Northeastern University

PROGRAM STRUCTURES & ALGORITHMS
INFO – 6205

Husky Benchmarking

Final Project

[GitHub] | [Excel Data]

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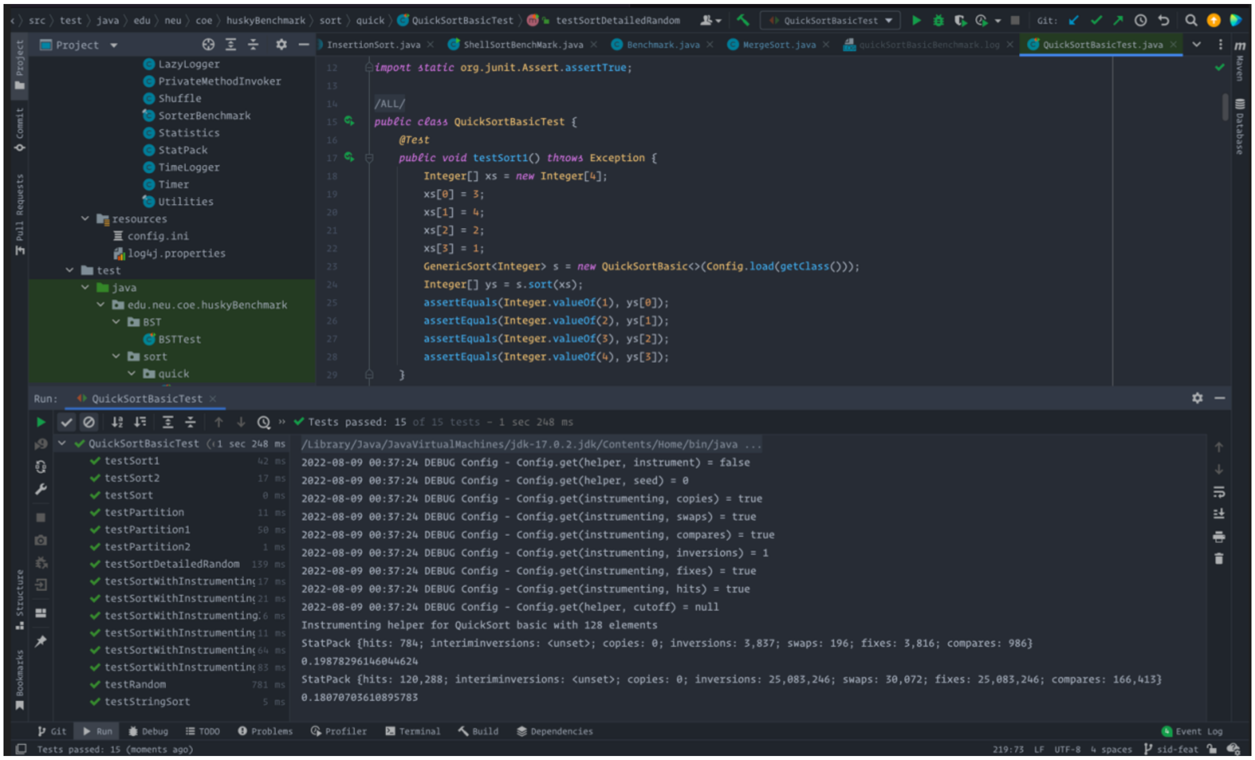
1.0 Quicksort

Do you agree that the number of swaps in "standard" quicksort is 1/6 times the number of comparisons? Is this figure correct? If not, why not. How do you explain it?

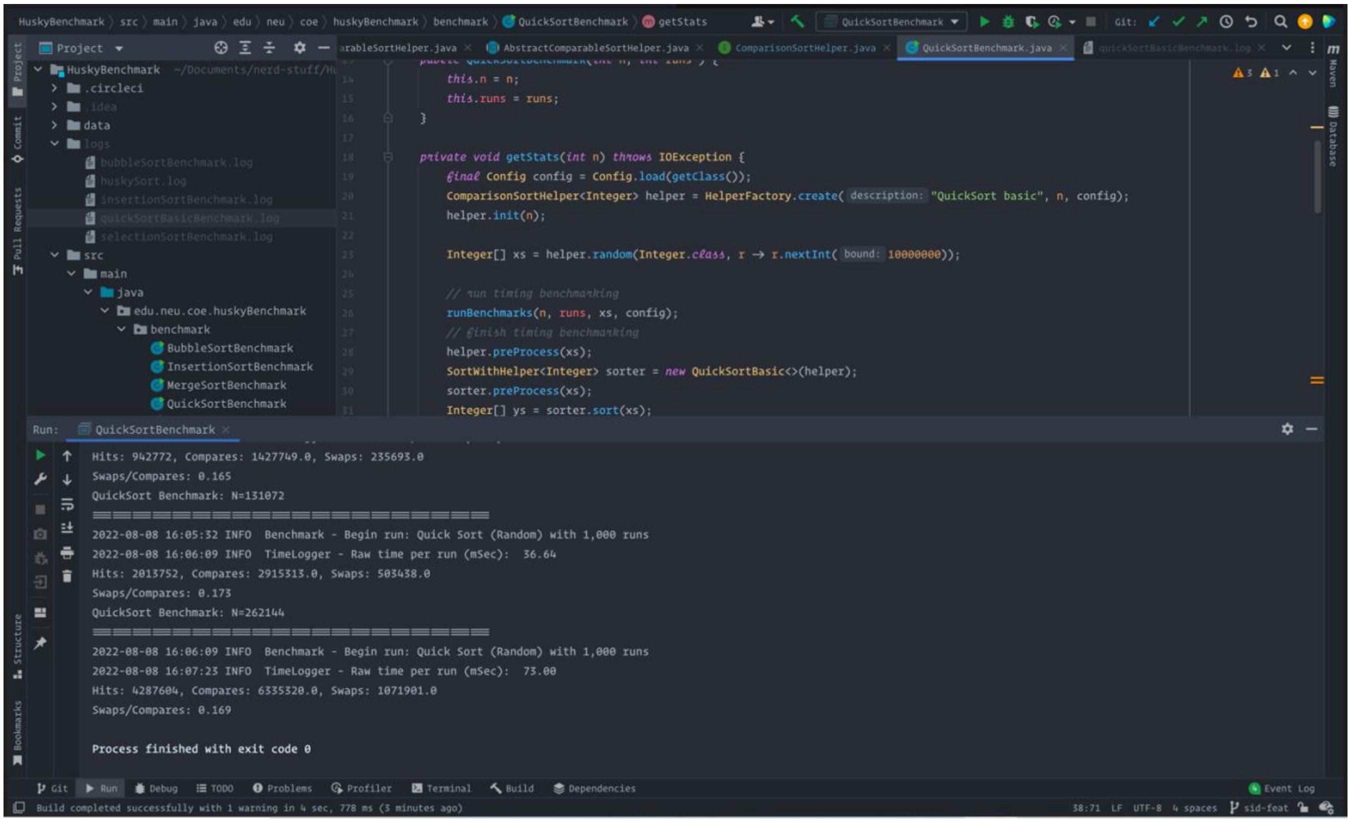
1.1 Approach:

