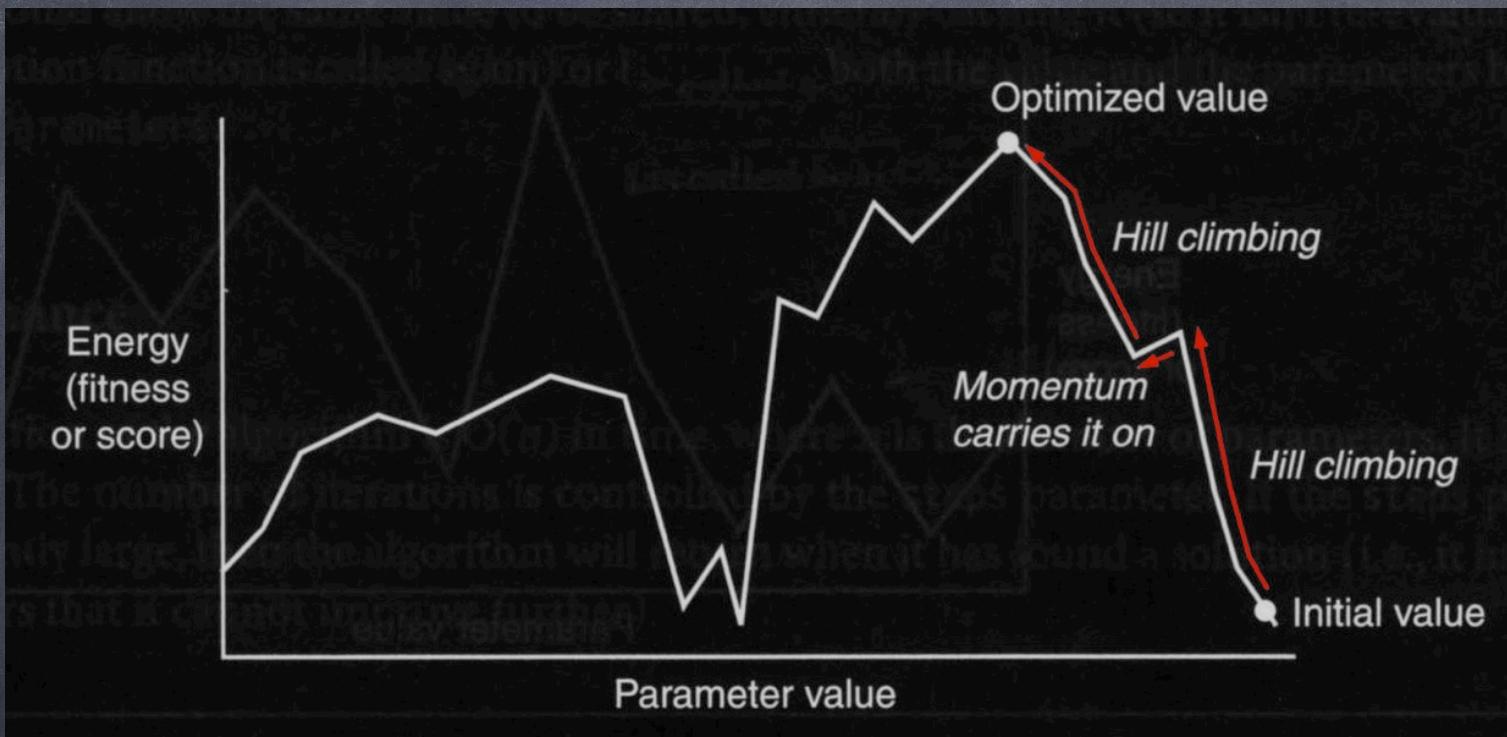
AI for Games, Millington & Funge: Figure 7.5

Introduce Momentum...



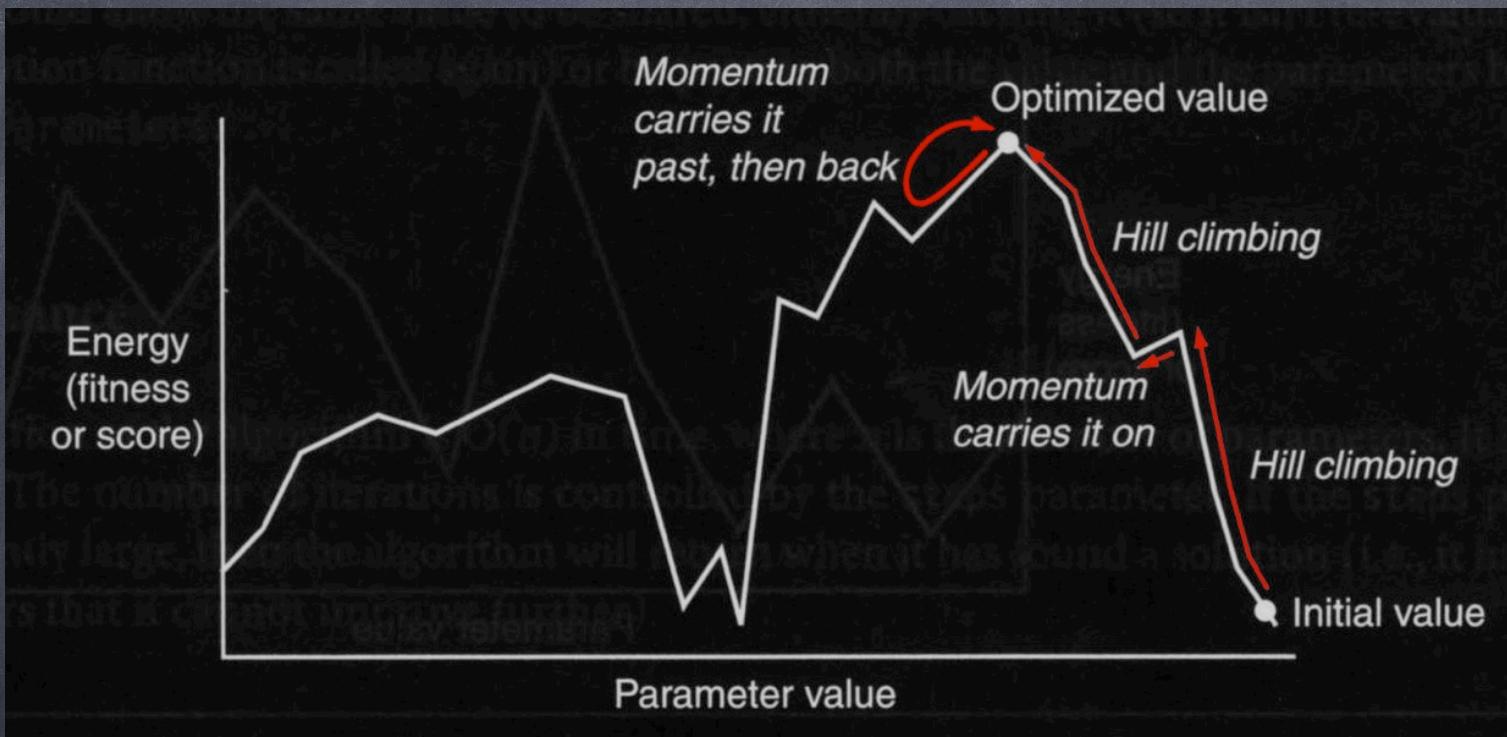
AI for Games, Millington & Funge: Figure 7.5

Introduce Momentum...



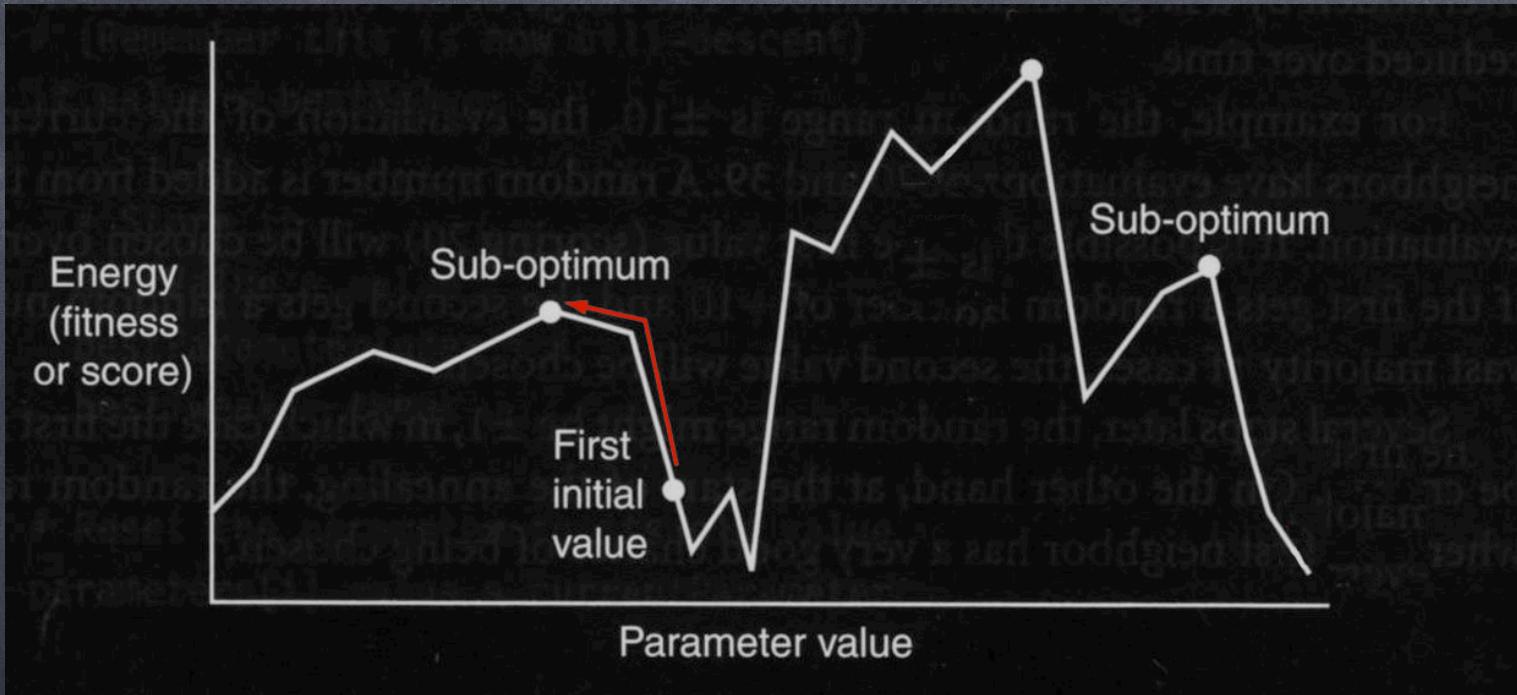
AI for Games, Millington & Funge: Figure 7.5

Introduce Momentum...



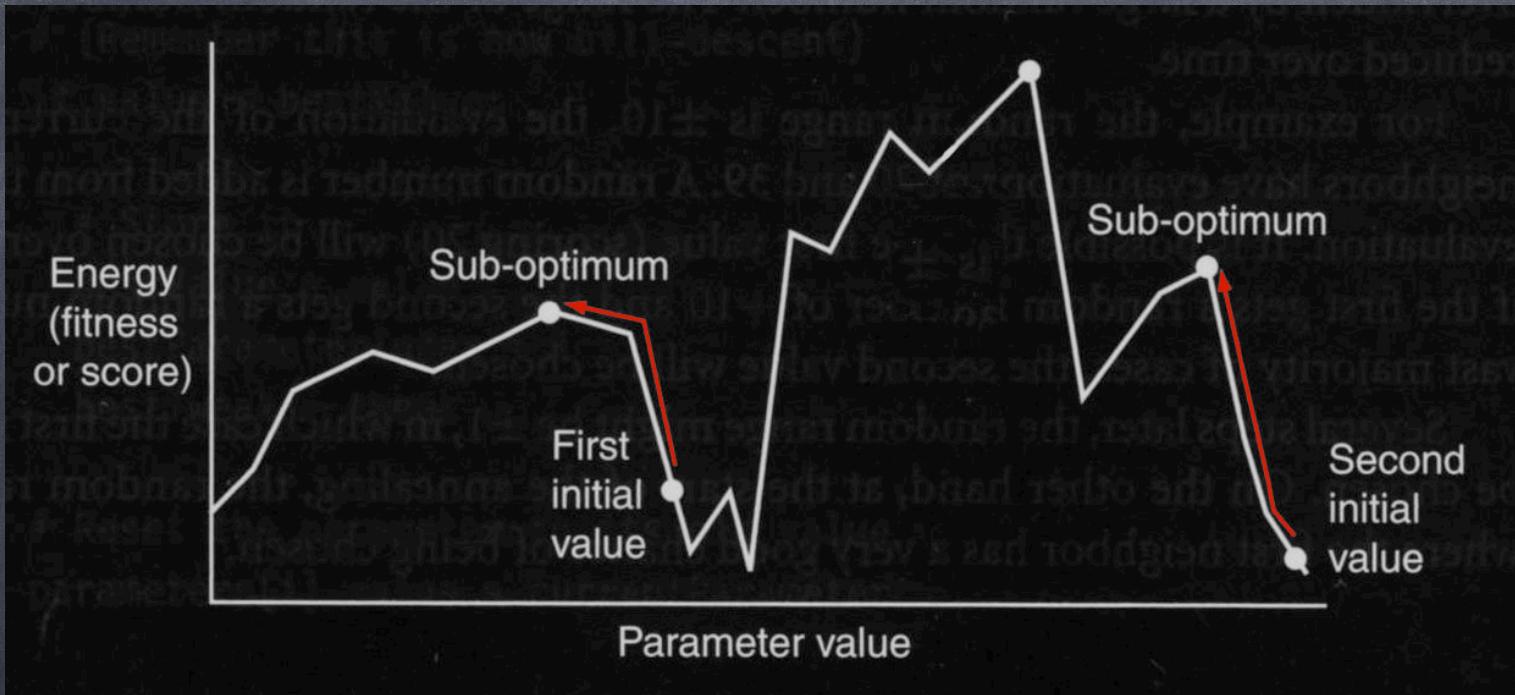
AI for Games, Millington & Funge: Figure 7.5

'Best of Three' Heuristic



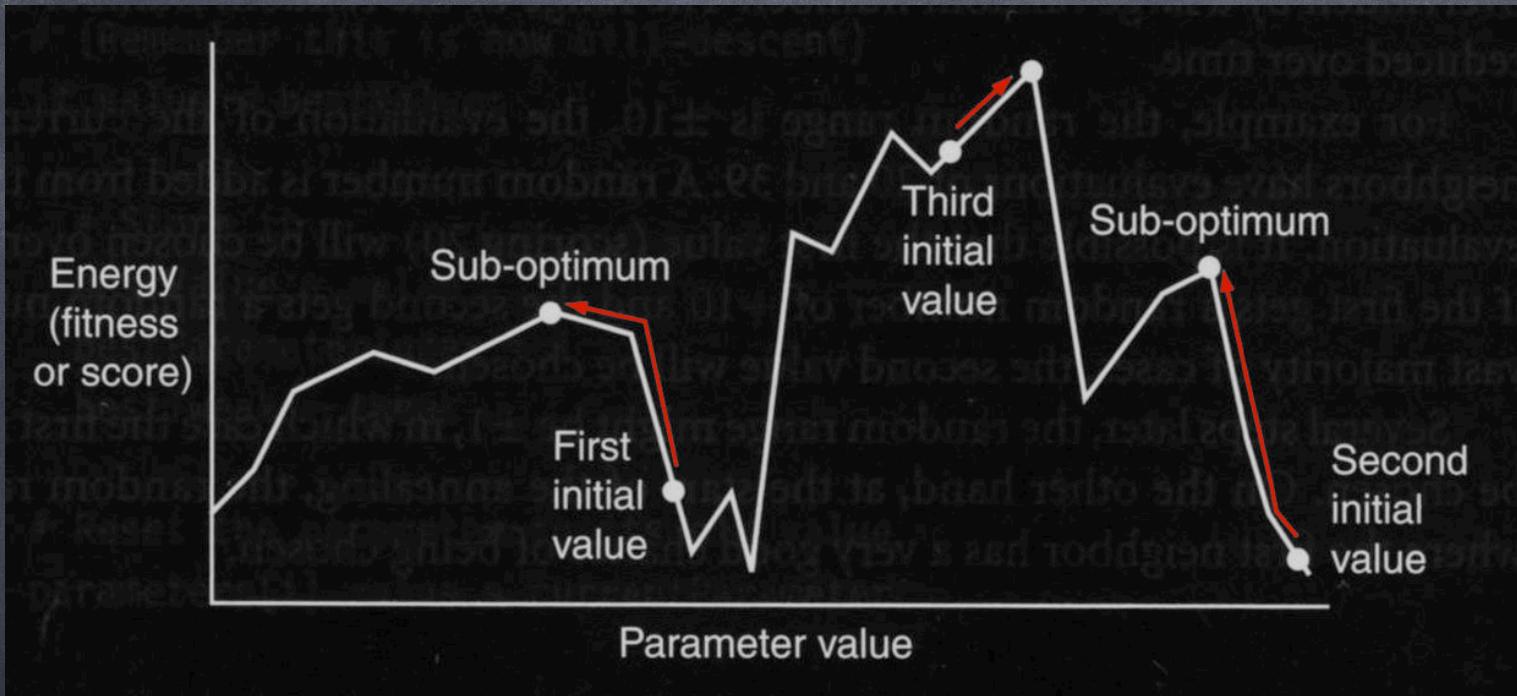
AI for Games, Millington & Funge: Figure 7.6

'Best of Three' Heuristic



AI for Games, Millington & Funge: Figure 7.6

'Best of Three' Heuristic



AI for Games, Millington & Funge: Figure 7.6



Beam Search

Beam Search

- Uses a heuristic evaluation function
- Like breadth-first search
but explores only M best nodes
prunes nodes not explored
- Not optimal, not complete

```
function beam-search
    open := [Start]
    done := [ ]
    while open != [ ] do
        X := open.dequeue()
        if (X = Goal) return success
        else
            generate children states of X
            discard any children in open, done
            for best M children C
                open.enqueue(C)
                done.add(X)
    return failure
```



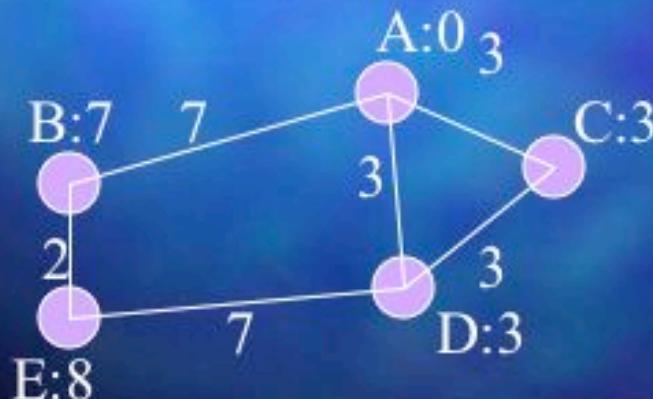
Best First Search
save all the children!

