CS170 Computation Theory

Lecture 7

September 26, 2023

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Tufts University | School of Engineering Computer Science

Review of Last Lecture

- Robustness of TMs
- Church-Turing Thesis

Today’s Topics

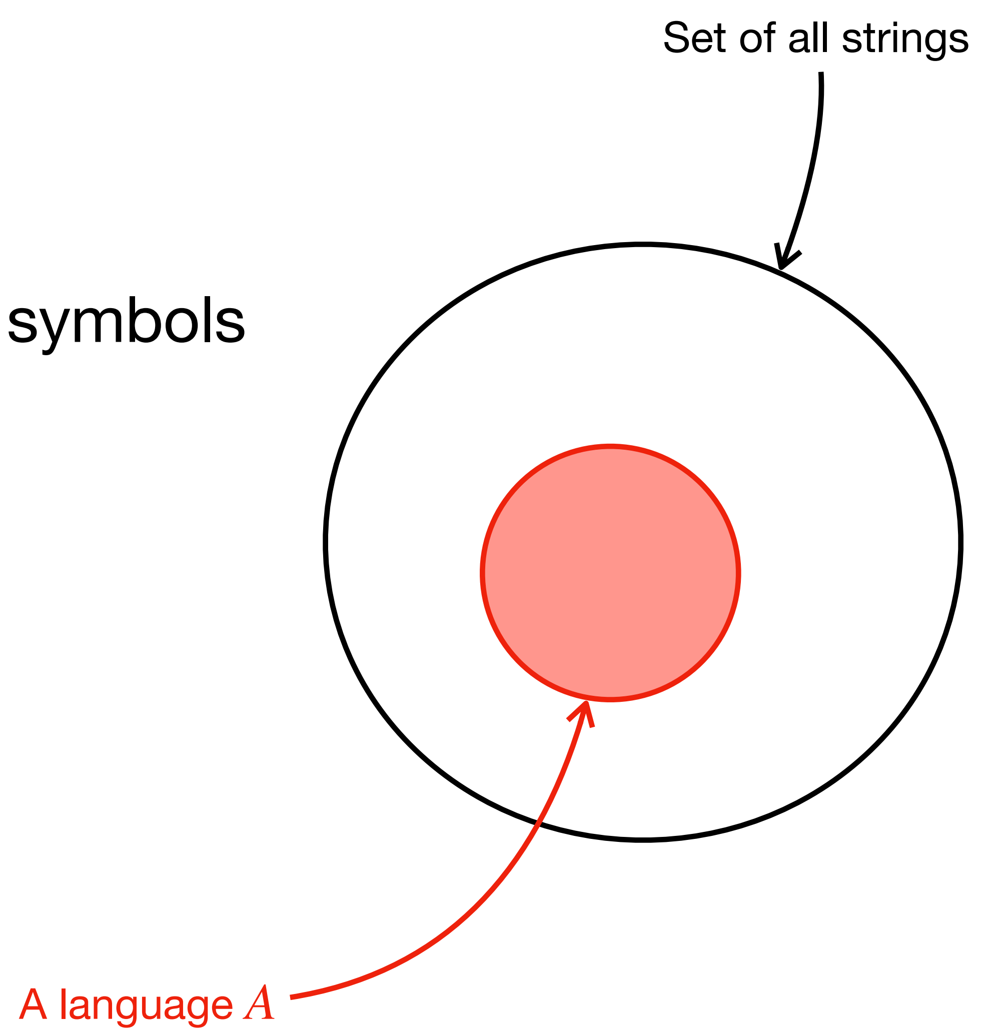
- ~~Robustness of TMs~~
- ~~Church-Turing Thesis~~

- Notation for Encodings and TMs
- Decision procedures for DFAs

Recall Definition of a Language

Definitions:

- A string is a finite sequence of symbols
- A language is a set of strings



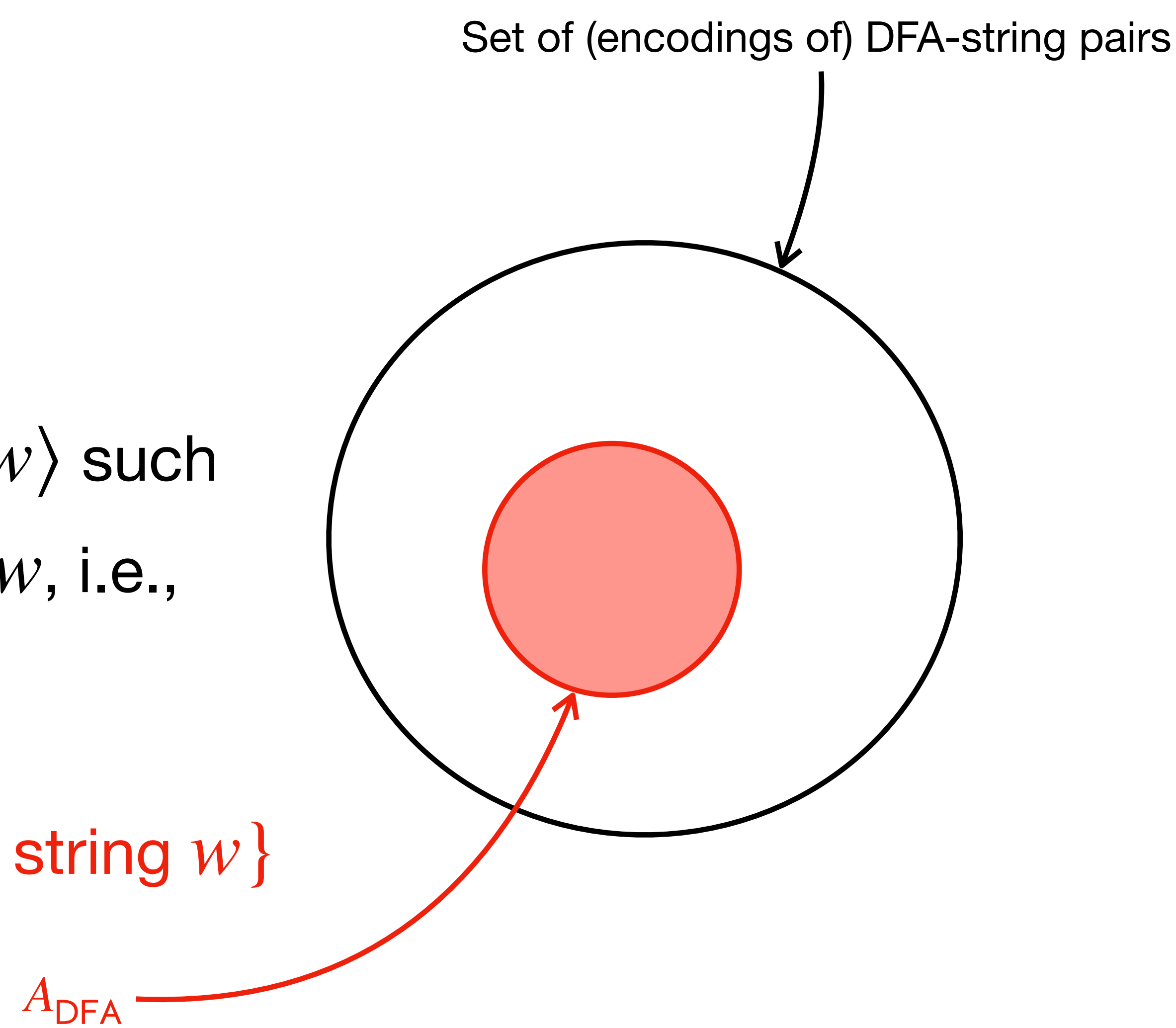
Language Can Represent a Computational Problem