While performing this task, we started by understanding that the number of swaps is $\sim\frac{1}{3}Nlog_e(N)$ and the number of compares is $\sim 2Nlog_e(N)$ for Quick Sort, according to the lecture notes. We started with the Test-Driven Development (TDD), and created several unit tests, based on which we create the code for the Quick Sort algorithm. We do not cutoff to insertion sort in this approach because the number of swaps and compares would vary due to this. Also, the Benchmarking of the code for a data set varying from size 2^7 to 2^{17} using doubling method for 1000 runs was performed. We have re-used the code from the class repository as well as the Husky sort repository and made necessary modifications to achieve the results.

1.2 Unit Test Cases:



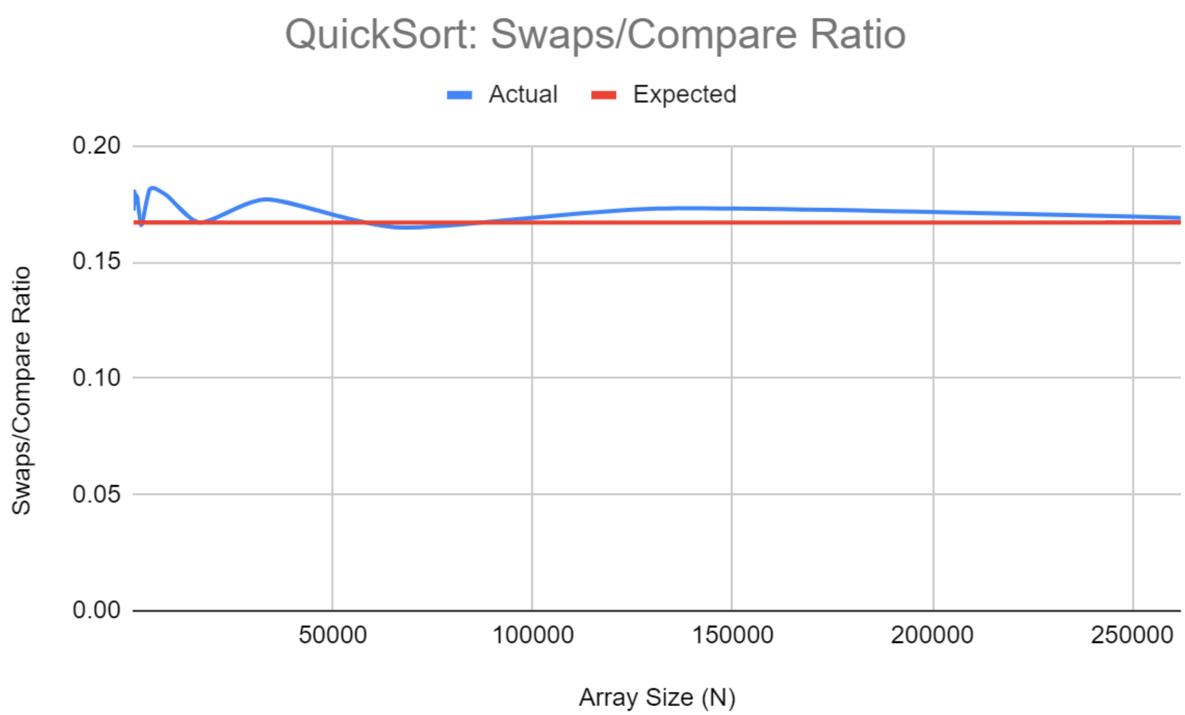
1.3 Output:



The output can be found [here].

1.4 Observations:

N	No Of Swaps	No of Compare	Actual	Expected
128	193	1125	0.172	0.167
256	427	2430	0.18	0.167
512	1033	5764	0.179	0.167
1024	2296	12911	0.178	0.167
2048	4972	29951	0.166	0.167
4096	11079	61334	0.181	0.167
8192	23955	133795	0.179	0.167
16384	50967	305105	0.167	0.167
32768	111437	630297	0.177	0.167
65536	235693	1427749	0.165	0.167
131072	503438	2915313	0.173	0.167
262144	1071901	6335320	0.169	0.167



The evidence can be found here.

1.5 Conclusion:

From the graph and the observations, it can be seen that the ratio of number of swaps to number of compares for a given size of array is approximately 0.1667.

Hence it can be concluded that the number of swaps in "standard" quicksort is approximately $\frac{1}{6}$ times the number of comparisons.

2.0 Hibbard deletion of BST

According to the course lecture notes, after a number of (Hibbard) deletions have been made, the average height of the tree is $\sqrt{N}$. Do you agree with this? How does it look after modifying the deletion process to either :

(a) randomly choose which direction to look for the node to be deleted, or

(b) choose the direction according to the size of the candidate nodes.

