Consider a point mass in the 3D workspace which is required to move to its target under its own instantaneous accelerations. Assume that there is one spherical obstacle within the workspace. Derive the acceleration components to enable the point mass to move to its target, where it will stop, whilst successfully avoiding the spherical obstacle.

System of ODE

$$\begin{aligned}\dot{x} &= v_1, \quad \dot{y} = v_2, \quad \dot{z} = v_3,\\ \dot{v}_1 &= \sigma_1, \quad \dot{v}_2 = \sigma_2, \quad \dot{v}_3 = \sigma_3.\end{aligned}$$

where (x, y, z) is the position of the point mass, (v_1, v_2, v_3) are its velocities and $(\sigma_1, \sigma_2, \sigma_3)$ are the acceleration components.

Target Attraction

The target with center (τ_1, τ_2, τ_3) and radius r_T. For the target attraction, we consider the potential function

$$V = \frac{1}{2}\left[(x-\tau_1)^2 + (y-\tau_2)^2 + (z-\tau_3)^2 + v_1^2 + v_2^2 + v_3^2\right].$$

Avoidance of fixed Obstacle

A spherical obstacle with center (o_1, o_2, o_3) and radius r_O. For its avoidance, we consider the avoidance function

$$W = \frac{1}{2}\left[(x-o_1)^2 + (y-o_2)^2 + (z-o_3)^2 - r_O^2\right].$$

Lyapunov Function

$$L = V + \frac{\beta F}{W}$$

where $\beta > 0$ is the control parameter while

$$F = \frac{1}{2}\left[(x-\tau_1)^2 + (y-\tau_2)^2 + (z-\tau_3)^2\right]$$

is an auxiliary function required for the Lyapunov function to vanish at the target.

Controllers

$$\begin{aligned}\dot{L} &= \frac{\partial L}{\partial x}\dot{x} + \frac{\partial L}{\partial y}\dot{y} + \frac{\partial L}{\partial z}\dot{z} + \frac{\partial L}{\partial v_1}\dot{v}_1 + \frac{\partial L}{\partial v_2}\dot{v}_2 + \frac{\partial L}{\partial v_3}\dot{v}_3\\ &= \frac{\partial L}{\partial x}v_1 + \frac{\partial L}{\partial y}v_2 + \frac{\partial L}{\partial z}v_3 + v_1\sigma_1 + v_2\sigma_2 + v_3\sigma_3\\ &= \left(\frac{\partial L}{\partial x} + \sigma_1\right)v_1 + \left(\frac{\partial L}{\partial y} + \sigma_2\right)v_2 + \left(\frac{\partial L}{\partial z} + \sigma_3\right)v_3\end{aligned}$$

$\dot{L}$ can be made non-positive by letting

$$\begin{aligned}\frac{\partial L}{\partial x} + \sigma_1 &= -\delta_1 v_1\\ \frac{\partial L}{\partial y} + \sigma_2 &= -\delta_2 v_2\\ \frac{\partial L}{\partial z} + \sigma_3 &= -\delta_3 v_3\end{aligned}$$

where $\delta_1 > 0,\ \delta_2 > 0,\ \delta_3 > 0$ are called the convergence parameter. Then the acceleration controllers are

$$\begin{aligned}\sigma_1 &= -\delta_1 v_1 - \frac{\partial L}{\partial x}\\ \sigma_2 &= -\delta_2 v_2 - \frac{\partial L}{\partial y}\\ \sigma_3 &= -\delta_3 v_3 - \frac{\partial L}{\partial z}\end{aligned}$$