Best First Search

- Expands tree heuristically
- Nodes to expand in best-first order
 - can be like depth or breadth first
 - Keeps all nodes (unlike beam search)

Best-First Searches

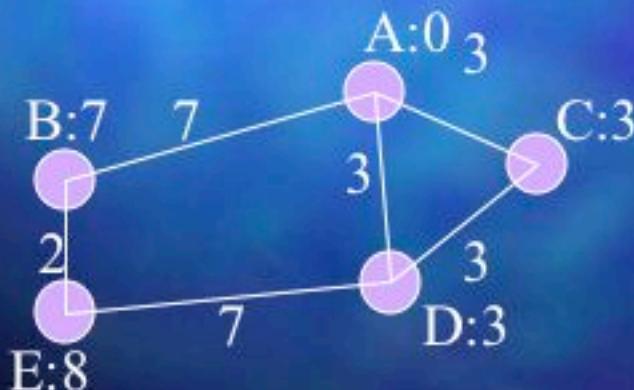
- More accurately, '**estimated best-first searches**'.
- Estimate the **cost** of the next nodes from a given node: e.g., $h(n)$ = the straight-line distance to goal, or something more complex.
- Are **complete**: eventually finds the goal if its in state space
- Now, however, they **queue** all the nodes **in order** of the estimated best first, where 'best' is defined according to some heuristic. E.g. consider this graph:



Node distance	Straight Line Dist. from A
A: C:3, D:3, B:7	B: A:0, E:8
B: E:2, A:7	C: A:0, D:3
C: A:3, D:3	D: A:0, C:3, E:8
D: C:3, A:3, E:7	E: D:3, B:7
E: B:2, D:7	

Greedy Search

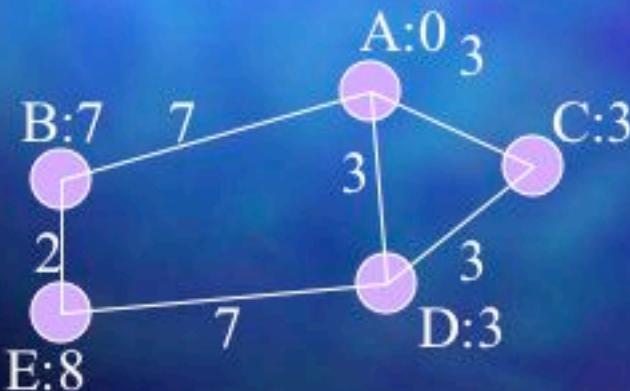
- Choose the least cost node first, here using $h(n_i) = \text{slid}$.
- If we start from E, and the goal is A, then the route is E, D, A:
 - The nodes available from E are B (cost 7) and D (cost 3), so lower cost D is chosen. Remembers B's cost, in case the route via D is bad.
 - The nodes available from D are A (cost 0), C (cost 3) and E (cost 8), so lowest cost A is chosen and the goal is reached.



Node distance	Straight Line Dist. from A
A: C:3, D:3, B:7	
B: E:2, A:7	B: A:0, E:8
C: A:3, D:3	C: A:0, D:3
D: C:3, A:3, E:7	D: A:0, C:3, E:8
E: B:2, D:7	E: D:3, B:7

Problem with Greedy Search

- The problem with greedy search is that it doesn't take notice of the distance it travels to get to a low cost node and can end up with sub-optimal solutions.
- In this case the route E,D,A is a little longer (10) than route E,B,A (9).

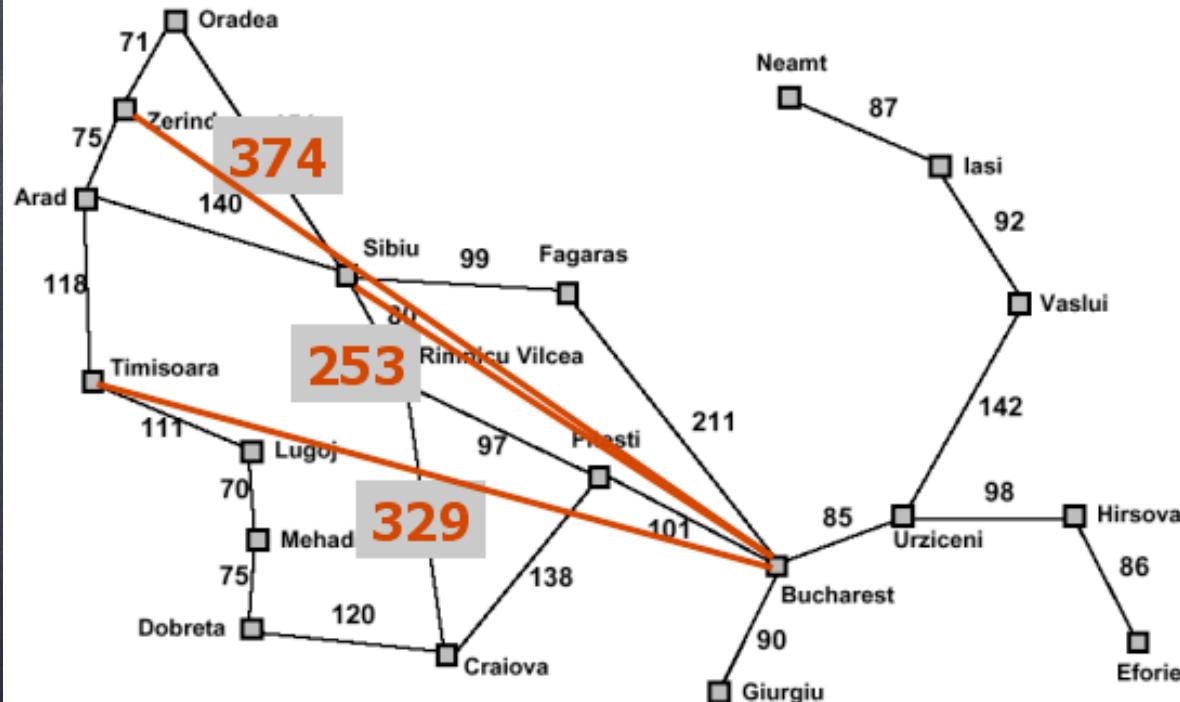


Node distance	Straight Line Dist. from A
A: C:3, D:3, B:7	B: A:0, E:8
B: E:2, A:7	C: A:0, D:3
C: A:3, D:3	D: A:0, C:3, E:8
D: C:3, A:3, E:7	E: D:3, B:7
E: B:2, D:7	

Another example...

let's travel from Arad to Bucharest

Romania with step costs in km



Straight-line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search



- Estimation function:

$h(n)$ = estimate of cost from n to goal (heuristic)

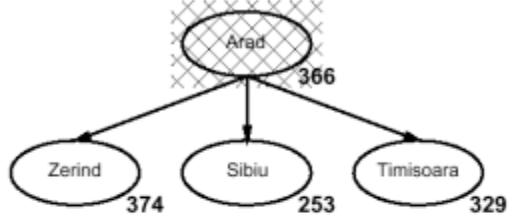
- For example:

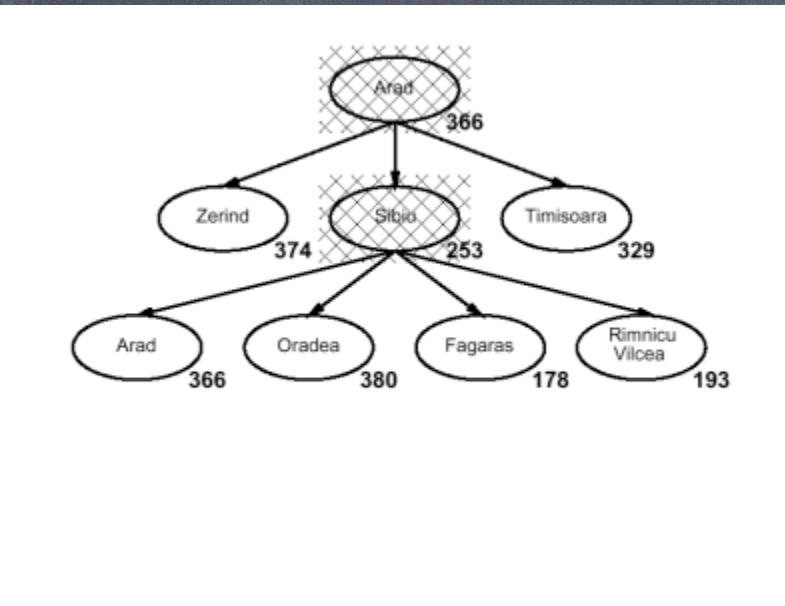
$h_{SLD}(n)$ = straight-line distance from n to Bucharest

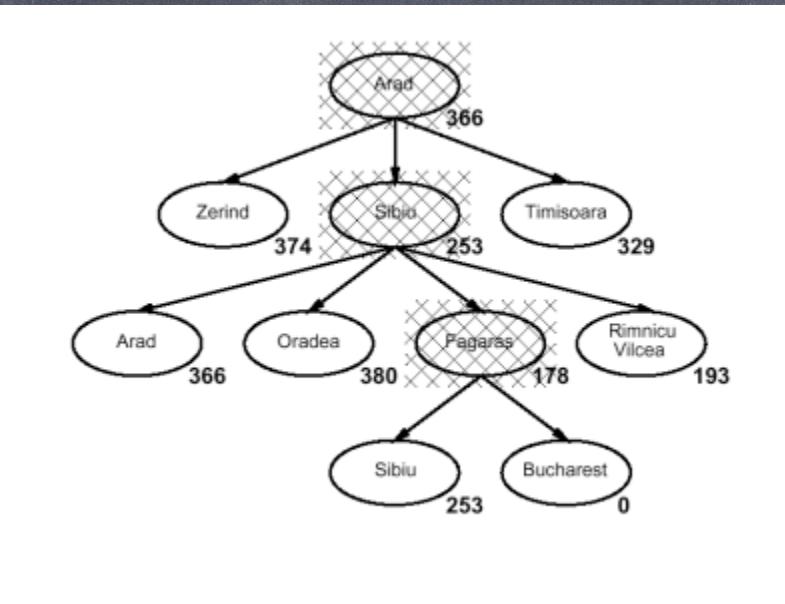
- Greedy search expands first the node that appears to be closest to the goal, according to $h(n)$.

Arad

366





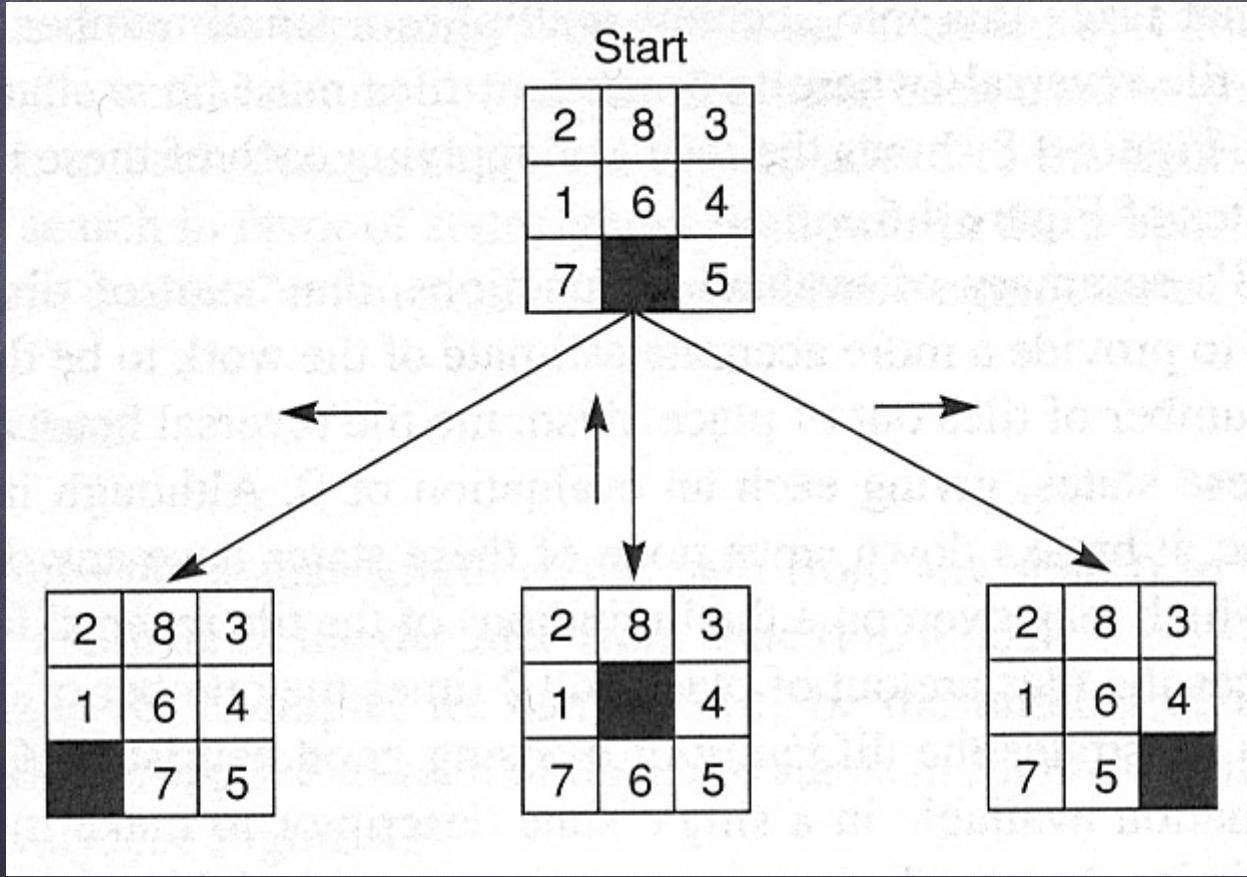


Properties of Greedy Search

- Complete ?
 - Can get stuck in loops without repeat checking
 - Complete in finite space with repeat checking
- Time ?
 - $O(b^m)$, but good heuristic improves performance
- Space ?
 - $O(b^m)$, keeps all nodes in memory
- Optimal ? - No

8 PUZZLE





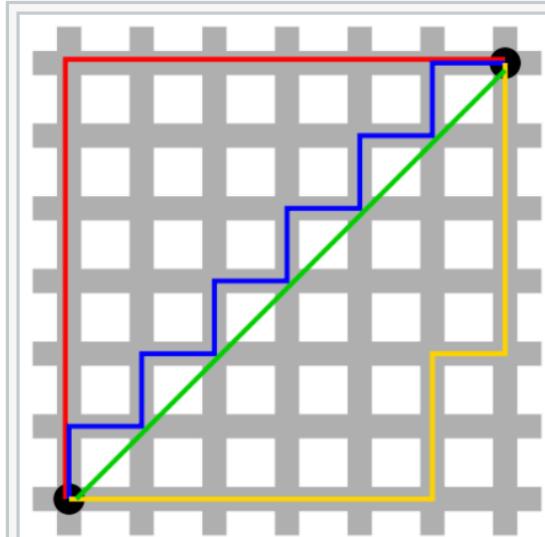
Possible Heuristics

- ⦿ Count number of tiles out of place
- ⦿ Sum all the distances the tiles must move

Taxicab geometry

From Wikipedia, the free encyclopedia

A **taxicab geometry** is a form of **geometry** in which the usual distance function or **metric** of Euclidean **geometry** is replaced by a new metric in which the **distance** between two points is the sum of the **absolute differences** of their **Cartesian coordinates**. The **taxicab metric** is also known as **rectilinear distance**, L_1 **distance**, L^1 **distance** or ℓ_1 **norm** (see L^p **space**), **snake distance**, **city block distance**, **Manhattan distance** or **Manhattan length**, with corresponding variations in the name of the geometry.^[1] The latter names allude to the **grid layout** of most **streets** on the island of **Manhattan**, which causes the shortest path a car

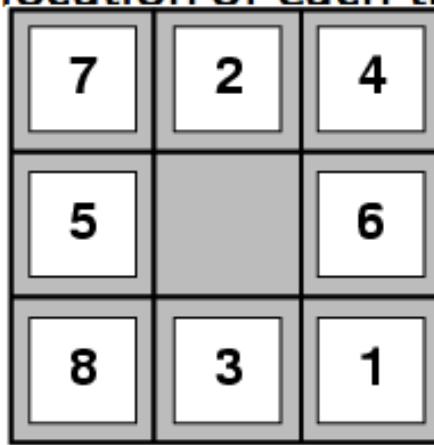


Taxicab geometry versus Euclidean distance: In taxicab geometry, the red, yellow, and blue paths all have the same shortest path length of 12. In Euclidean geometry, the green line has length $6\sqrt{2} \approx 8.49$ and is the unique shortest path.

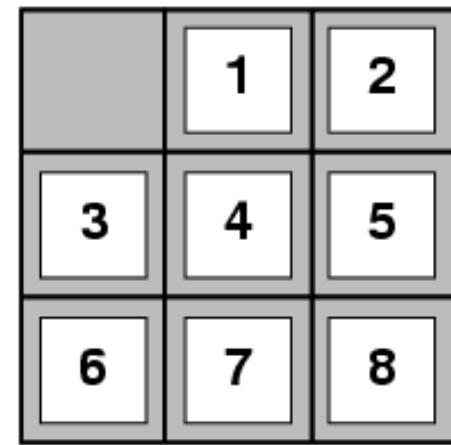


E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State



Goal State

- $h_1(S) = ?$ 8
- $h_2(S) = ?$ $3+1+2+2+2+3+3+2 = 18$

2	1	3
8		4
7	5	6

1	2	3
8		4
7	6	5

Goal

difficult reversals!