2.1 Approach:

We started by creating our own implementation of Binary Search Tree referencing Sedgewick and Wayne. The BinarySearchTree implements an API called BST which accepts a Key-Value pair and has the following functions:

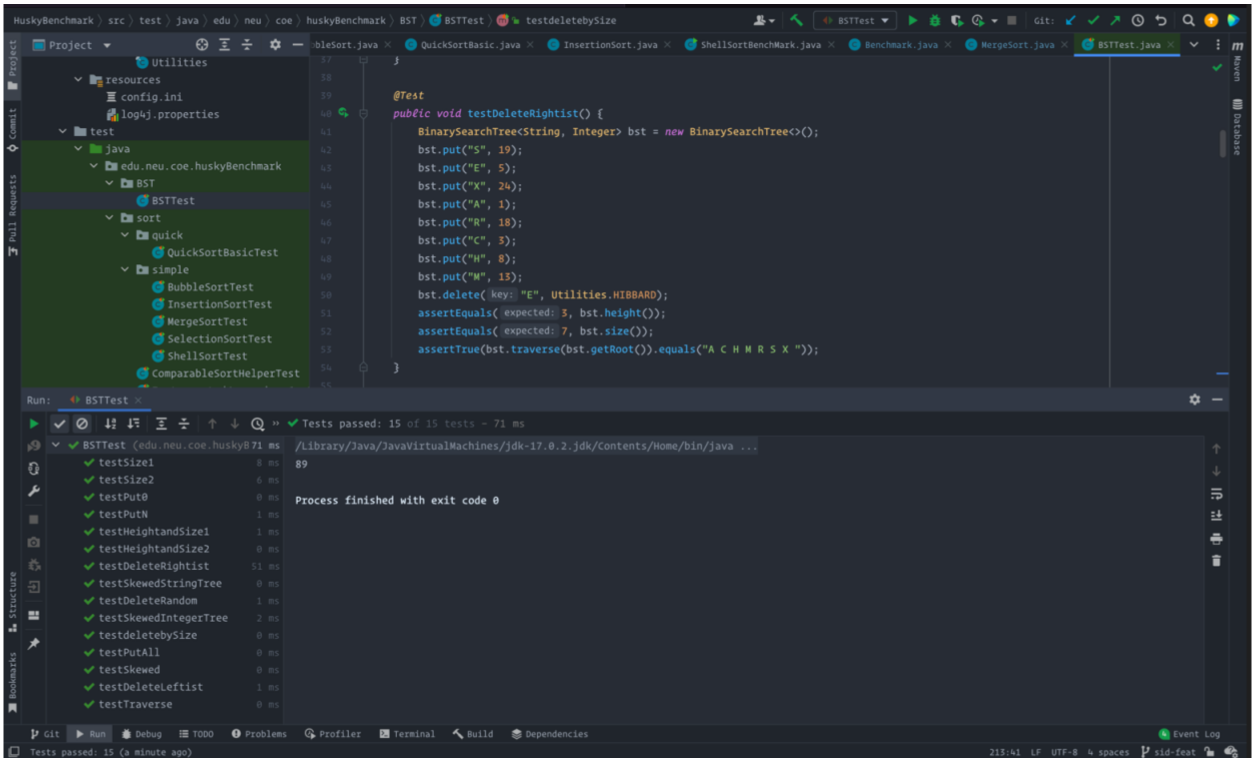
1. get()
2. put()
3. delete()
4. size()
5. height()
6. putAll()

Our approach here was to create a delete function which works well with different deletion strategies. Using TDD approach we created various test cases that checks the BST created for its height, size and the InOrder Traversal of the same.

For random number of deletions that have been made on the BST, we performed benchmarking where the size of the BST varies from 20 to 1600 nodes and are deleting N/2 elements at random. We performed this experiment for 100 iterations and the results are written into CSV file.

To perform the various deletion strategies on the BST we define constants viz., “Hibbard”, “Leftist” and “SizedDeletion”.

2.2 Unit Test Cases:



2.3 Output:

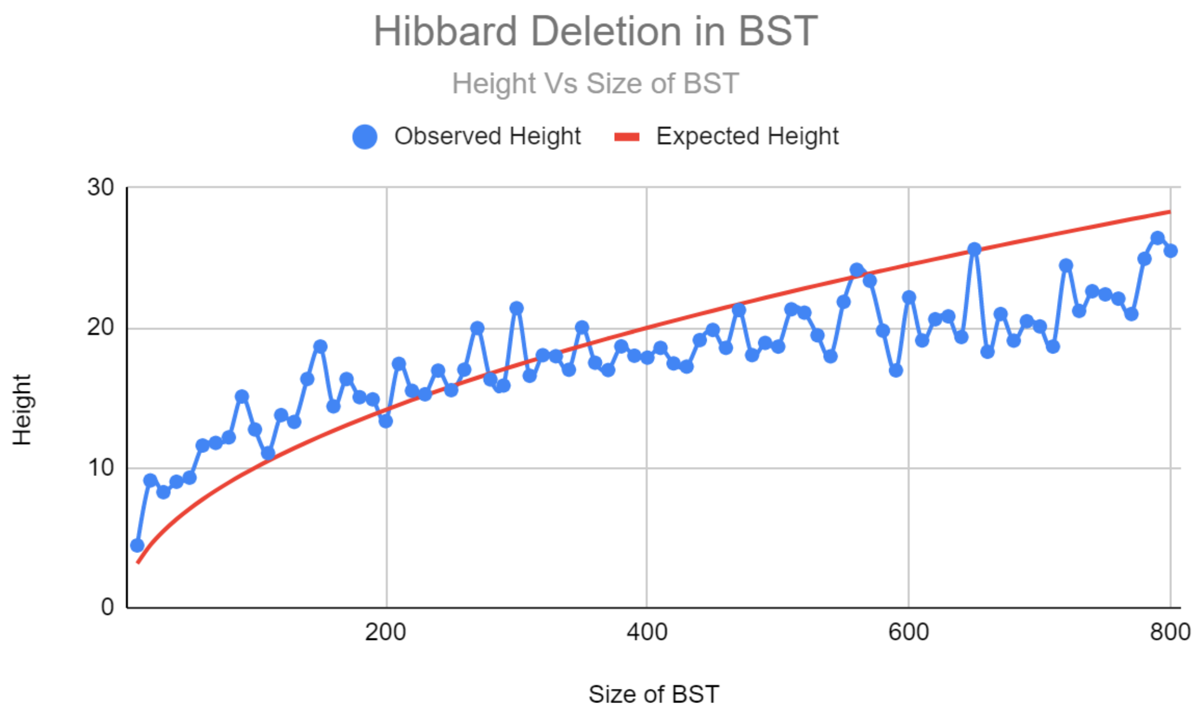
The output can be found [here](#) for all three cases.

