# CS170 Computation Theory

## Lecture 6

September 21, 2023

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# Review: Formal Definition of TM

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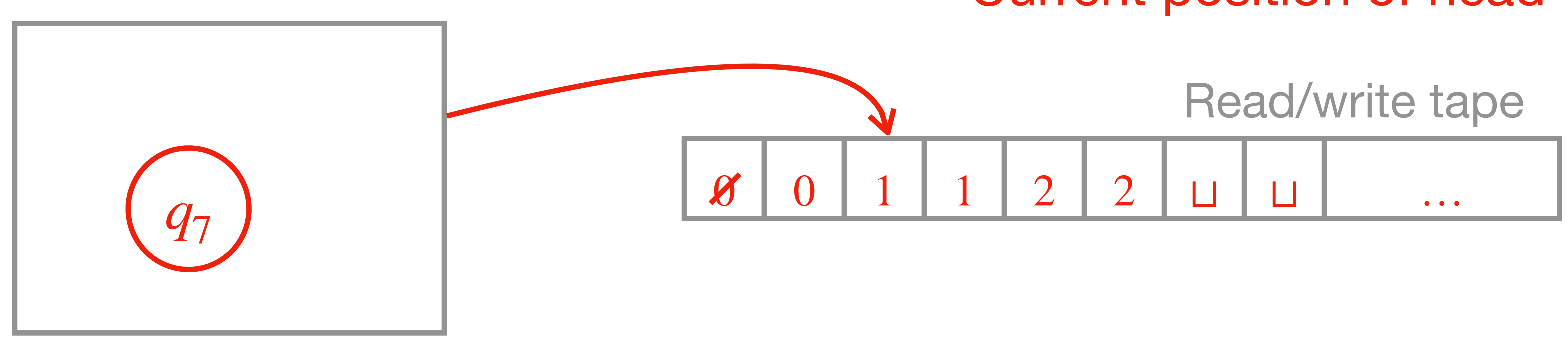
Configuration =
- Current state,
- Current tape contents,
- Current position of head




Control

"$\emptyset 0 q_7 1122$"

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E.g., $\delta(q_7, 1) = (q_4, \cancel{1}, \text{R})$

$\boxed{q_{\text{accept}} \in Q\text{, an accept state}}$

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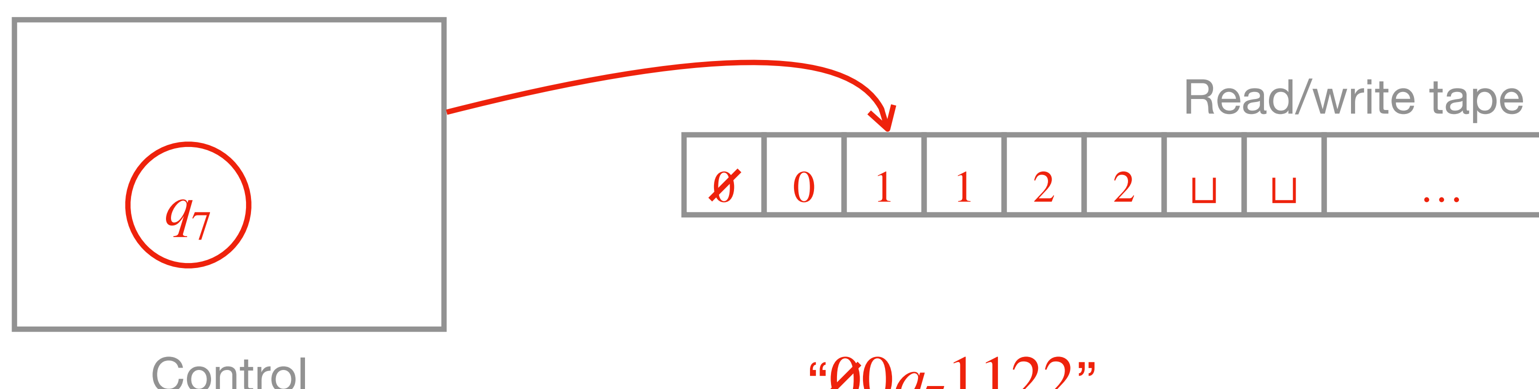


“$\cancel{0}0q_71122$”

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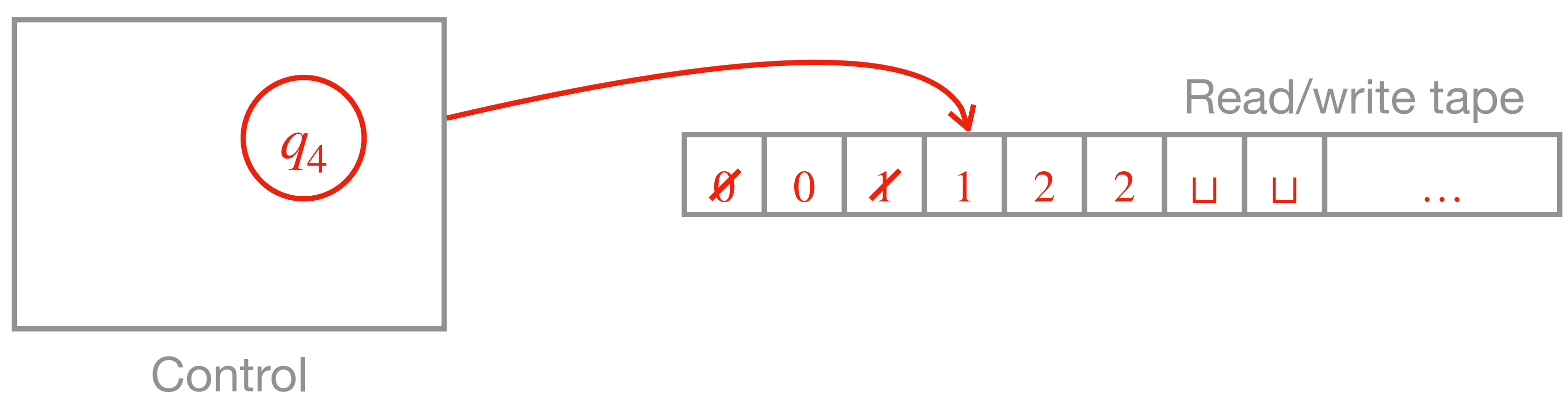

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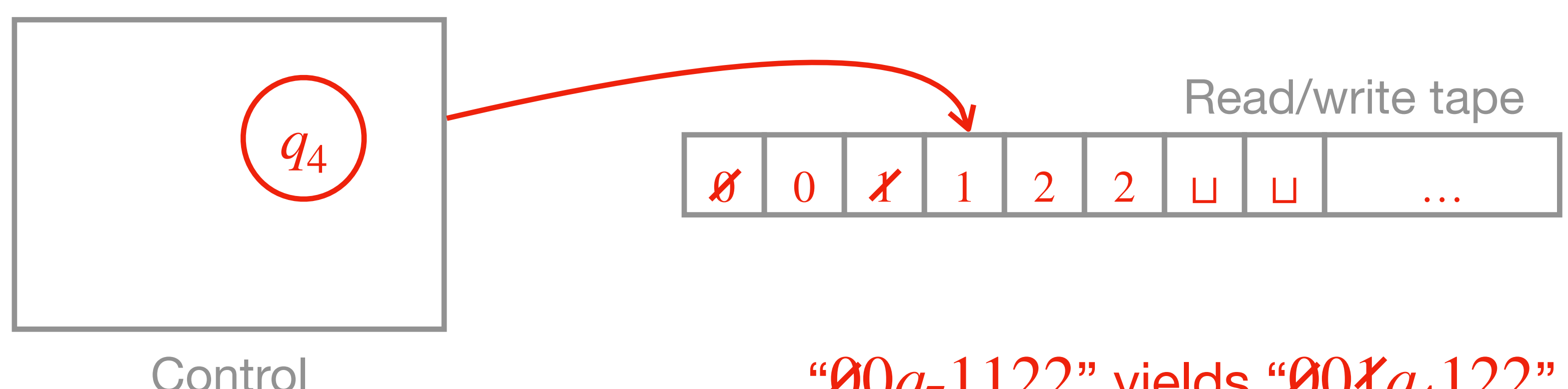


"$\cancel{0}0q_7 1122$" yields "$\cancel{0}0\cancel{1}q_4 122$"

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**Definition (p. 170):** A language is <u>T-recognizable</u> if there is a TM that recognizes it.

# Review: T-decidability

The TM $M$ has 3 possible outcomes for input $w$:

- Accept (by entering $q_{\text{accept}}$)
- Reject by entering $q_{\text{reject}}$
- Reject by looping (running forever)

**Definition (p. 170):** A TM $M$ is a decider if it halts on all inputs.

**Definition (p. 170):** A language $A$ is T-decidable (or just decidable) if $A = L(M)$ for some TM decider $M$.

# Today’s Topics

- ~~Pumping Lemma for CFLs~~
- ~~Turing Machines (TMs)~~
- ~~T-recognizability and T-decidability~~

- Robustness of TMs
- Church-Turing Thesis

# Robustness of TM Model

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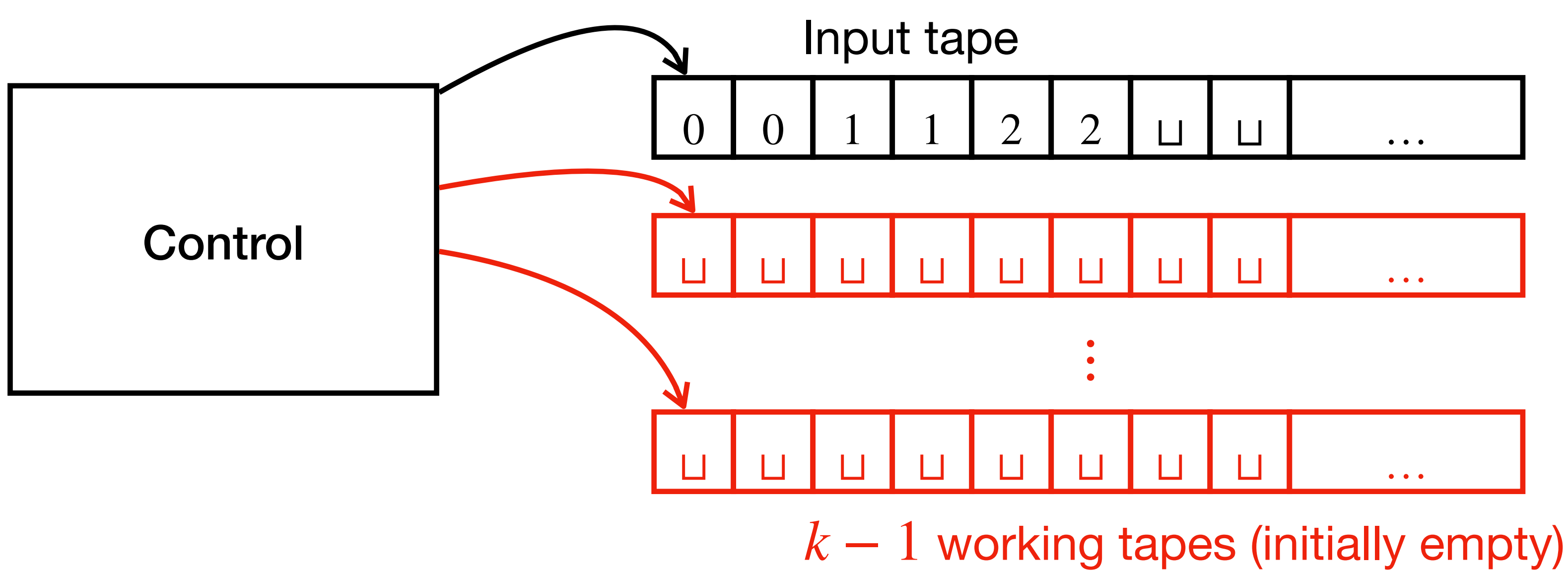
- Captures our intuition of computation
- Mathematically precise
- Simple
- <u>Robust</u>: equivalent to other TM variants (e.g., TMs with multiple tapes, Nondeterministic TMs)

# Multi-tape TMs




**Definition (p. 176):** A multi-tape Turing Machine is a TM with $k$ tapes. The transition function $\delta : Q \times \Gamma^k \mapsto Q \times \Gamma^k \times \{\mathsf{L}, \mathsf{R}\}^k$ allows reading, writing, and moving the heads on some or all of the tapes.

**Theorem:** The language $A$ is T-recognizable iff some multi-tape TM recognizes it.

# Proof of Equivalence of Multi-tape TMs and TMs

We must prove both directions:

- $A$ is T-recognizable $\Longrightarrow$ multi-tape TM recognizes $A$.
- Multi-tape TM recognizes $A \Longrightarrow A$ is T-recognizable.

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  Need to show.