Possible Heuristics

- ⦿ Count number of tiles out of place
- ⦿ Sum all the distances the tiles must move
- ⦿ Recognize very hard to swap adjoining tiles

2	8	3
1	6	4
7	5	

5

6

0

2	8	3
1		4
7	6	5

3

4

0

2	8	3
1	6	4
7	5	

5

6

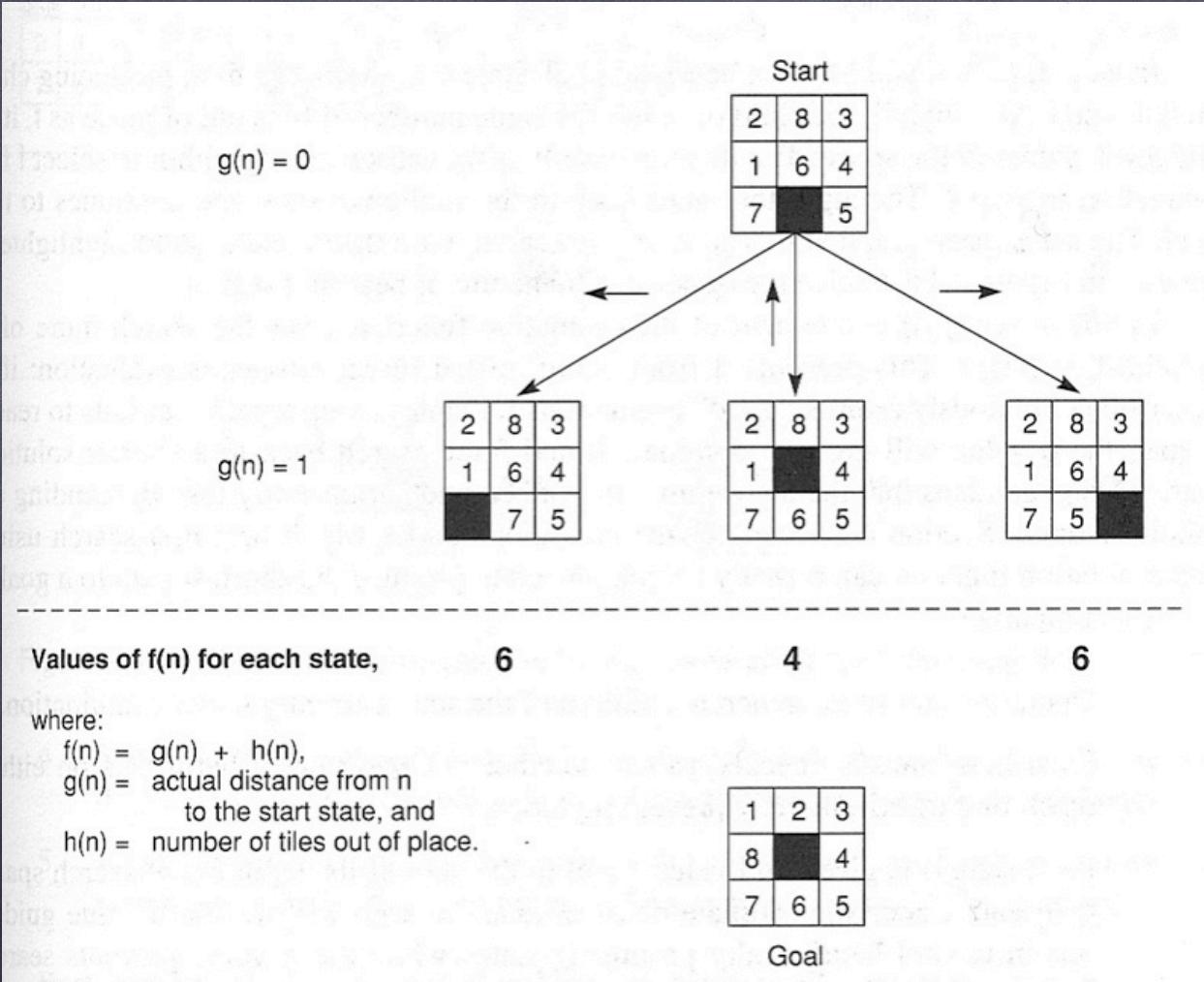
0

[Luger 2002]

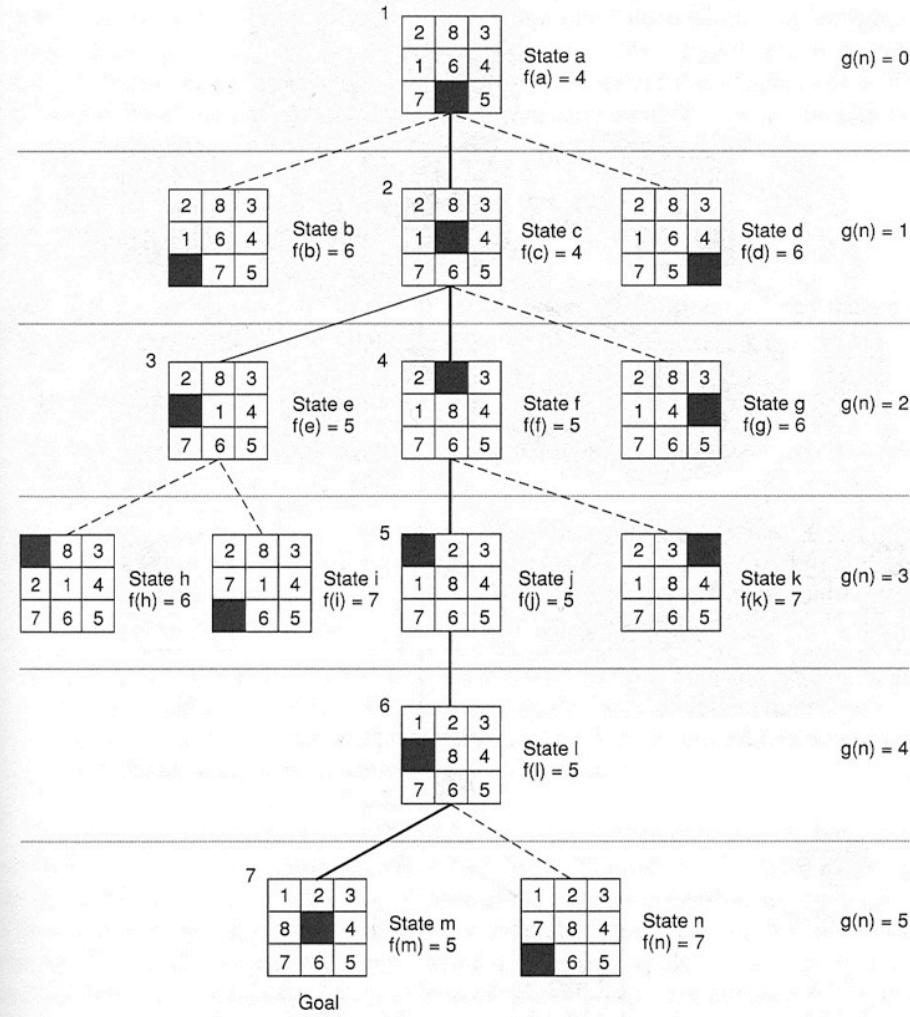
Tiles out of
placeSum of
distances out
of place2 x the number
of direct tile
reversals

Evaluation function

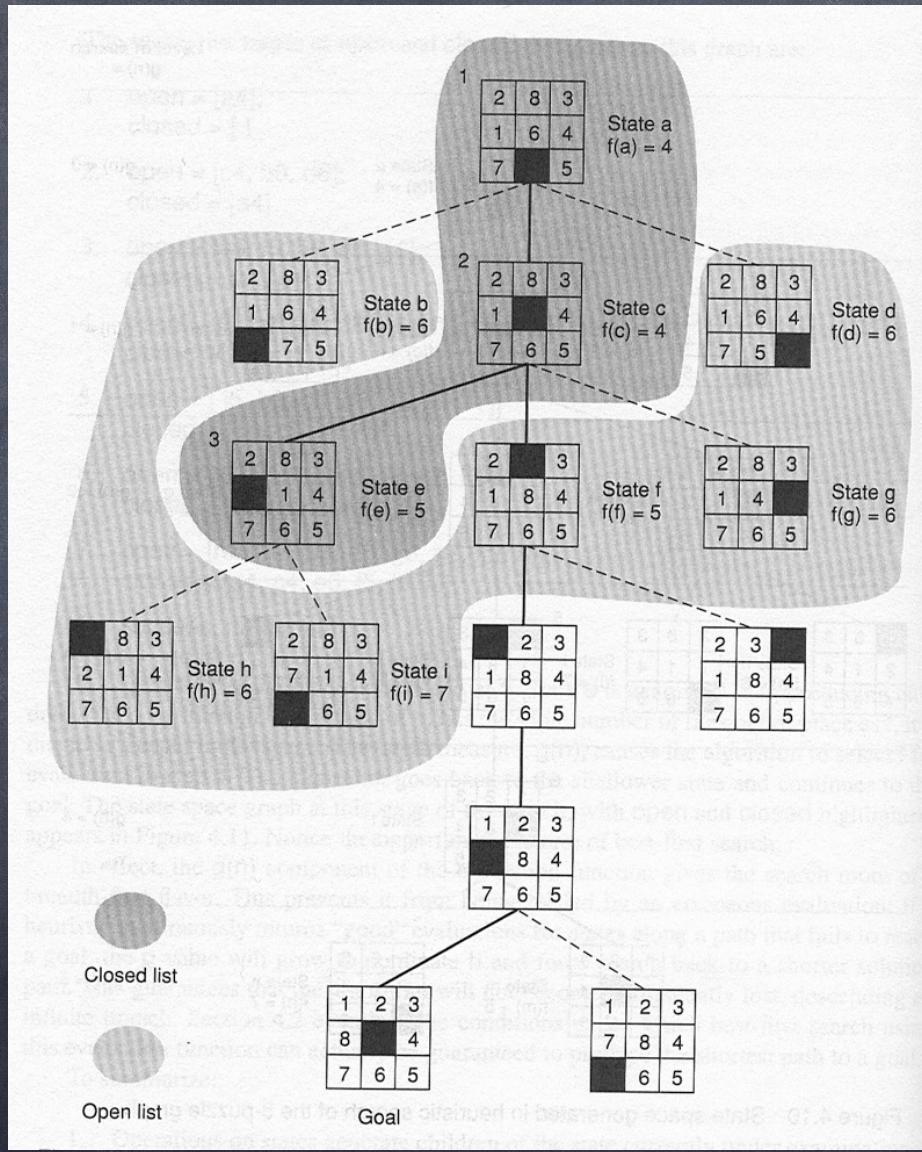
- ⦿ Can combine multiple heuristics
 - e.g. distance + (2 x reversals)
- ⦿ Force exploration of shallower solutions by adding a path-length cost



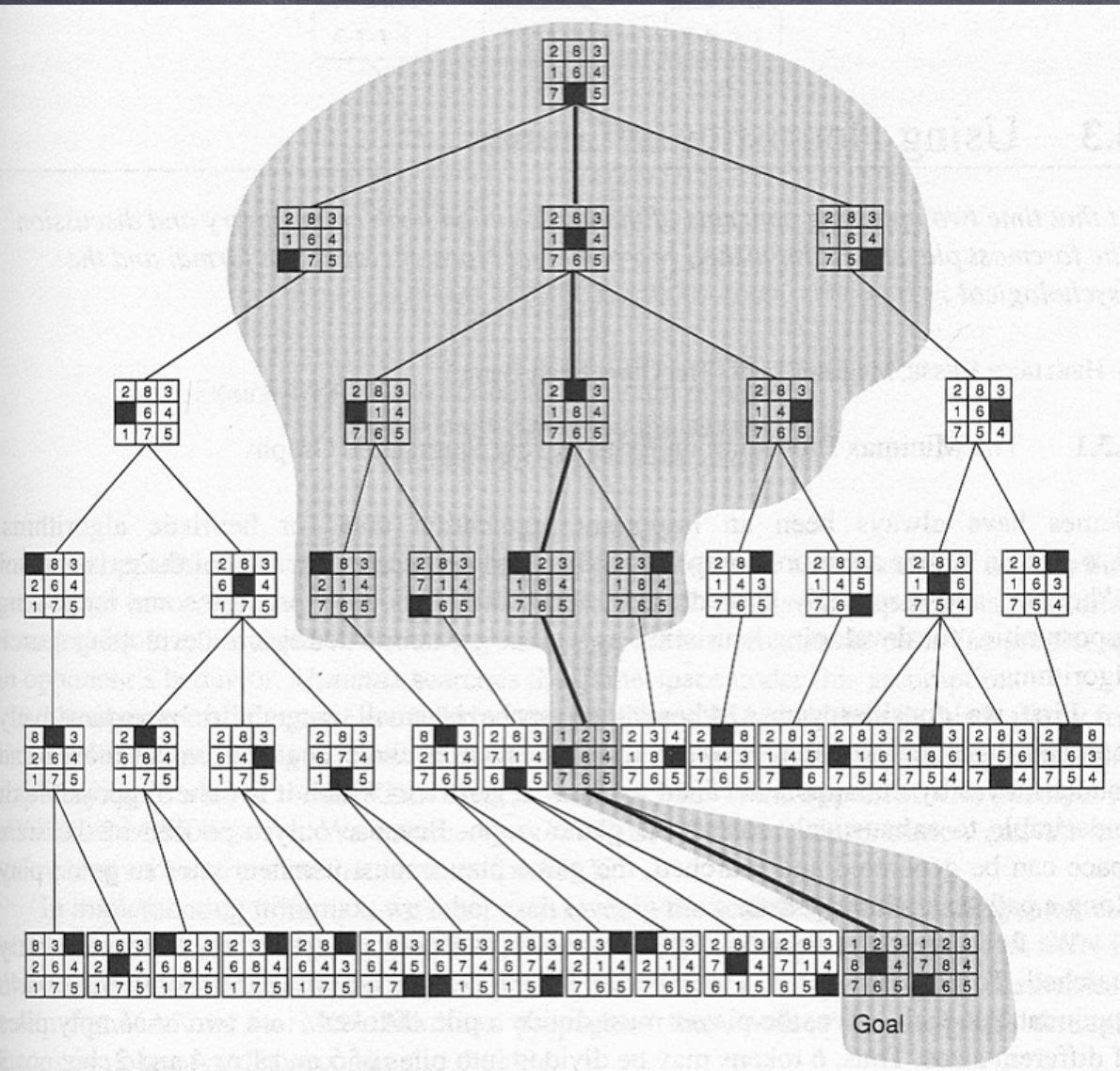
Level of search
 $g(n) =$



```
function best-first-search
    open := [Start]
    done := [ ]
    while open != [ ] do
        X := best state in open
        if (X = Goal) return success
        else
            generate children states of X
            discard any children in open, done
            for each child
                compute heuristic value
                add to open (keeping sorted)
            done.add(X)
    return failure
```



[Luger 2002]



[Luger 2002]

Admissibility

a **search algorithm** is admissible if it is guaranteed to find a minimal path when it exists.

breadth-first search is admissible

is best-first search?

a **heuristic** is admissible when it finds the shortest path.

Our heuristic:

$$f(n) = g(n) + h(n)$$

takes into account path length and 'goodness'

'goodness' is really an estimated distance to goal

as long as heuristic does not OVERESTIMATE distance,
it is admissible.

(an overestimate might cause the best path to be ignored.)

2	8	3
1	6	4
7	5	

5

6

0

2	8	3
1		4
7	6	5

3

4

0

2	8	3
1	6	4
7	5	

5

6

0

Tiles out of place

Sum of distances out of place

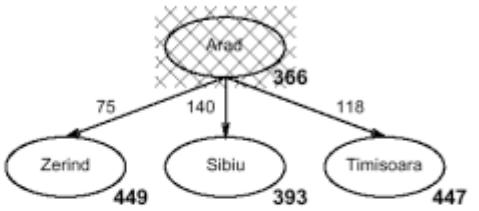
2 x the number of direct tile reversals

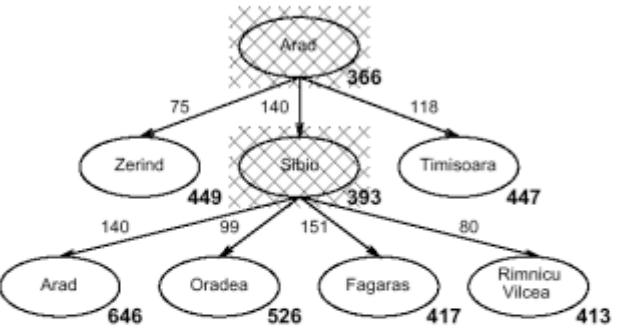
- A problem with fewer restrictions on the actions is called a **relaxed problem**
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

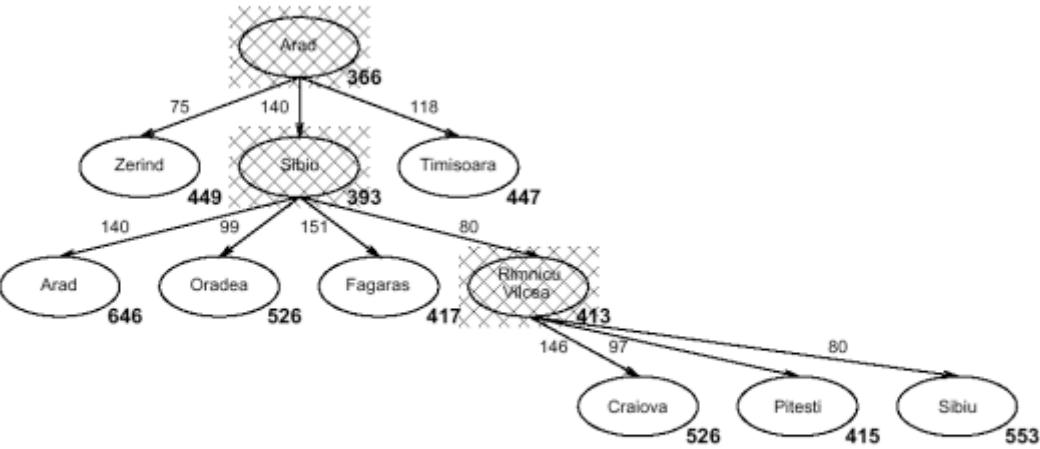


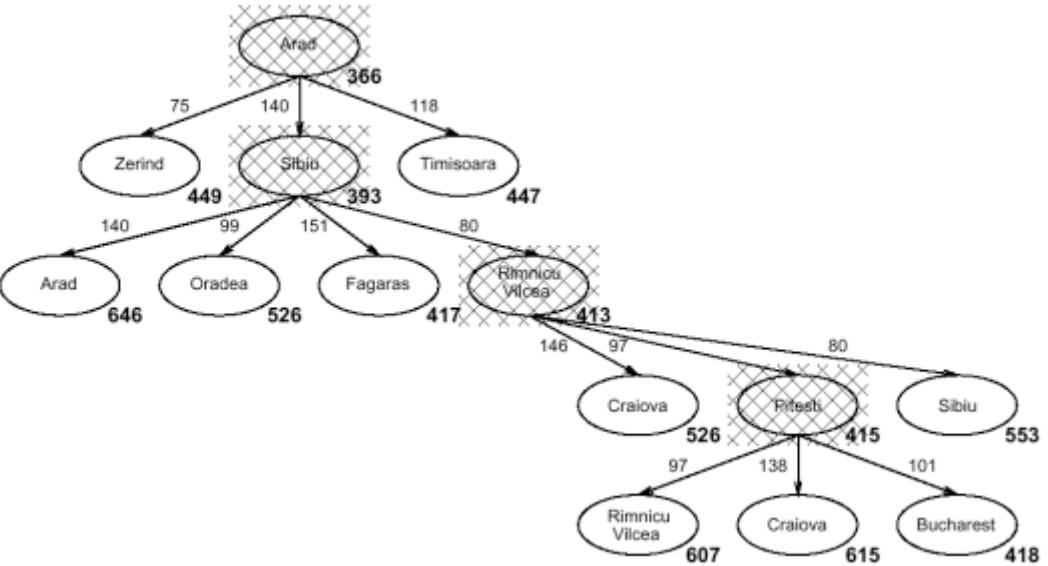
Best-first search with an
admissible heuristic is
called the A* Algorithm