2.4 Observations:

The excel sheet with observations can be found here:

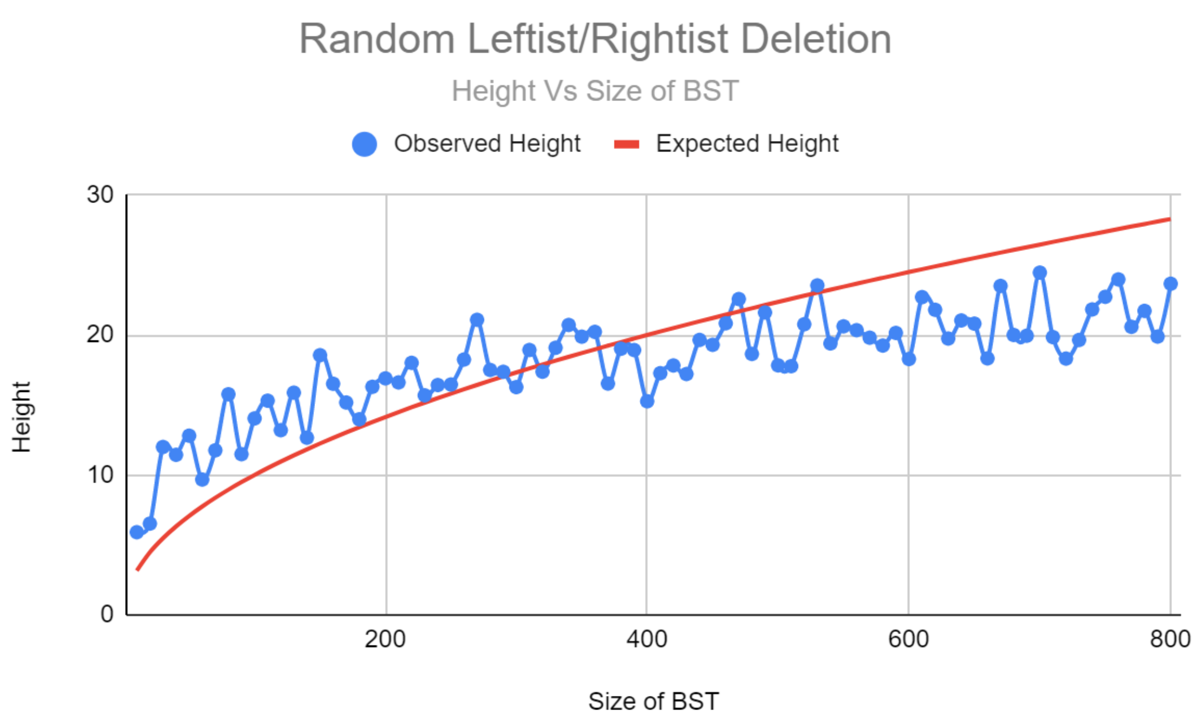
(a) BST Random Deletion
(b) BST Hibbard Deletion
(c) BST Sized Deletion

2.4.1 Hibbard Deletion:



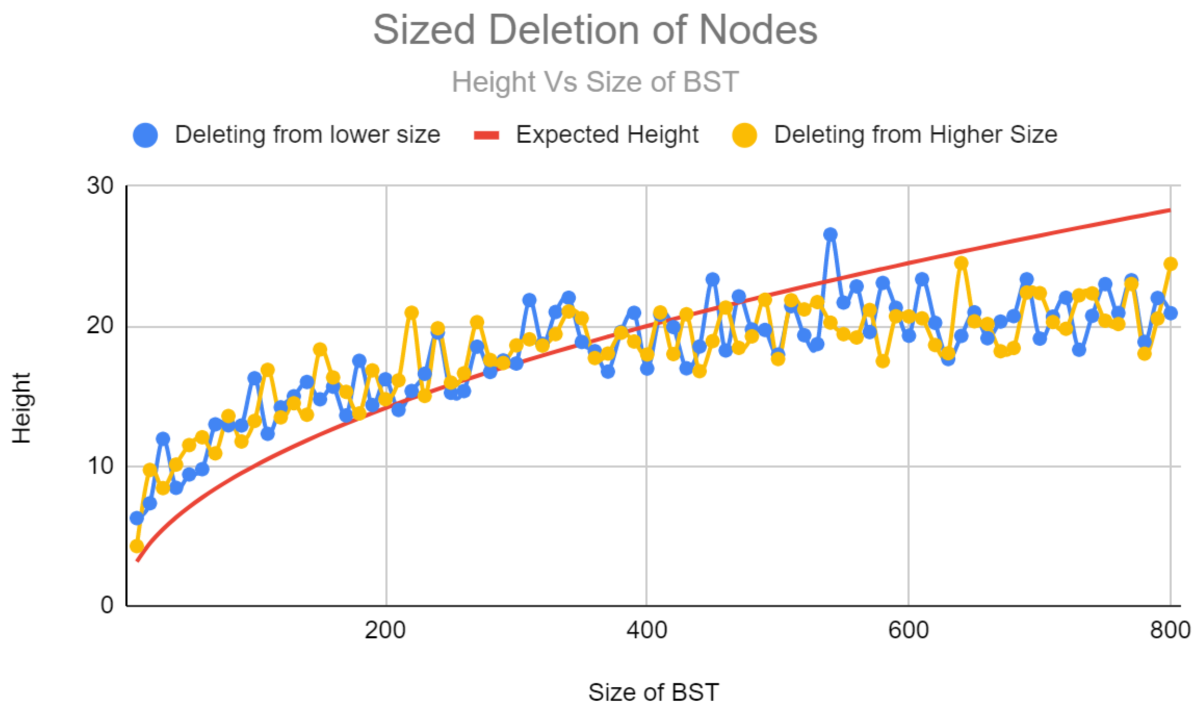
2.4.2 Random Deletion:

Random deletion of nodes from BST either by Leftist or Rightist(Hibbard) Strategy.



2.4.3 Sized Deletion:

Deletion of nodes from BST by comparing the size of the candidate nodes.



2.5 Conclusion:

We performed random deletion of nodes from BST using 3 strategies viz., Hibbard, Random(Hibbard or Leftist) and Sized Deletion.

We delete N/2 random nodes from BST using Hibbard's deletion for a BST whose size varies from 20 to 1600. We observed that the trees with size between 300 and 700 nodes, the height of the tree tends to root n after the deletions have been made.

Similarly, when we performed deletion on BST using Random and SizedDeletion. We could observe that they follow the same trend as Hibbard's deletion, where the height of the tree tends to root n

Hence based on our observations, we conclude that the height of BST after N/2 random deletions is approximately proportional to root n and it does not depend upon the strategy used for deletion.

