Ximan Liu
CSC400 - 802 Loop
Assignment #6

Section 8.2

53. $T^t = \{(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (2,3), (3,0), (3,1), (3,2), (3,3)\}$

Section 8.3

10. In same equivalence class as -5, $(-5)^2 - a^2 = 25 - a^2$
when $(-5, a) \in R$, $25 - a^2$ is divisible by 3.
Then we found that $-4, 4, -2, 2, -1, 1$ and 5 are in same equivalence class as -5, thus the first equivalence class is $\{-5, 5, -4, 4, -2, 2, -1, 1\}$
Then there is the same equalence class as -3.
When $(-3, a) \in R$, $(-3)^2 - a^2 = 9 - a^2$ is divisible by 3.
Then we found that ~~3~~ 3, 0 are in same equivalence class as -3,
the equivalence class ~~are~~ is $\{-3, 0, 3\}$
$\therefore$ All in all, the equivalence classes are $\{-5, 5, -4, 4, -2, 2, -1, 1\}$, $\{-3, 0, 3\}$.

22.(1) Proof: to proof R is an equivalence relation.
For reflexive:
Let $x \in A$, since a statement is always logically equivalent with itself, a statement always has the same ~~truth table~~ ~~as~~ as itself and thus $(x, x) \in R$.
Since $(x, x) \in R$ for all $x \in A$, therefore R is reflexive.

For symmetric:
Assume $(x, y) \in R$. As x and y have the same truth table, then y and x have the same truth table, $\therefore (y, x) \in R$.
Therefore, $(x, y) \in R$, $(y, x) \in R$, R is symmetric.

For transitive;
Assume $(x, y) \in R$ and $(y, z) \in R$. As x and y have the same truth table, y and z have the same truth table, however, ~~it~~ implies x, y and z all have same truth table and x and z have the same truth table as well. $\therefore (x, z) \in R$.
Therefore, $(x, y) \in R$ and $(y, z) \in R$ and $(x, z) \in R$, $\therefore$ R is transitive. ~~Q.E.D.~~
~~(2)~~ Since R is reflexive, symmetric and transitive, R is an equivalence relation.
Q.E.D.

(2) Hence two statement forms have the same truth table, $\therefore$ they are related, and they are logically equivalent.
Each ~~equivalentce~~ equivalence class contains all logically equivalent statements.

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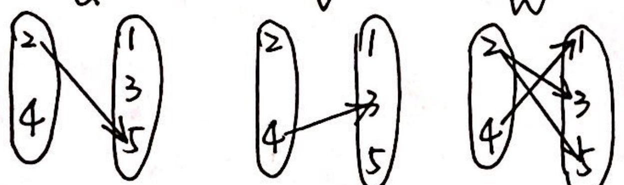
Section 1.3

8. U V W



a. arrow diagrams for U, V and W.

b. None of U, V and W are function.
For U and V, there isn't $y \in B$ for every $x \in A$. For W, there are two y values for the value 2 in X.

12. T is not a function. There are several outputs for some inputs. It doesn't follow the rule of being a function.

Section 4.8

I.e $= \frac{41}{48}$

7. $a=59$, $d=13$ Trace table:

Variable	0	1	2	3	4 iteration
a	59				
d	13				
r	59	46	33	20	7
q	0	1	2	3	4

16. gcd (4131, 2431):

iteration	gcd(,)	a	b	r
1	gcd (4131, 2431)	4131	2431	1700
2	gcd (2431, 1700)	2431	1700	731
3	gcd (1700, 731)	1700	731	238
4	gcd (731, 238)	731	238	17
5	gcd (238, 17)	238	17	0
6	gcd (17, 0)	17 gcd	0	

∴ the greatest common divisors as known as gcd (4131, 2431) = 17

23b. Input: A, B [integers with $A > B \geq 0$]
Algorithm Body:

```
a := A
b := B
r := B
while (b≠0)
    r := a - ⌊a/b⌋·b
    a := b
    b := r
end while
gcd := a
Output: gcd
```

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Section 5.6

6. $t_0=-1$, $t_1=2$, $t_2=t_1+2t_0=2+2\cdot(-1)=0$, $t_3=t_2+2t_1=0+2\cdot2=4$

8. $v_1=1$, $v_2=3$, $v_3=v_2+v_1+1=3+1+1=5$, $v_4=v_3+v_2+1=5+3+1=9$

16. $m_7=2m_6+1=2\cdot63+1=127$

$m_8=2m_7+1=2\cdot127+1=255$

33. $F_n=\frac{1}{\sqrt5}\left[\left(\frac{1+\sqrt5}{2}\right)^{n+1}-\left(\frac{1-\sqrt5}{2}\right)^{n+1}\right]$

Explicit:

$F_0=\frac{1}{\sqrt5}\cdot\left[\left(\frac{1+\sqrt5}{2}\right)^1-\left(\frac{1-\sqrt5}{2}\right)^1\right]=1$

$F_1=\frac{1}{\sqrt5}\left[\left(\frac{1+\sqrt5}{2}\right)^2-\left(\frac{1-\sqrt5}{2}\right)^2\right]=1$

$F_2=\frac{1}{\sqrt5}\left[\left(\frac{1+\sqrt5}{2}\right)^3-\left(\frac{1-\sqrt5}{2}\right)^3\right]=2$

$F_3=\frac{1}{\sqrt5}\left[\left(\frac{1+\sqrt5}{2}\right)^4-\left(\frac{1-\sqrt5}{2}\right)^4\right]=3$

$F_4=\frac{1}{\sqrt5}\left[\left(\frac{1+\sqrt5}{2}\right)^5-\left(\frac{1-\sqrt5}{2}\right)^5\right]=5$

Recurrence:

$F_0=1$

$F_1=1$

$F_2=F_1+F_0=1+1=2$

$F_3=F_2+F_1=2+1=3$

$F_4=F_3+F_2=3+2=5$

$\therefore$ For spot-check $F_2.F_3.F_4$ of Fibonacci sequence, the explicit and reccurrence forms are the same.

Extra Credit:

Section 4.8

28. For $\gcd(a,b)$, we have $\gcd(a,b)|a$.

Proof: For $\text{lcm}(a,b)$, we have $a|\text{lcm}(a,b)$.

Therefore, $\gcd(a,b)|\text{lcm}(a,b)$. Q.E.D.