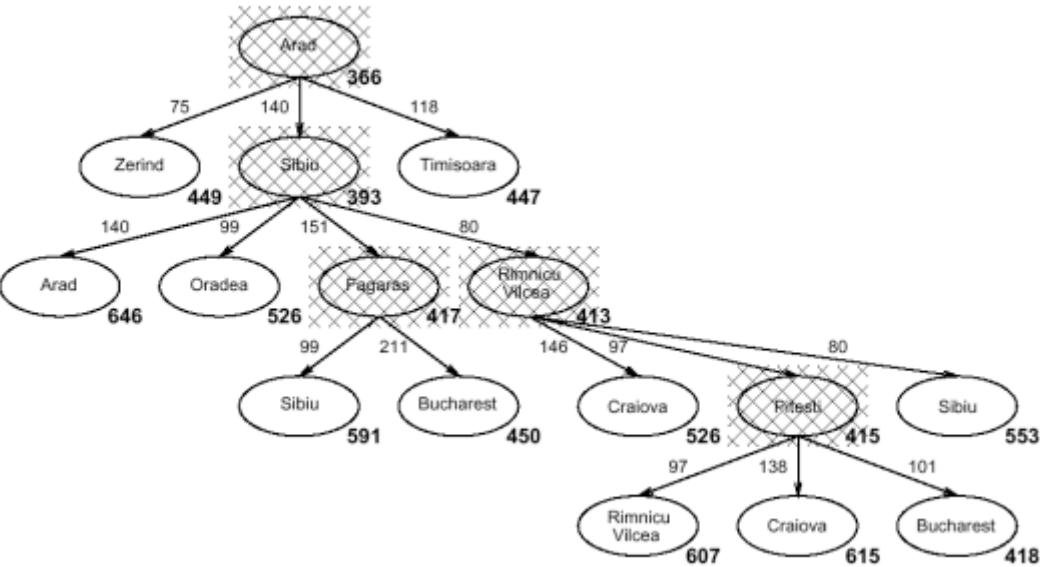
Arad
366



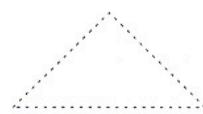
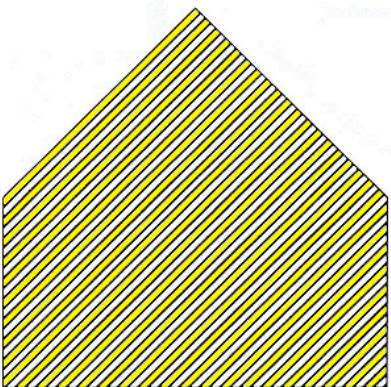
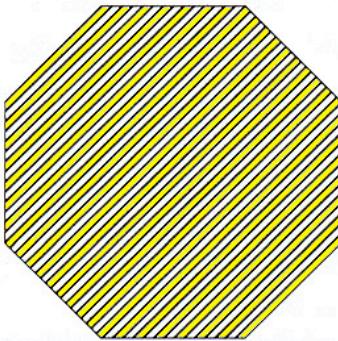
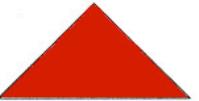




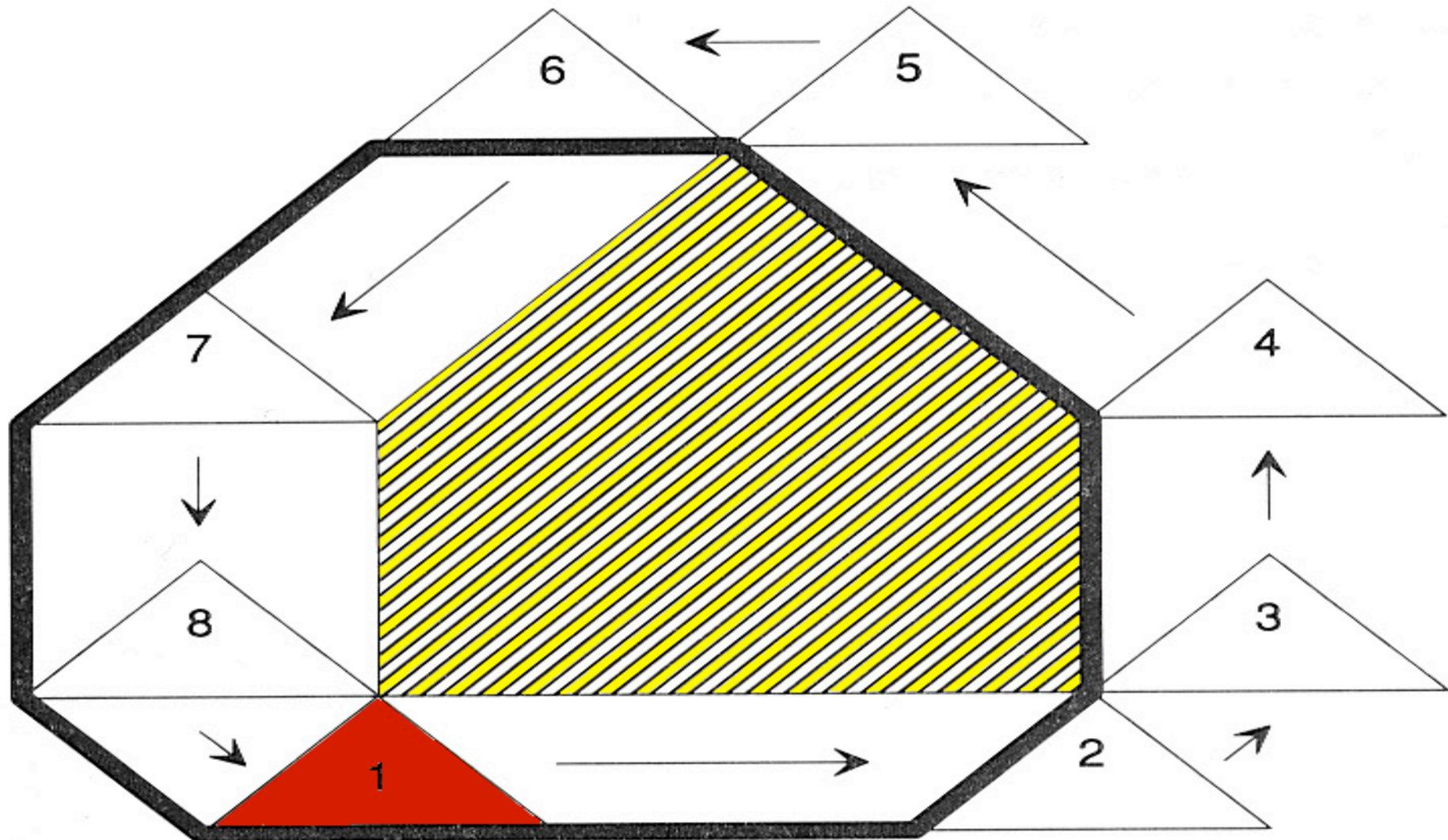




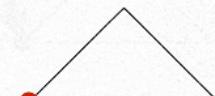
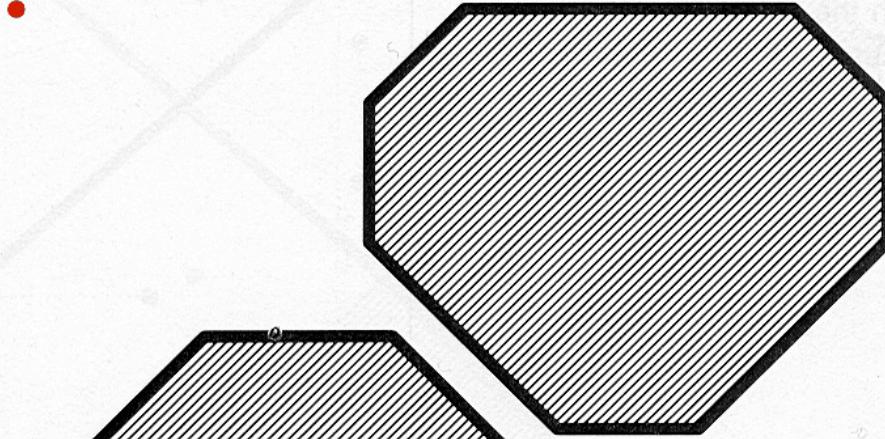
Initial position