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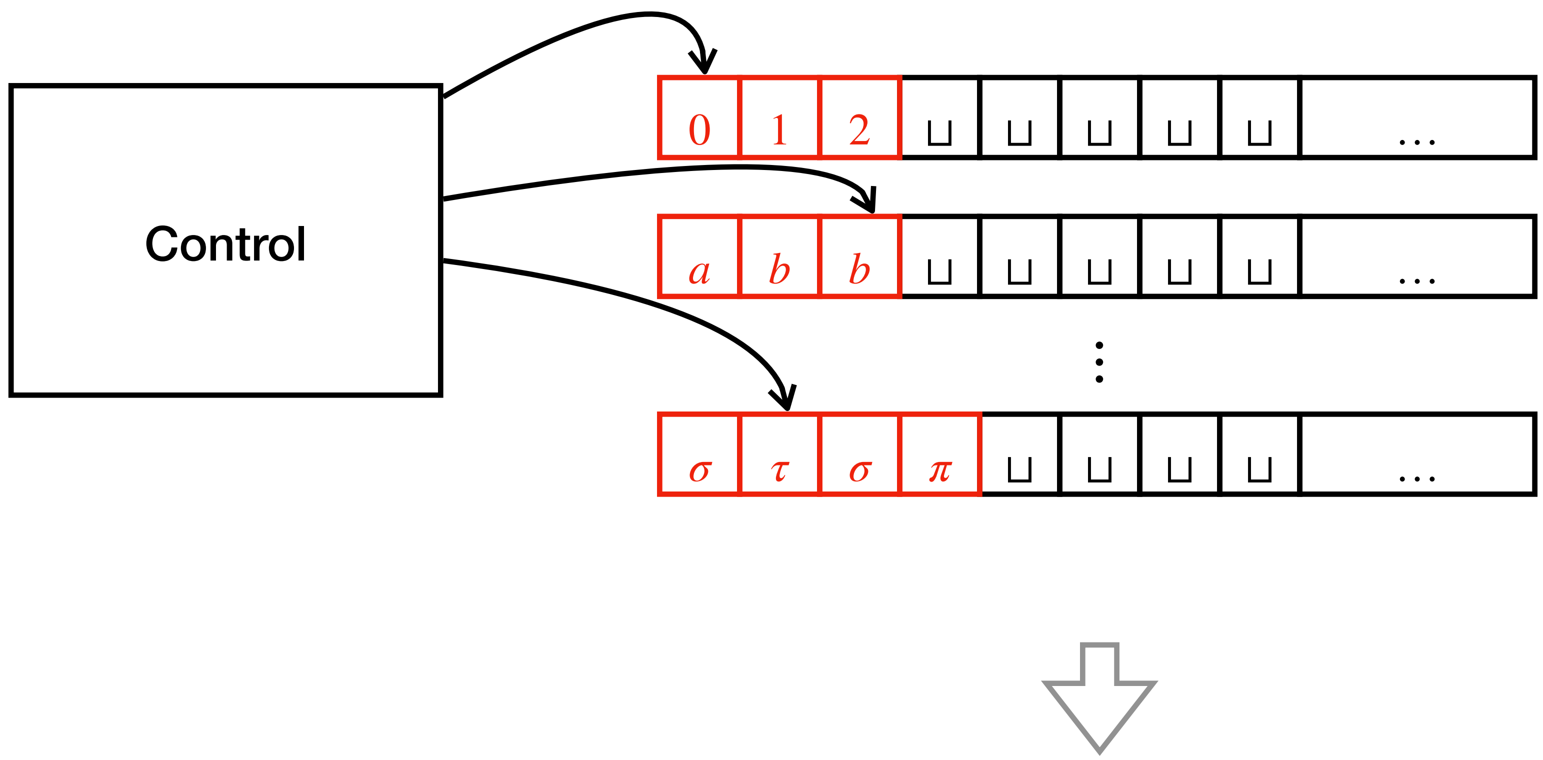
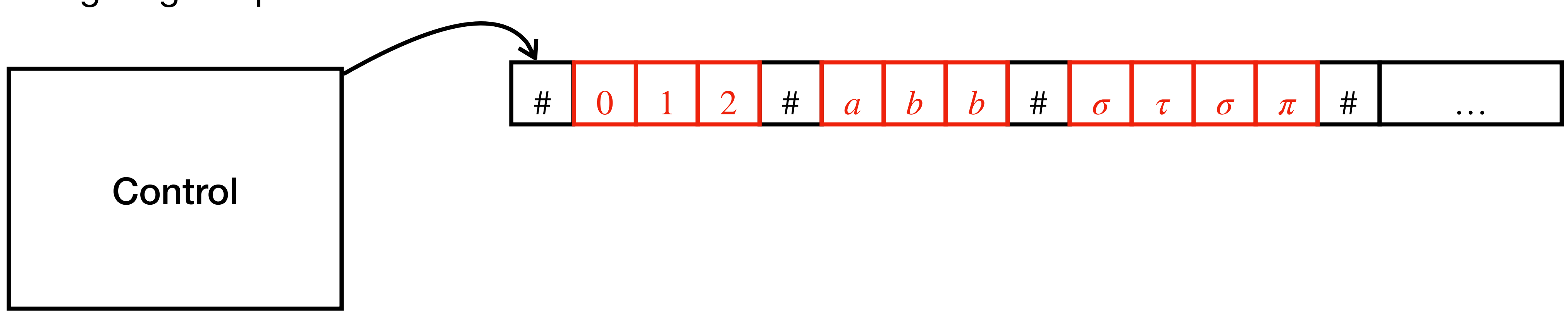
Multi-tape TM $M$:




"Simulate" $M$ using single-tape TM $M'$:



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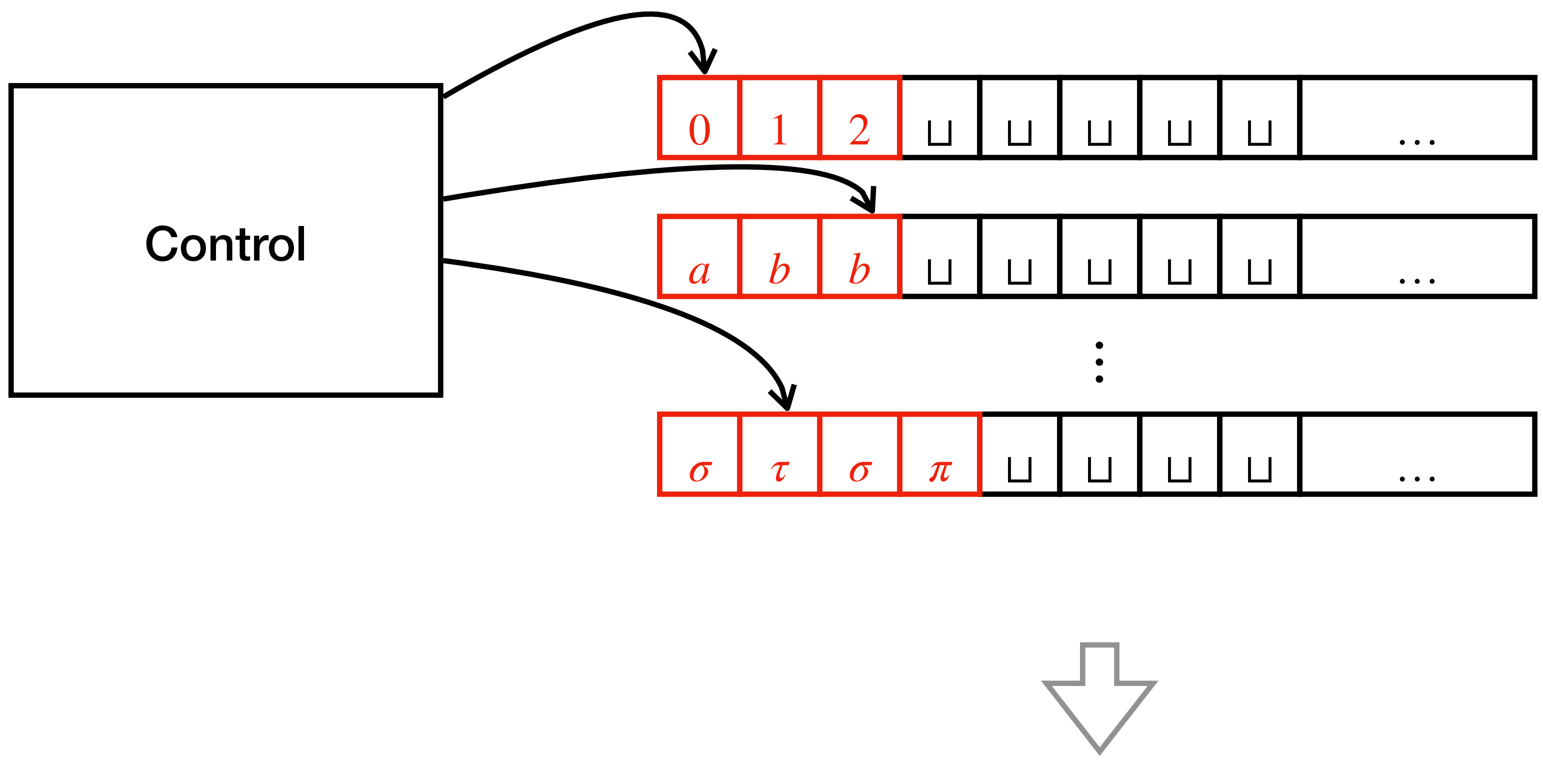


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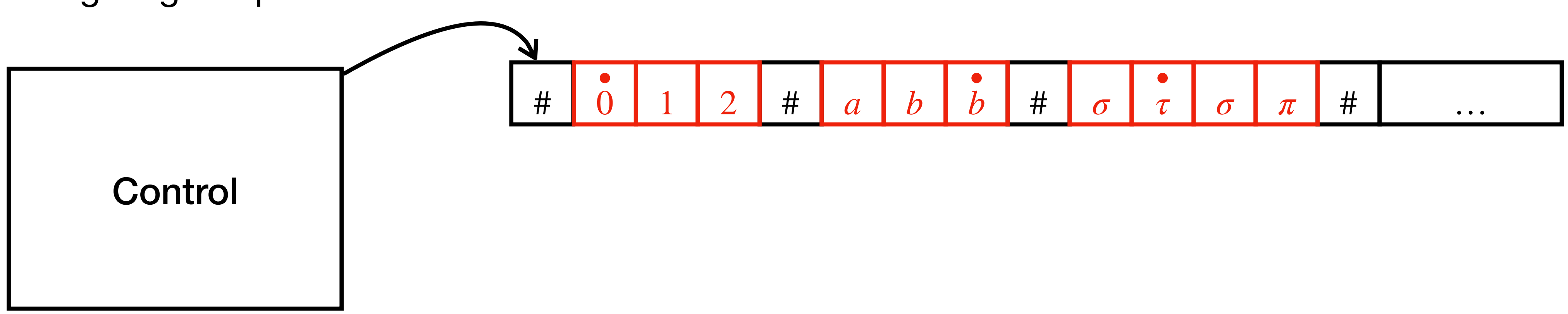
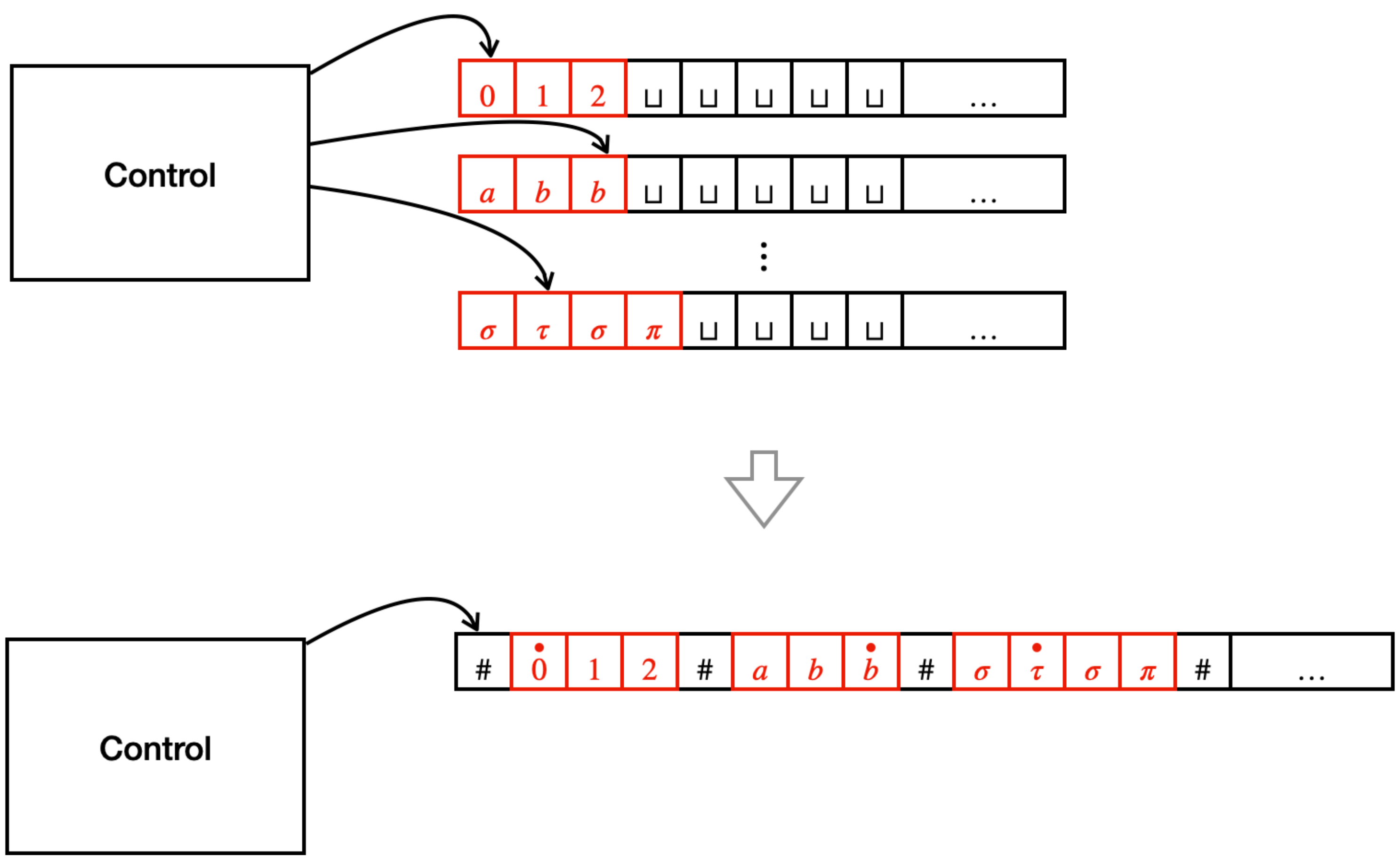


