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CSC453 Homework 3

Part A: Functional Dependencies

A-1 Transitive Dependency and Keys

You have a relation R(L,M,N,O,P,Q) and a set of functional dependencies F = {LNO→M, MN→LOP, N→O, OP→LN}.

$NP \to LM \Rightarrow NP \to L,\ NP \to M$

- [2pt] Can we infer NP → LM from F ?

$N \to O$, $OP \to LN \Rightarrow OP \to L$ $\Rightarrow NP \to L$

$LNO \to M$, $N \cdot OP \cdot O \to M$, $N \to O$ $\Rightarrow NP \to M$

$\Rightarrow NP \to LM$ Yes! we can infer it.

- [3pt] Can we infer NQ → LO from F ?

$NQ \to LO \Rightarrow NQ \to L,\ NQ \to O$

No, there isn't Q within any relations.

A-2 Keys

(i) [5pt] Find **all** the candidate keys of the Relation R(ABCDE) with FD's:
D → C, CE → A, D → A, and AE → D

$(ABE)^+ = ABEDC$ ✓

$(BDE)^+ = BDECA$ ✓

$(BE)^+ = BE$

$(BCE)^+ = BCEAD$ ✓

∴ $(ABE)^+$. $(BDE)^+$. $(BCE)^+$ are all the candidate keys.

(ii) [5pt] Determine **all** the candidate and superkeys of the relation R(ABCDEF) with FD's:

AEF → C, BF → C, EF → D, and ACDE → F

All candidate keys:

$(ABE)^+ = ABE$ $(ABCDE)^+ = ABCDEF$

Super key: $(ABEF)^+ = ABEFCD$

$(ABEF)^+ = ABEFCD$

$(ABEFC)^+ = ABEFCD$

$(ABEFD)^+ = ABEFCD$

$(ABDE)^+ = ABDE$

$(ABCE)^+ = ABCE$

A-3 Minimal Cover

[5pt] Find a minimal cover for the following set F of functional dependencies.

A→BC

AB→D

C→AD

D→B

Show your working clearly. Points will be deducted if you do not show the extraneous attributes, and their elimination.

Step 1:

× A→B
A→C
× AB→D
C→A
C→D
D→B

⇒ Step 2:

A→B
A→C
√ A→D
√ C→A
√ C→D
D→B

⇒ Step 3:

A→B
A→C
C→A
C→D
D→B

⇒ Step 4:

A→C
C→A
C→D ×
D→B

⇒ Step 5:

A→C
C→A
D→B

∴ $(ACD)^+ = ABCD$

A-4 Equivalence

[15pt] Consider the following set of F.Ds. Determine if FD1 is equivalent to FD2 or to FD3:

FD1:

{BC->D,ACD->B,CG->B,CG->D,AB->C,C->A,D->E,BE->C,D->G,CE->A,CE->G}

FD2:

{AB->C,C->A,BC->D,CD->B,D->E,D->G,BE->C,CG->D}

FD3:

{AB->C,C->A,D->G,BE->C,CG->D,CE->G,BC->D,CD->B,D->E}

FD1:

BC→D, ACD→B, CG→B, CG→D, AB→C, C→A, D→E, BE→C, D→G, CE→A, CE→G

⇒ AB→D, AE→B, AG→B, AG→D, AB→C, C→A, D→E, BE→C, D→G, CE→A, CE→G

⇒ D→E, E→B, B→C, C→A, A→D, D→G, G→D

⇒ D→E, E→B, B→C, C→A, D→G, G→D

FD2:

AB→C, C→A, BC→D, CD→B, D→E, D→G, BE→C, CG→D

⇒ B→A, C→A, ~~B→G~~, A→D, D→B, D→E, D→G, E→A, C→G

⇒ C→A, A→D, D→B, B→A, D→E, E→A, ~~C→A~~, D→G, C→G

E→B ✓, B→C ✓, G→D ✓

∴ FD1 ≡ FD2

FD3:

AB→C, C→A, D→G, BE→C, CG→D, CE→G, BC→D, CD→B, D→E

⇒ A→D, D→E, E→G, E→A, B→A, C→A

∴ FD1 ≡ FD3

∴ FD1 ≡ FD2 ≡ FD3

They all are equivalent.

Part B: Normalization

B-1. Dependency Preservation (7 points)

For the relation R(w,x,y,z), consider the decomposition D consisting of R_1(w,y,z) and R_2(x,y), and the set of functional dependencies F ={y→xz; yz→w; x→w}. Recall that the projection of set of functional dependences G on relation R_x consists of every functional dependency in $(G)^+$ that contains only attributes from R_x.

a. Compute the projection of F on R_1.

$y \rightarrow z$

$yz \rightarrow w$

b. Compute the projection of F on R_2.

$y \rightarrow x$

c. Does the decomposition D preserve the set of dependencies F? Why or why not?

No. Because $x \rightarrow w$ is invalid.

