Assignment w/ Latin Hypercube Sampling (LHS)

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Introduction

For this assignment, we are interested in estimating vegetation or crop water by first estimating atmospheric conductance, which is how easily water diffuses into the air. Atmospheric conductance depends on factors such as windspeed (you get more evaporation in windier conditions), the vegetation itself, and the turbulence it creates.

1. Code a function to compute atmospheric conductance C_{at} (how easily vapor diffuses from vegetation surfaces)

$$C_{at} = \frac{v_m}{6.25 * ln(\frac{z_m - z_d}{z_0})^2}$$

$$z_d = k_d * h$$

$$z_0 = k_0 * h$$

Figure 1: Equation for atmospheric conductance C_{at}, how easily vapor diffuses from vegetation surfaces

Note that:

- *`zm`: height at which windspeed is measured (usually 200cm above the vegetation)*
- *`h`: vegetation height (cm)*

- *`v`: windspeed (cm/sec)*
- *`kd`: 0.7*
- *`ko`: 0.1*

```
#source in atmospheric conductance (cat) function
source(here("functions", "compute_cat.R"))
```

2. Run your model

You are estimating the atmospheric conductance for a forest that is 10 m high (the accuracy of that measurement is +/- 0.5 m) Windspeeds (`v`) in this region are normally distributed with a mean of 250 cm/s with a standard deviation of 30 cm/sec.

Come up with a single estimate of atmospheric conductance for this forest.

Set up the C_{at} model parameters:

- number of samples
- `h`: vegetation height (m)
- `v`: windspeed (cm/sec)

```
#parameters
nsamples = 100

#convert m to cm to match the windspeed units
h_default <- 10 * 100
h_deviation = 0.5 * 100

#sample from the uniform distribution of plant heights based on the given forest height accuracy of +/-
h <- runif(min = h_default - h_deviation,
           max = h_default + h_deviation,
           n=nsamples)

v_mean <- 250
v_sd <- 30

#sample the normal distribution of windspeeds based on the given mean and SD values
v <- rnorm(mean = v_mean, sd = v_sd, n = nsamples)

#bind vegetation height (h) and windspeed (v) into a parameters dataframe
parameters <- cbind.data.frame(h, v)
```

Run the model

```
results <- compute_cat(h = parameters$h, v = parameters$v)

mean_cat <- round(mean(results), digits = 2)
```

The model estimates that the mean atmospheric conductance for this forest is approximately 15.14 cm/sec.

3. Now do a sensitivity analysis as follows

Consider the sensitivity of your estimate to uncertainty in the following parameters and inputs

- *`h`*: vegetation height (cm)

- ***kd***
- ***ko***
- ***v***: windspeed (cm/sec)

Windspeeds (v) are normally distributed with a mean of 250 cm/sec with a standard deviation of 30 cm/sec.

For vegetation height assume that height is somewhere between 9.5 and 10.5 m (but any value in that range is equally likely).

*For the **kd** and **ko** parameters you can assume that they are normally distributed with standard deviation of 1% of their default values.*

```
#set default parameters
kd_default = 0.7
k0_default = 0.1

#calculate SD
kd_sd = 0.01 * kd_default
k0_sd = 0.01 * k0_default
```

a) *Use LHS to generate parameter values for the 4 parameters*

Note the following sample distribution types:

1. v - normally distributed
2. h - uniform distribution
3. k0 - normally distributed
4. kd - normally distributed

```
#consider the 4 parameters
factors = c("v", "h", "k0", "kd")

#decide how many parameter sets to generate
nsets=100

#note which parameter uses which type of distribution
q = c("qnorm", "qunif", "qnorm", "qnorm")

#generate the distributions for each parameter
q.arg = list(list(mean = 250, sd = 30),                   #v
             list(min = 9.5 * 100, max = 10.5 * 100), #h
             list(mean = k0_default, sd = k0_sd),       #k0
             list(mean = kd_default, sd = kd_sd))       #kd

# generate samples from LHS
sens_cat = LHS(NULL, factors, nsets, q, q.arg)
sens_parameters = get.data(sens_cat)

head(sens_parameters, n = 10) #check that we have 100 parameter sets
```

```
##           v      h         k0        kd
## 1  252.6353  970.5 0.09965487 0.6947121
## 2  258.7712 1025.5 0.10181191 0.6881322
## 3  315.1027 1046.5 0.10037186 0.6984971
## 4  327.2749 1000.5 0.10125357 0.6981358
## 5  234.6978  973.5 0.09978530 0.6953881
```

```
## 6  218.2564 1006.5 0.09988696 0.7035705
## 7  276.8942  994.5 0.10159819 0.6972080
## 8  202.0542 1029.5 0.10056805 0.6999123
## 9  204.5769 1034.5 0.09946116 0.6894013
## 10 254.1491 1045.5 0.09975957 0.7060173
```