3.0 ShellSort

A. When shell-sorting a sorted array, the number of comparisons is logarithmic. Do you agree? What is the base of the logs? Explain your observations.
B. Can you develop an expression for the average number of comparisons for shell-sort (choose any one of the "common" gap sequences). Recall that this is "unknown."
C. Can you develop an expression for the worst-case number of comparisons for the gap sequence you choose?

3.1 Approach:

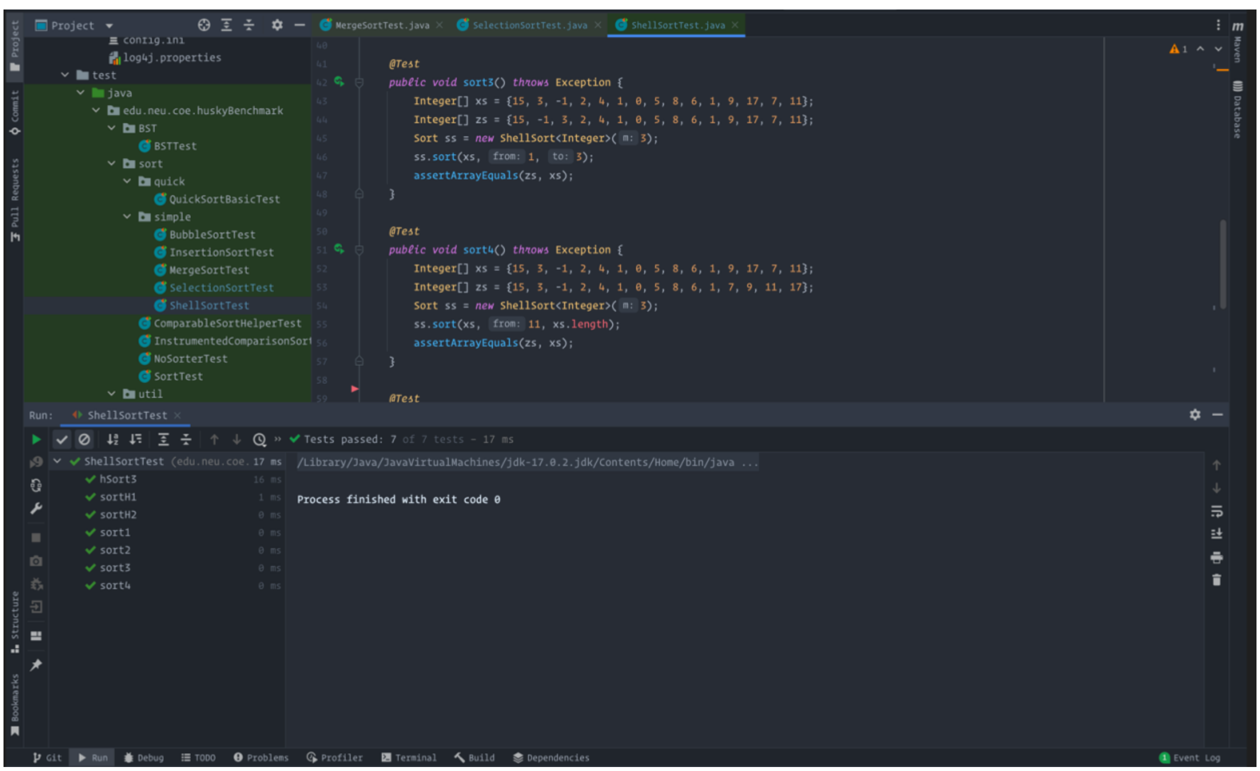
Using the TDD approach, we initially created unit tests for shell sort. For this algorithm, we follow Knuth's sequence to generate the gap(h). Knuth's sequence is

h=3*h+1

After creating the unit test cases for the above algorithm, we created the algorithm such that it passes all the unit tests.
To run benchmarking, we use array size varying from 5000 to 30000 and used the following loggers for logging the results:

1. Benchmark
2. TimeLogger
3. LazyLogger

3.2 Unit Test Cases:



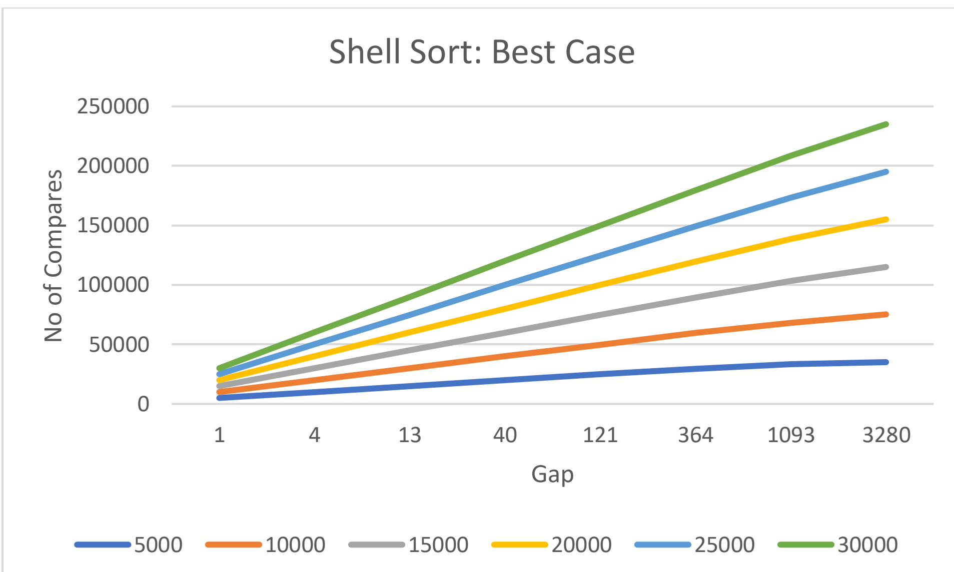
3.3 Output :

The output can be found here.