E.g.,

Let B be a Deterministic Finite Automaton.

Let A_{DFA} be the set of pairs $\langle B, w \rangle$ such that w is a string and B accepts w, i.e.,

$$A_{\text{DFA}} = \{\langle B, w \rangle \mid B \text{ is a DFA that accepts input string } w\}$$



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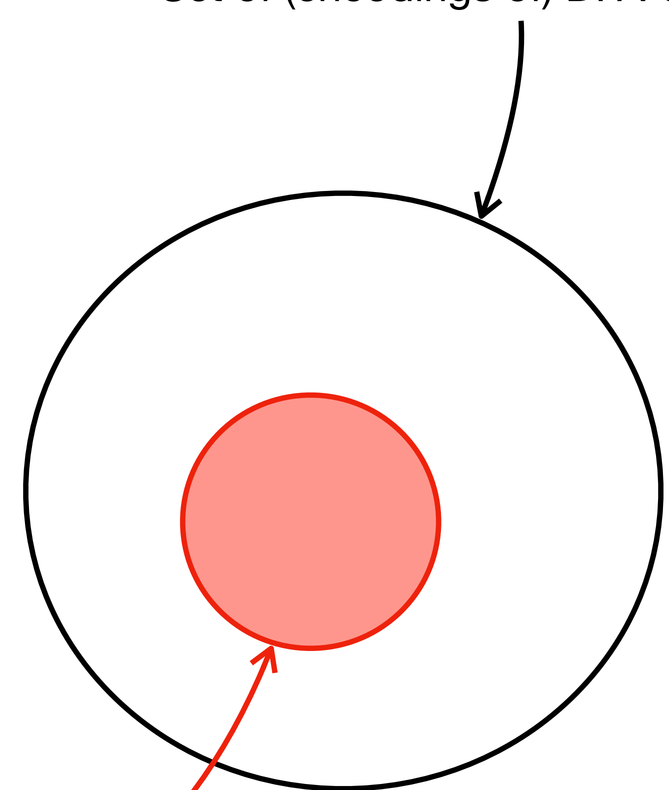

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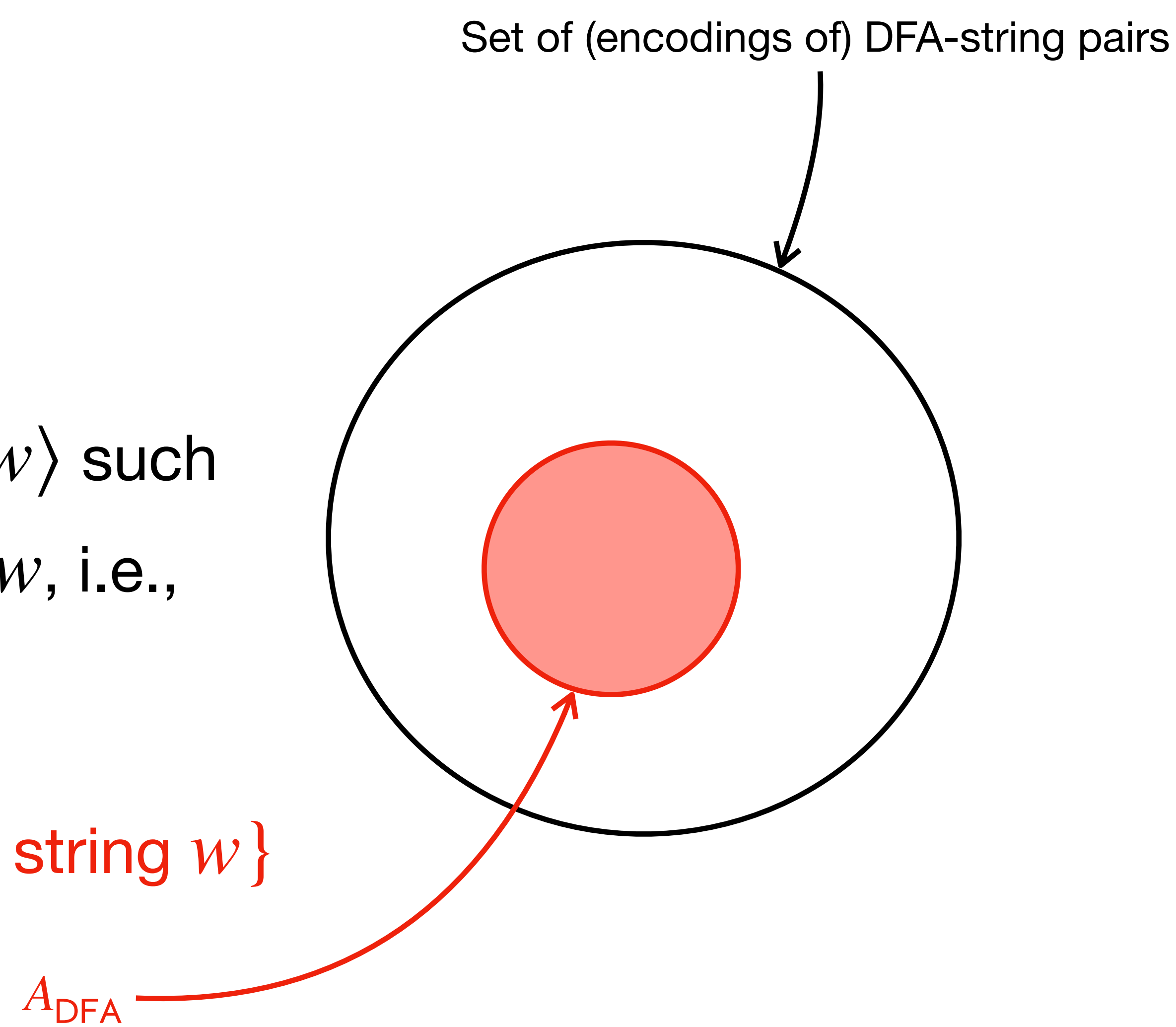
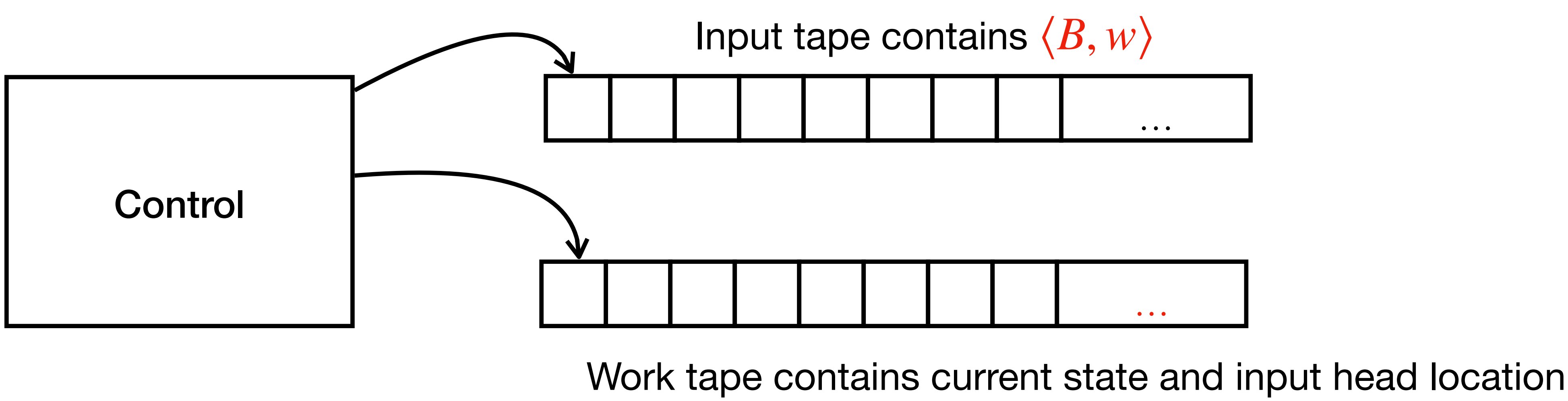