# Description of $M'$



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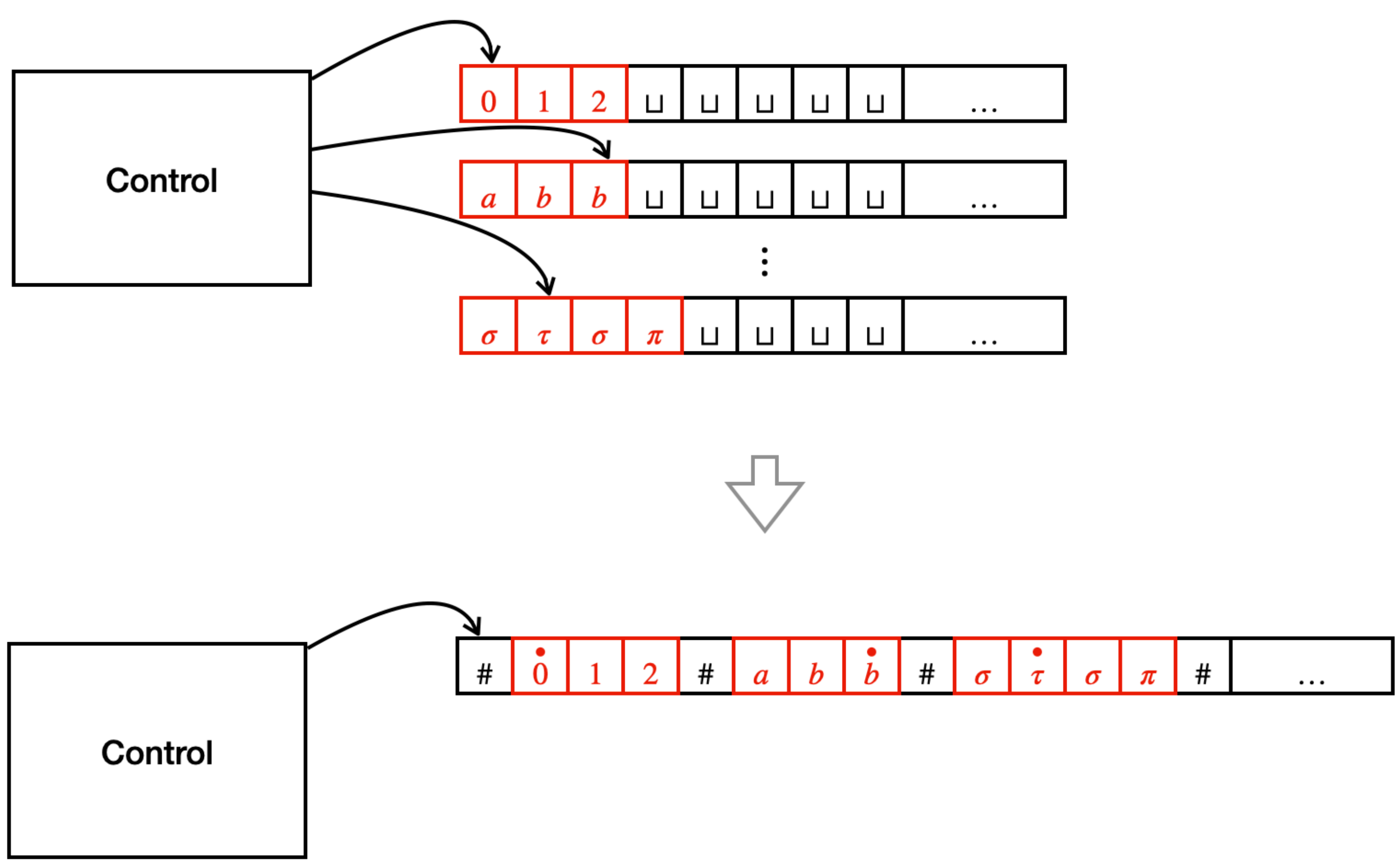

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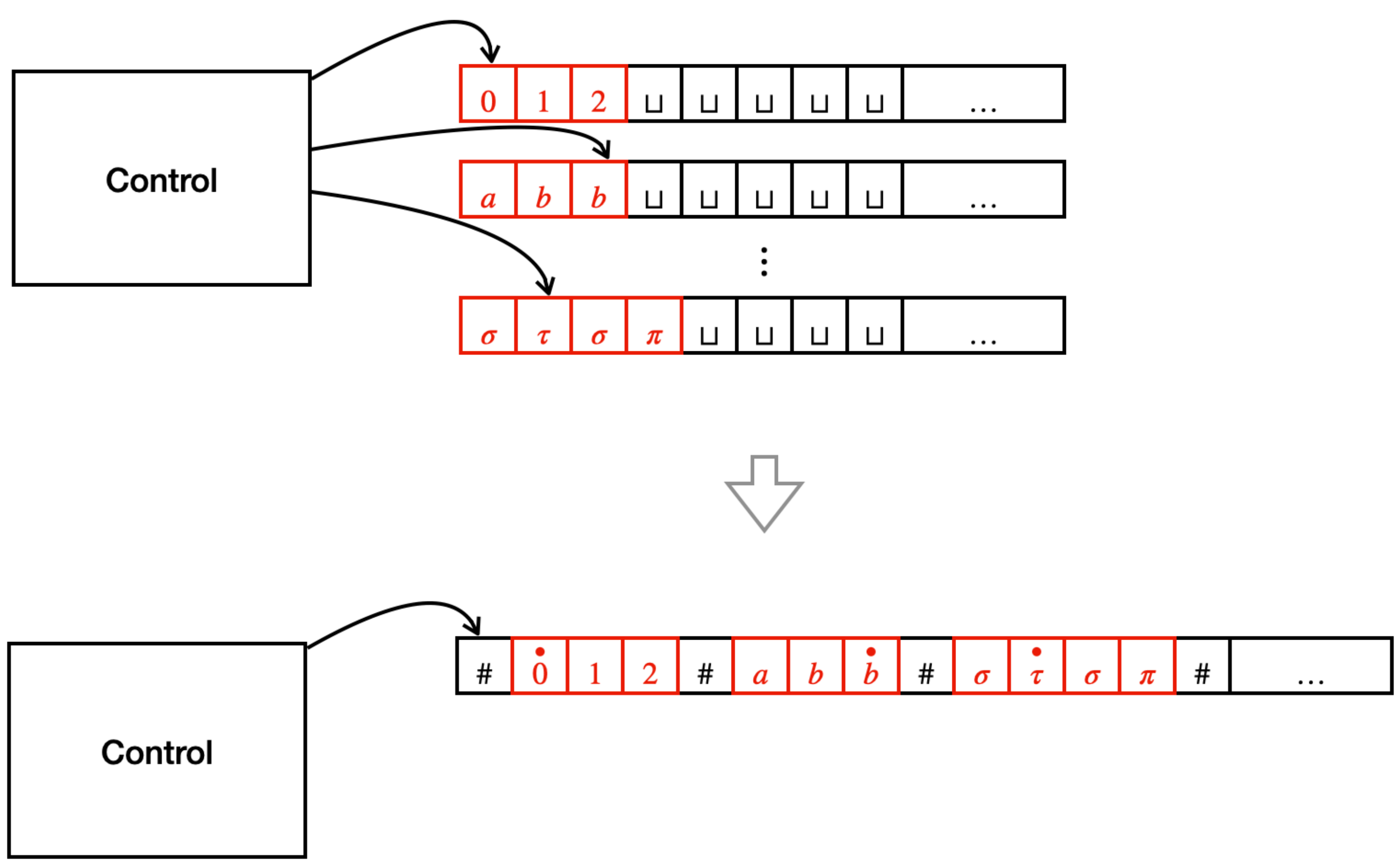


$M' =$ "On input $w = w_1 w_2 \ldots w_n$:

1. Initiate tape: $\# \dot{w}_1 w_2 \ldots w_n \# \dot{\sqcup} \# \dot{\sqcup} \# \ldots \#$

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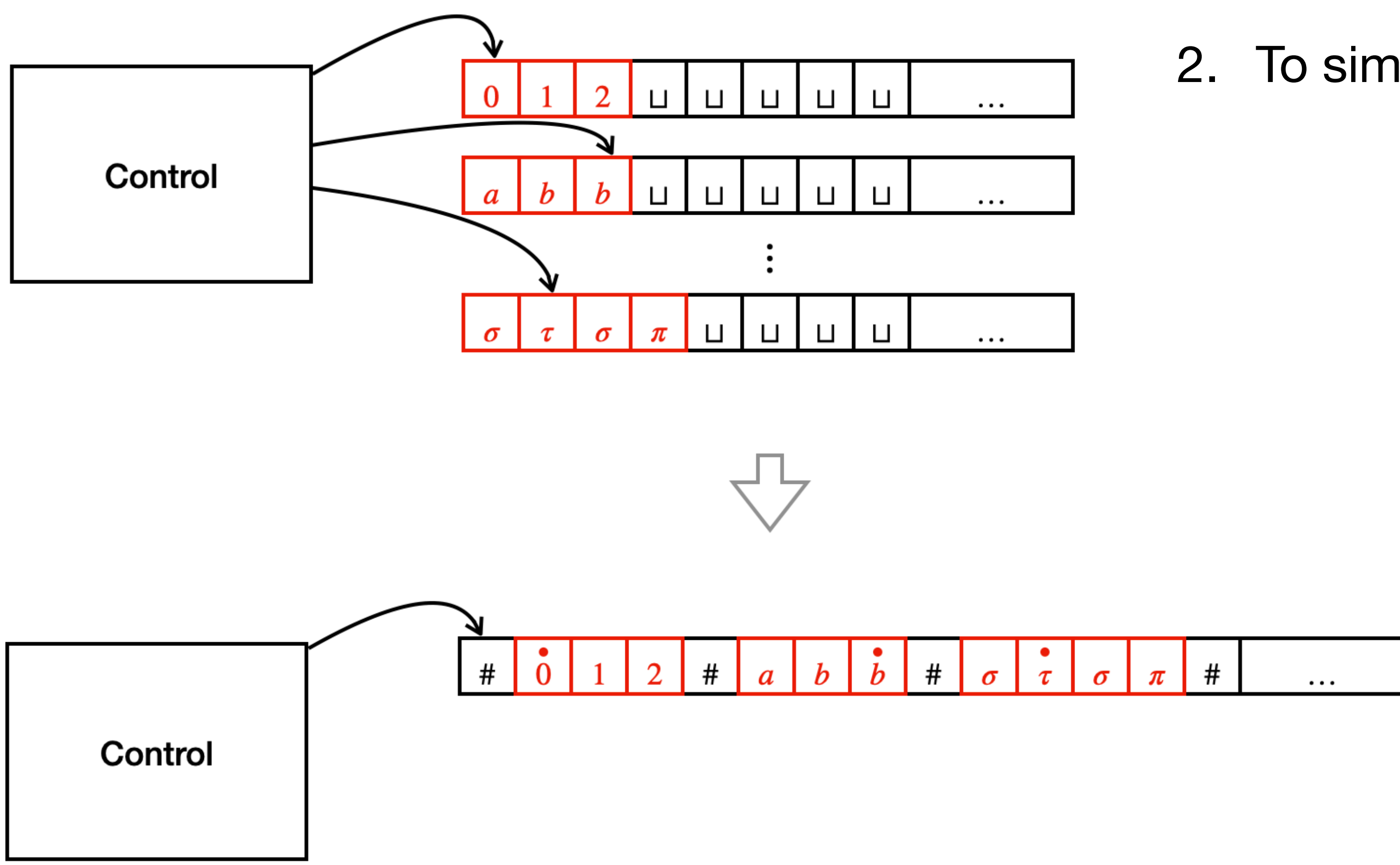

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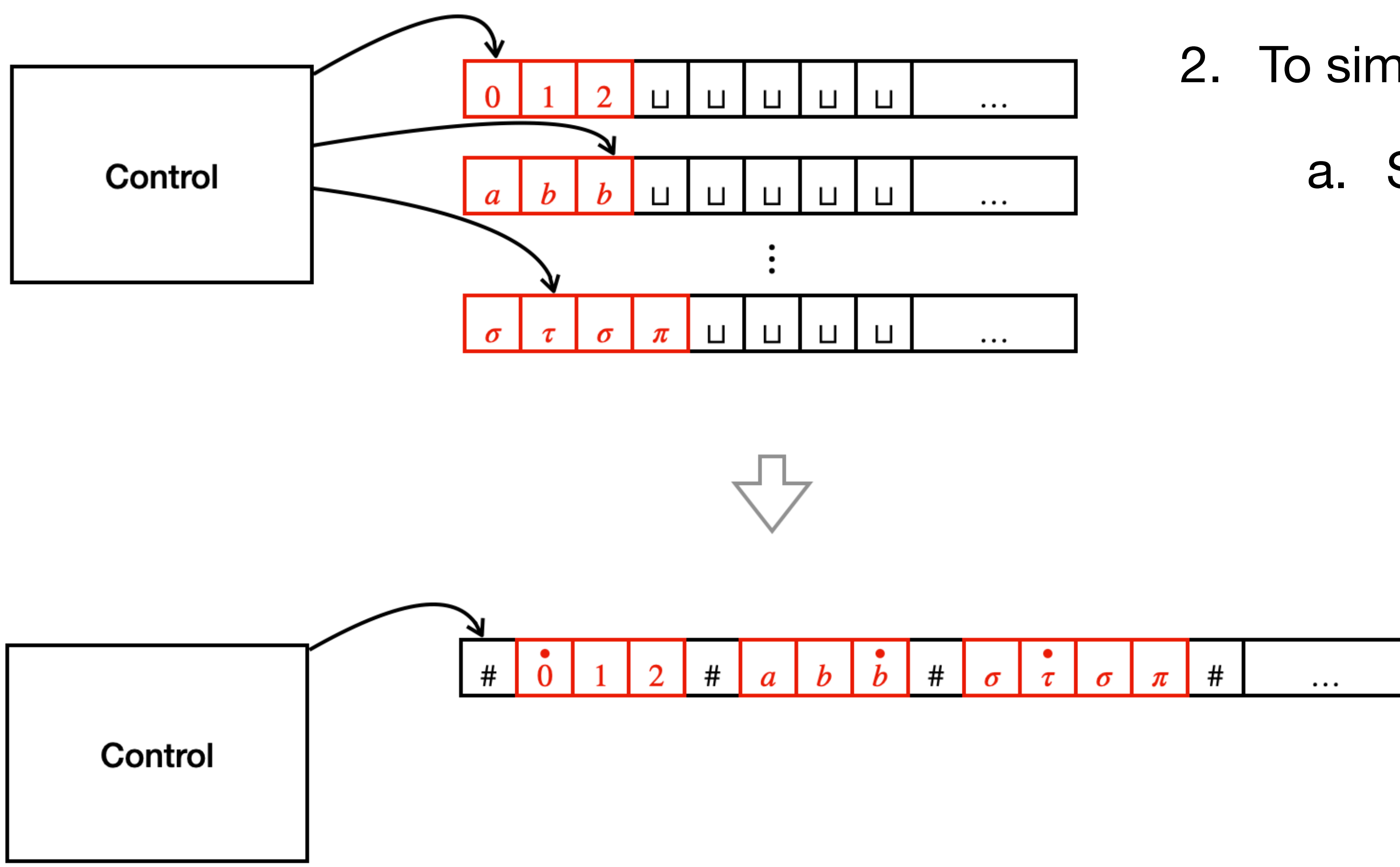