3.4 Observations :

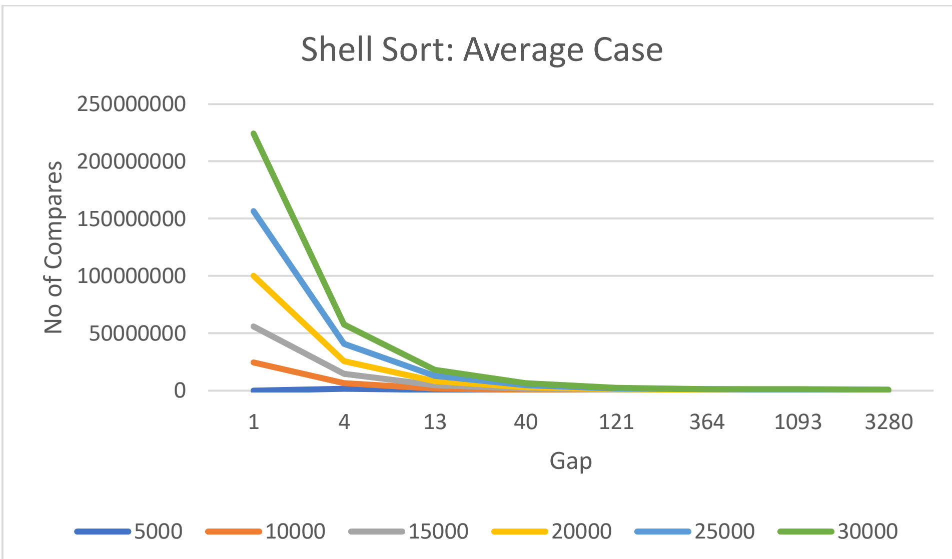
3.4.1 Best Case:

	h							
N	1	4	13	40	121	364	1093	3280
5000	4999	9995	14982	19942	24821	29457	33364	35084
10000	9999	19995	29982	39942	49821	59457	68364	75243
15000	14999	29995	44982	59942	74821	89457	103364	115084
20000	19999	39995	59982	79942	99821	119457	138364	155084
25000	24999	49995	74982	99942	124821	149457	173364	195084
30000	29999	59995	89982	119942	149821	179457	208364	235084



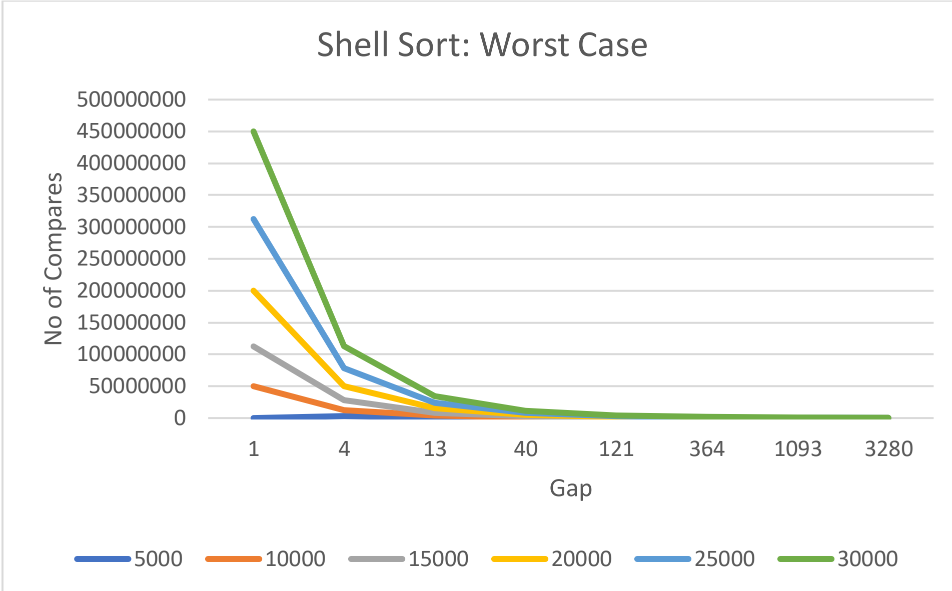
3.4.2 Average Case:

	h							
N	1	4	13	40	121	364	1093	3280
5000	6273560	1663385	584636	234546	131735	104868	105042	101896
10000	24627616	6548882	2139477	822933	397670	284586	238656	244578
15000	56017117	14839555	4670372	1734309	775092	485038	429945	387519
20000	1E+08	25714413	8420446	2973827	1278703	736322	587467	529751
25000	1.57E+08	40863541	12778699	4545871	1863261	1049604	782282	715562
30000	2.24E+08	57589929	18361852	6474799	2577899	1435103	1069223	901767



3.4.2 Worst Case:

N	h							
	1	4	13	40	121	364	1093	3280
5000	12497500	3134996	976724	341449	137727	72502	53716	51491
10000	49995000	12519996	3873453	1307949	480870	208230	142610	121986
15000	1.12E+08	28154996	8699412	2899449	1032672	421887	231004	183374
20000	2E+08	50039838	15436042	5115809	1774900	703838	373467	278113
25000	3.12E+08	78174761	24133406	7957245	2731444	1054041	521758	350928
30000	4.5E+08	1.13E+08	34701677	11423659	3905920	1453038	709463	458992



3.5 Conclusion:

The algorithm runs for 1000 iterations, 3 times:

a. Sorted Array – We observed that the number of comparisons is logarithmic and the base for the log is "e".

$$\text{Number of compares} \approx N * log_e(h)$$

where N is array size and h>1

For h=1 the number of compares is N-1

b. Average Case – Using Knuth's sequence for shell sort, we could not develop an expression for average case. This is still unknown.
For h=1, Number of Compares is ¼(N^2 - N)

c. Worst Case – Following the same sequence we used above we could develop an expression for worst case where the array is sorted in reverse

$$\text{Number of compares} \approx e^2 * N * (log_h N)^2 + 1.414 * \left(\frac{N}{h}\right)^2$$

where N is array size and h>1

For h=1, Number of Compares is ½(N^2 - N)

4.0 General Benchmarking

Run benchmarks on at least five independent algorithms and analyze the numbers of array accesses. To what extent can you predict the execution time based solely on the number of array accesses?

4.1 Approach:

Using the TDD approach, we initially created unit tests for following independent algorithms:

1. Bubble Sort
2. Insertion Sort
3. Selection Sort
4. Merge Sort
5. Quick Sort
6. Shell Sort

After creating the unit test cases for the above algorithms, we created the algorithms such that they pass all respective unit tests.

To perform benchmarking on these algorithms, we created specific benchmarking files that run for at least 1000 times for varying values of N, where N is the size of the array.