Desired position

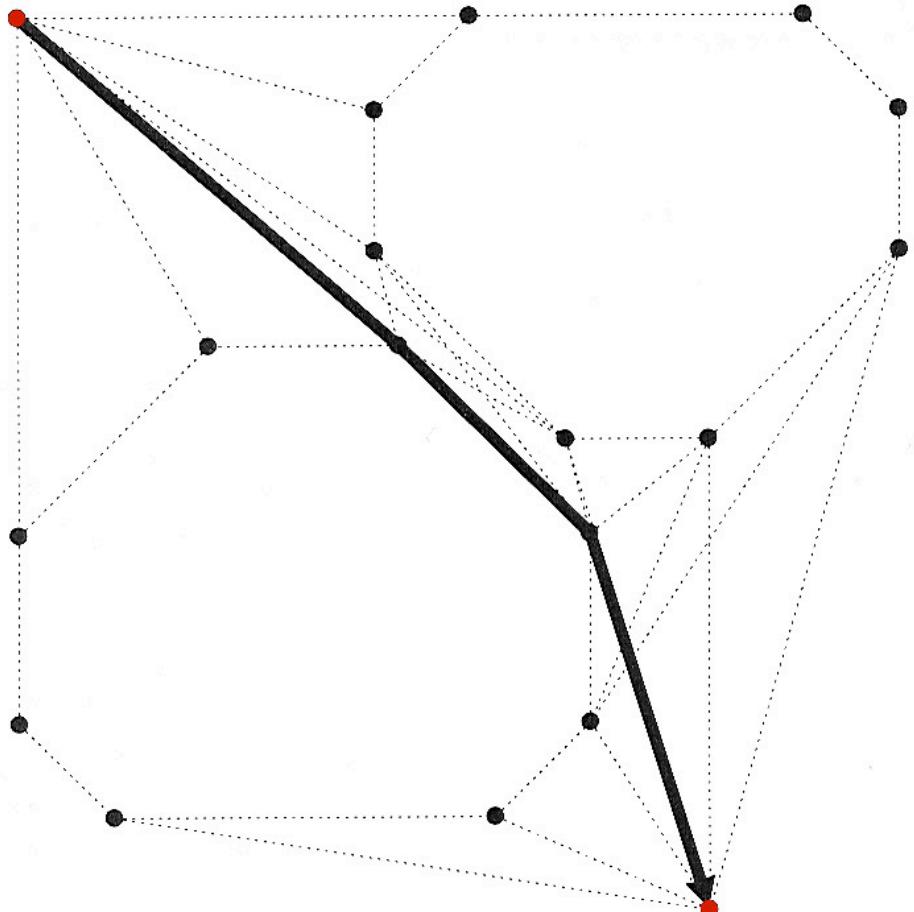


Initial position

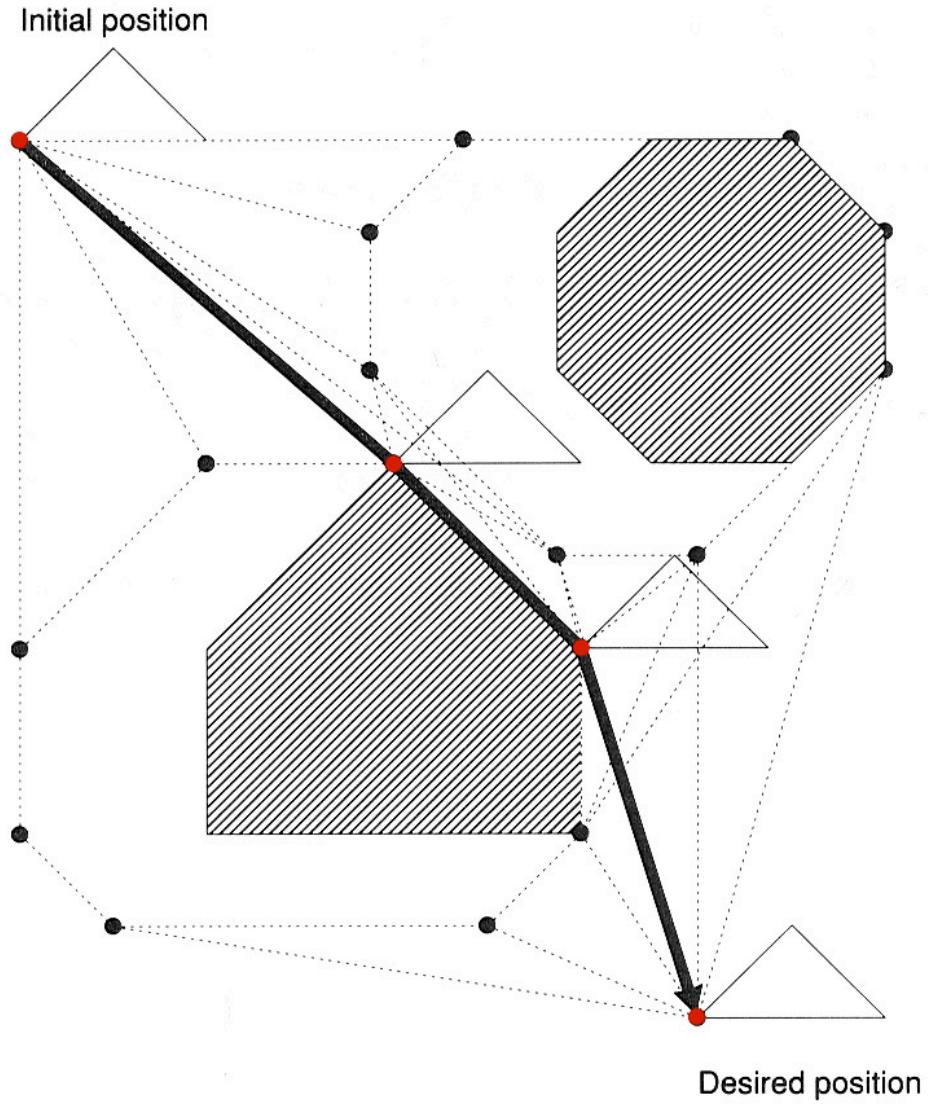


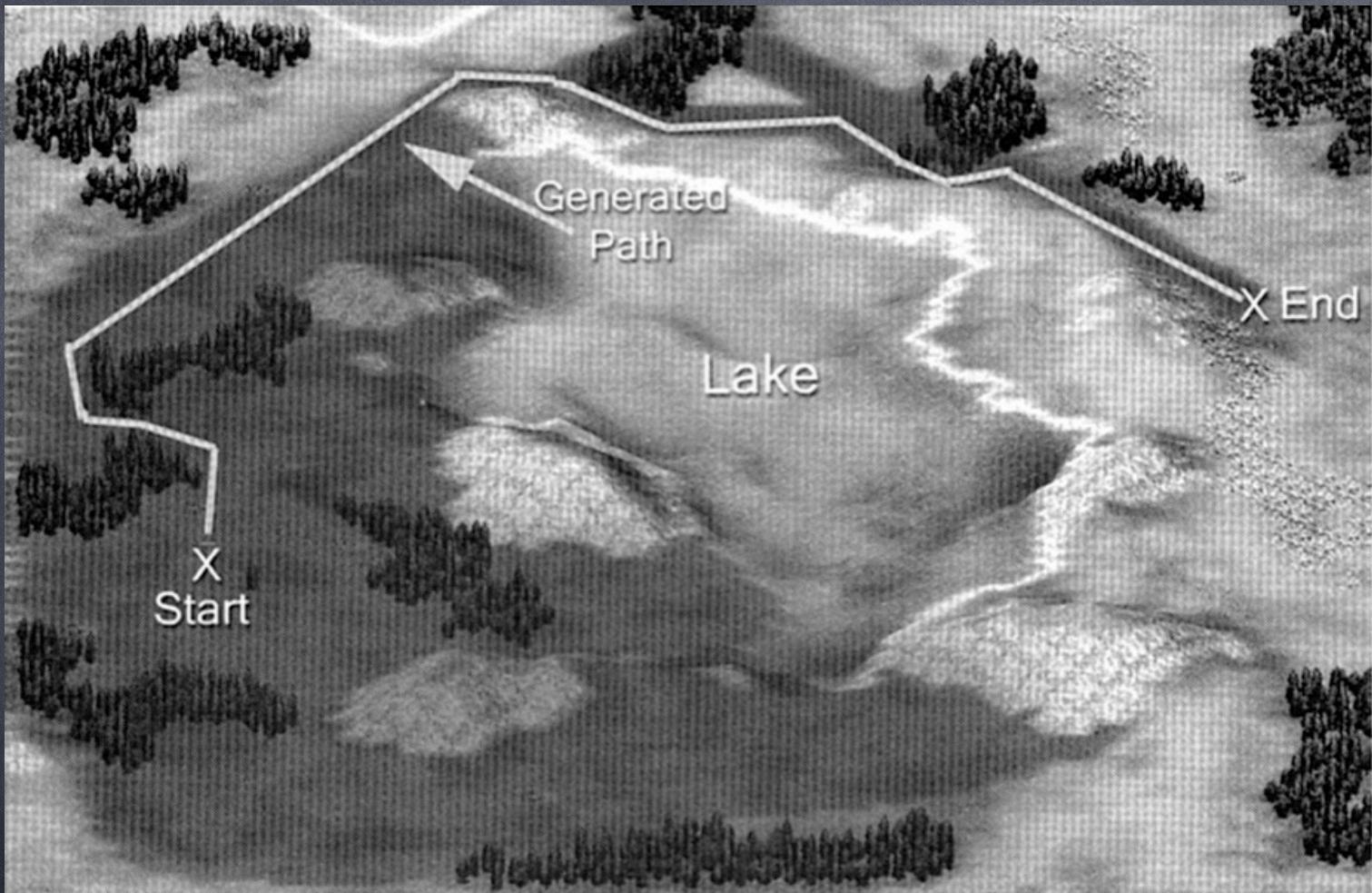
Desired position

Initial position



Desired position





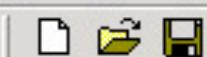
Example: A* explorer

www.generation5.org

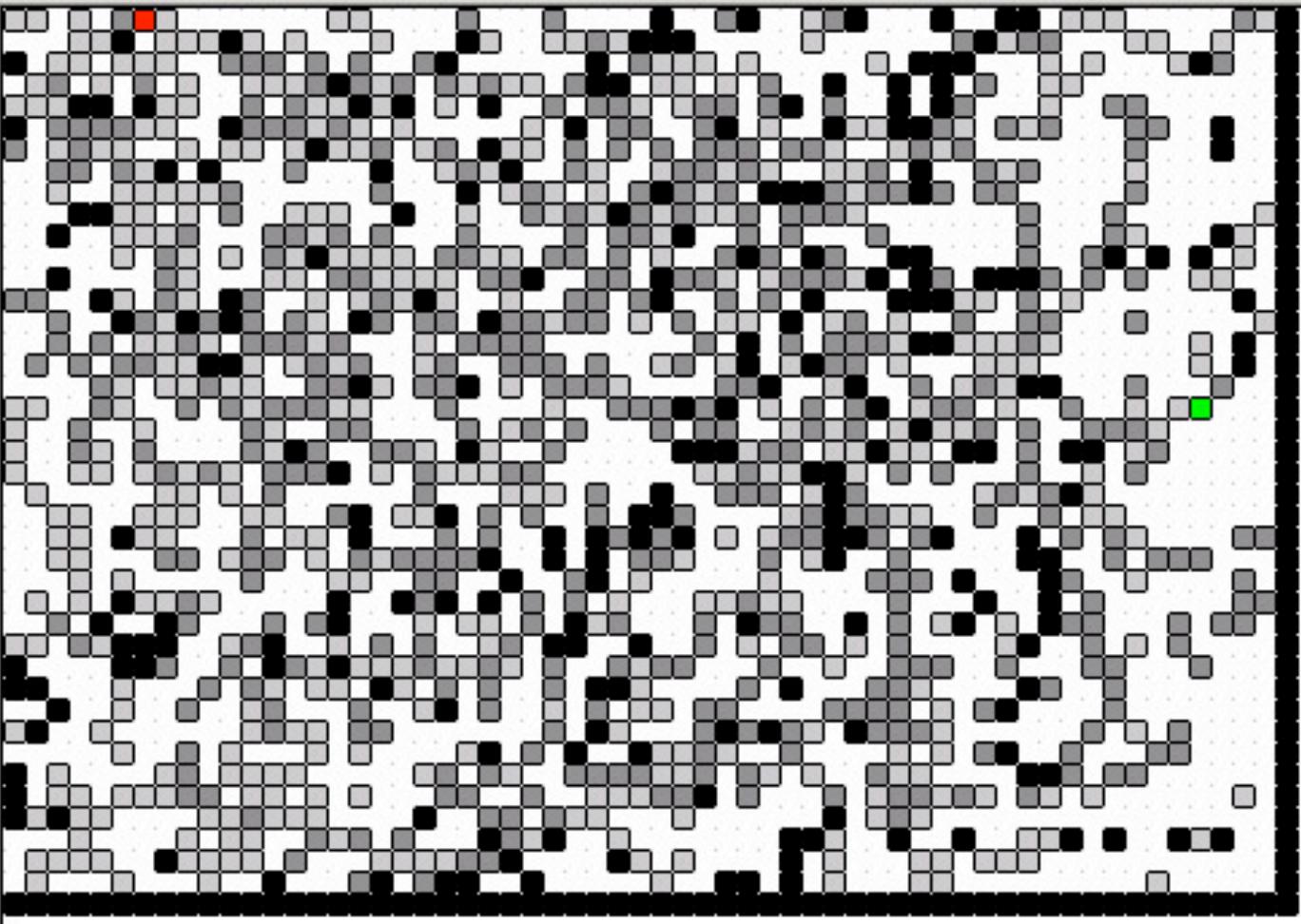
provides A* implementation in C++

RANDOM.ASE - A* Explorer

File Pathing View Help

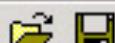


- ... A* Tree
- T_t Open List
 - 6,0; f = 67
- T_c Closed List



RANDOM.ASE - A* Explorer

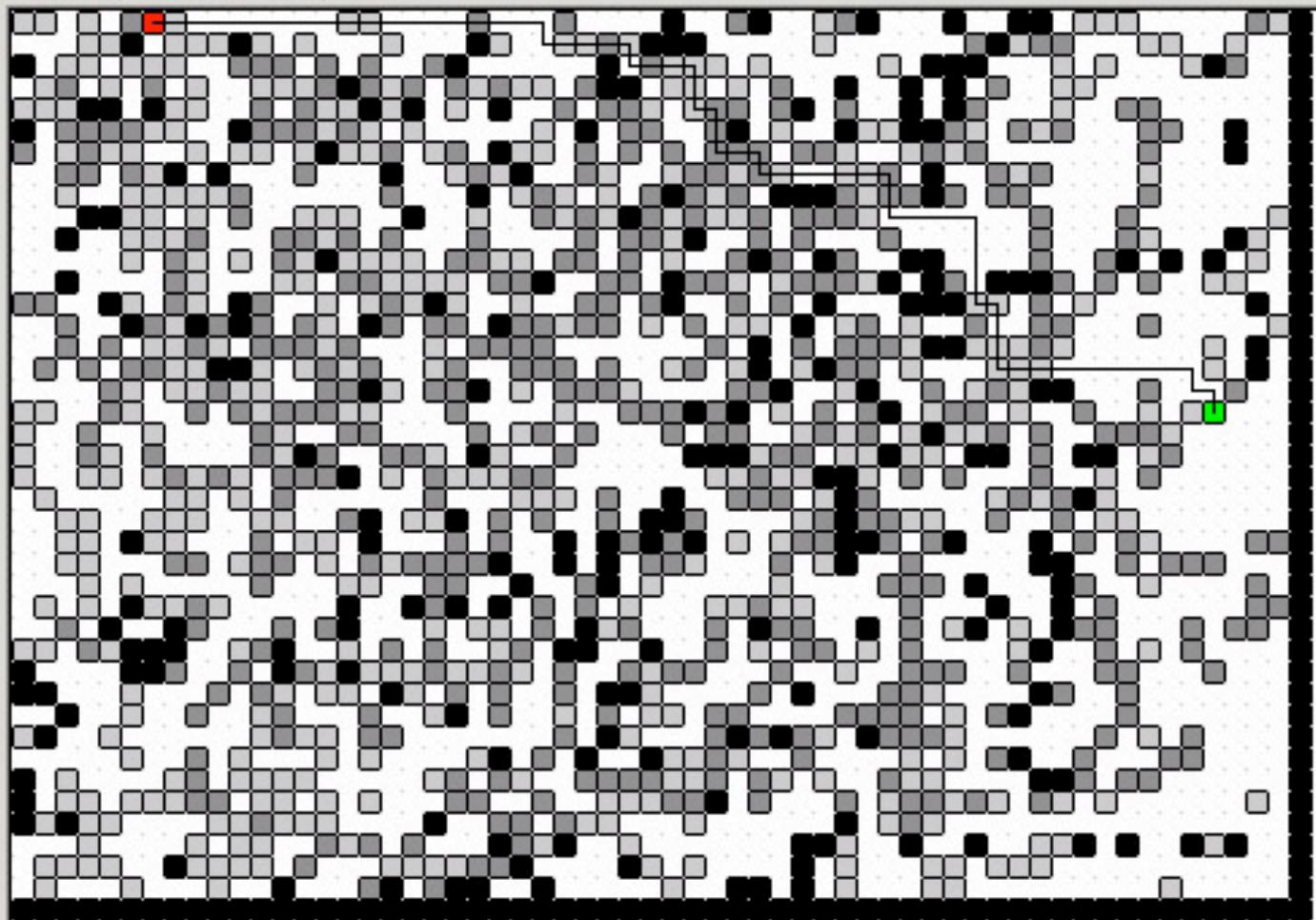
File Pathing View Help



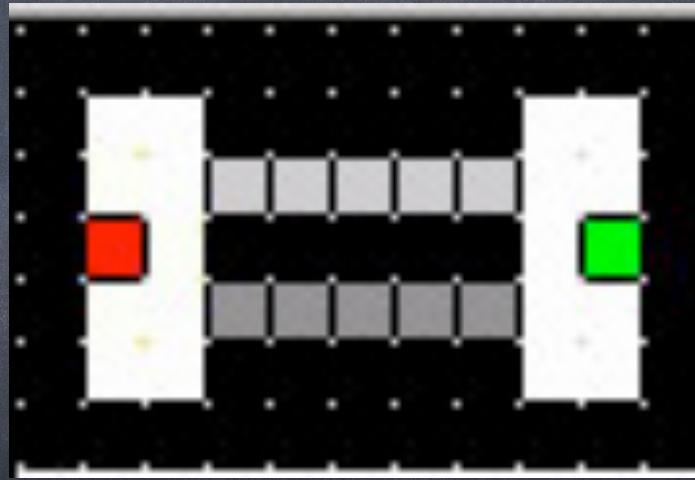
A* Tree

Open List

- 45,13; f = 88
- 39,19; f = 91
- 42,1; f = 89
- 43,1; f = 89
- 44,14; f = 89
- 45,14; f = 89
- 46,13; f = 89
- 46,14; f = 89
- 38,20; f = 90
- 39,20; f = 88
- 46,15; f = 90
- 45,15; f = 90
- 44,15; f = 90
- 40,13; f = 89
- 40,14; f = 89
- 37,9; f = 89
- 41,0; f = 91
- 42,0; f = 91
- 44,1; f = 91

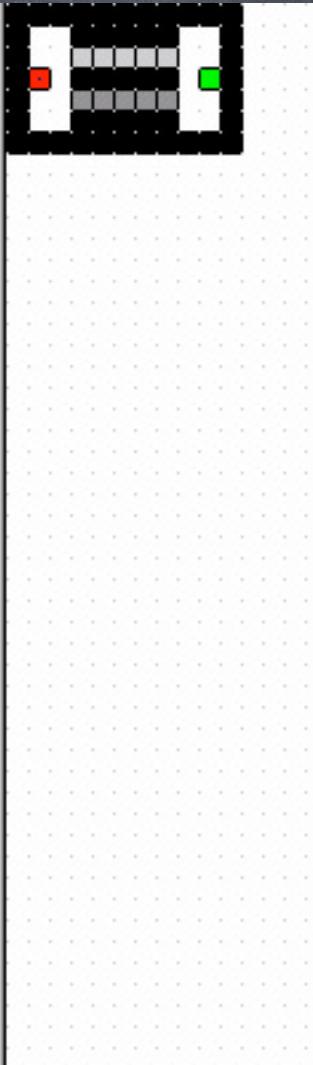


■ A* Tree
■ Open List
■ 1,3; f = 8
■ Closed List



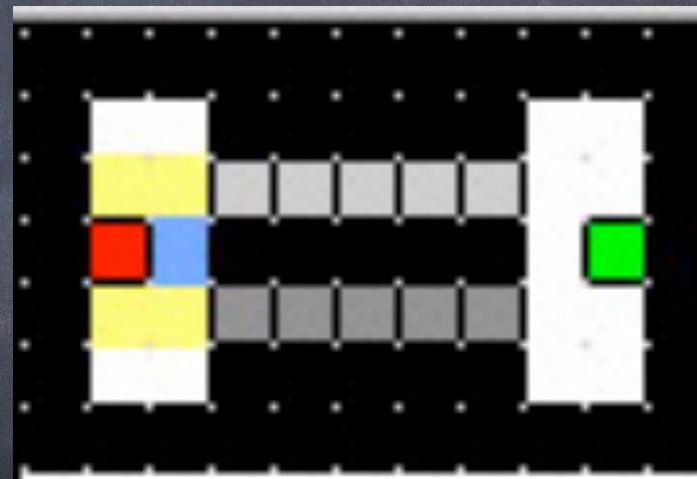
A* Tree

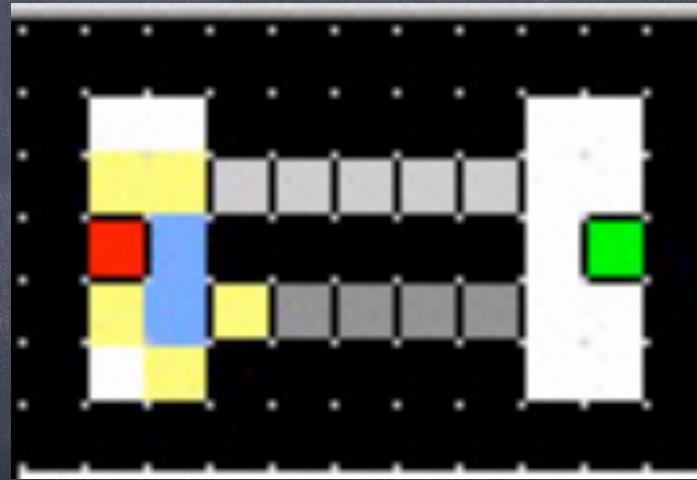
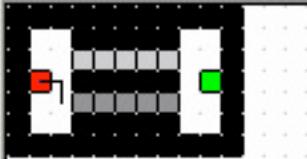
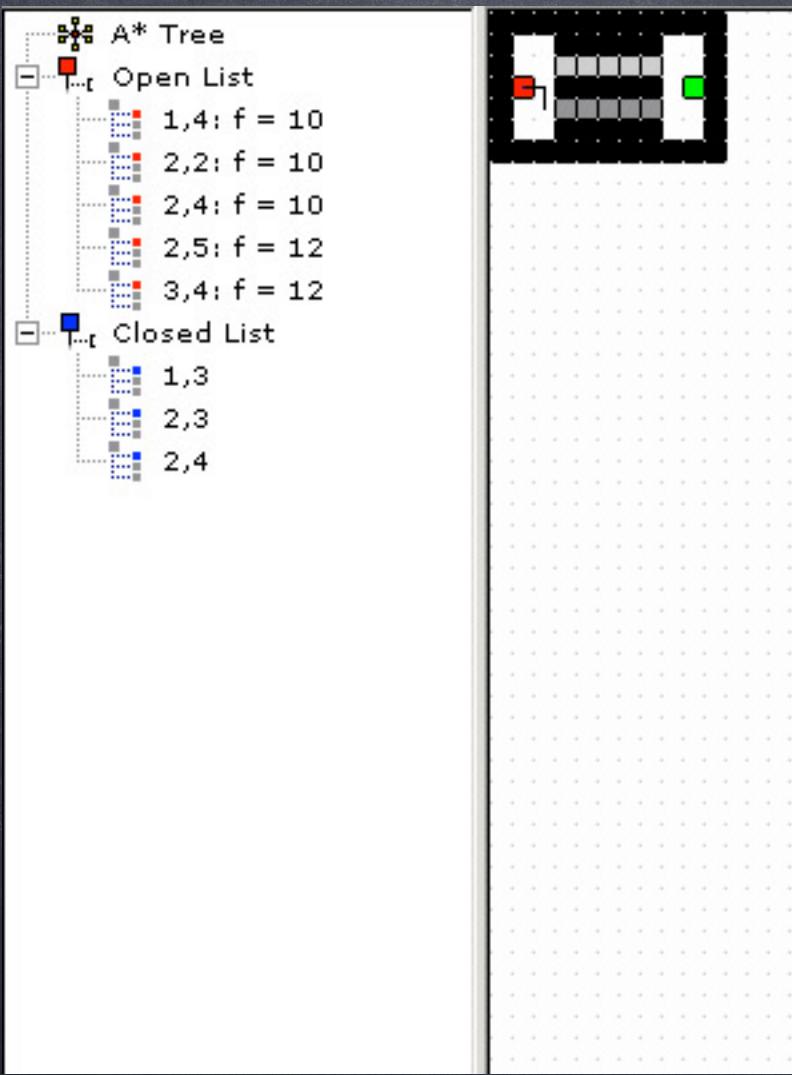
- Open List
 - 2,3; f = 8
 - 1,2; f = 10
 - 1,4; f = 10
- Closed List
 - 1,3

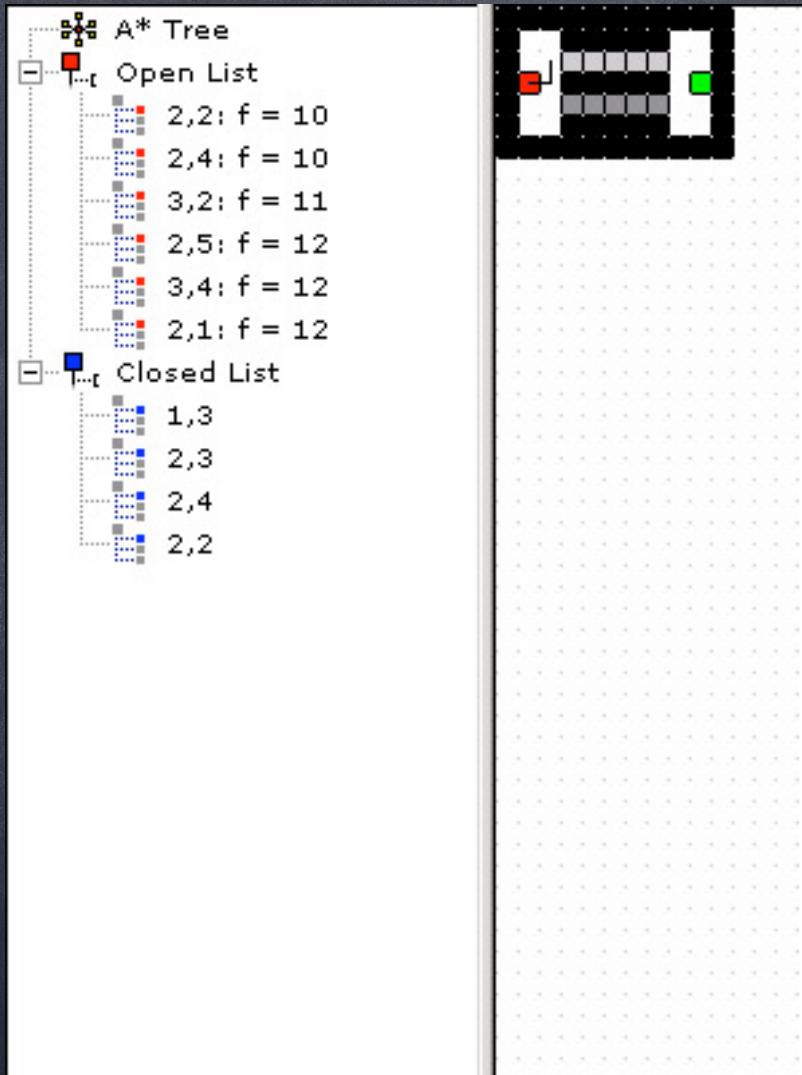


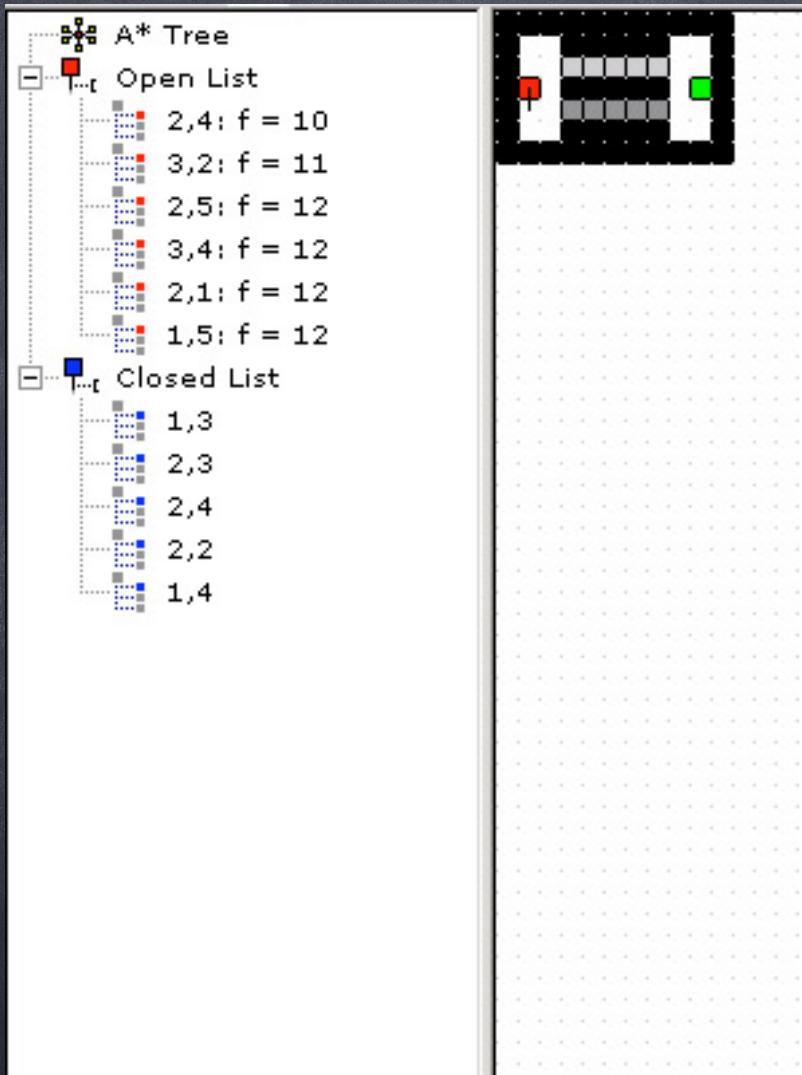
❖ A* Tree

- T-t Open List
 - 1,2; f = 10
 - 1,4; f = 10
 - 2,2; f = 10
 - 2,4; f = 10
- T-t Closed List
 - 1,3
 - 2,3



