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CSC 400 – 802 Loop

Assignment HW#3

General Questions:

1. Proof: The equation (to be proven) can be written using sequence notation as: $\sum_{i=1}^{n} i(i+1) = [n\cdot(n+1)\cdot(n+2)]/3$

We show by induction on n.

1) Basis step: When $n=1$,

LHS (Left-hand side) $= \sum_{i=1}^{1} i(i+1) = 1\cdot(1+1) = 2$

RHS (Right-hand side) $= [1\cdot(1+1)\cdot(1+2)]/3 = 2$

Therefore, we have LHS = RHS ··· (A)

2) Inductive step:

[Inductive hypothesis]

Assume the equation holds for some integer k which is $>= 1$.

That is, $\sum_{i=1}^{k} i(i+1) = [k\cdot(k+1)\cdot(k+2)]/3$

[Inductive ~~hypo~~ statement]

We show that the equation holds for $k+1$, that is $\sum_{i=1}^{k+1} i(i+1) = [(k+1)\cdot(k+2)\cdot(k+3)]/3$.

LHS

$= \sum_{i=1}^{k+1} i(i+1)$

$= \sum_{i=1}^{k} i(i+1) + (k+1)(k+1+1)$ ··· by ~~the~~ definition of this sequence

$= [k\cdot(k+1)(k+2)]/3 + (k+1)\cdot(k+2)$ ··· by inductive hypothesis

$= [k(k+1)(k+2) + 3(k+1)(k+2)]/3$ ··· by algebra

$= [(k+1)\cdot(k+2)\cdot(k+3)]/3$

RHS $= [(k+1)\cdot(k+2)\cdot(k+3)]/3$ ··· as to be shown in inductive statement

Thus we get LHS = RHS ··· (B)

By (A) and (B), we can conclude that the statement is true (for all integers $n >= 1$). Q.E.D.

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2. Proof: The equation (to be proven) can be written using sequence notation as: $\sum_{i=1}^{n} 4i = 2n^2 + 2n$

We show by induction on n.

1) Basis step: When $n=1$,

$LHS = \sum_{i=1}^{1} 4i = 4 \cdot 1 = 4$

$RHS = 2 \cdot 1^2 + 2 \cdot 1 = 4$

Therefore, we have $LHS = RHS$ ... (A)

2) Inductive step:

[Inductive hypothesis]

Assume the equation holds for some integer k which is $>= 1$.

That is, $\sum_{i=1}^{k} 4i = 2k^2 + 2k$

[Inductive statement]

We show that the equation holds for $k+1$, that is $\sum_{i=1}^{k+1} 4i = 2(k+1)^2 + 2(k+1)$

LHS

$= \sum_{i=1}^{k+1} 4i$

$= \sum_{i=1}^{k} 4i + 4(k+1)$ ... by definition of this sequence

$= 2k^2 + 2k + 4(k+1)$ ... by inductive hypothesis

$= 2k^2 + 6k + 4$ ... by algebra

$RHS = 2(k+1)^2 + 2(k+1) = 2k^2 + 6k + 4$ ... as to be shown in inductive statement

Thus we get $LHS = RHS$ ... (B)

By (A) and (B), we can conclude that the statement is true (for all integers $n >= 1$). Q.E.D.

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Textbook Questions:

Section 4.3

13. Yes. $n^2-1=(4k+3)^2-1=16k^2+24k+8=8(2k^2+3k+1)=8(2k+1)(k+1)$, and $2k^2+3k+1$ is an integer because k is an integer and sums and products of integers are integers.

28. The statement is false.

Counterexample: Let $a=8$, $b=2$ and $c=4$. Then $a|bc$ because $8|8$ but $a\nmid b$ and $a\nmid c$ because $8\nmid 2$ and $8\nmid 4$.

Section 4.6

4. Proof: By contradiction, suppose there is an integer n such that $7m+4$ is divisible by 7. By definition of divisibility, $7m+4=7k$ for some integer k. Subtracting $7m$ from both sides gives that $4=7k-7m=7(k-m)$.

By the definition of divisibility, $7|4$.

But by Theorem 4.3.1 it implies that $7\le 4$, which contradicts the fact that $7>4$.

Therefore the original statement was true.

Thus for all integers m, $7m+4$ is not divisible by 7. Q.E.D.