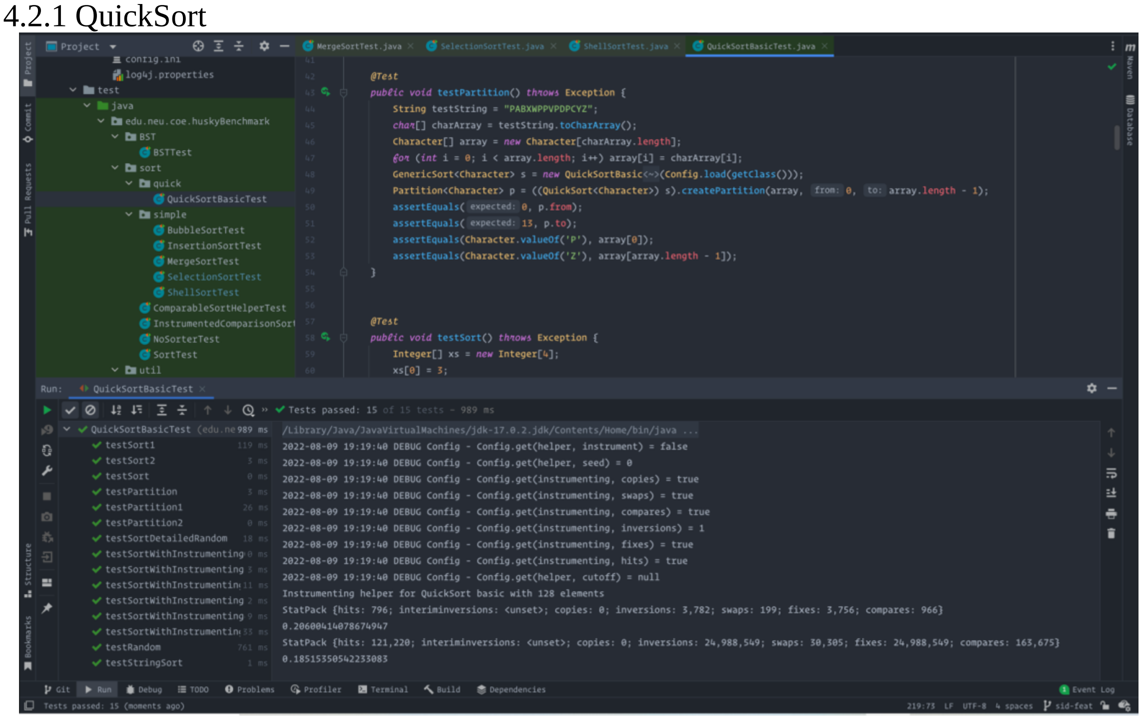
For logging the benchmarking results, we use the following classes:

4. Benchmark
5. TimeLogger
6. LazyLogger

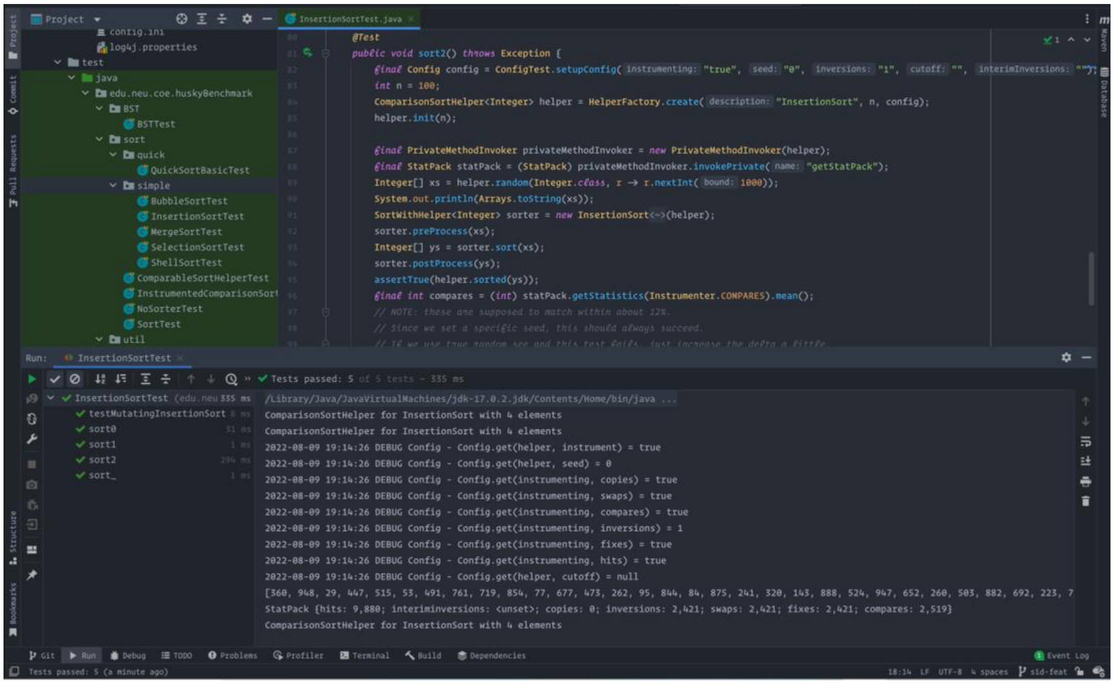
All of the above logs are automatically written into "logs/HuskyBenchmark.log"