Is there a TM that recognizes this language A_{DFA}? that <u>decides</u> this language A_{DFA}?

Notation for Encodings

- Let $O_1, O_2, \ldots, O_k$ be “objects,” e.g., TMs, graphs, etc.
- We denote the encoding of these objects as $\langle O_1, O_2, \ldots, O_k \rangle$.
- In example in previous slide, $\langle B, w \rangle$ is the encoding of the pair, consisting of the DFA B and the string w.



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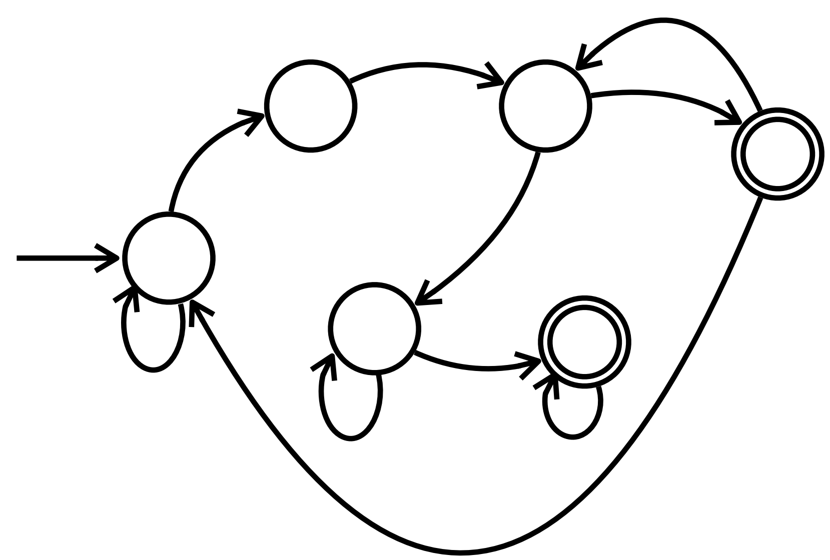
Why does $D_{A_{\text{DFA}}}$ always halt?

$D_{A_{DFA}}$ Always Halts

- Let $r_0 = q_{\text{start}}$ (in the DFA).

- The simulation (step 2) always completes in $|w|$ sub-steps because each transition “consumes” an input symbol w_i and takes us to the next state r_{i+1}.



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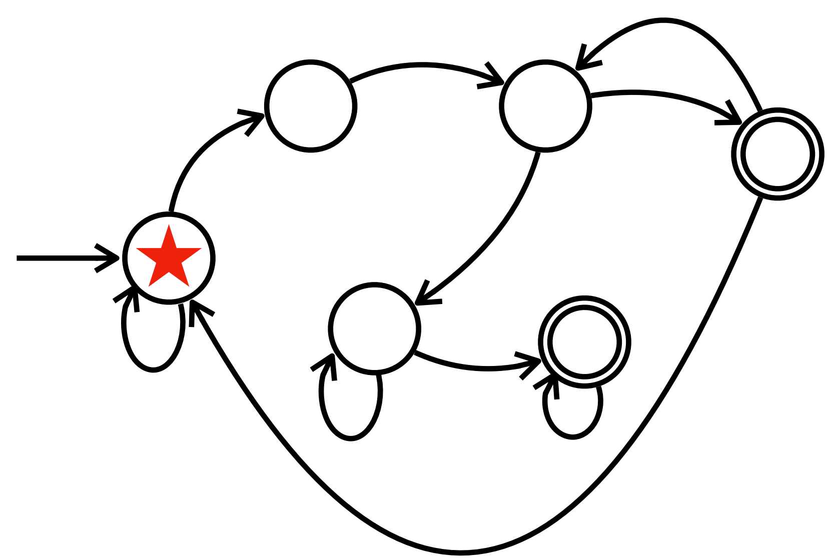