b) *Run you atmospheric conductance model for these parameters and return aerodynamic conductances*

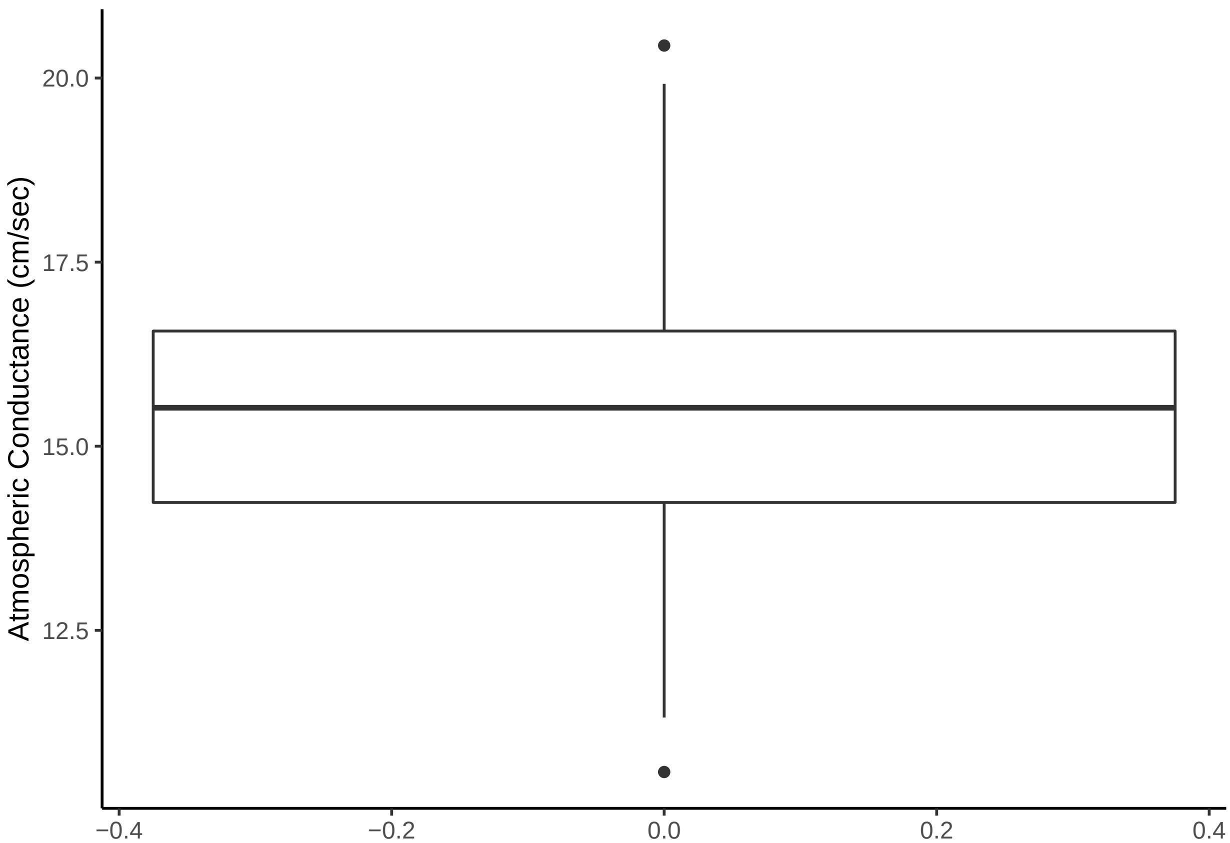
```
conductances <- sens_parameters %>% pmap(compute_cat)
conductances_df <- as.data.frame(unlist(conductances)) %>%
  rename(conductances = "unlist(conductances)")

sens_cat = pse::tell(sens_cat, t(as.matrix(conductances_df)),
                        res.names=c("conductance"))
```