Section 5.1

4. First four terms: $2, \frac{3}{2}, \frac{5}{4}, \frac{9}{8}$

7. For a_k, first four terms: 1, 3, 5, 7

For b_k, first four terms: 1, 3, 5, 13

13. $a_n=\frac{1}{n}-\frac{1}{n+1}$, where n is an integer and $n\ge 1$

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Section 5.2

11. Proof: For the given statement, the property is the equation to be proven as: $1^3+2^3+\cdots+n^3=\left[\frac{n(n+1)}{2}\right]^2$

We show by induction on n.

1) Basis step: When $n=1$,

$LHS = 1^3 = 1$

$RHS = \left[\frac{1\cdot(1+1)}{2}\right]^2 = 1$

Therefore, we have $LHS = RHS$ ---(A)

2) Inductive step:

[Inductive hypothesis]

Assume the equation holds for some integer k which is $>=1$.

That is, $1^3+2^3+3^3+\cdots+k^3=\left[\frac{k(k+1)}{2}\right]^2$

[Inductive statement]

We show that the equation holds for $k+1$, that is $1^3+2^3+\cdots+(k+1)^3=\left[\frac{(k+1)(k+2)}{2}\right]^2$

LHS

$= (1^3+2^3+3^3+\cdots+k^3)+(k+1)^3$ ... by definition of this sequence

$= \left[\frac{k(k+1)}{2}\right]^2+(k+1)^3$ ... by inductive hypothesis

$= \frac{k^2(k^2+2k+1)+4(k^3+3k^2+3k+1)}{4}$ ... by algebra

$= \frac{k^4+6k^3+13k^2+12k+4}{4}$

$RHS = \left[\frac{(k+1)(k+2)}{2}\right]^2 = \frac{(k^2+2k+1)(k^2+4k+4)}{4} = \frac{k^4+6k^3+13k^2+12k+4}{4}$ ... as to be shown in inductive statement

Thus we get $LHS = RHS$ ---(B)

By (A) and (B), we can conclude that the statement is true (for all integers $n>=1$). Q.E.D.

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12. Proof: For the given statement, the property is the equation to be proven as: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

We show by induction on n.

1) Basis step: When $n=1$,

$LHS=\frac{1}{1\cdot(1+1)}=\frac{1}{2}$

$RHS=\frac{1}{1+1}=\frac{1}{2}$

Therefore, we have $LHS=RHS$ ⋯ (A)

2) Inductive step:

[Inductive hypothesis]

Assume the equation holds for some integer k which is $>=1$.

That is, $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{k(k+1)}=\frac{k}{k+1}$

[Inductive statement]

We show that equation holds for $k+1$, that is $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

LHS

$=\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{k(k+1)}\right)+\frac{1}{(k+1)(k+2)}$ ⋯ by definition of this sequence

$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$ ⋯ by inductive hypothesis

$=\frac{k(k+2)+1}{(k+1)(k+2)}$ ⋯ by algebra

$=\frac{(k+1)^2}{(k+1)(k+2)}$

$=\frac{k+1}{k+2}$

$RHS=\frac{k+1}{k+2}$ ⋯ as to be shown in inductive statement

Thus we get $LHS=RHS$ ⋯ (B)

By (A) and (B), we can conclude that the statement is true (for all integers $n>=1$). Q.E.D.

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Extra Credit

Section 2.3

40. Muscles killed Shorky.

Only d, what Muscles said is true, others ~~are~~ were all lying.

44. (d) $\sim q \vee s$

(e.) $\sim s$

(*) $\therefore \sim q$ by Elimination and (d) and (e)

(a.) $p \to q$

(**) $\therefore \sim p$ by Modus Tollens and (a.) and (*)

(e.) $\sim s$

(b.) $r \vee s$

(***) $\therefore r$ by Elimination and (b.) and (e)

(e.) $\sim s$

(c.) $\sim s \to \sim t$

(****) $\therefore \sim t$ by Modus ponens and (c.) and (e.)

(g.) $w \vee t$

(*****) $\therefore w$ by Elimination and (g) and (****)

(f.) $\sim p \wedge r \to u$

(******) $\therefore u$ by (f) and (**) and (***)

(h.) $\therefore u \wedge w$ by conjunction of (*****) and (******)

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