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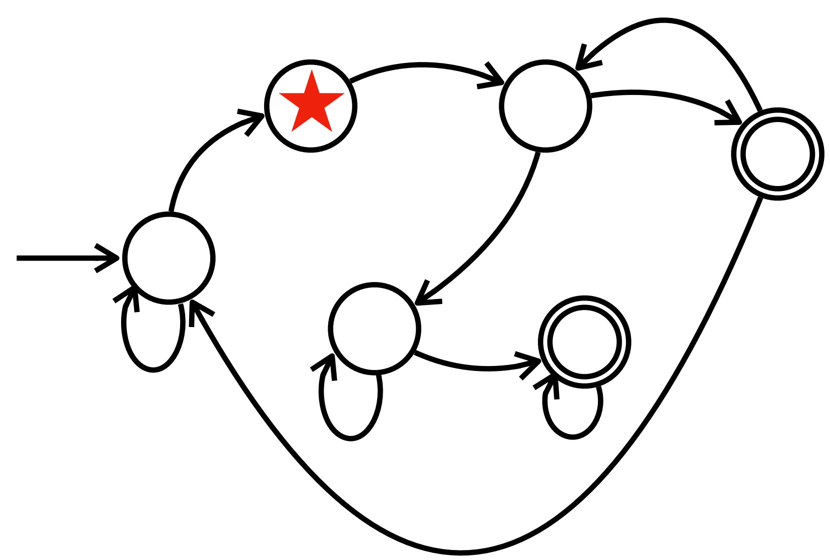

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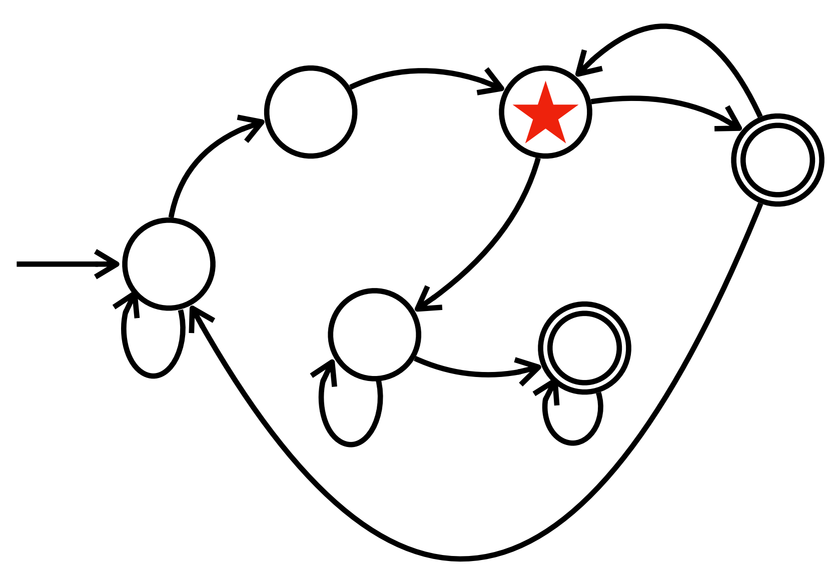

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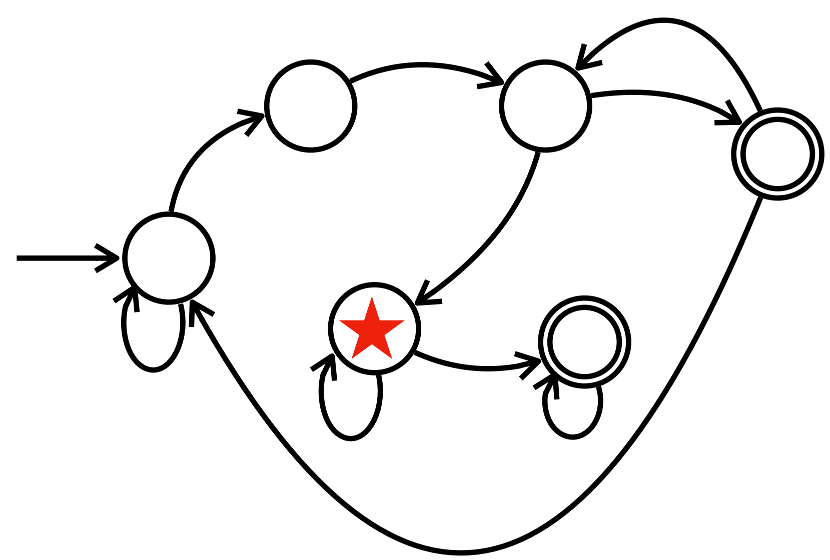

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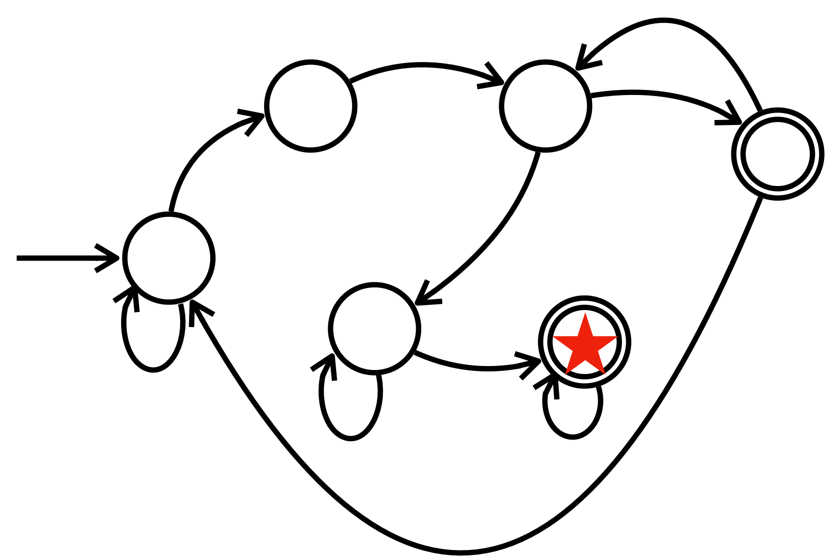
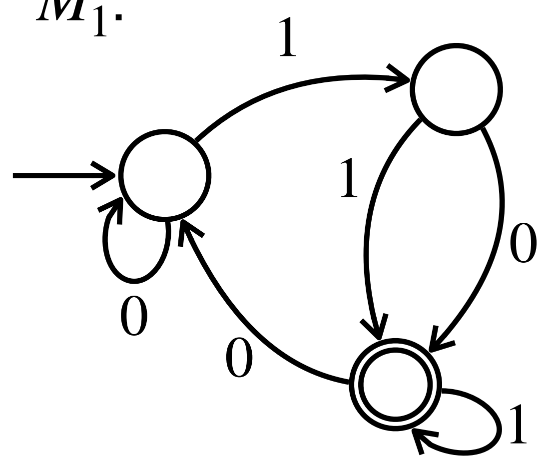