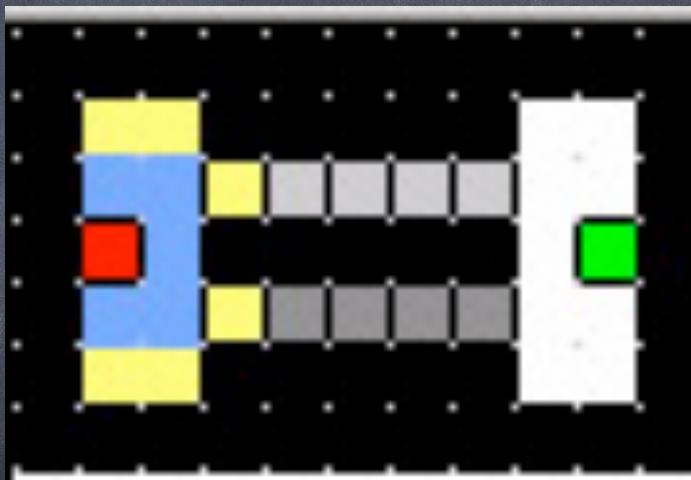




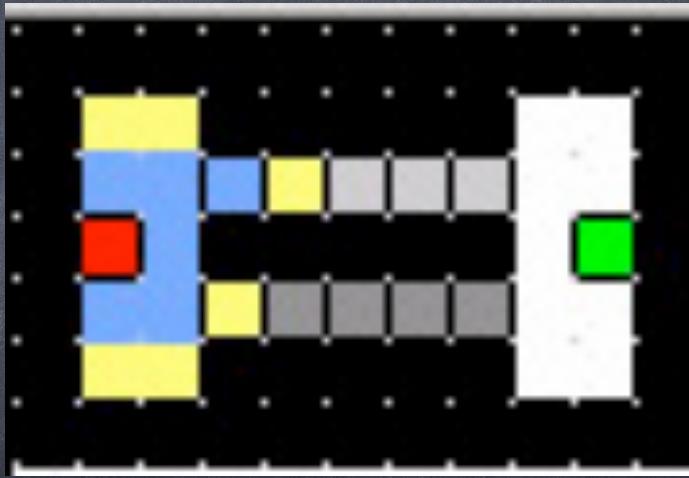


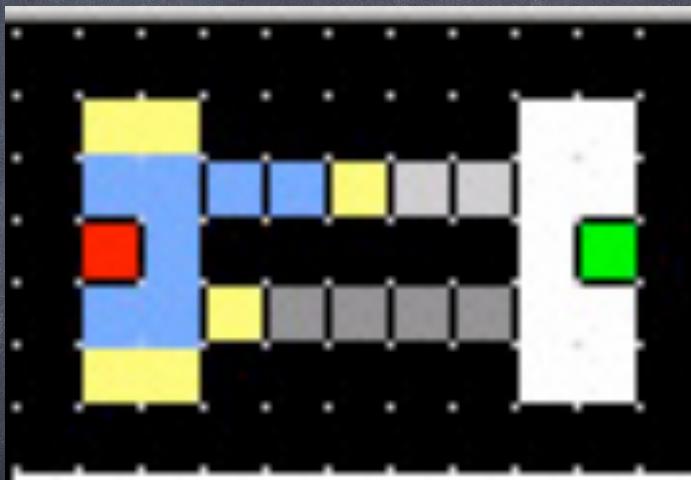
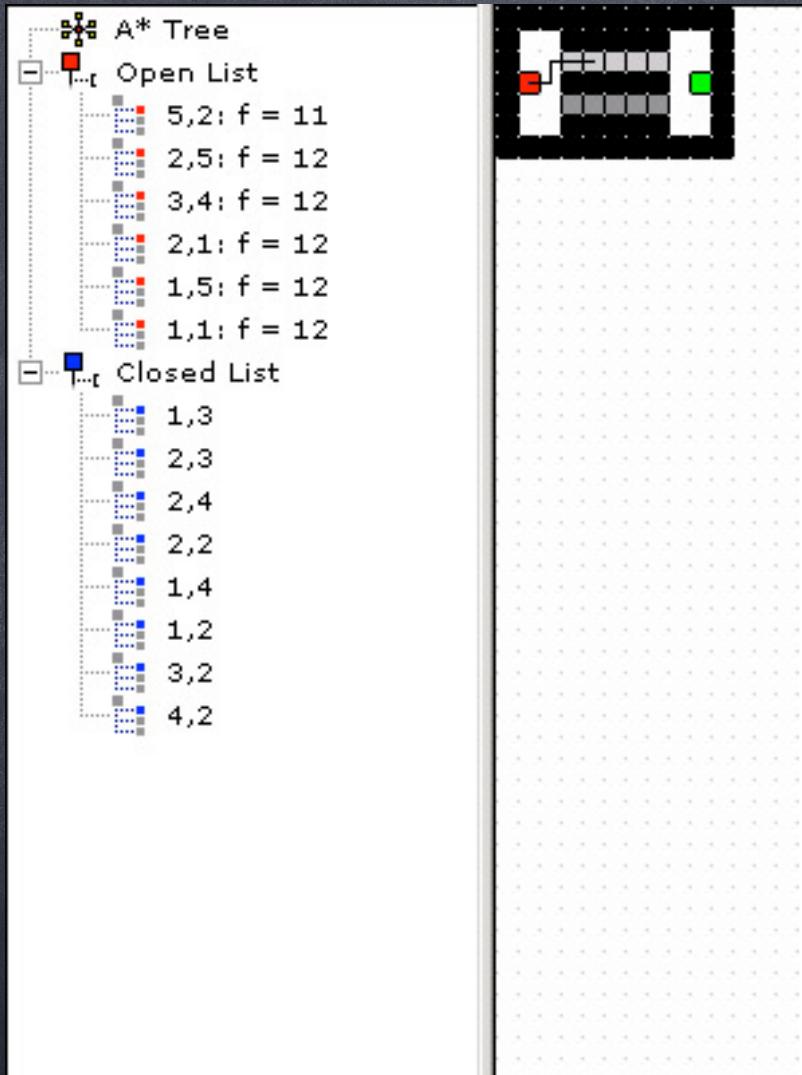
❖ A* Tree

- T_t Open List
 - 3,2; f = 11
 - 2,5; f = 12
 - 3,4; f = 12
 - 2,1; f = 12
 - 1,5; f = 12
 - 1,1; f = 12
- T_t Closed List
 - 1,3
 - 2,3
 - 2,4
 - 2,2
 - 1,4
 - 1,2

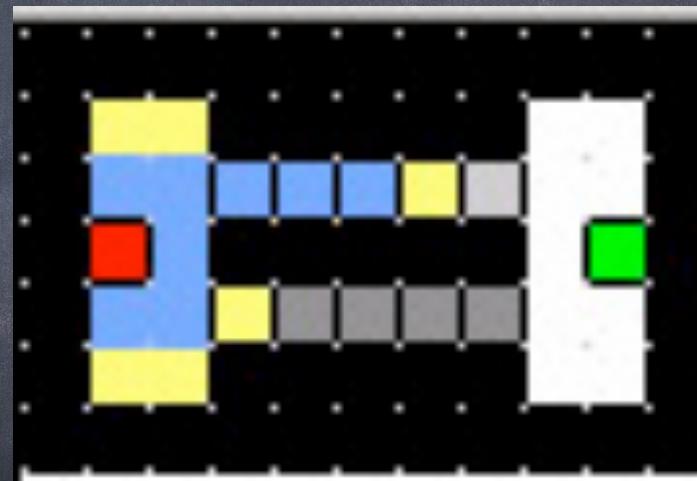


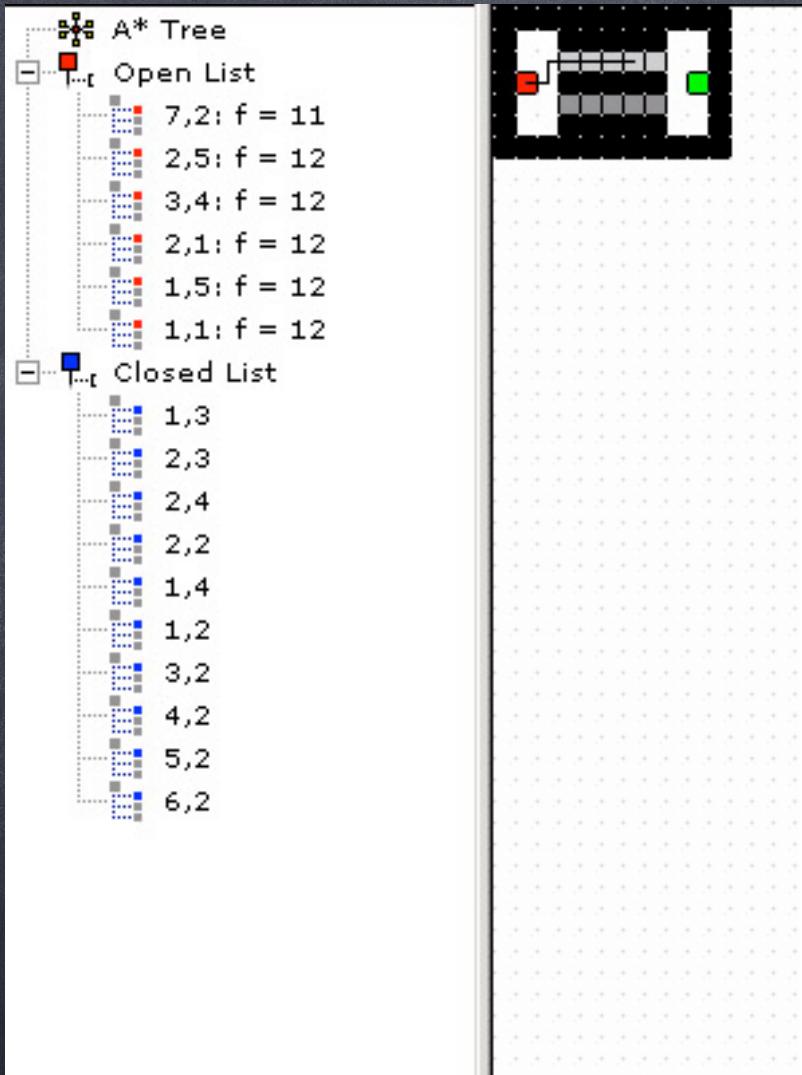
A* Tree	
■	Open List
■	4,2: f = 11
■	2,5: f = 12
■	3,4: f = 12
■	2,1: f = 12
■	1,5: f = 12
■	1,1: f = 12
■	Closed List
■	1,3
■	2,3
■	2,4
■	2,2
■	1,4
■	1,2
■	3,2

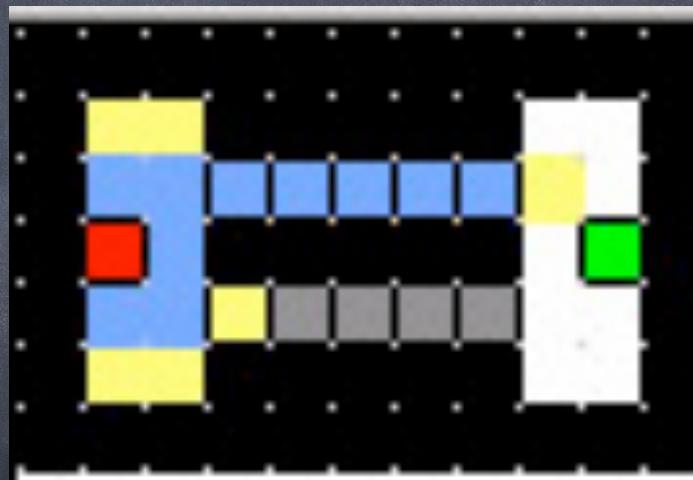
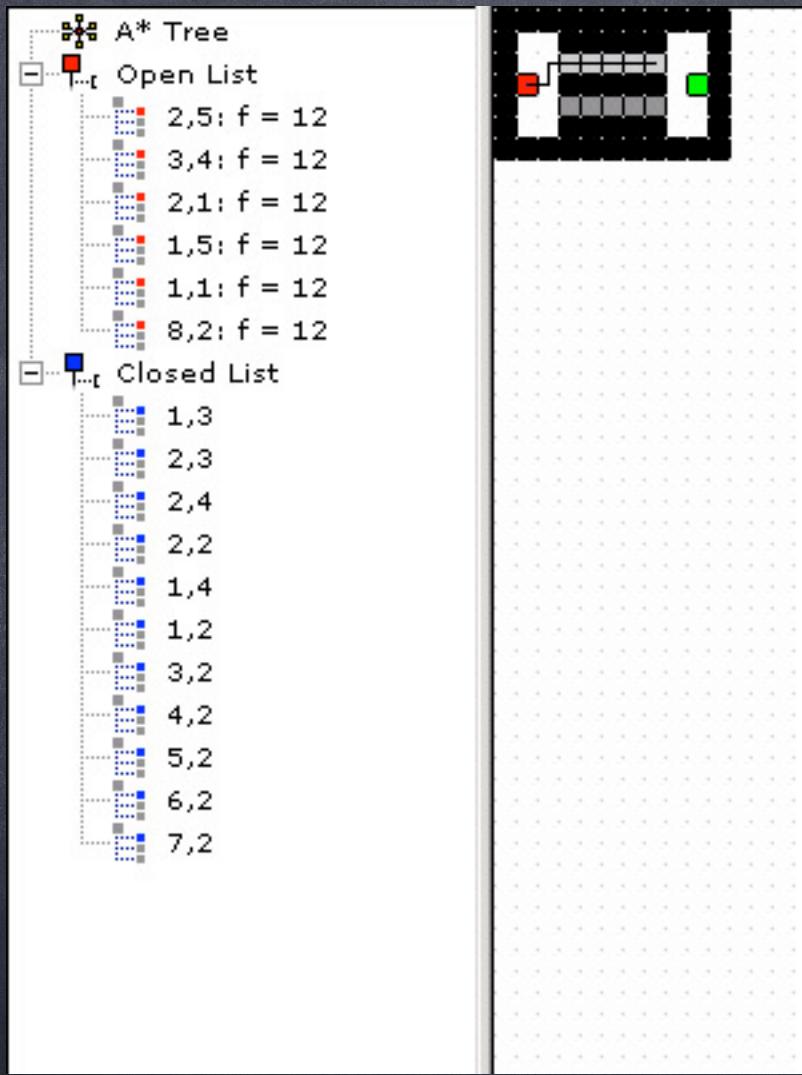


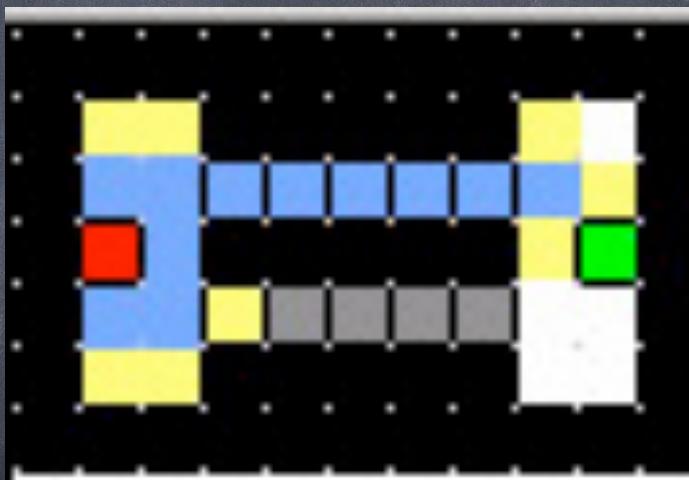
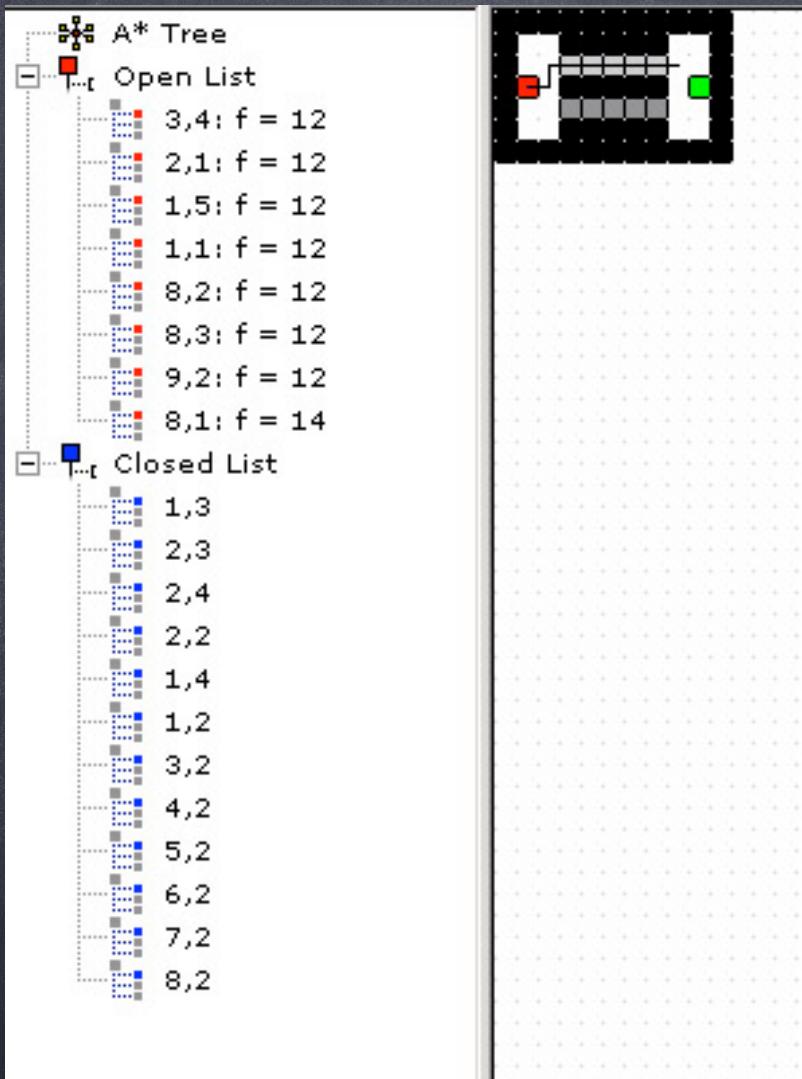