c) *Plot conductance estimates in a way that accounts for parameter uncertainty*

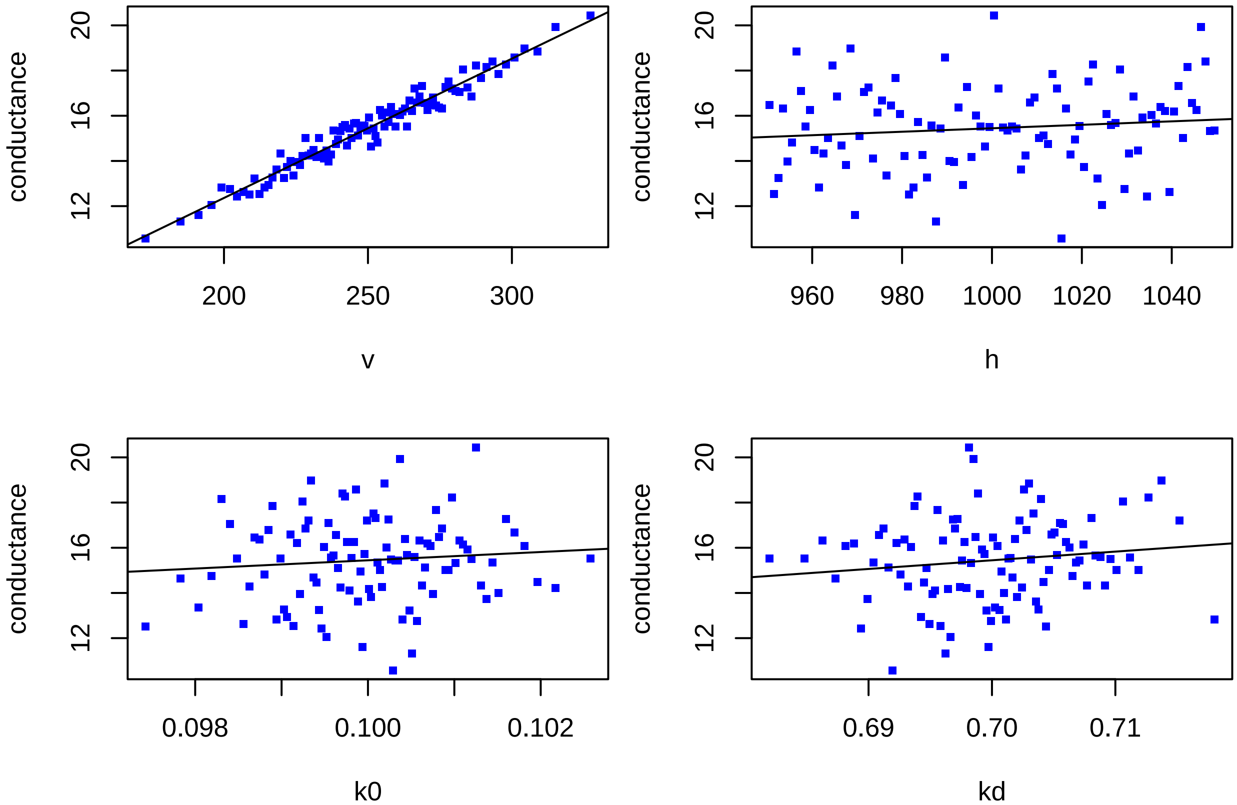
```
tmp = conductances_df %>% gather(value="conductances", key="conductances")

ggplot(tmp, aes(y = conductances)) +
  geom_boxplot() +
  theme(axis.title.x=element_blank(), axis.title.y=element_blank()) +
  labs(y="Atmospheric Conductance (cm/sec)") +
  theme_classic()
```