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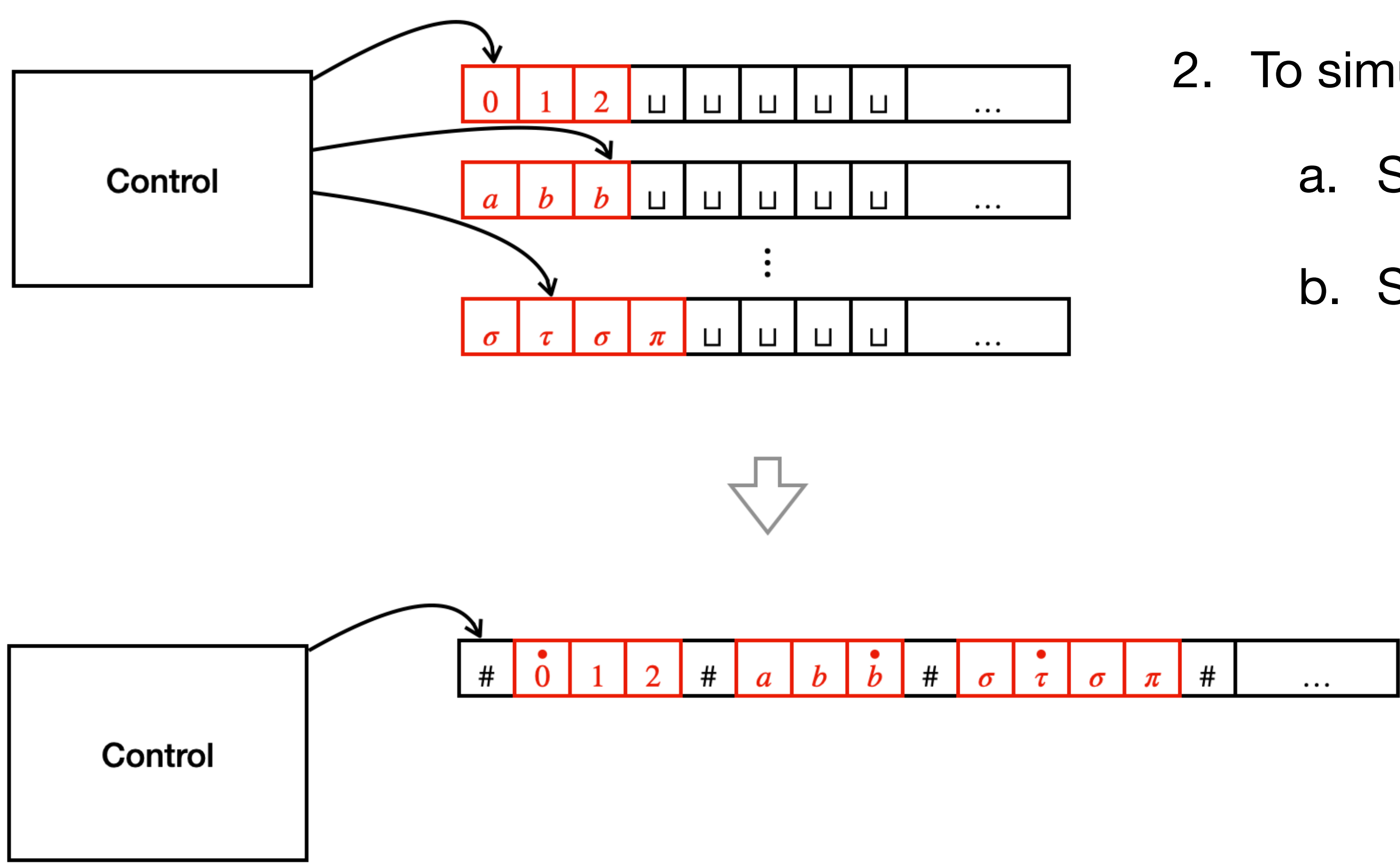


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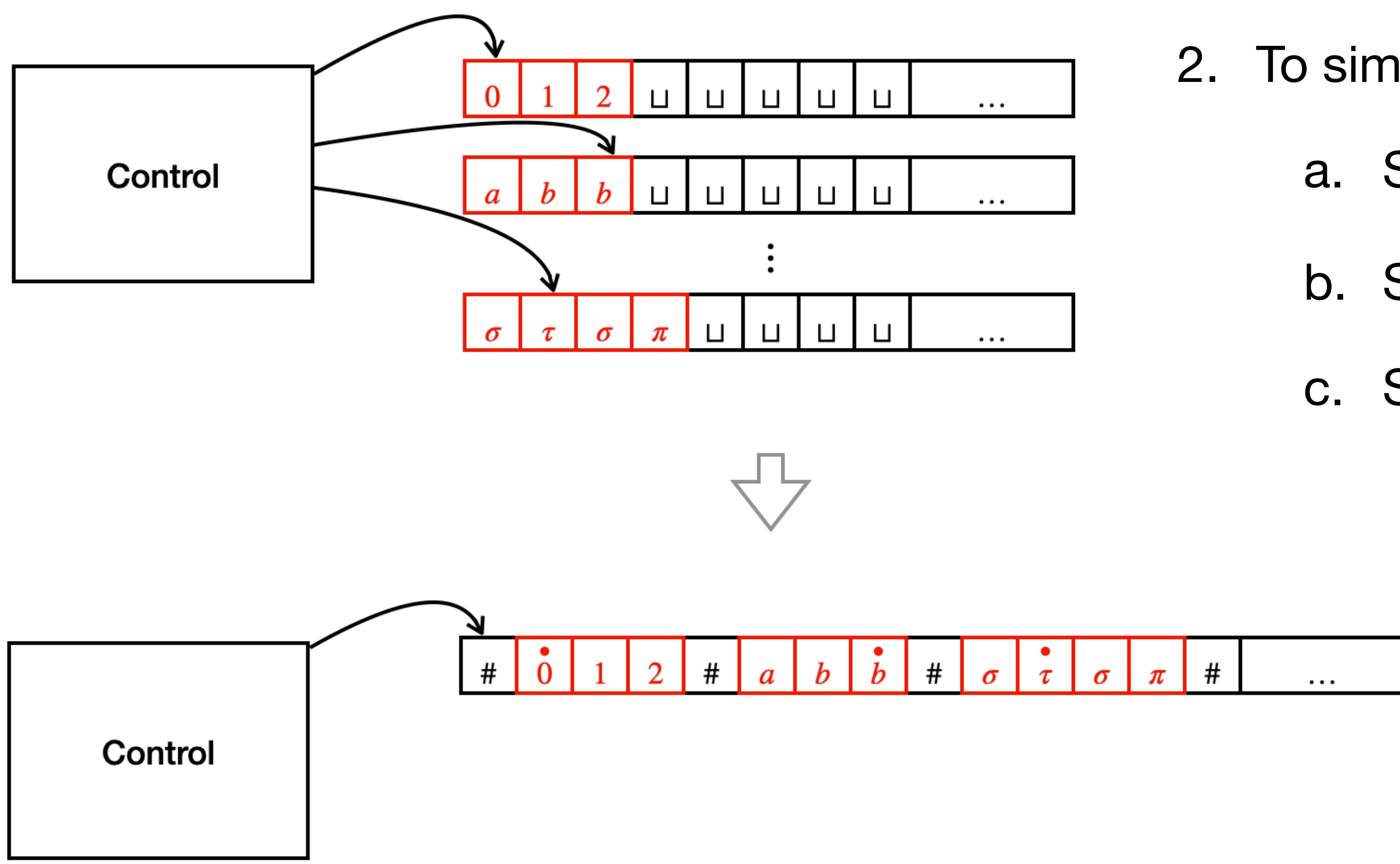


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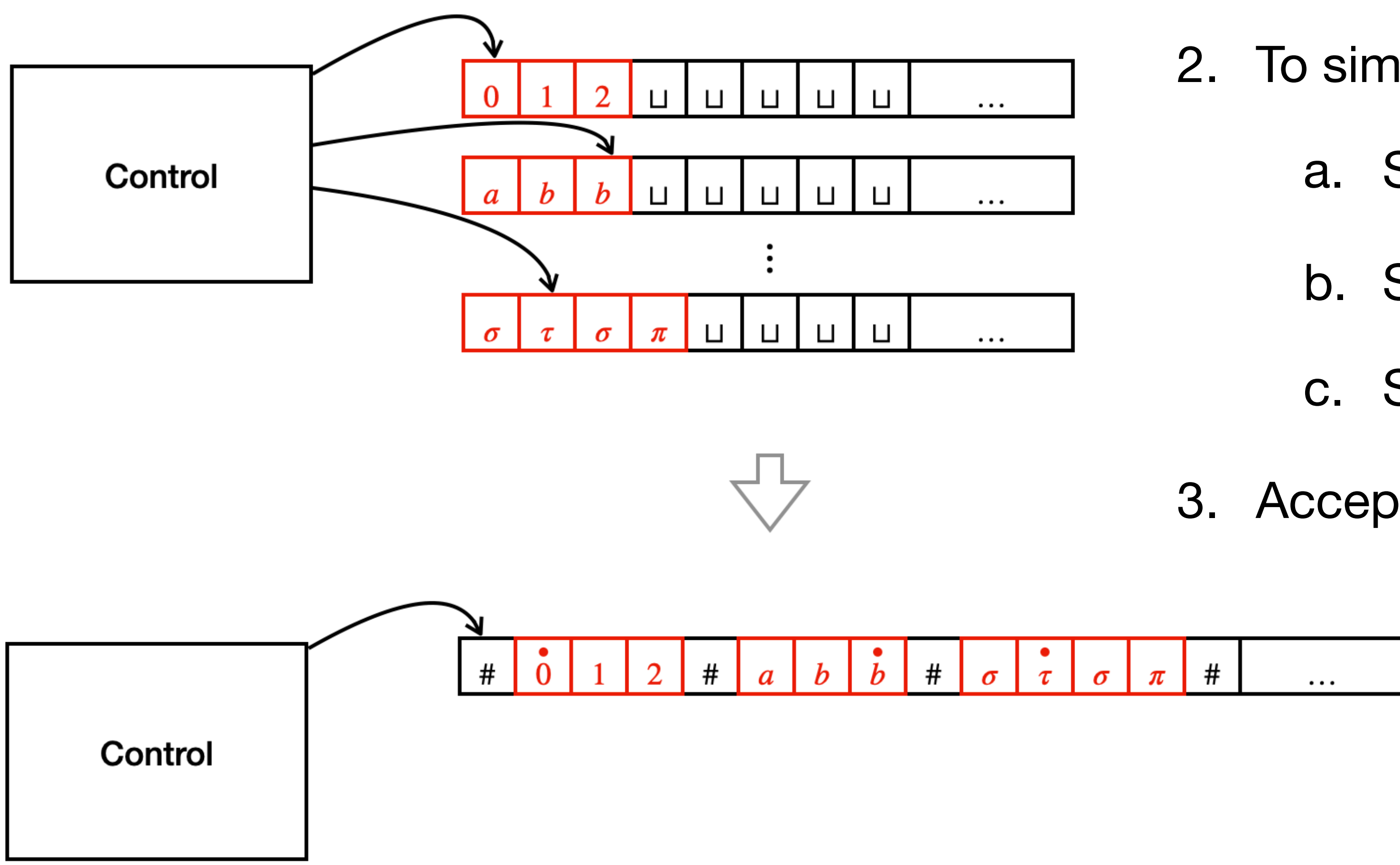