B-2. Lossless Decomposition (8 points)

Perform the test for the non-additive join property (lossless join) for the relation R(A_1, A_2, A_3, A_4, A_5), and the decompositions D_a, D_b, D_c, D_d and set of functional dependencies F given below. You can ignore attributes that are not mentioned in each particular subsection (e.g., you can ignore absence of A_4 in D_d, just test the join between R_1 and R_2):

F = {A_1→A_4;A_4→A_5;A_3→A_4}

$A_1 \rightarrow A_4$

$A_4 \rightarrow A_5$

$A_3 \rightarrow A_4$

D_a = {R_1(A_1, A_2), R_2(A_3, A_4, A_5)}
D_b = {R_1(A_3, A_4), R_2(A_4, A_5)}
D_c = {R_1(A_1, A_5), R_2(A_4, A_5)}
D_d = {R_1(A_1, A_2, A_3), R_2(A_1, A_2, A_5)}

a. Does the decomposition D_a have the non-additive join property? Explain why or why not. No. $A_1 \to A_4$ is lost, and it's not part of any decomposition

$R_1(A_1, A_2)$

$R_2(A_3, A_4, A_5)$

b. Does the decomposition D_b have the non-additive join property? Explain why or why not. No. $A_1 \to A_4$ gets lost.

$R_1(A_3, A_4)$

$R_2(A_4, A_5)$

c. Does the decomposition D_c have the non-additive join property? Explain why or why not.

$R_1(A_1, A_5)$ $A_1 \to A_4$ Yes.

$A_4 \to A_5$

$R_2(A_4, A_5)$ $A_4 \to A_5$

d. Does the decomposition D_d have the non-additive join property? Explain why or why not.

$R_1(A_1, A_2, A_3)$ $A_1 \to A_4$ No.

$R_2(A_1, A_2, A_5)$ $A_4 \to A_5$

B-3. Normalization (15 points)

Consider the universal relation

F L T D S

EMPLOYEE(ID, First, Last, Team, Dept, Salary)

with the following set F of functional dependencies:

ID → First F
ID → Last L
F,L First, Last → ID
L Last → Team T
ID → Dept D
ID → Salary S
S Salary → Dept D

a. Identify candidate keys of EMPLOYEE.

$(F)^+ = F$ $\quad$ $(F,L)^+ = (ID)^+ = (ID, F, L, T, D, S)$

$(L)^+ = L, T$ $\quad$ $(ID)^+ = (ID, F, L, T, D, S)$

$(T)^+ = T$ $\quad$ ∴ Candidate key is $(ID)^+$, $(\text{First}, \text{Last})^+$

$(D)^+ = D$

$(S)^+ = \cancel{S}\, S, D$

b. Construct a decomposition of EMPLOYEE into relations in 3NF that preserves dependencies. Show full working. How can you be sure that your decomposition is dependency-preserving?

D minimal cover: Projection on R1: Projection on R3: L→T
Projection on R4: S→D

R1(ID, F, L, S) CK=ID ID→F
R2(F, L, ID) CK=F,L ID→L
R3(L, T) CK=L ID→S
R4(S, D) CK=S F,L→ID

Projection on R2:
F,L→ID
ID→F
ID→L

Test lossless property:
$R_1 \cap R_2 \cap R_3 \cap R_4 = \emptyset$ empty set.
∴ The lossless property is not upheld

c. Are all of the relations in your decomposition in BCNF? Either explain why they are, or identify one that is not and explain why it is not. (Note that for a relation to be in BCNF, the determinants of all functional dependencies in the relation must be superkeys *of that relation* – not superkeys of the original universal relation.)

Yes. Each determinant of the functional dependencies is a super key of a relation (not the universal relation).

B-4. 3NF (15 points)
Which of the following relations is in Third normal form (3NF)? Give sufficient reasoning if not in 3NF.

(a) R(ABCD) F = {ACD → B; AC → D; D → C; AC → B}

keys: AC, AD

prime attributes: A, C, D

∵ D→C, left side is not a superkey, but right side is a prime attribute. Other ~~3~~ three relations are all left sides as superkeys.

∴ R(ABCD) is in 3NF.

(b) R(ABCD) F = {AB → C; BCD → A; D → A; B → C}

keys: BD

prime attributes: B, D.

∵ AB→C left-hand side is not super key, right side is not prime attribute

∴ Not in 3NF

(c) R(ABCD) F = {AB → C; ABD → C; ABC → D; AC → D}

keys: ~~A, B~~ AB

prime: A, B

∵ AC→D left side is not a super key, right side is not a prime attribute

∴ R(ABCD) is not in 3NF

(d) R(ABCD) F = {C → B; A → B; CD → A; BCD → A}

keys: ~~C, D~~ CD

prime attribute: C, D

∵ C→B Left-hand side is not superkey, right-hand side is not prime

∴ R(ABCD) is not in 3NF

B-5. (BCNF) (15 points)
Which of the following relations is in BCNF? Give sufficient reasoning if not in BCNF.

(a) R(ABCD) F = {BC → A; AD → C; CD → B; BD → C}

$(BC)^+ = BCA$ ∴ Not in BCNF

$(AD)^+ = ADCB$

$(CD)^+ = ACDB$

$(BD)^+ = ABDC$

(b) R(ABCD) F = {BD → C; AB → D; AC → B; BD → A}

$(BD)^+ = BDCA$ ∵ For each given FD, the closure of left side attribute is ABCD.

$(AB)^+ = ABDC$

$(AC)^+ = ACBD$ Thus, the left side attribute of each FD contains a key,

$(BD)^+ = BDAC$ and R(ABCD) is in BCNF.

(c) R(ABCD) F = {A → C; B → A; A → D; AD → C}

$(A)^+ = ACD$ Only b is prime, no violation of the rule

$(B)^+ = BACD$ ∴ R(ABCD) is in BCNF.

$(AD)^+ = ADC$