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CSC 400-802 Loop

Assignment HW#2

Section 2.3

11.

p	q	r	$p \to q \vee r$	$\sim q \vee \sim r$	$\sim p \vee \sim r$
			premises		conclusion
T	T	T	T	F	F
T	F	T	T	T	F
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

← This row shows that it is possible for an argument of this form to have ~~the~~ true premises and a false conclusion. Thus this argument form is invalid.

Section 3.2

8. Informal negation:

There is at least one simple solution to life's problems.

Formal version of statement:

$\forall$ solutions x, if x is simple, then x is not to life's problems.

19. $\exists n \in \mathbb{Z}$, such that n is prime and n is ~~even~~ not odd and $n \neq 2$.

29. Exercise 19:

Statement: $\forall n \in \mathbb{Z}$, if n is prime then n is odd or $n=2$.

Contrapositive: $\forall n \in \mathbb{Z}$, if n is not odd and $n \neq 2$, then n is not prime.

Converse: $\forall n \in \mathbb{Z}$, if n is odd or $n=2$, then n is prime.

Inverse: $\forall n \in \mathbb{Z}$, if n is not prime, then n is not odd and $n \neq 2$.

The statement and its contrapositive are both true.

The converse and inverse are also false.

38.

Proof: The statement is false.

There is not any letter "u" in Discrete Mathematics.

Therefore, there is no such letter u with lowercase in these words. Q.E.D.

47. It is the case that if there is a reasoneble [program] correctness, then it is the absence of error messages during translation of a computer program.

Section 3.4.

13. Valid by universal modus ponens.

19. ~~a~~. $\forall x$, if x is a good car, then x is not cheap.

c. Valid, universal modus tollens.

d. Invalid, ~~converse~~ inverse error.

Section 4.1

13. Counterexample: Let $m=2$ and $n=1$. Then $2m+n = 2\times2+1 = 5$ is odd, but m is not odd. So the conclusion of the statement is ~~a~~ false.

41. The mistake in the "Proof" is, there exists an integer r such that $m\cdot n = r$, not "$m\cdot n = 2r$" as it showed, otherwise we cannot prove whethere r is odd or even. When $m=2p$, $n=2q+1$, ~~then~~ $(m\cdot n)$ will be like $m\cdot n = 2p\cdot(2q+1) = 2[p\cdot(2q+1)]$. ~~there~~ Therefore $r = 2[p\cdot(2q+1)]$ is a even number, ~~The statement~~ the product is even.

In fact, that is exactly what is to be proved.

50. True.

Proof: Since $n-m$ is even and n, m are both integers, ~~let $n-m=2k$~~

by definition of even, there exists an integer k such that $n-m=2k$.

By the definition of factoring a difference of cubes, $n^3-m^3=(n-m)(n^2+mn+m^2)$.

Therefore $n^3-m^3=(n-m)(n^2+mn+m^2)=2k(n^2+mn+m^2)$, n^3-m^3 is even.

Q.E.D.

Section 4.6

20. Proof (by contraposition):

[To go by contraposition, we must prove that, $\forall$ real numbers, a and b, if $a \geq 25$ and $b \geq 25$, then $a+b \geq 50$.]

Suppose a and b are real numbers and $a \geq 25$ and $b \geq 25$.

By the alge of inequalities, $a+b \geq 50$. ~~Q.E.D.~~

Therefore the original statement was true. Q.E.D.