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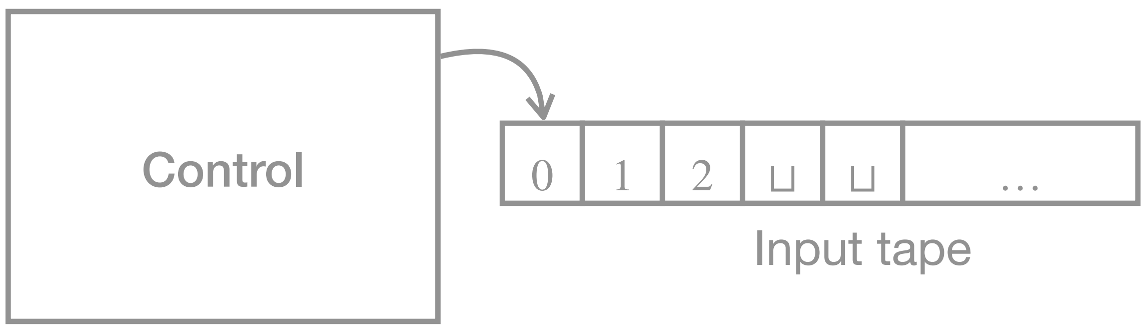
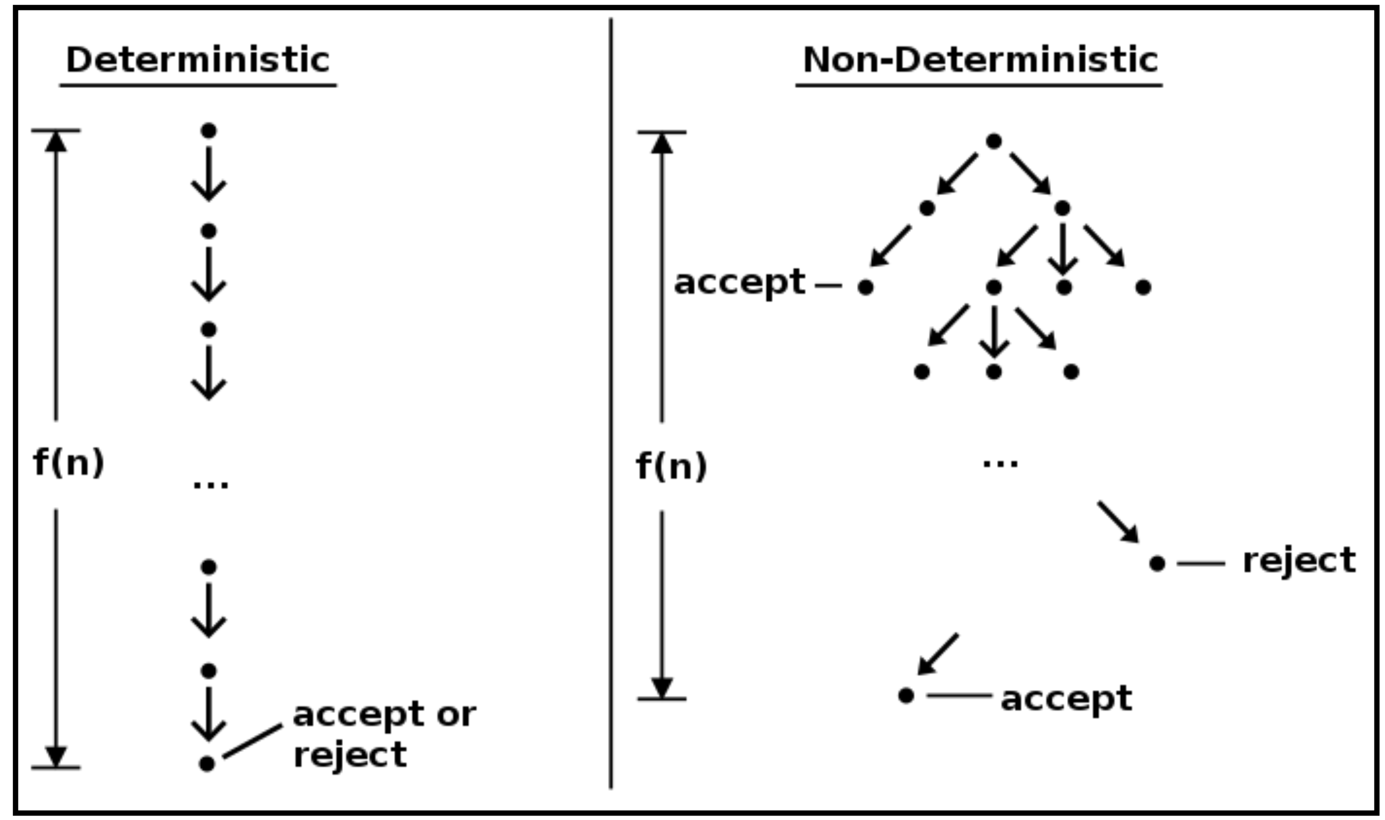
2. To simulate a step in $M$:
   a. Scan entire tape to find dotted symbols.
   b. Scan again to update according to $M$.
   c. Shift to add room as needed.
3. Accept if $M$ does."



**Q.E.D.**

# Nondeterministic TMs




**Definition (p. 178):** A <u>Nondeterministic Turing Machine</u> is like a Deterministic TM, except for the transition function $\delta : Q \times \Gamma \mapsto \mathscr{P}(Q \times \Gamma \times \{\mathsf{L}, \mathsf{R}\})$.

**Theorem:** The language $A$ is T-recognizable iff some Nondeterministic TM recognizes it.

Figure comparing of deterministic and nondeterministic computation taken from https://en.wikipedia.org/wiki/Nondeterministic_Turing_machine

# Proof of Equivalence of Nondeterministic TMs and TMs

We must prove both directions:

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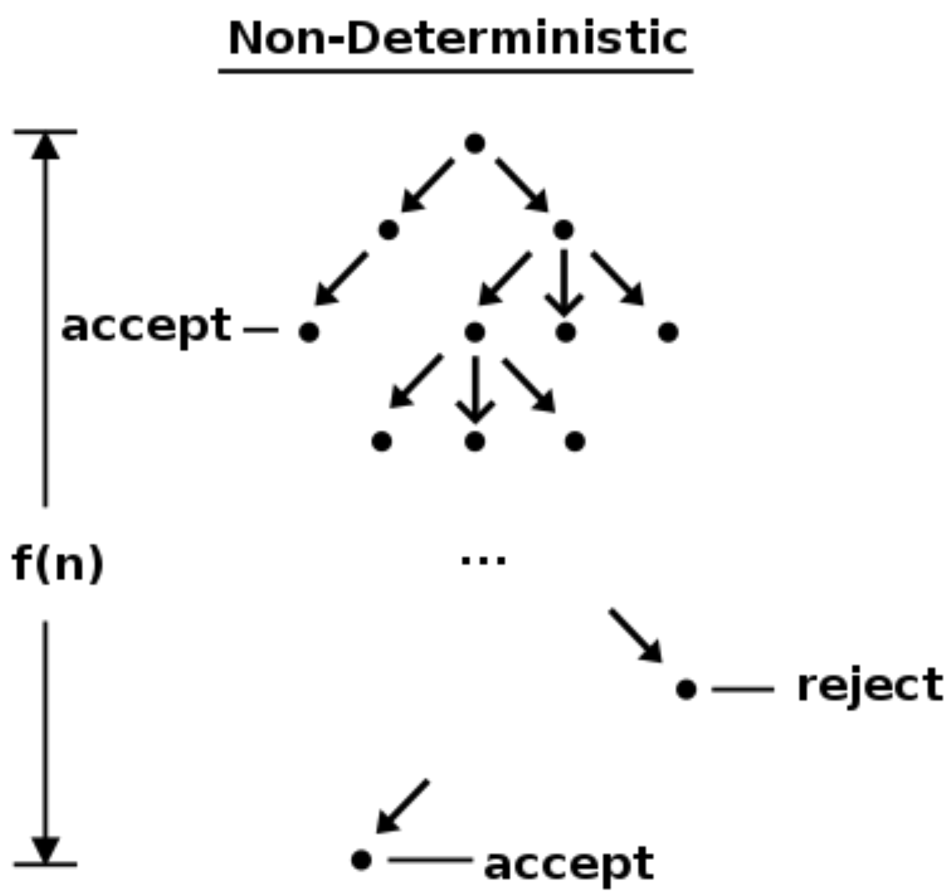

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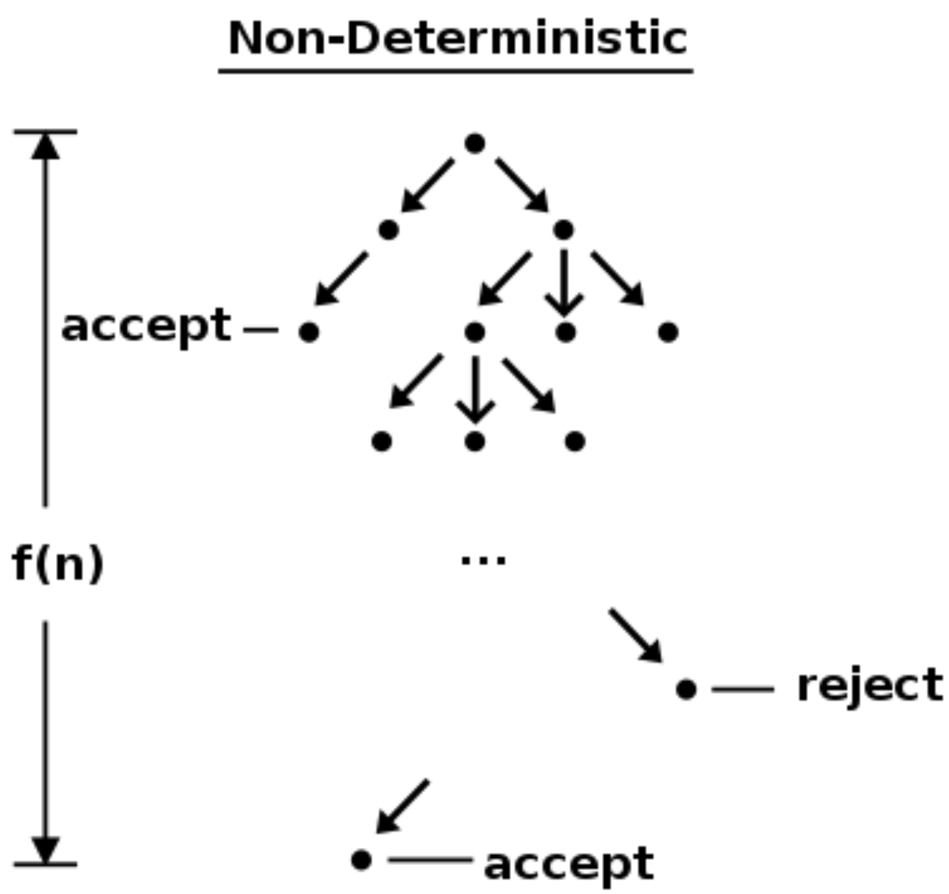


- Each vertex represents a configuration

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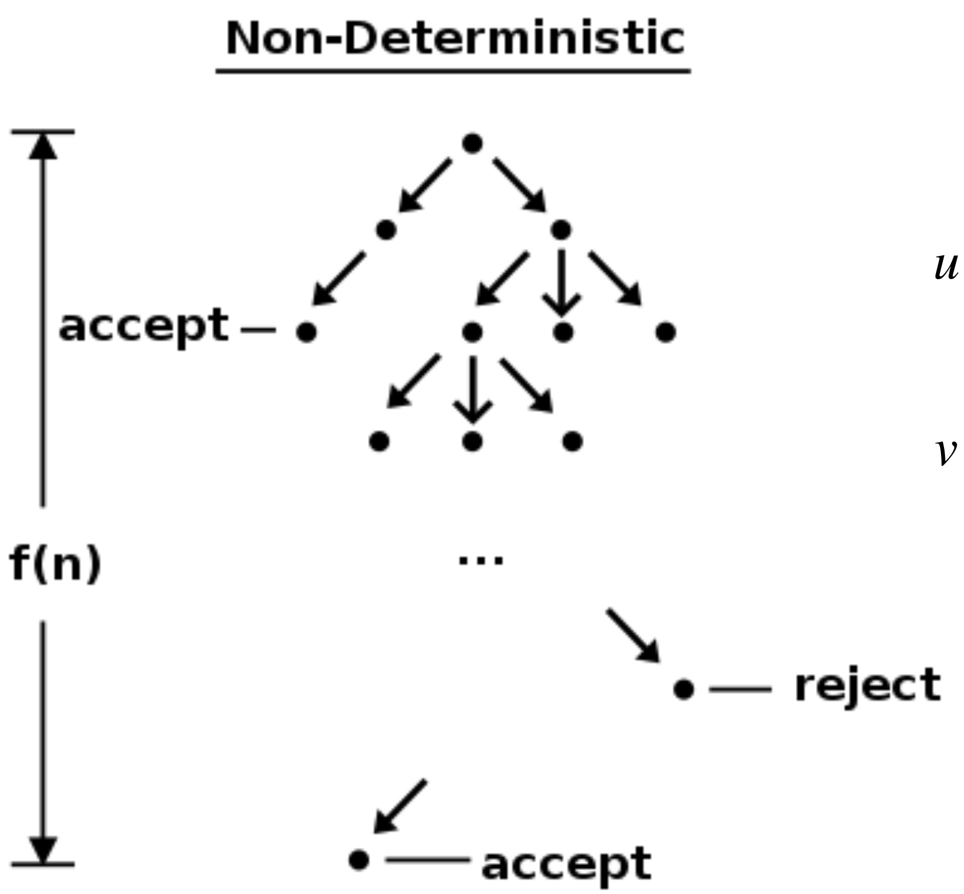


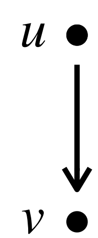


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An edge between $u$ and $v$ represents “$u$ yields $v$”

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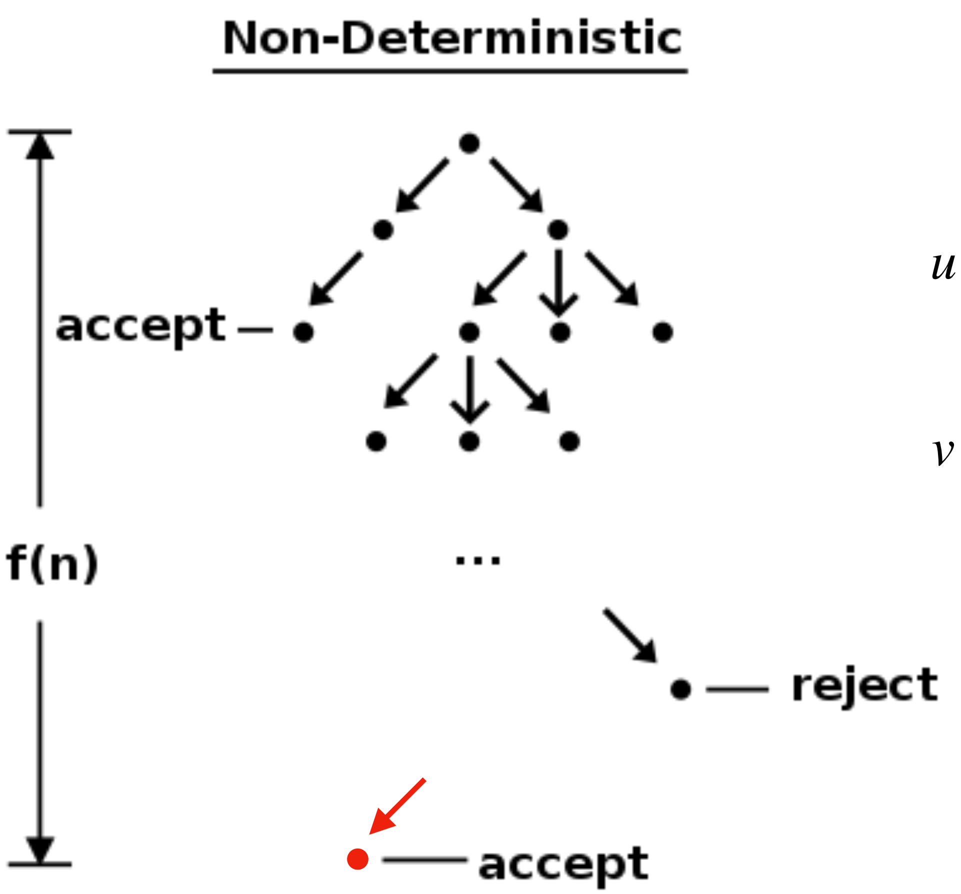