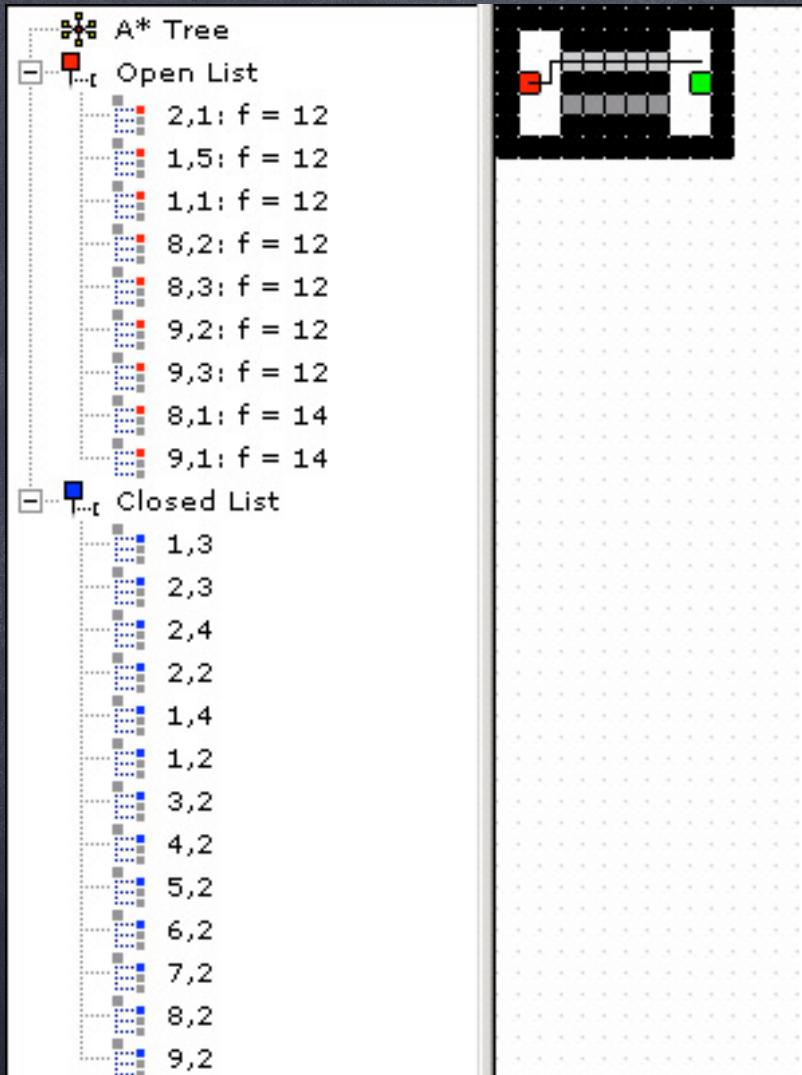
A* Tree	
[-]	T _t Open List
[■]	6,2; f = 11
[■]	2,5; f = 12
[■]	3,4; f = 12
[■]	2,1; f = 12
[■]	1,5; f = 12
[■]	1,1; f = 12
[-]	T _t Closed List
[■]	1,3
[■]	2,3
[■]	2,4
[■]	2,2
[■]	1,4
[■]	1,2
[■]	3,2
[■]	4,2
[■]	5,2

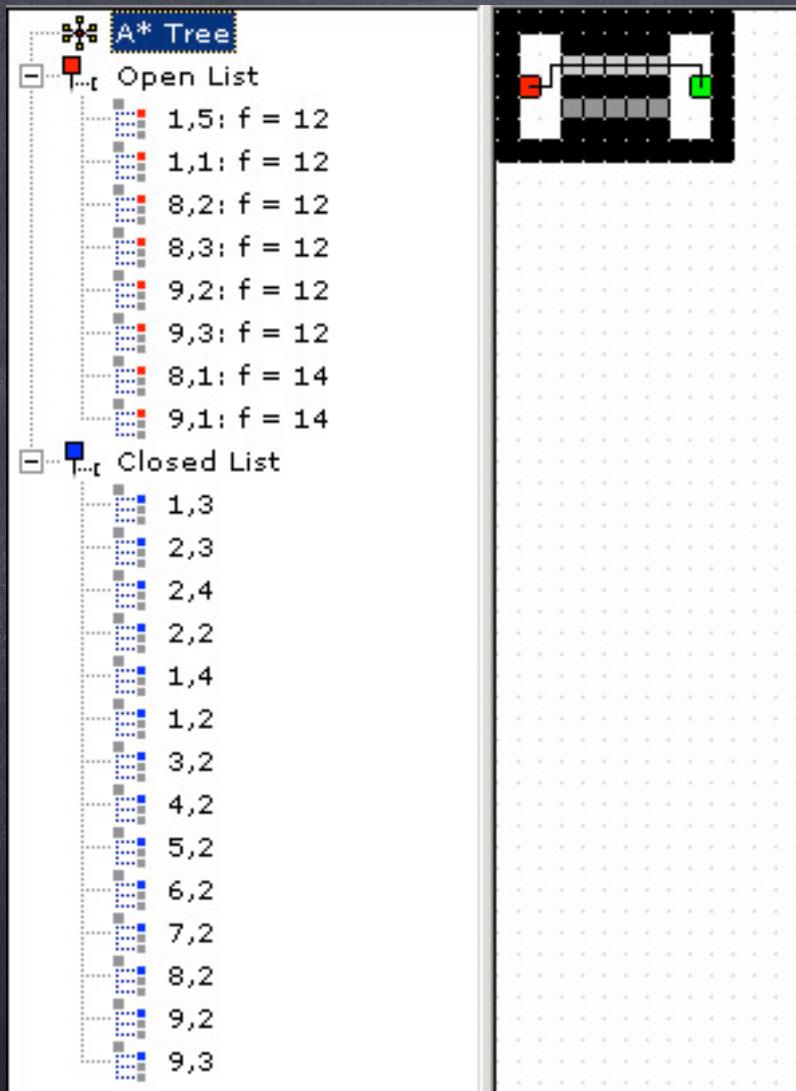








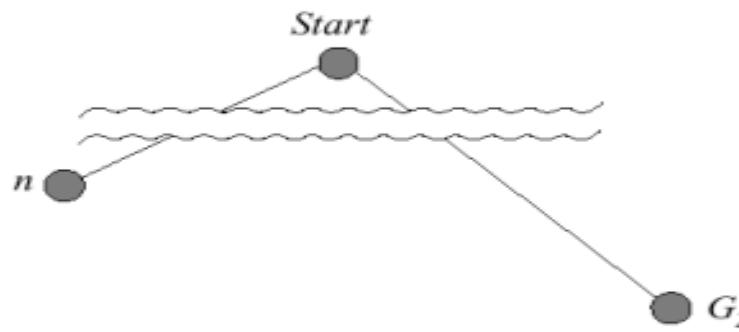




<https://www.youtube.com/watch?v=huJEgJ82360>

Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1 .



$$\begin{aligned}f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\&\geq f(n) && \text{since } h \text{ is admissible}\end{aligned}$$

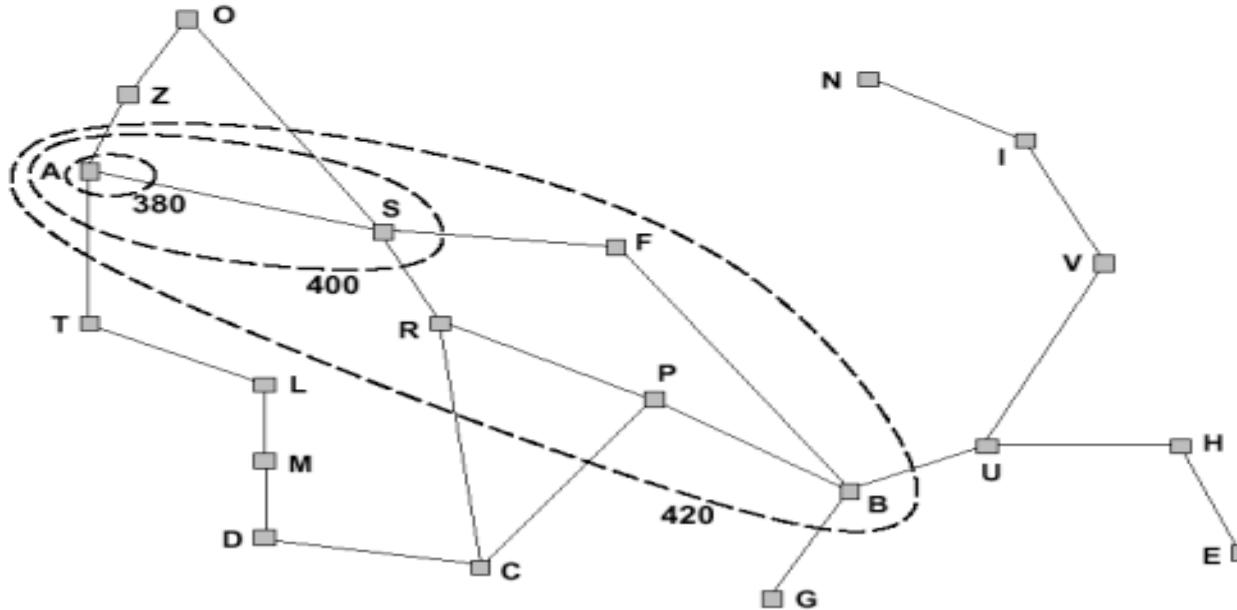
Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful proof)

Lemma: A* expands nodes in order of increasing f value

Gradually adds “ f -contours” of nodes (cf. breadth-first adds layers)

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Memory Problems?

Implement A* with Iterative deepening.

Sizes of Problem Spaces

Problem	Nodes	Brute-Force Search Time (10 million nodes/second)
• 8 Puzzle:	10^5	.01 seconds
• 2^3 Rubik's Cube:	10^6	.2 seconds
• 15 Puzzle:	10^{13}	6 days
• 3^3 Rubik's Cube:	10^{19}	68,000 years
• 24 Puzzle:	10^{25}	12 billion years

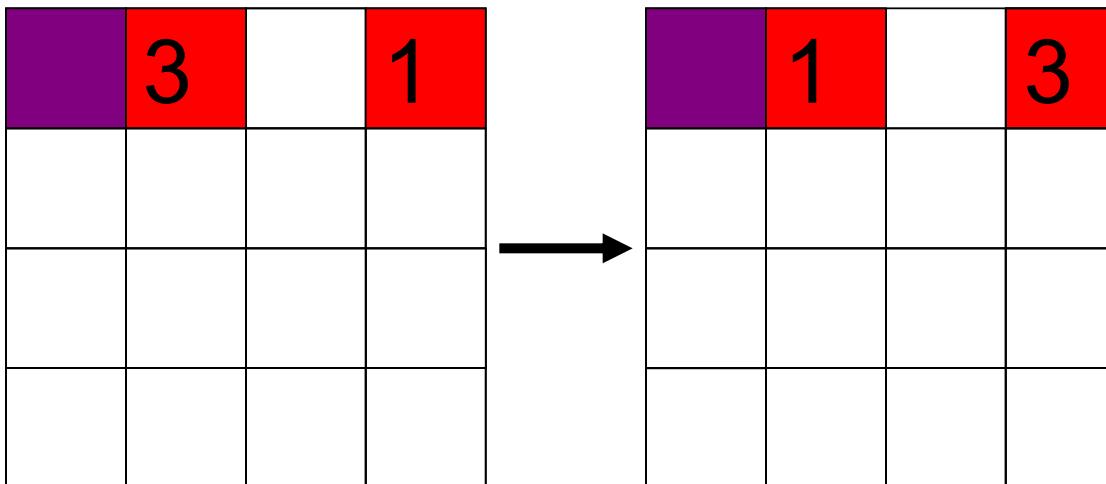
Performance of IDA* on 15 Puzzle

- Random 15 puzzle instances were first solved optimally using IDA* with Manhattan distance heuristic (Korf, 1985).
- Optimal solution lengths average 53 moves.
- 400 million nodes generated on average.
- Average solution time is about 50 seconds on current machines.

Limitation of Manhattan Distance

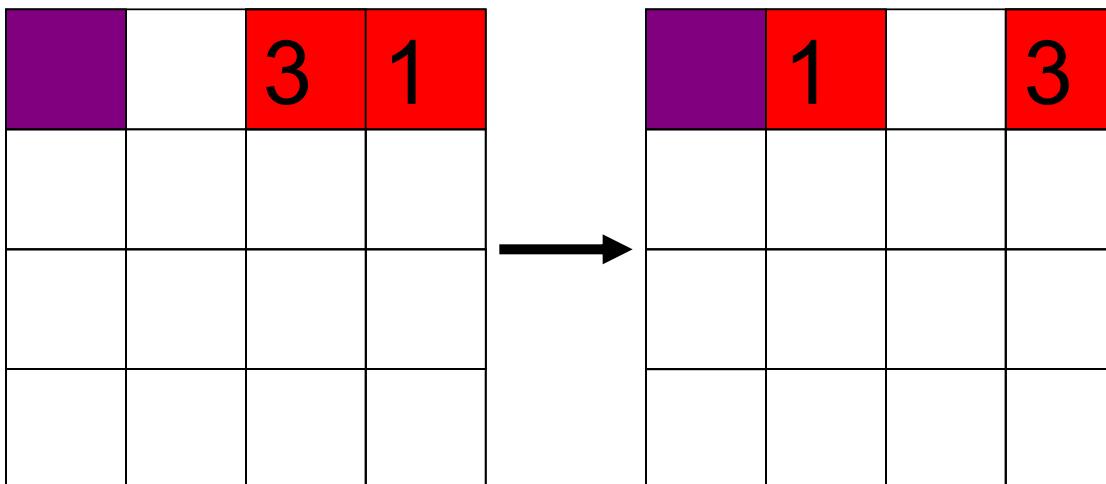
- To solve a 24-Puzzle instance, IDA* with Manhattan distance would take about 65,000 years on average.
- Assumes that each tile moves independently
- In fact, tiles interfere with each other.
- Accounting for these interactions is the key to more accurate heuristic functions.

