Ximan Liu

CSC400-802 Loop

Assignment HW#7

Section 5.7

2b. $3^{n-1}+3^{n-2}+\cdots+3^2+3+1$ (n is an integer and $n\geq 1$)

$$= \frac{(3^n-1)\cdot 3^0}{3-1}$$

$$= \frac{3^n-1}{2}$$

4. $\because b_k = \frac{b_{k-1}}{1+b_{k-1}}$, for all integers $k\geq 1$, $b_0=1$,

$\therefore b_1 = \frac{1}{1+1} = \frac{1}{2}$, $b_2 = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$, $b_3 = \frac{\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{4}$

Therefore, the explicit formula for the sequence is $b_k = \frac{1}{k+1}$ for all $k\geq 0$.

13. $\because t_k = t_{k-1}+3k+1$, for all integers $k\geq 1$, $t_0=0$.

$\therefore t_1 = t_0+3\cdot 1+1 = 0+3+1 = 4$

$t_2 = t_1+3\cdot 2+1 = 4+3\cdot 2+1 = 11$

$t_3 = t_2+3\cdot 3+1 = 11+9+1 = 21$

$t_4 = t_3+3\cdot 4+1 = 21+12+1 = 34$

Therefore, $t_k = 0+3\cdot 1+3\cdot 2+3\cdot 3+\cdots+3\cdot k+k$

$= 3\cdot[1+2+3+\cdots+k]+k$

$= 3\cdot\frac{k(1+k)}{2}+k$

$= \frac{3k^2+5k}{2}$ holds for all integers $k\geq 1$

However, when $k=0$, $\frac{3\cdot 0+5\cdot 0}{2} = 0 = t_0$, $\therefore$ formula also holds for $k=0$.

Therefore, $t_k = \frac{3}{2}k^2+\frac{5}{2}k$ when $k\geq 0$

29. Proof: The equation (to be proven) can be written as $b_n = \frac{1}{n+1}$ for all integers ~~$k\geq 0$~~ $n\geq 0$.

We show by induction on n.

1) Basis Step: when $n=0$,

~~$LHS = b_1 = \frac{b_0}{1+b_0} = \frac{0}{1+0} = 1$~~ $LHS = b_0 = 1$

$RHS = \frac{1}{n+1} = \frac{1}{0+1} = 1$

Therefore, we have $LHS = RHS$ ⋯ (A)

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2) Inductive Step:

[Inductive hypothesis]:

Assume the equation holds for some integer k which is $>= 0$.

That is, $b_k = \frac{1}{k+1}$

[Inductive statement]:

We show that the equation holds for $k+1$, that is $b_{k+1} = \frac{1}{k+2}$

$LHS = b_{k+1} = \frac{b_k}{1+b_k} = \frac{\frac{1}{k+1}}{1+\frac{1}{k+1}} = \frac{1}{k+2}$

$RHS = \frac{1}{k+2}$

Thus we get $LHS = RHS$ ... (B)

By (A) and (B), we can conclude that the statement, $b_n = \frac{1}{n+1}$ for all integers $n \geq 0$ is true. Q.E.D.

Section 9.1

14. b) $C_3^2 (\frac{1}{2})^2 \cdot \frac{1}{2} + C_3^3 (\frac{1}{2})^3 = \frac{1}{2} = 50\%$

c) $C_3^0 (\frac{1}{2})^3 = \frac{1}{8} = 12.5\%$

Section 9.2

7. b) $A_4^2 + A_4^2 = 4 \cdot 3 + 4 \cdot 3 = 24$

c) $P = \frac{A_3^2}{A_4^2} + \frac{A_2^2}{A_4^2} = \frac{3}{12} + \frac{1}{12} = \frac{1}{3} \approx 33.3\%$

22. b) $n^m = 2^5 = 32$

c) Function: set $m \Rightarrow$ set n $\underbrace{\underline{n}\ \underline{n}\ \underline{n}\ \underline{n}\ \underline{n}}_{m \text{ positions}}$ $\therefore$ There are n^m possible functions.

38. b. $P(8,2) = A_8^2 = \frac{8!}{6!} = 8 \times 7 = 56$

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Assignment HW #7

Section 9.3

8\. b. $50 \times 49 \times 48 + 50 \times 49 \times 48 \times 47 + 50 \times 49 \times 48 \times 47 \times 46 = 259896000$

c. $50 \times 50 \times 50 + 50 \times 50 \times 50 \times 50 + 50 \times 50 \times 50 \times 50 \times 50 - 259896000$

$= 318875000 - 259896000$

$= 58979000$

$\therefore$ 259896000 passwords contain no repeated symbols.

58979000 passwords have at least one repeated symbol.

19\. $39 \times 38 \times 38 = 56316$

$\therefore$ There are 56316 possible selections for the combinations.

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