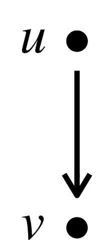


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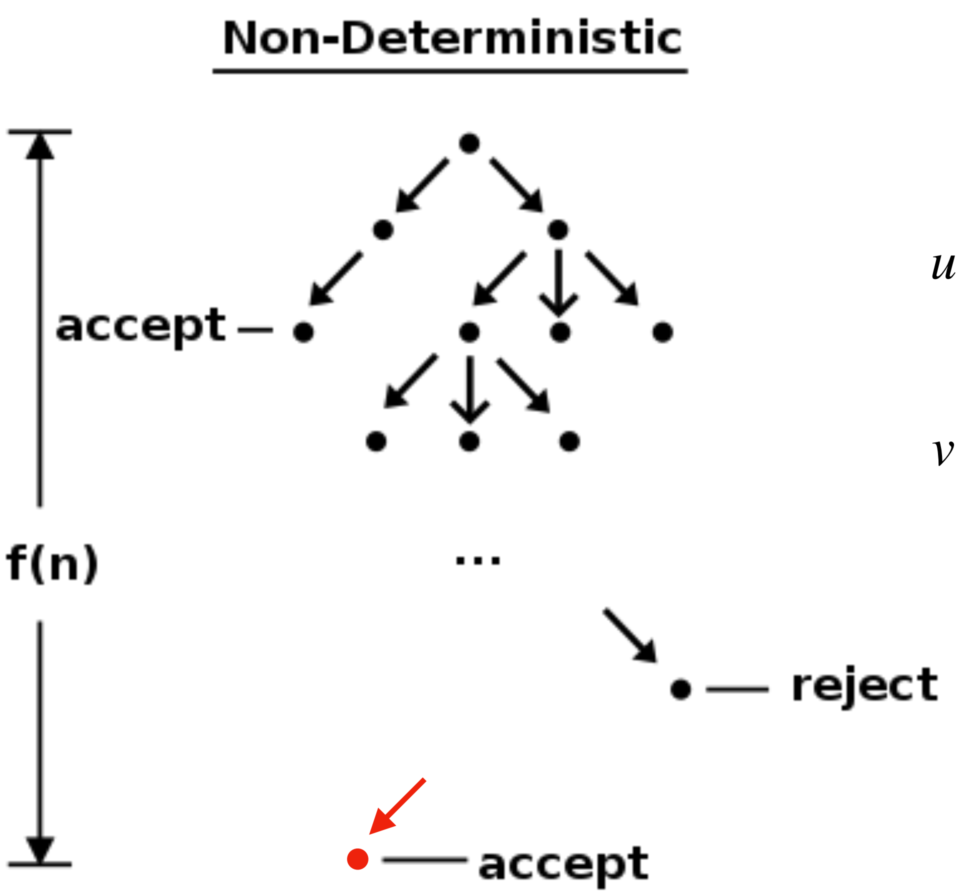


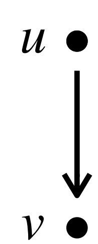


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**Attempt #1** at simulating NTM $N$.

Look for an accepting configuration via a depth-first search on the computation tree. (Doesn't work. Why?)

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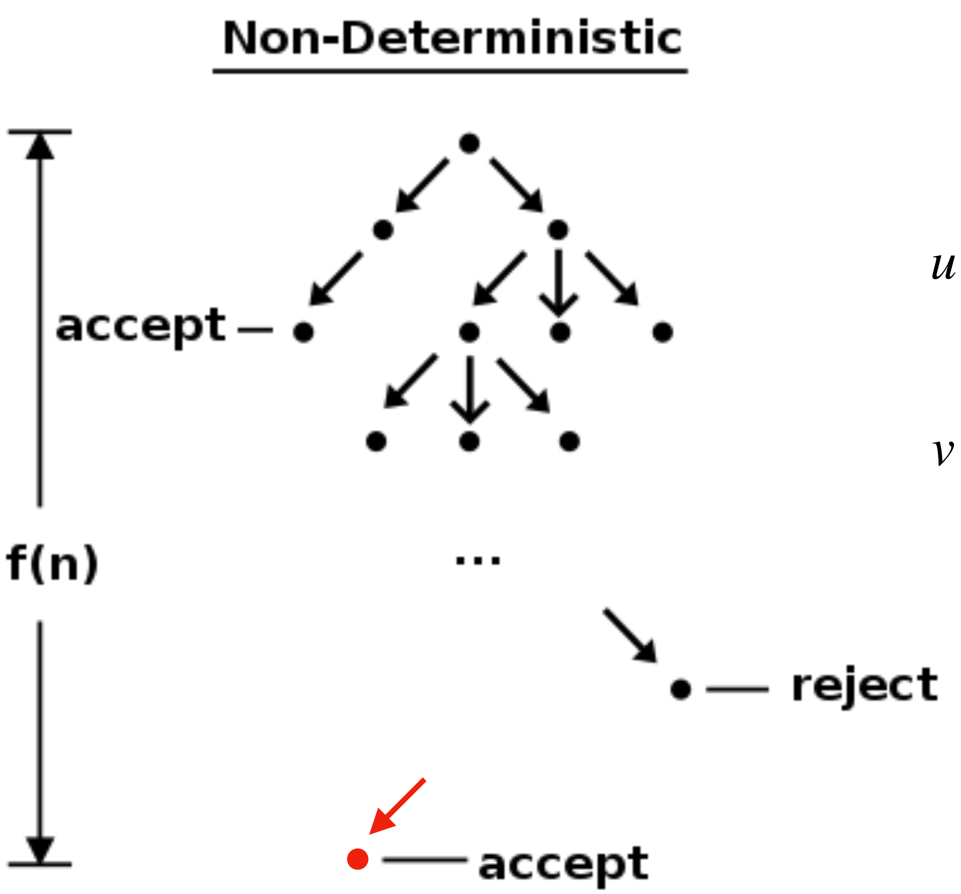


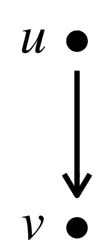


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**Proof Idea:**

Look for an accepting configuration via a breadth-first search on the computation tree.

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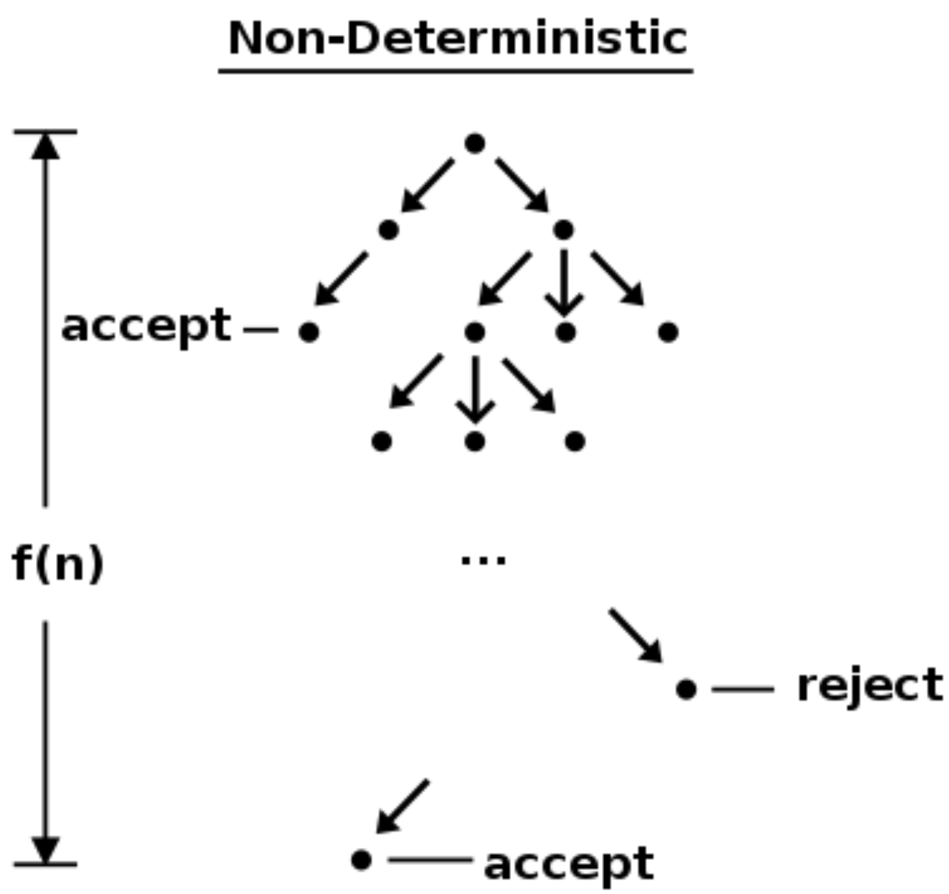

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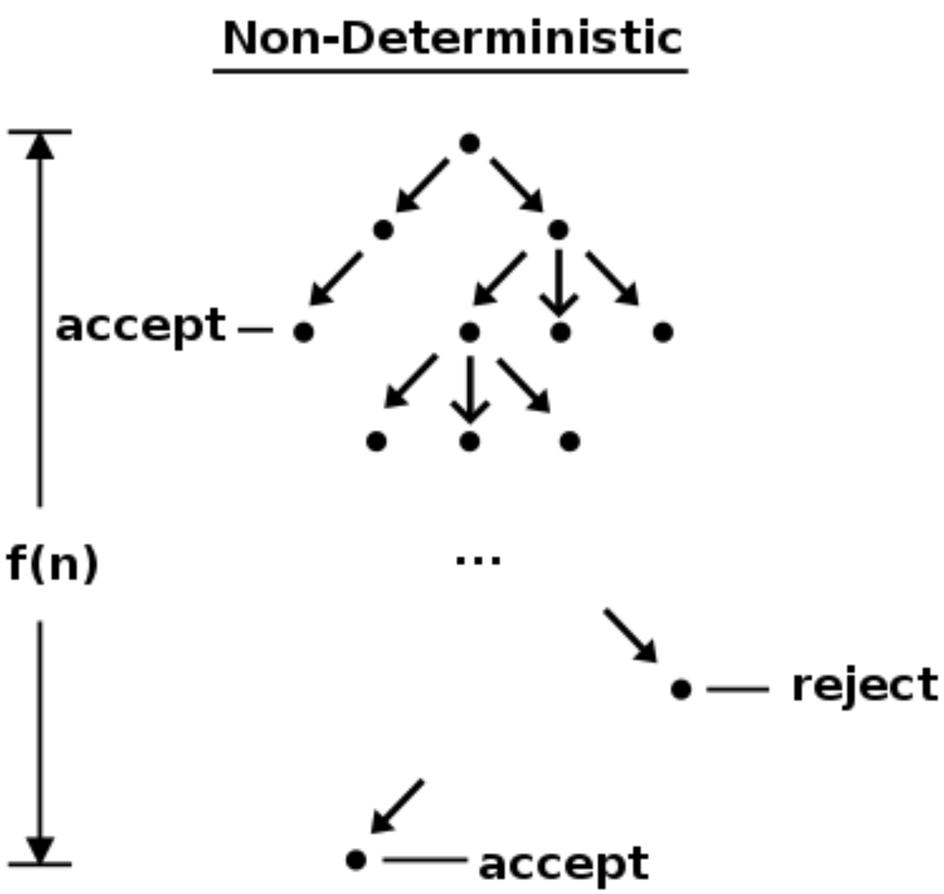


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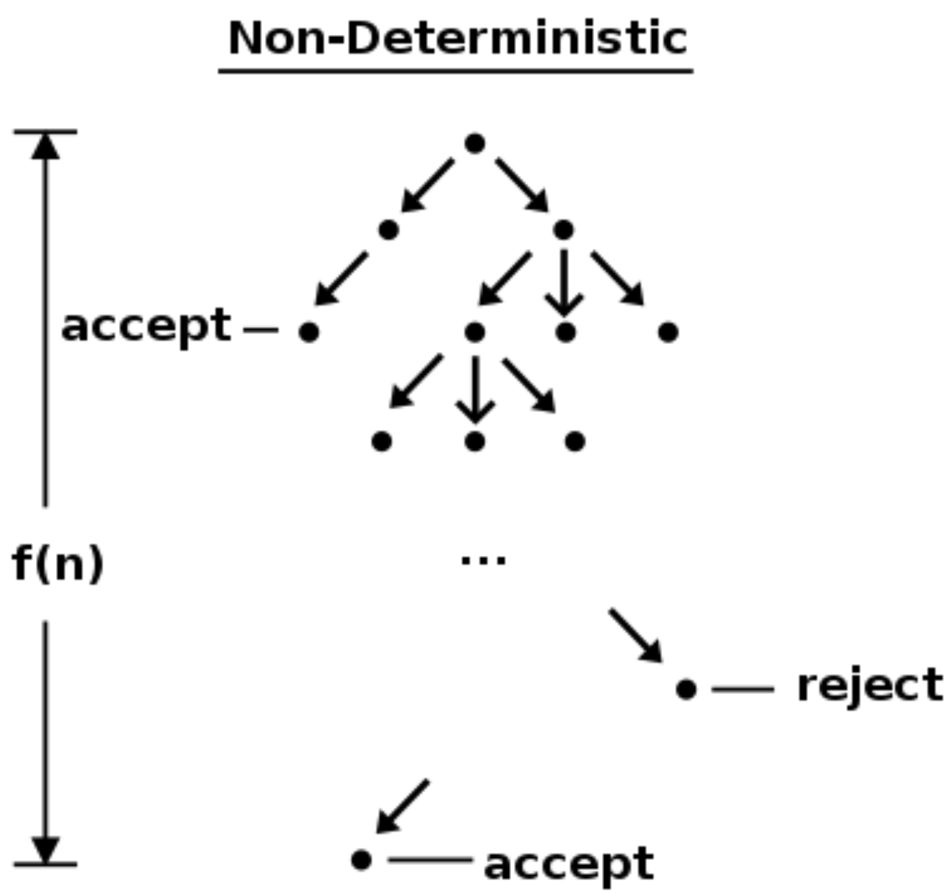