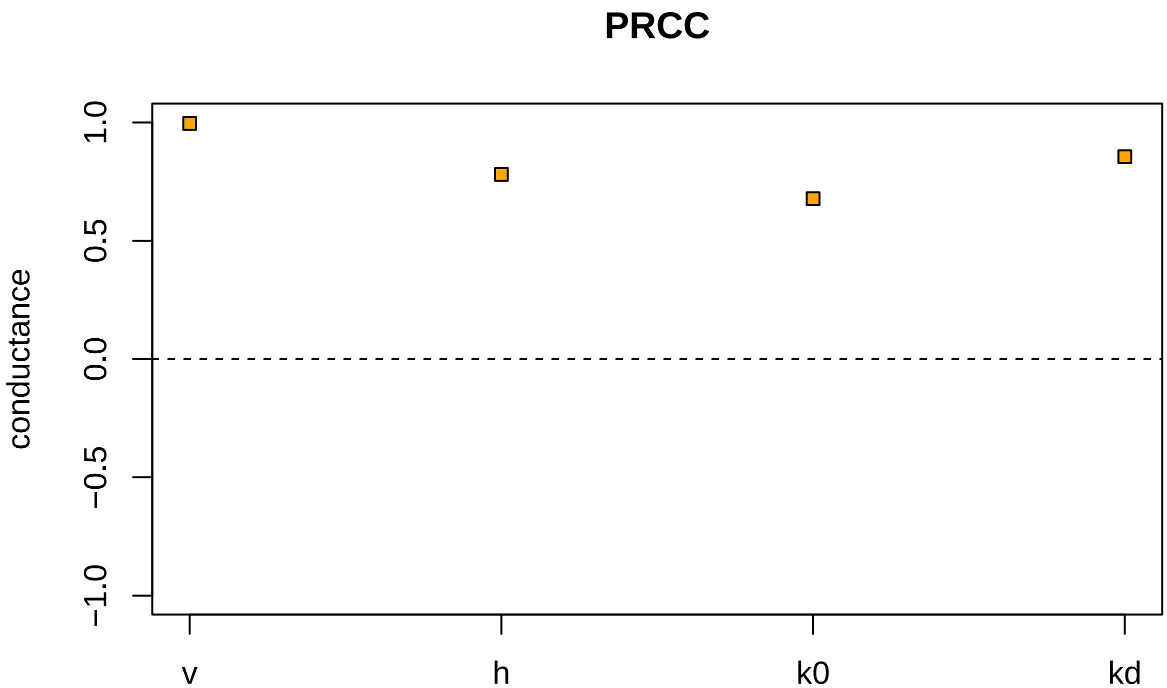
d) Plot conductance estimates against each of your parameters

```r
pse::plotscatter(sens_cat, col="blue", cex=5)
```



e) Estimate the Partial Rank Correlation Coefficients

```r
pse::plotprcc(sens_cat)
```




```r
# list PRCC value for each parameter
sens_cat$prcc
```

```
## [[1]]
##
## Call:
## pcc.default(X = L, y = r, rank = T, nboot = nboot)
##
## Partial Rank Correlation Coefficients (PRCC):
##     original
## v  0.9954193
## h  0.7801499
## k0 0.6775988
## kd 0.8550923
```

```r
# compare PRCC with parameter values
# head(sens_cat)
```

f) Discuss what your results tell you about how aerodynamic conductance varies with the different parameters.

What does it suggest about what you should focus on if you want to reduce uncertainty in aerodynamic conductance estimates? Does this tell you anything about the sensitivity of plant water use to climate change?

Aerodynamic conductance has the strongest positive relationship to the windspeed (v) parameter, as an increase in windspeed leads to an increase in aerodynamic conductance. Although aerodynamic conductance varies with vegetation height (h), kd, and k0 parameters, there does not appear to be any strong trends or relationships based on exploratory plots. Windspeed has the largest PRCC (0.99) of all parameters, indicating a strong correlation to aerodynamic conductance, whereas the other parameters have PRCC's ranging from 0.63 to 0.72, indicating moderate correlations. These results make sense, as windspeed likely impacts plant water use the most (due greater evaporation in windier conditions), compared to plant height, kd, and k0 parameters. Therefore, based on our sensitivity analysis results, we should focus on the windspeed parameter (v) to reduce uncertainty in aerodynamic conductance estimates.