Example: Linear Conflict



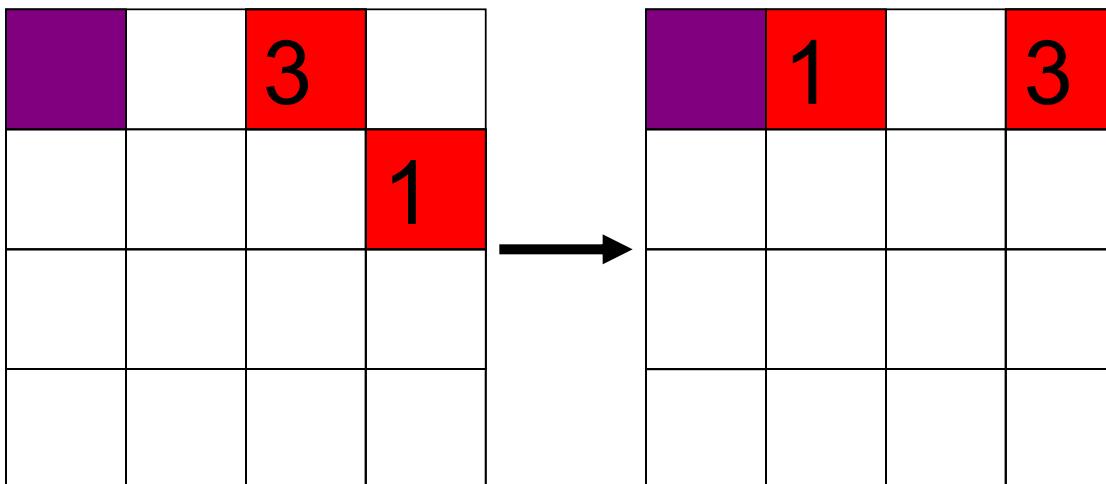
Manhattan distance is $2+2=4$ moves

Example: Linear Conflict



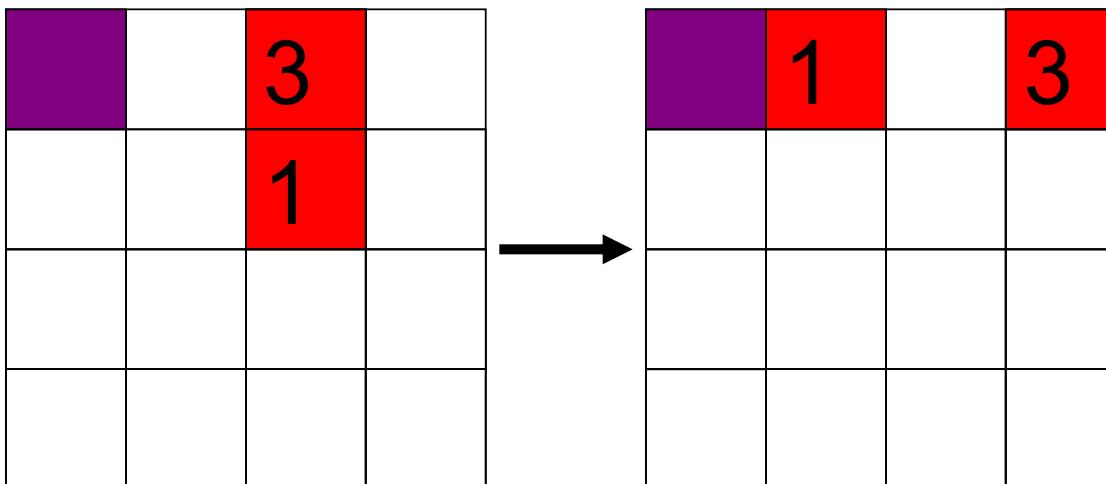
Manhattan distance is $2+2=4$ moves

Example: Linear Conflict



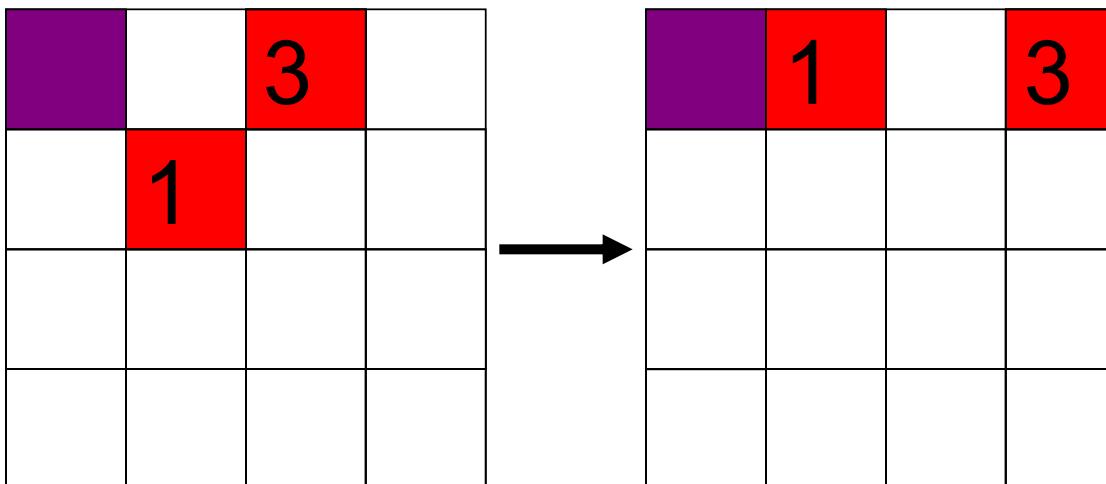
Manhattan distance is $2+2=4$ moves

Example: Linear Conflict



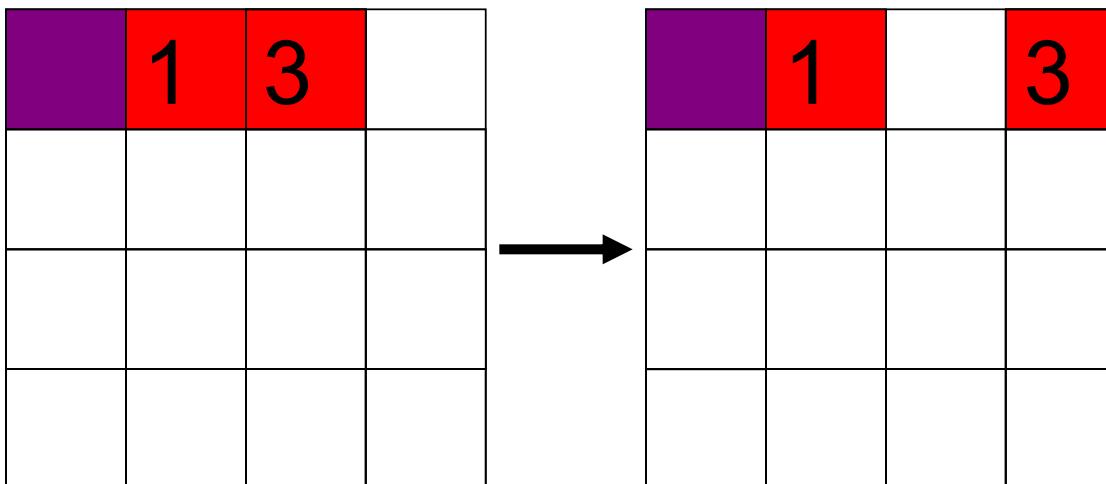
Manhattan distance is $2+2=4$ moves

Example: Linear Conflict



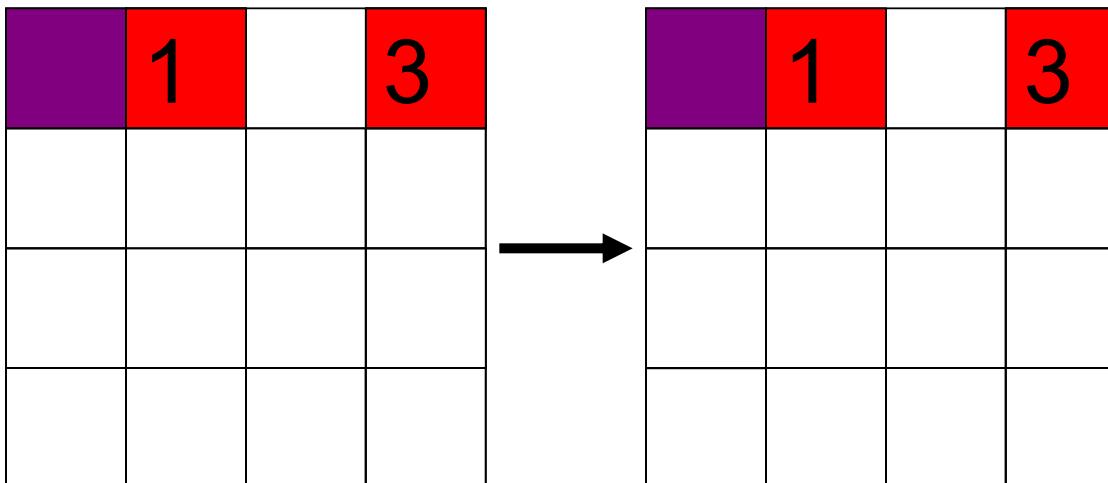
Manhattan distance is $2+2=4$ moves

Example: Linear Conflict



Manhattan distance is $2+2=4$ moves

Example: Linear Conflict



Manhattan distance is $2+2=4$ moves, but linear conflict adds 2 additional moves.

Linear Conflict Heuristic

- Hansson, Mayer, and Yung, 1991
- Given two tiles in their goal row, but reversed in position, additional vertical moves can be added to Manhattan distance.
- Still not accurate enough to solve 24-Puzzle
- We can generalize this idea further.

More Complex Tile Interactions

			14	7
	3			
15		12		
	11		13	



				3
				7
				11
12	13	14	15	

M.d. is 19 moves, but 31 moves are needed.

7	13		
		12	
	15		3
11		14	



			3
			7
			11
12	13	14	15

M.d. is 20 moves, but 28 moves are needed

12			11
	7		14
13		3	
	15		



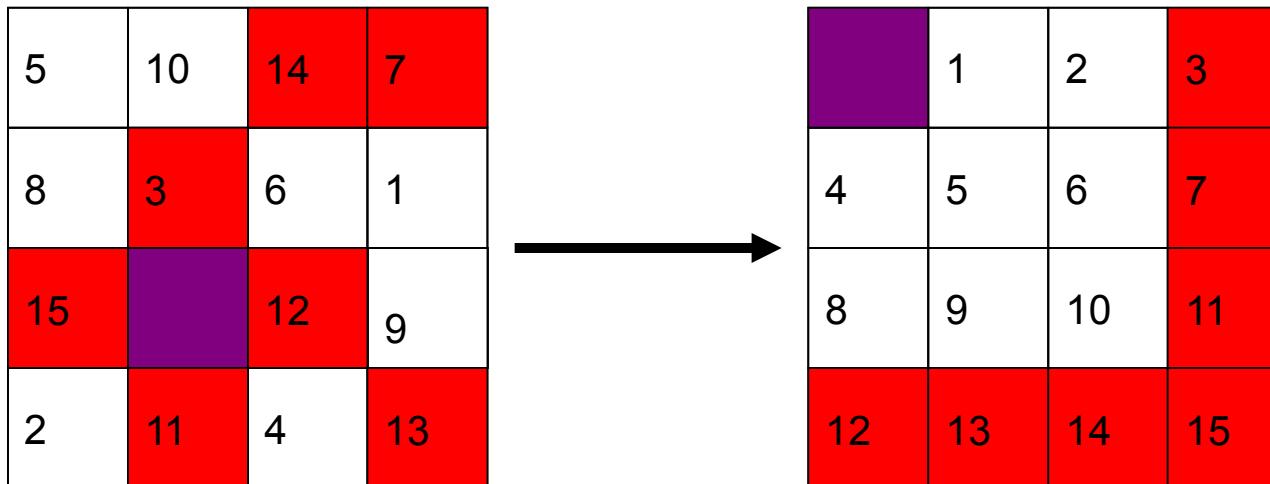
			3
			7
			11
12	13	14	15

M.d. is 17 moves, but 27 moves are needed

Pattern Database Heuristics

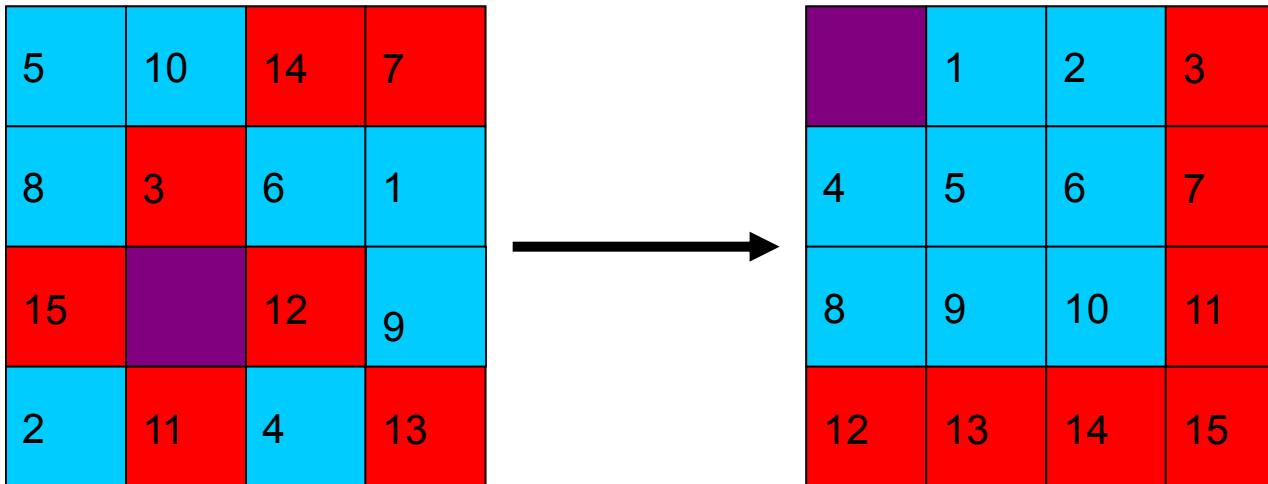
- Culberson and Schaeffer, 1996
- A pattern database is a complete set of such positions, with associated number of moves.
- e.g. a 7-tile pattern database for the Fifteen Puzzle contains 519 million entries.

Heuristics from Pattern Databases



31 moves is a lower bound on the total number of moves needed to solve this particular state.

Combining Multiple Databases



31 moves needed to solve red tiles

22 moves need to solve blue tiles

Overall heuristic is maximum of 31 moves

Additive Pattern Databases

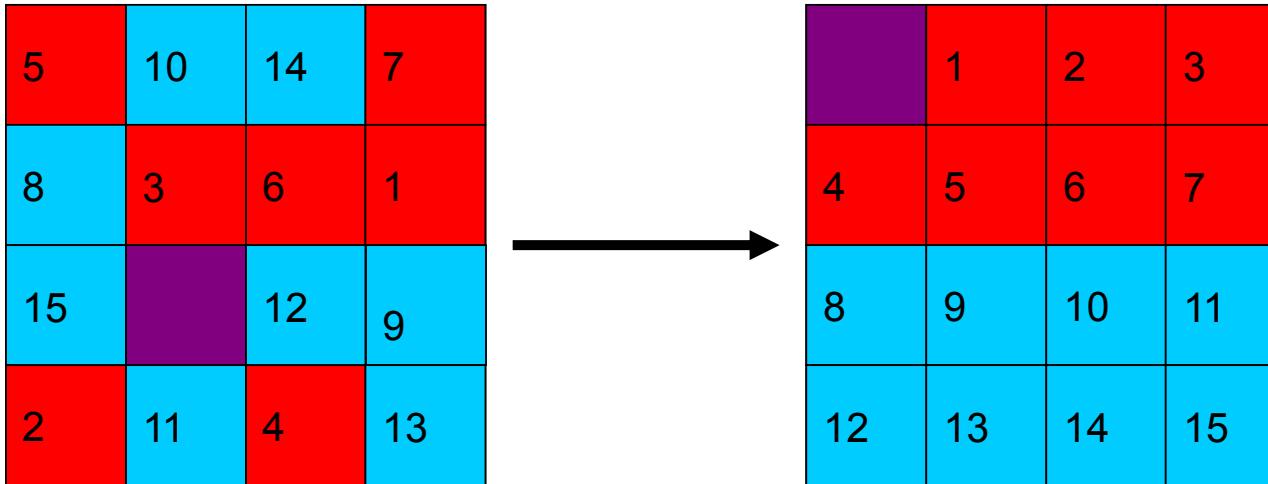
- Culberson and Schaeffer counted all moves needed to correctly position the pattern tiles.
- In contrast, we count only moves of the pattern tiles, ignoring non-pattern moves.
- If no tile belongs to more than one pattern, then we can add their heuristic values.
- Manhattan distance is a special case of this, where each pattern contains a single tile.

Example Additive Databases

	1	2	3
4	5	6	7
8	9	10	11
12	13	15	14

The 7-tile database contains 58 million entries. The 8-tile database contains 519 million entries.

Computing the Heuristic



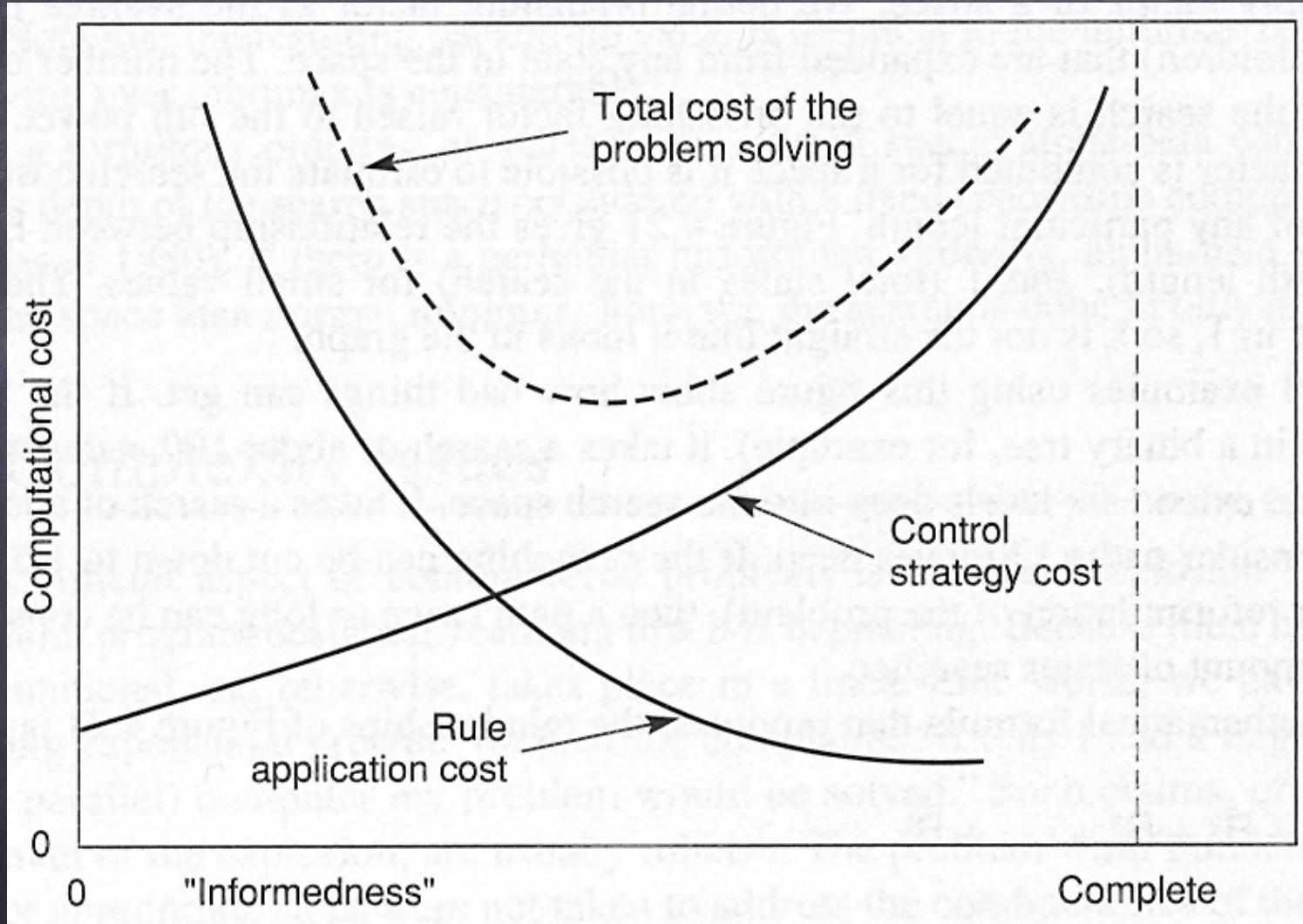
20 moves needed to solve red tiles

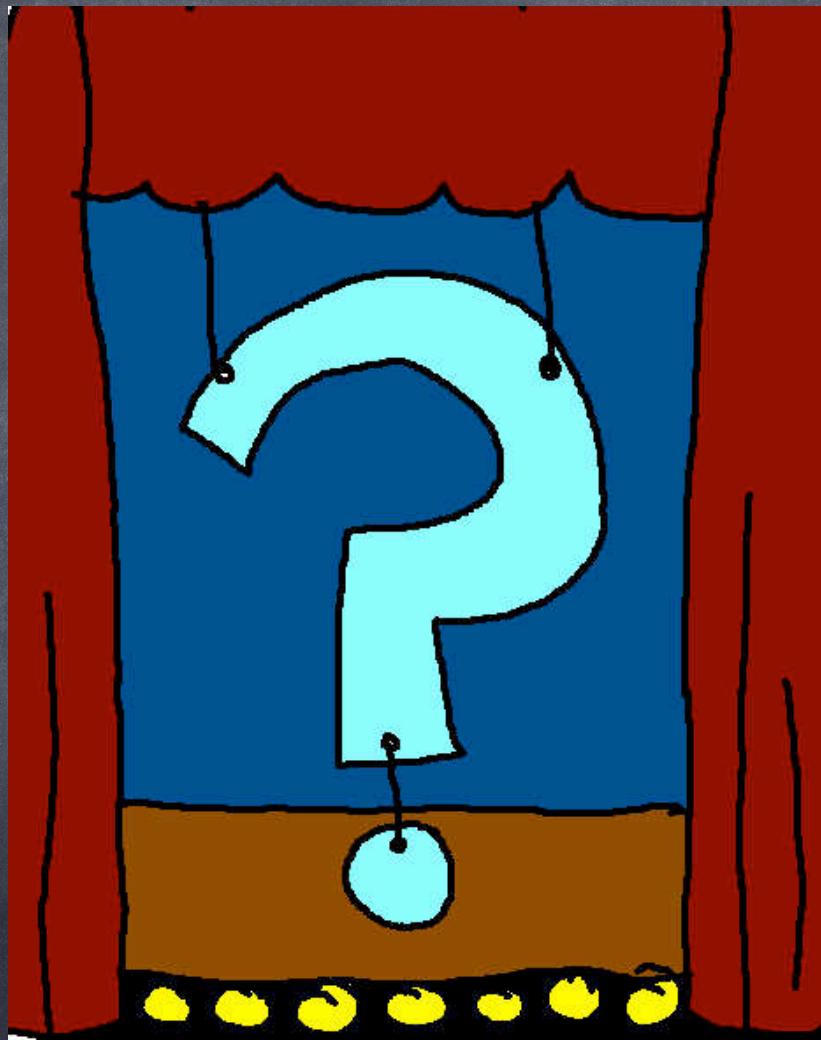
25 moves needed to solve blue tiles

Overall heuristic is sum, or $20+25=45$ moves

Performance on 15 Puzzle

- IDA* with a heuristic based on these additive pattern databases can optimally solve random 15 puzzle instances in less than 29 milliseconds on average.
- This is about 1700 times faster than with Manhattan distance on the same machine.







Read Basic A* essay

Try A*

Study Questions

- ⦿ Distinguish between blind and informed search algorithms.
- ⦿ Explain the three main problems with Hill Climbing. How are they solved by A*?
- ⦿ What is an admissible heuristic?