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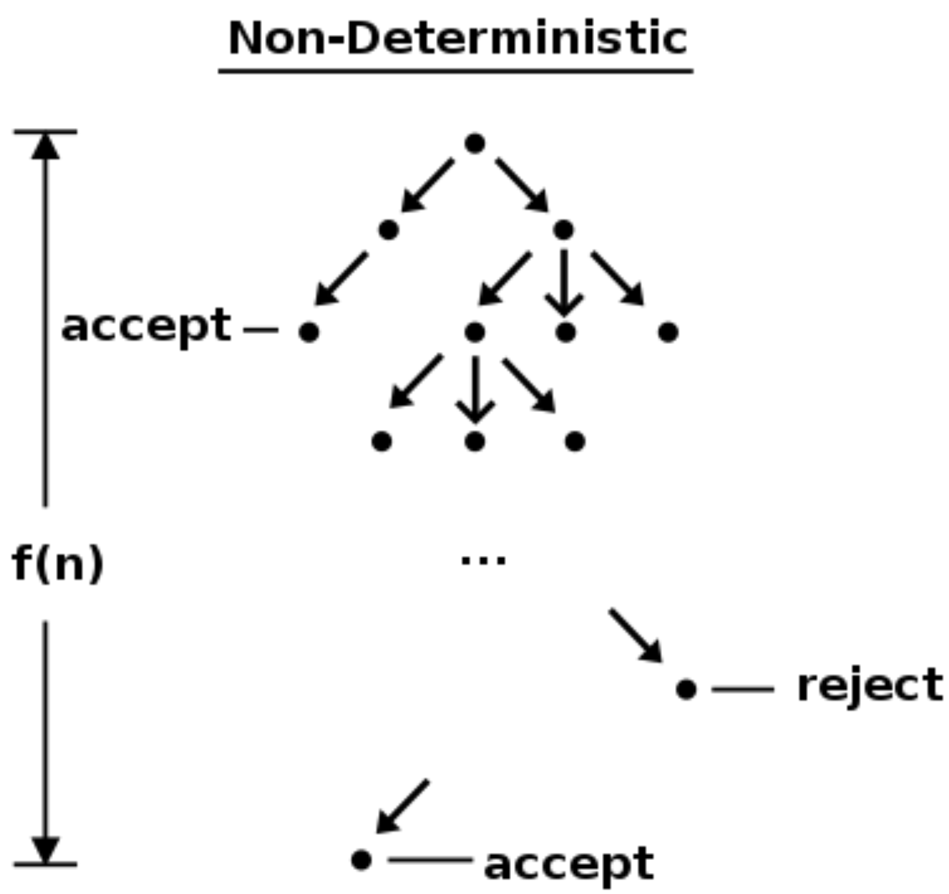


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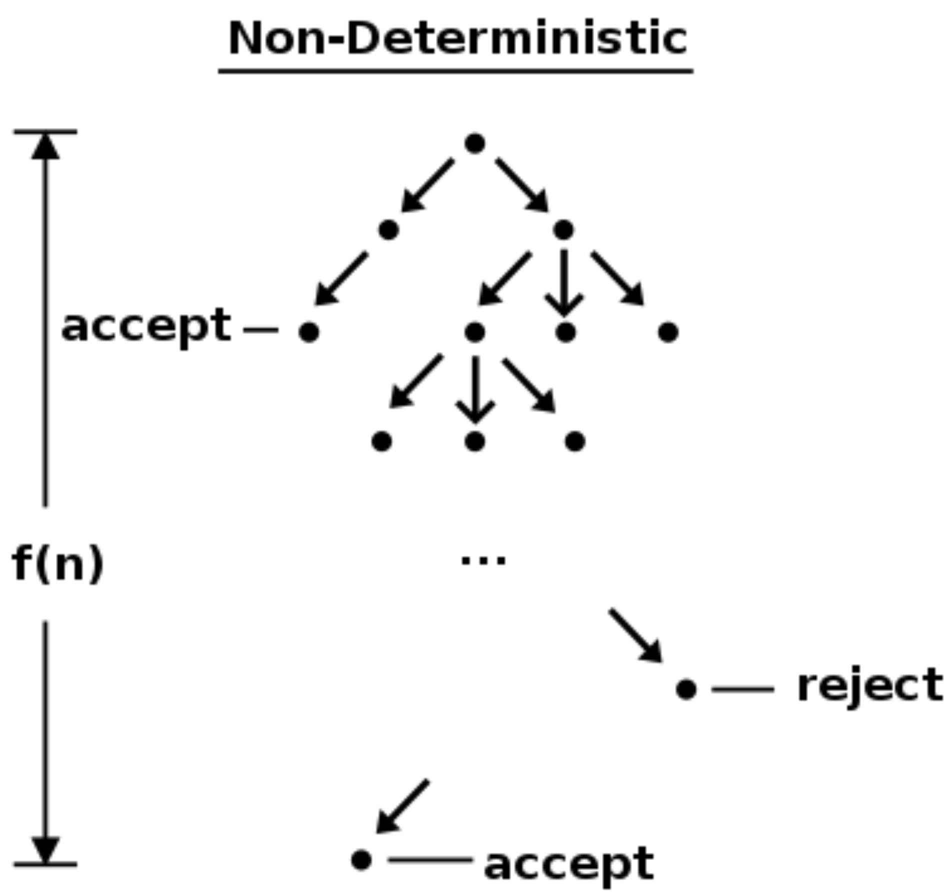


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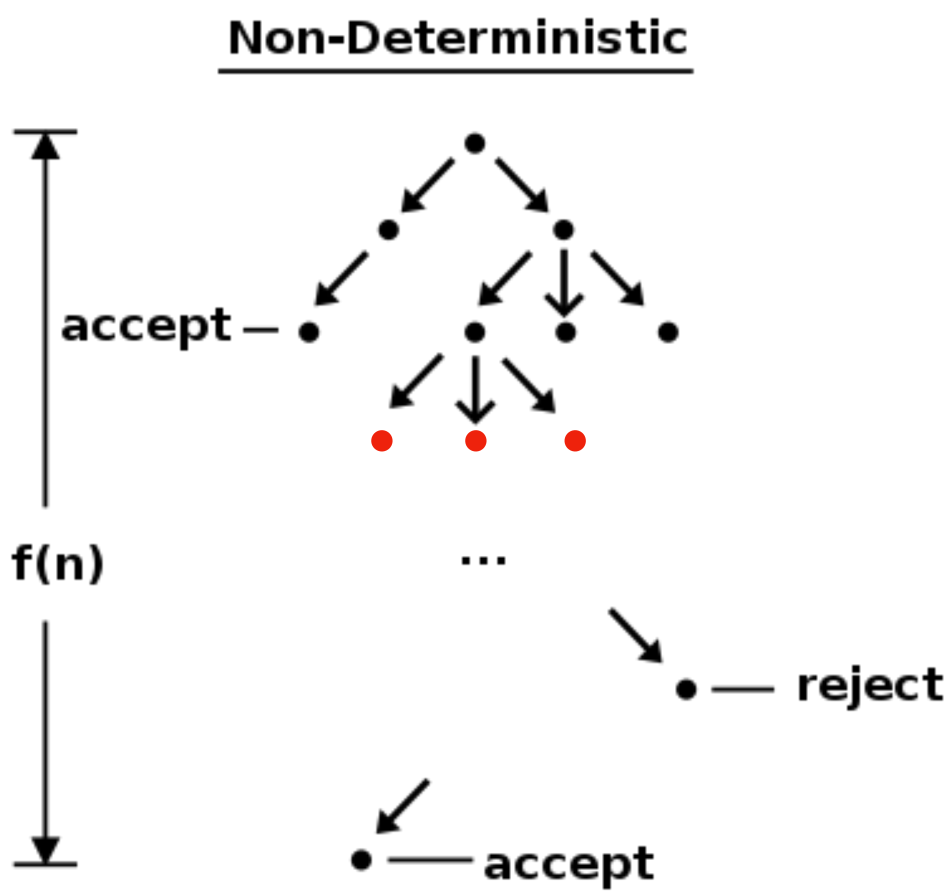


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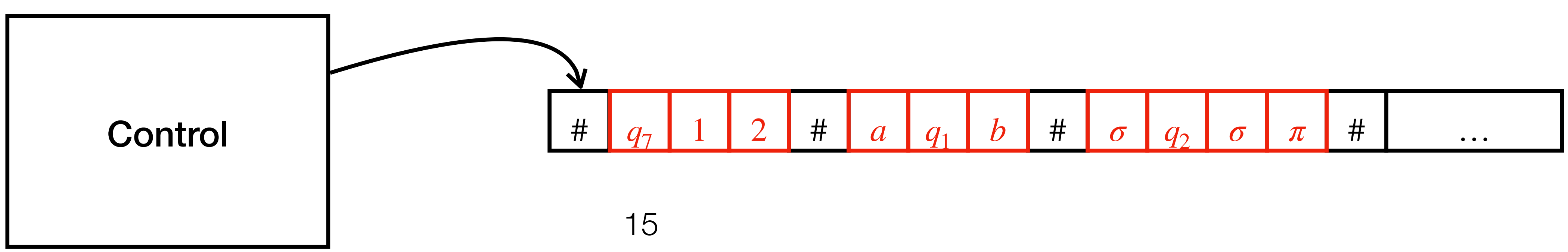

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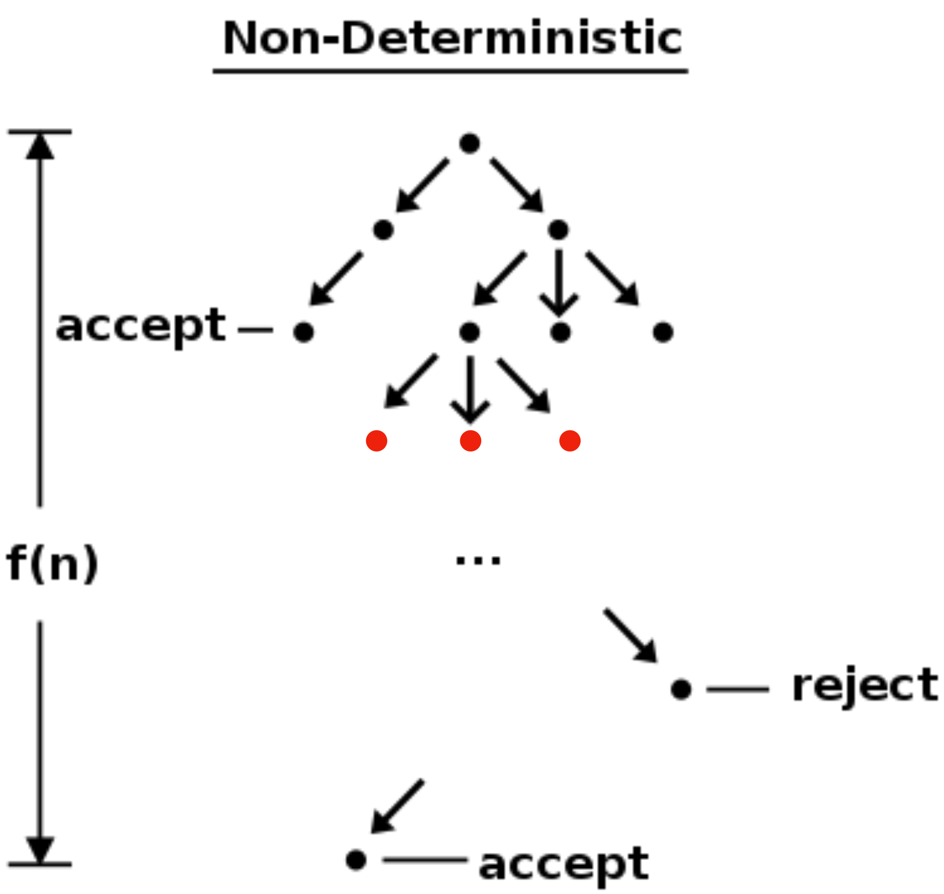


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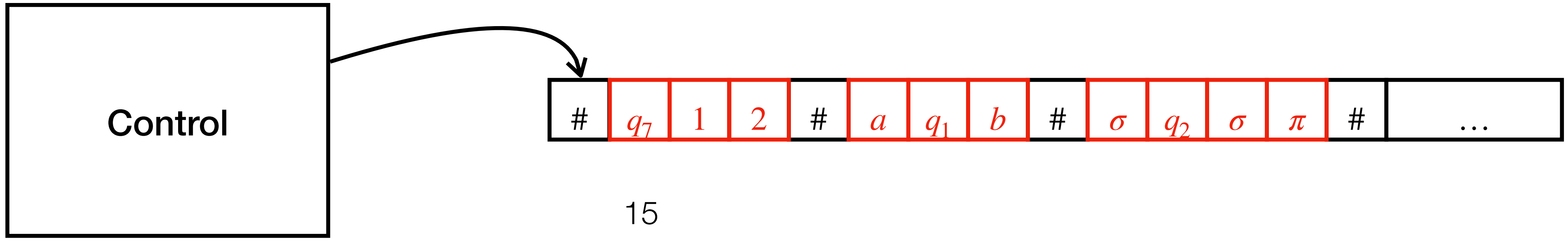

