for simplicity + neatness, I write just $\sum$ for $\sum_{i=1}^{N}$

ANALYTICS HW 1 Q6

1. $m(a+bX) =$

$\frac{1}{N}\sum(a+bx_i) =$

$\frac{1}{N}(Na + \sum bx_i) =$

$\frac{1}{N}(Na + Nb\sum x_i) =$

$a + b\sum x_i =$

$a + b\,m(X)$

2. $cov(X,X) =$

$\frac{1}{N}\sum(x_i - m(X))(x_i - m(X)) =$

$\frac{1}{N}\sum(x_i - m(X))^2 =$

$s^2$

3. $cov(X, a+bY) =$

$\frac{1}{N}\sum(x_i - m(X))(a+by_i - m(a+bY)) =$

$\frac{1}{N}\sum(x_i - m(X))(a+by_i - (a+bm(Y)) =$

$\frac{1}{N}\sum(x_i - m(X))(by_i - bm(Y)) =$

$\frac{1}{N}\sum b(x_i - m(X))(y_i - m(Y)) =$

$b \cdot \frac{1}{N}\sum(x_i - m(X))(y_i - m(Y)) =$

$b\,cov(X,Y)$

4. $cov(a+bX, a+bY) =$

$\frac{1}{N}\sum(a+bx_i - m(a+bX))(a+by_i - m(a+bY)) =$

$\frac{1}{N}\sum(a+bx_i - (a+bm(X))(a+by_i - (a+bm(Y)) =$

$\frac{1}{N}\sum b(x_i - m(X))b(y_i - m(Y)) =$

$b^2 \cdot \frac{1}{N}\sum(x_i - m(X))(y_i - m(Y)) =$

$b^2\,cov(X,Y)$

5. Yes. It is true that $med(a+bX) = a+b\,med(X)$. No. It is not true that $IQR(a+bX) = a+b\,IQR(X)$. That is to say that $IQR(a+bX) \neq a+b\,IQR(X)$.

6. let $X = \{1,4,9\}$

then $X^2 = \{1,16,81\}$

and $\sqrt{X} = \{1,2,3\}$

$m(X) = \frac{1}{3}(1+4+9) = \frac{14}{3}$

$m(X^2) = \frac{1}{3}(1+16+81) = \frac{98}{3}$

$(m(X))^2 = (14/3)^2 = 196/9$

$98/3 \neq 196/9$

$\therefore m(X^2) \neq (m(X))^2$

$m(\sqrt{X}) = \frac{1}{3}(1+2+3) = \frac{6}{3} = 2$

$\sqrt{m(X)} = \sqrt{14/3}$

$2 \neq \sqrt{14/3}$

$\therefore m(\sqrt{X}) \neq \sqrt{m(X)}$