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CSC 400 - 802 Loop

Assignment HW#5-2

Section 6.2

6(2). (a) $A \cap (B \cup C)$

(b) or

(c) and

(d) $A \cap (B \cup C)$

(e) $x \in A$ and $x \in C$

(3) (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Section 6.3

2. Counterexample:

Let $U = \{1, 2, 3, 4\}$, $A = \{1\}$, $B = \{2\}$. Hence $A \cup B = \{1, 2\}$, $(A \cup B)^c = \{3, 4\}$.
$\because A^c = \{2, 3, 4\}$, $B^c = \{1, 3, 4\}$, $\therefore A^c \cup B^c = \{1, 2, 3, 4\}$. Therefore, $\{3, 4\} \neq \{1, 2, 3, 4\}$
$(A \cup B)^c \neq A^c \cup B^c$, the statement is false. Q.E.D.

Section 7.2

8. a. H is not one-to-one. Because $y = H(b) = H(c)$.
H is not onto. Because for x, z in Y (co-domain of H) there isn't any corresponding connection from the element in X (domain of H).

b. K is one-to-one. K takes 3 different values on the 3 different elements of X.
K is not onto. K ~~never~~ never takes value z in the codomain.

22. a. No, D is not one-to-one.
Let $S_1 = 10$, $S_2 = 01$. Obviously $S_1 \neq S_2$. But $D(S_1) = D(S_2) = 1 - 1 = 0$.
Therefore, D is not one-to-one.

b. Yes. D is onto.
For any $n \in \mathbb{Z}$, if $n = 0$, $S = 01$, $\therefore D(S) = 0$.
If $n > 0$, $S = 1 \cdot 1 \cdots 1 = 1^n$, then $D(S) = n - 0 = n$.
If $n < 0$, $S = 0 \cdot 0 \cdots 0 = 0^{|n|}$, then $D(S) = 0 - (-n) = n$.
Hence for any $n \in \mathbb{Z}$, $\exists s \in S$ such that $D(S) = n$, $\therefore$ D is onto.

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29. a.) Let $(x, y), (a, b) \in R \times R$, in that case: $H(x, y) = H(a, b)$.

$\Rightarrow (x+1, 2-y) = (a+1, 2-b) \Rightarrow x+1 = a+1$ and $2-y = 2-b \Rightarrow x = a$ and $y = b$

$\Rightarrow (x, y) = (a, b)$

Therefore, H is one-to-one function. Q.E.D.

b.) Let $(x, y) \in R \times R$. Also ~~let~~ We want $(a, b) \in R \times R$ such that $H(a, b) = (x, y)$.

$\Rightarrow (a+1, 2-b) = (x, y) \Rightarrow \begin{cases} a+1 = x \\ 2-b = y \end{cases} \Rightarrow \begin{cases} a = x-1 \\ b = 2-y \end{cases} \Rightarrow (a, b) = (x-1, 2-y) \in R \times R$

Hence then we have $H(a, b) = (x-1+1, 2-(2-y)) = (x, y)$.

Therefore, H is onto function. Q.E.D.

49. (Exercise 12b).

The function is a one-to-one correspondence. (It's one-to-one and onto.)

Inverse function:

$G(n) = 2 - 3n$ ... the definition of G.

Let $x = n$, $y = G(n)$, $\cancel{\therefore y = 2-3x}$ $\therefore y = 2 - 3x$

Inverse x and y, $\therefore x' = 2 - 3y' \Rightarrow y' = \frac{2-x'}{3}$

$\therefore G^{-1}(x) = \frac{2-x}{3}$

Therefore, the inverse function of $G(x)$ is $G^{-1}: R \to R$, $G^{-1}(x) = \frac{2-x}{3}$

Section 8.1

20. $A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

$R = \{(-1,1), (1,1), (2,2)\}$

$S = \{(-1,1), (1,1), (2,2), (4,2)\}$

$R \cup S = \{(-1,1), (1,1), (2,2), (4,2)\}$

$R \cap S = \{(-1,1), (1,1), (2,2)\}$

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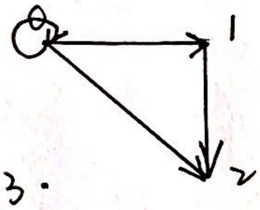
Section 8.2

5. $A=\{0,1,2,3\}$, $R_5=\{(0,0),(0,1),(0,2),(1,2)\}$

a) 

b.) Not reflexive.

Counterexample: $(1,1)$ $(2,2)$

c) Not symmetric.

Counterexample: $(1,0)$, $(2,0)$, $(2,1)$

d) Transitive.

14. $O=\{m,n \in Z \mid (m-n) \text{ is odd}\}$

① Since $m,n \in Z$, let $m=n$. If O is reflexive, it contains (m,m), (n,n) for $m,n \in Z$. But O doesn't contain (m,m), (n,n) because $m-m=n-n=0$ is even. Therefore, it is not reflexive.

② Assume there is $(m,n) \in O$ and $(m-n)$ is odd. By the definition of odd, there exists an integer k that: $m-n=2k+1$.
By algebra, $n-m=-(2k+1)=-2k-1=2(-k-1)+1$ $\therefore (n-m)$ is odd.
$\therefore (n-m) \in O$. Therefore, O is symmetric.

③ Let $(1,2) \in O$ and $(2,3) \in O$ since $1-2=-1$ is odd and $2-3=-1$ is odd.
But $(1,3) \notin O$ as $1-3=-2$ is even.
Therefore, O is not transitive. Q.E.D.