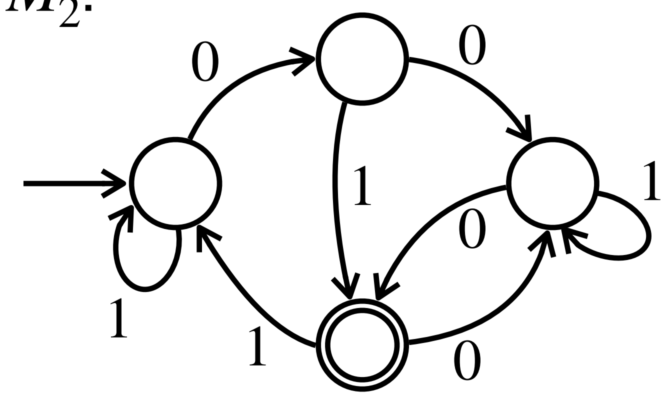
Check-In 1 (Break)

Consider the following DFAs.

M_1:



M_2:



1. Is $\langle M_1 \rangle$ in A_{DFA}?
2. Is $\langle 0100 \rangle$ in A_{DFA}?
3. Is $\langle M_1, 0101 \rangle$ in A_{DFA}?
4. Is $\langle M_2, 0101 \rangle$ in A_{DFA}?
5. Is $\langle M_2, 0011111111111110010 \rangle$ in A_{DFA}?
6. Is $\langle M_2, 00111111111111110011111111101 \rangle$ in A_{DFA}?

Acceptance Problem for NFAs

Theorem (p.195): The language
$A_{\text{NFA}} = \{\langle B, w\rangle \mid B \text{ is a NFA that accepts input string } w\}$ is decidable.

Proof Attempt 1: Let $D'_{A_{\text{NFA}}}$ be the following TM.

$D'_{A_{\text{NFA}}}$= "On input $\langle B, w\rangle$ where B is a NFA and w is a string,

1. Simulate the computation of B on w.
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$D'_{A_{\text{NFA}}}$ decides A_{NFA}.

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(ϵ-transitions make argument tricky.)

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Why does $D_{A_{\text{NFA}}}$ always halt?

(Because $D_{A_{\text{DFA}}}$ does.)

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$D_{A_{\text{REG}}}$ always halts because $D_{A_{\text{NFA}}}$ does.

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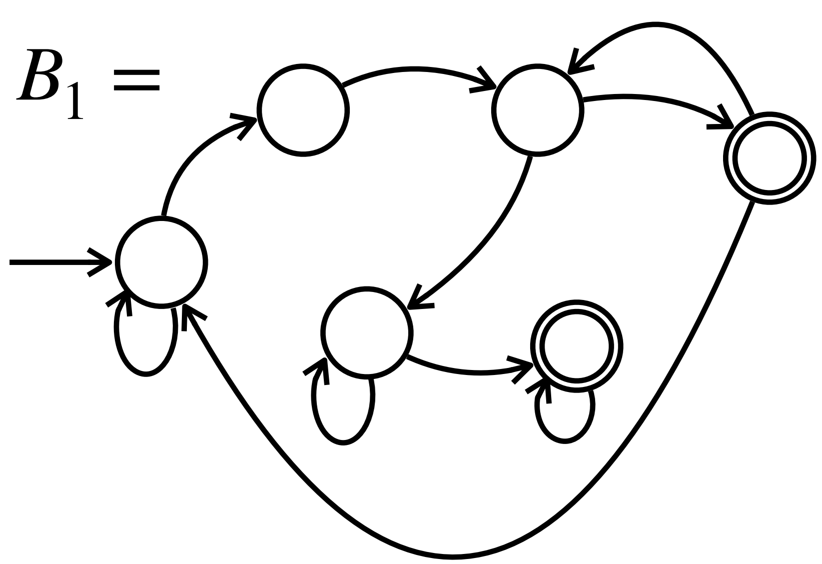

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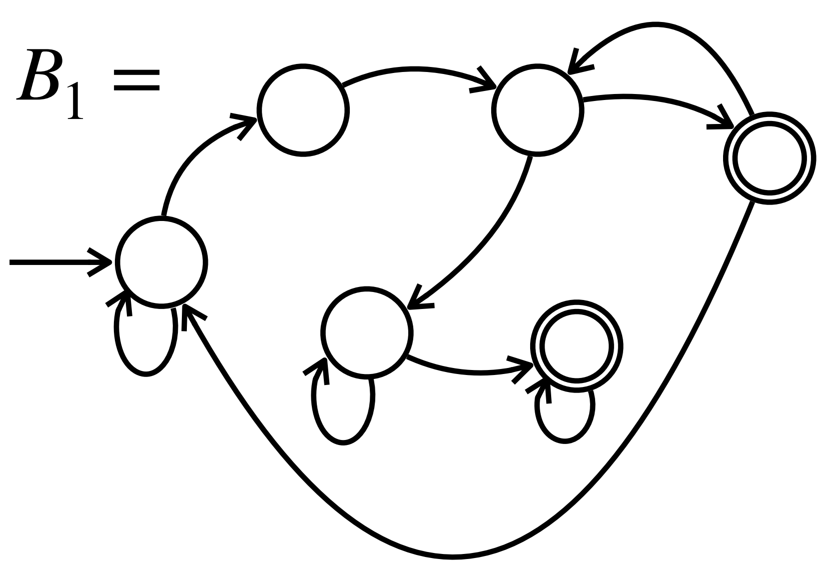