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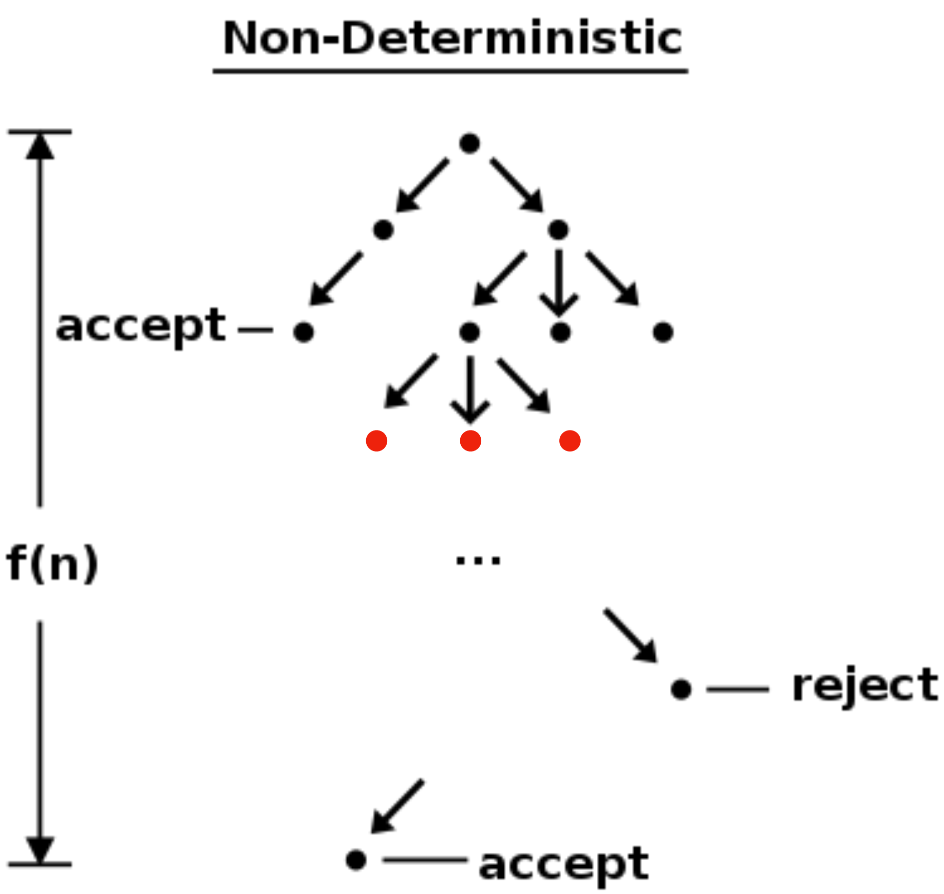


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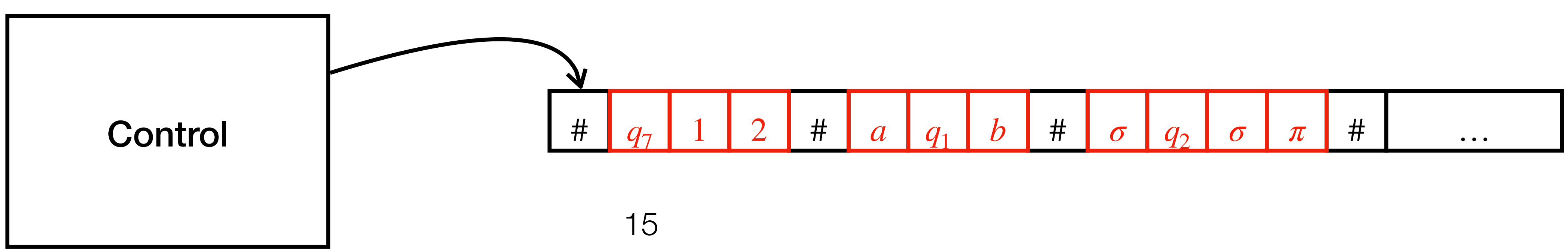
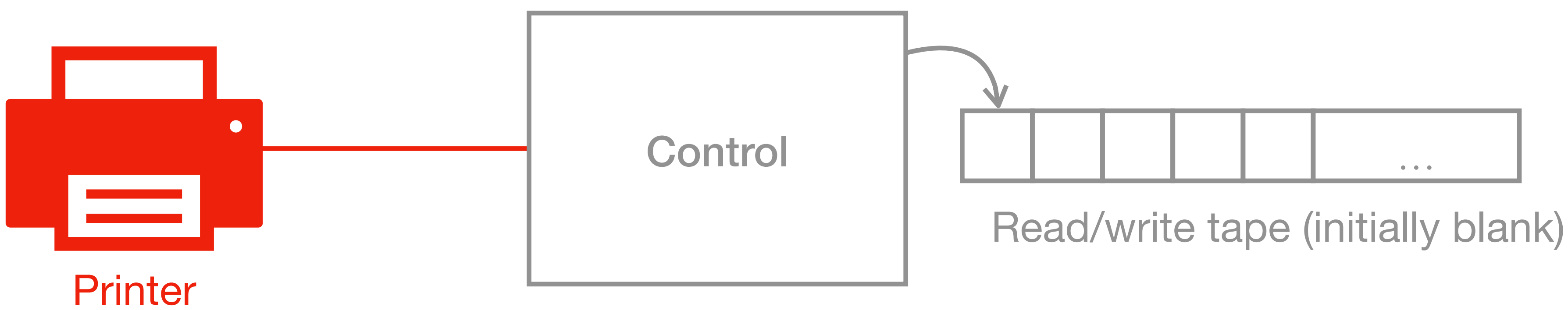
   c. **If a thread accepts, accept**." **Q.E.D.**



# Check In (Break)

**True or false:** A language is decidable iff some nondeterministic TM decides it.

# Enumerators




**Definition (p. 180):** An enumerator $E$ is like a Deterministic TM, except it has a "printer." It starts with a blank tape and eventually prints out all the strings $w_1, w_2, \ldots$ in the language $L(E)$ it generates.

**Theorem:** The language $A$ is T-recognizable iff $A = L(E)$ for some enumerator $E$.

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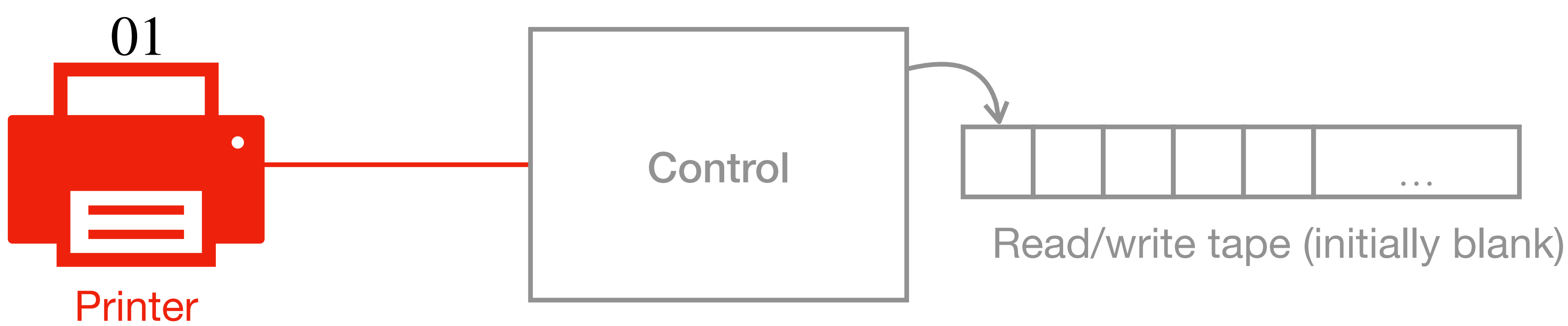


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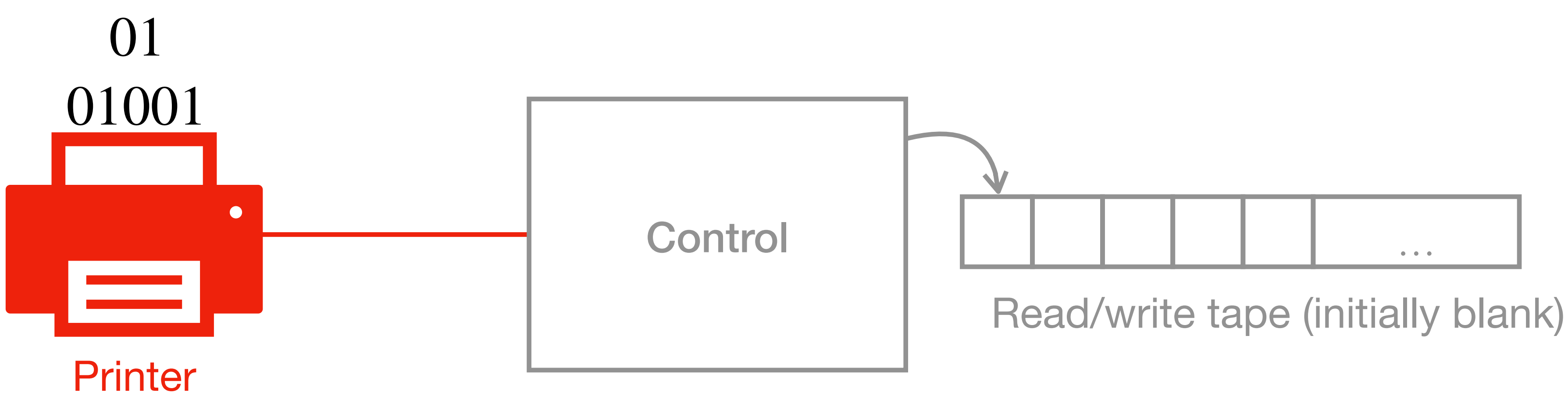


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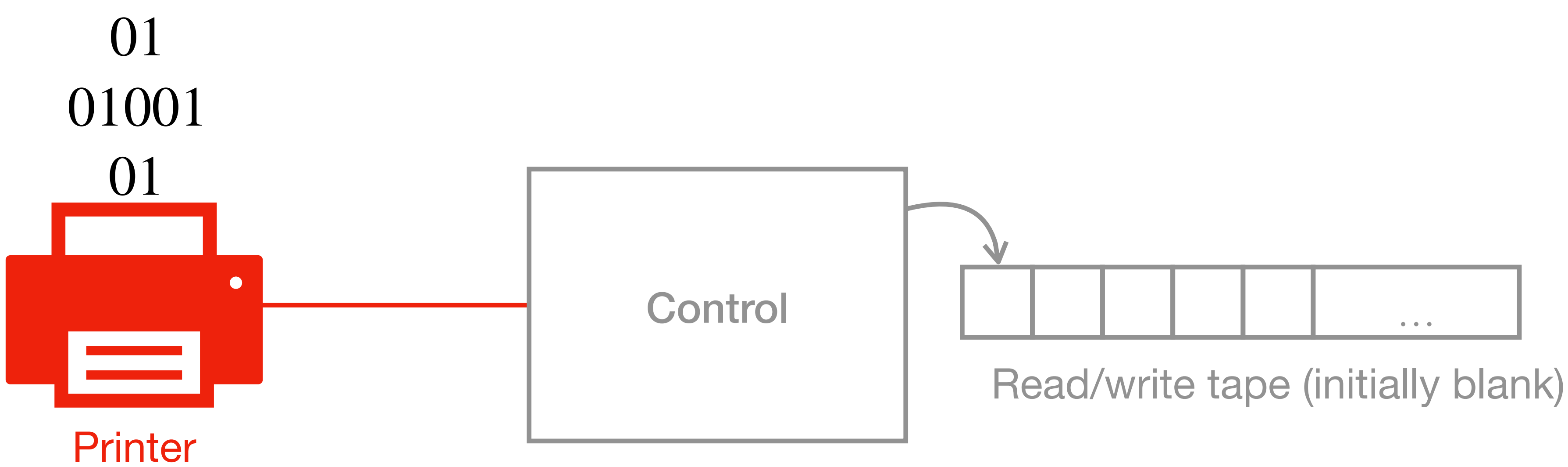


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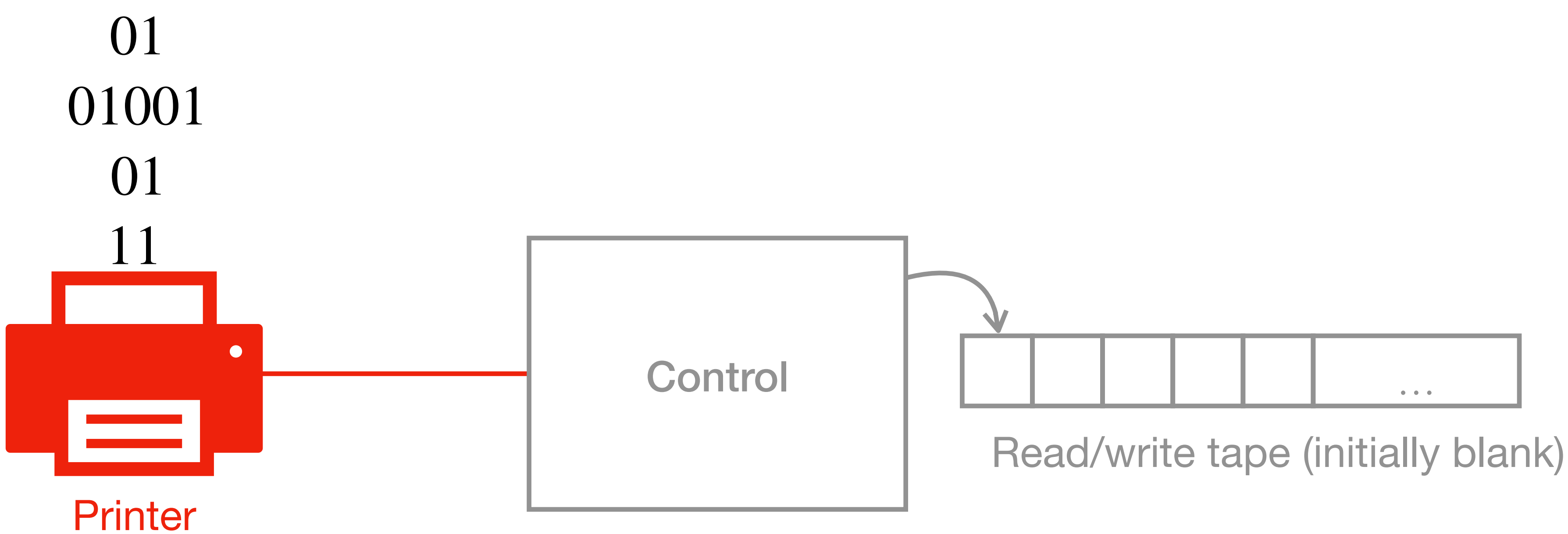


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**Q.E.D.**

# Church-Turing Thesis

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- Church-Turing Thesis: any (classical) real-world computation can be modeled as a TM

# Summary of Today's Lecture

- Robustness of TMs
- Church-Turing Thesis

# Acknowledgements


- These slides are based on lecture notes on Theory of Computation from other universities, namely Michael Sipser (MIT), Lorenzo De Stefani (Brown).

- **Errata:** If you let us know of any errors in the slides, we'll fix them and acknowledge you here!