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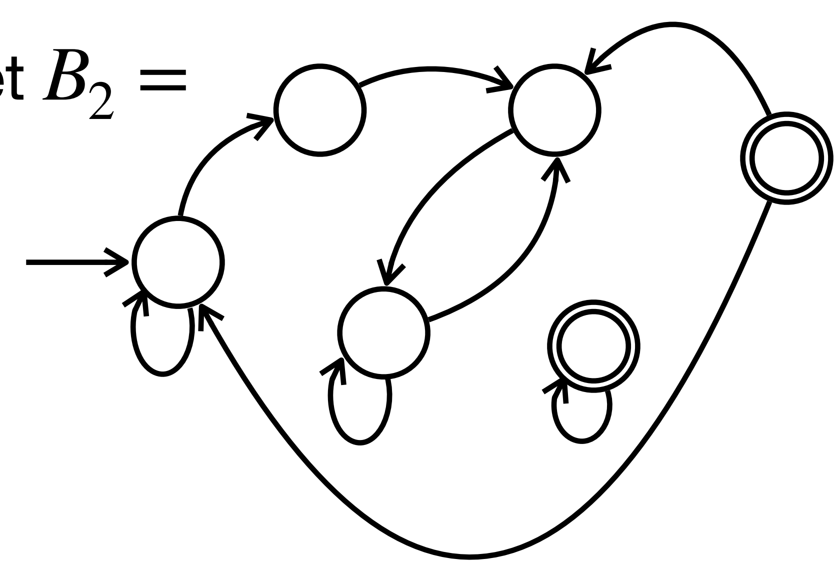

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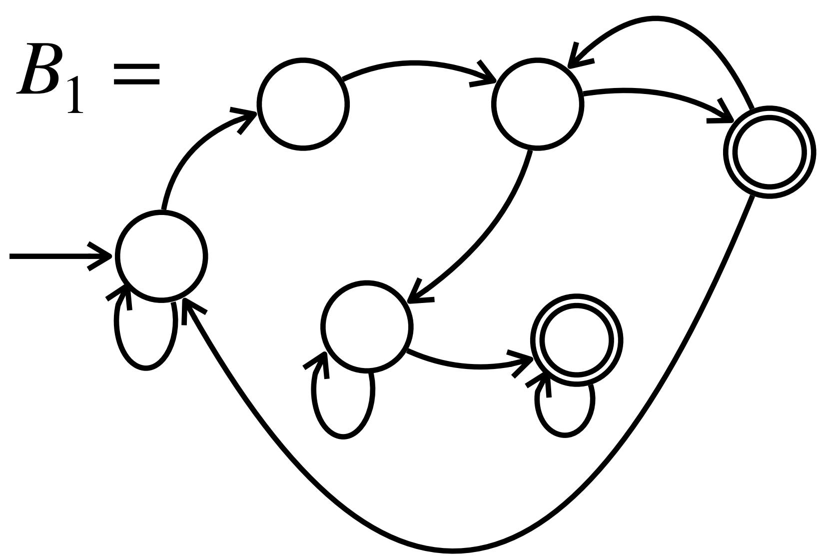
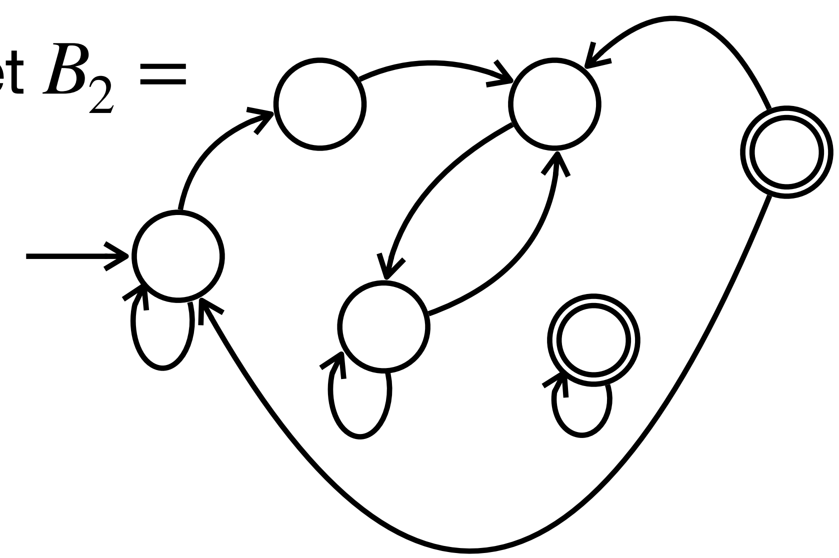


Let $B_2 =$



Is $B_1 \in E_{\text{DFA}}$? Is $B_2 \in E_{\text{DFA}}$?

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$D_{E_{\text{DFA}}}$ = "On input $\langle B \rangle$ where B is a DFA,

1. Mark the start state of B.
2. Repeat until no new states are marked: mark any unmarked state with a transition from a marked state.

Emptiness Problem for DFAs

Theorem (p.196): The language $E_{\text{DFA}} = \{\langle B \rangle \mid B \text{ is a DFA and } L(B) = \emptyset\}$ is decidable.

Proof: Let $D_{E_{\text{DFA}}}$ be the following TM that tests if there is a path from the start state to an accept state.

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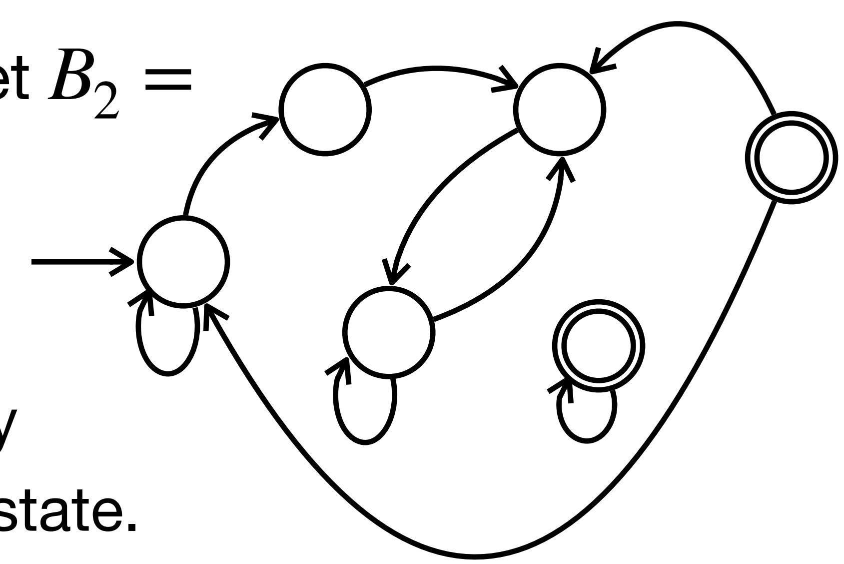

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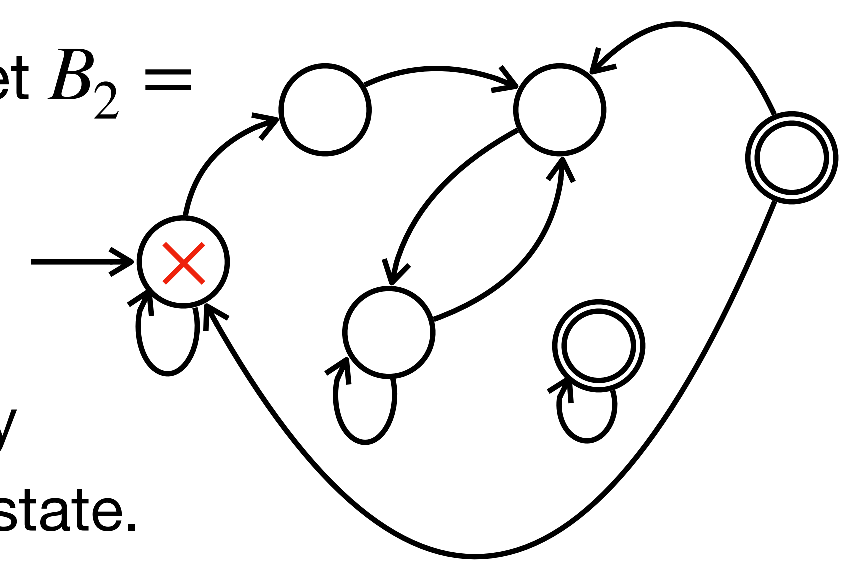

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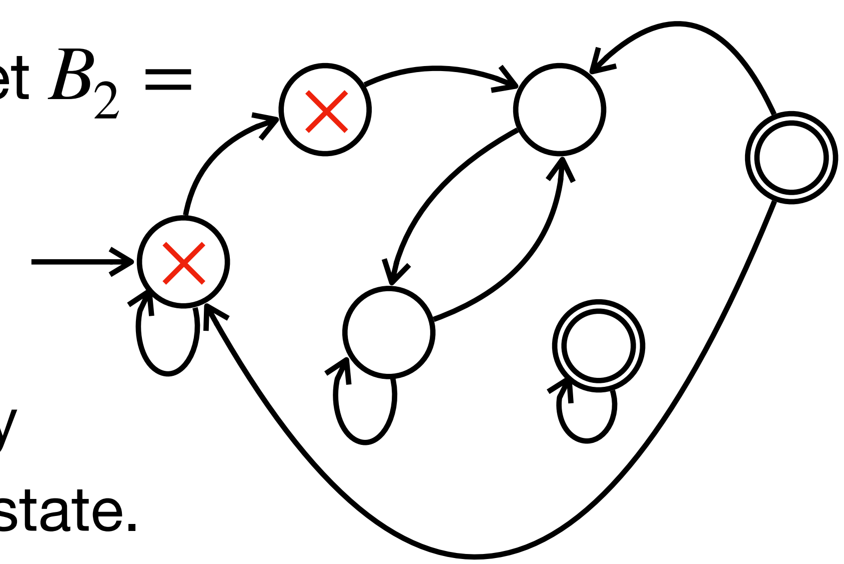

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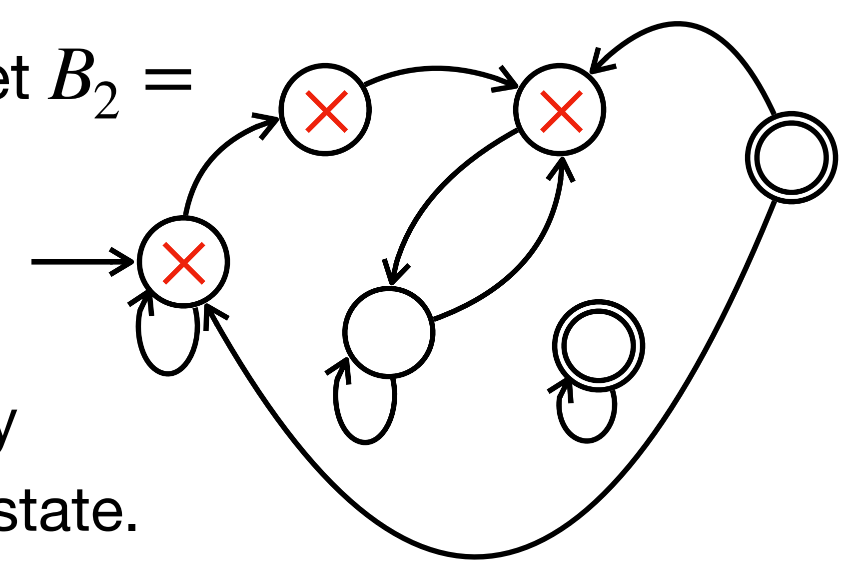

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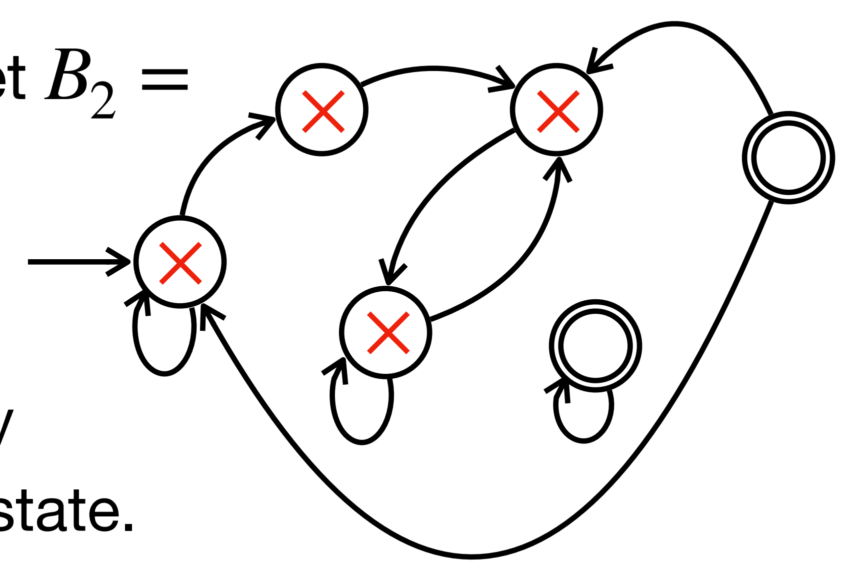

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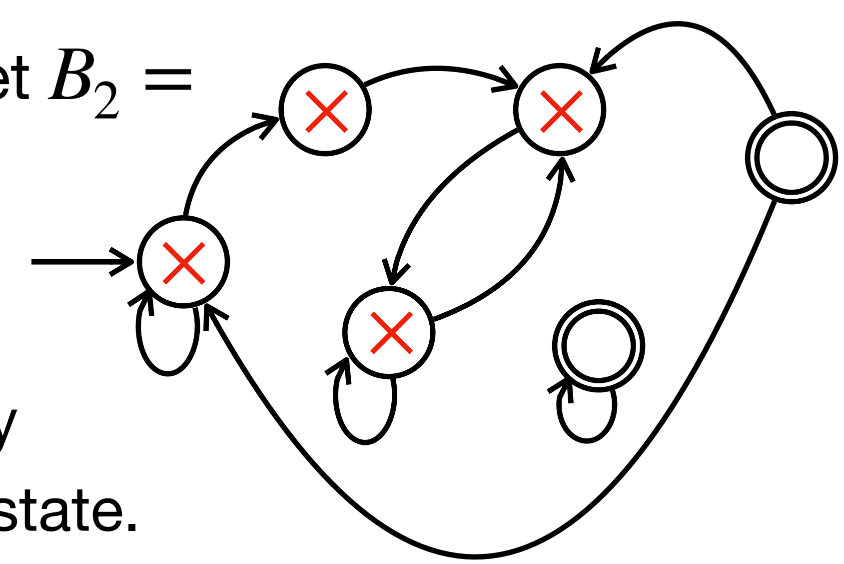

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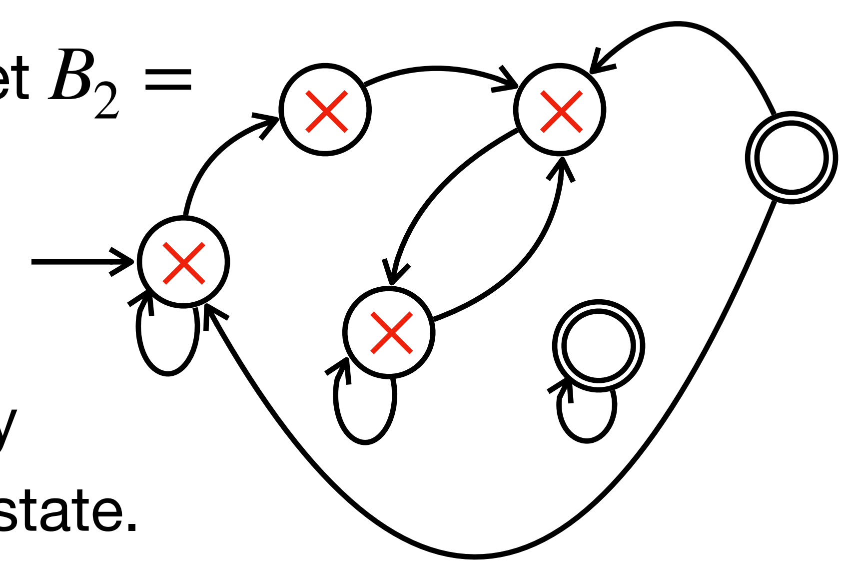

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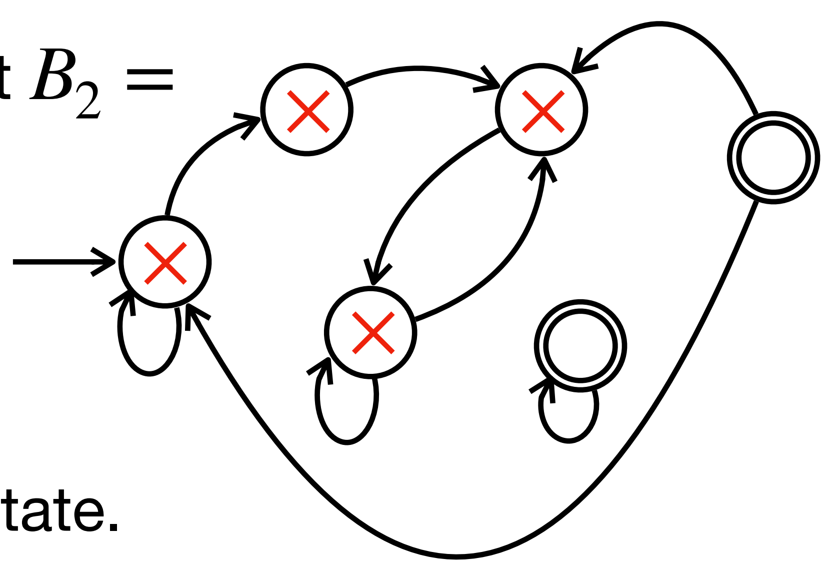

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$D_{E_{\text{DFA}}}$ decides E_{DFA}.

$D_{E_{\text{DFA}}}$ always halts within $|Q|$ sub-steps.

Q.E.D.

Check-In 2 (Break)

Let $ALL_{\text{DFA}} = \{\langle M \rangle \mid M \text{ is a DFA and } L(M) = \Sigma^*\}$.

Prove that ALL_{DFA} is decidable.

(*Hint*: Consider what we just saw in the last slide: E_{DFA}.)

Equivalence Problem for DFAs

Theorem (p.197): The language $EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A$ and B are DFAs and $L(A) = L(B)\}$ is decidable.

On the whiteboard.

Summary of Today's Lecture

- Notation for Encodings and TMs
- Decision procedures for DFAs

Acknowledgements

- These slides are based on lecture notes on Theory of Computation from other universities, namely Michael Sipser (MIT), Lorenzo De Stefani (Brown).

- **Errata:** If you let us know of any errors in the slides, we'll fix them and acknowledge you here!