4.2 Unit Test Cases:

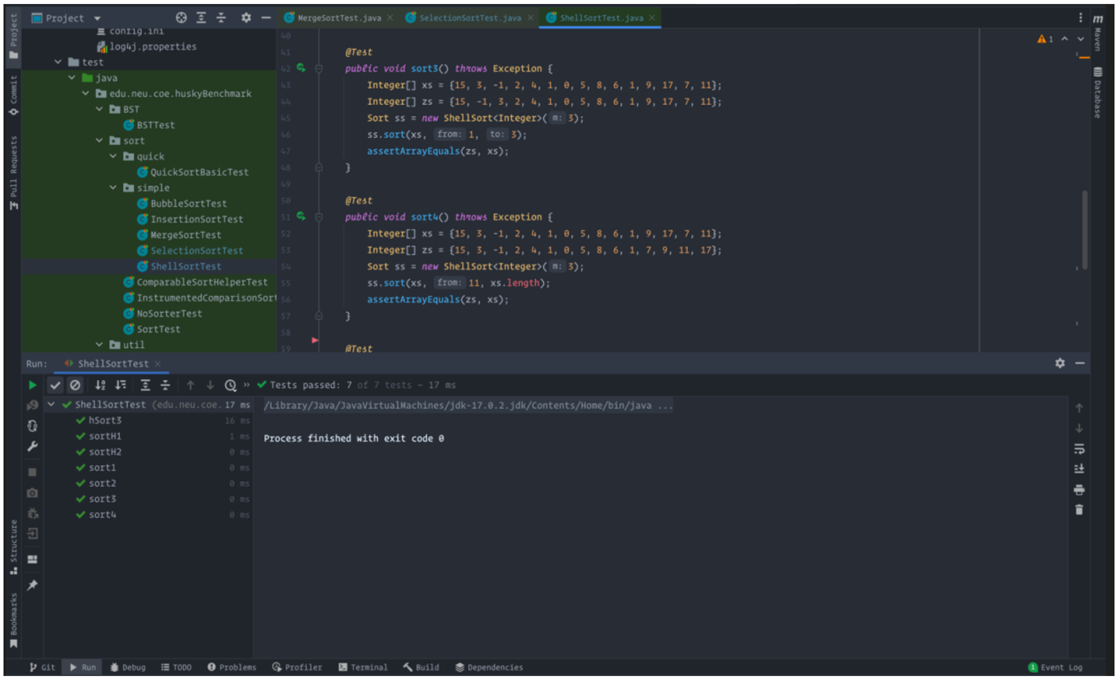
4.2.1 QuickSort



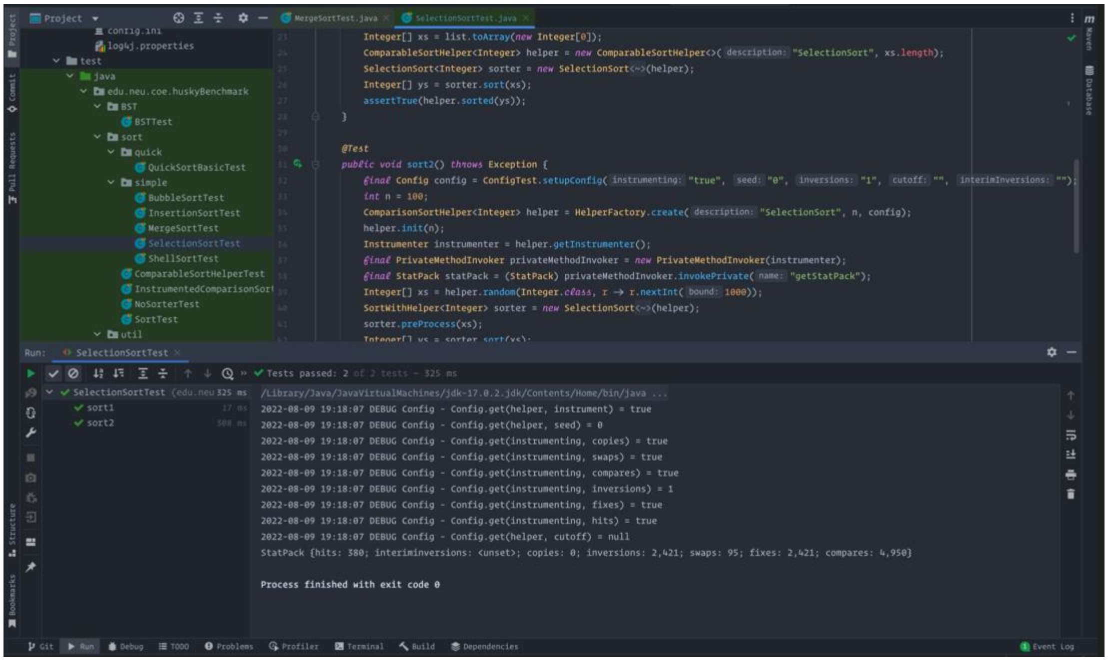
4.2.2 InsertionSort



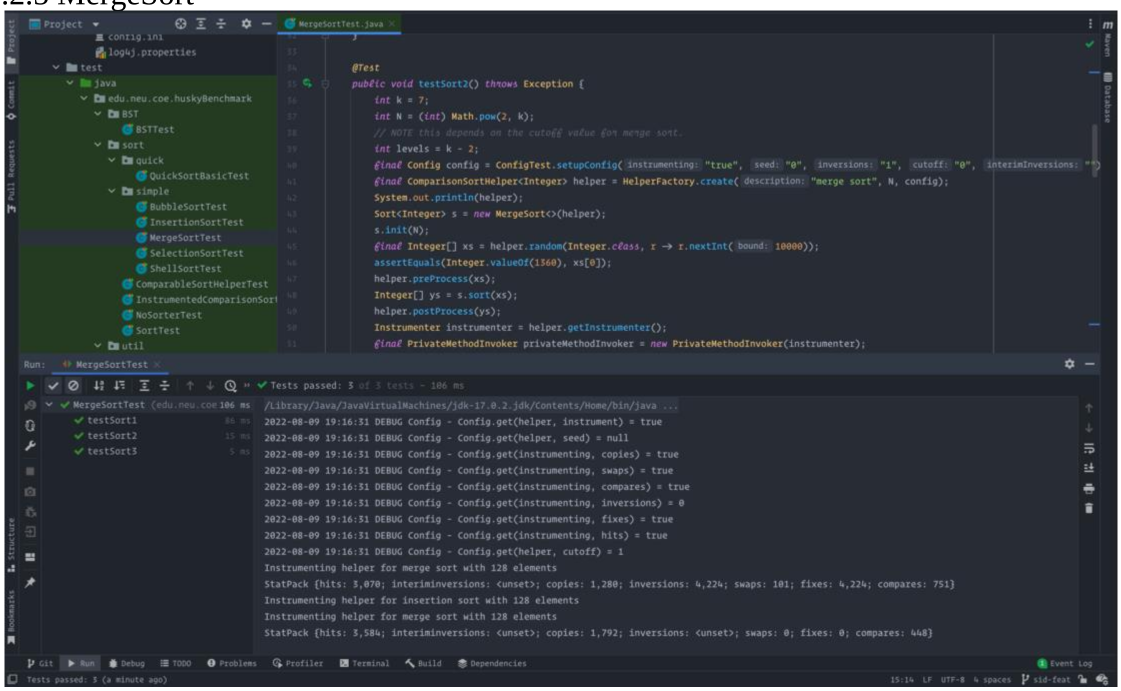
4.2.3 ShellSort



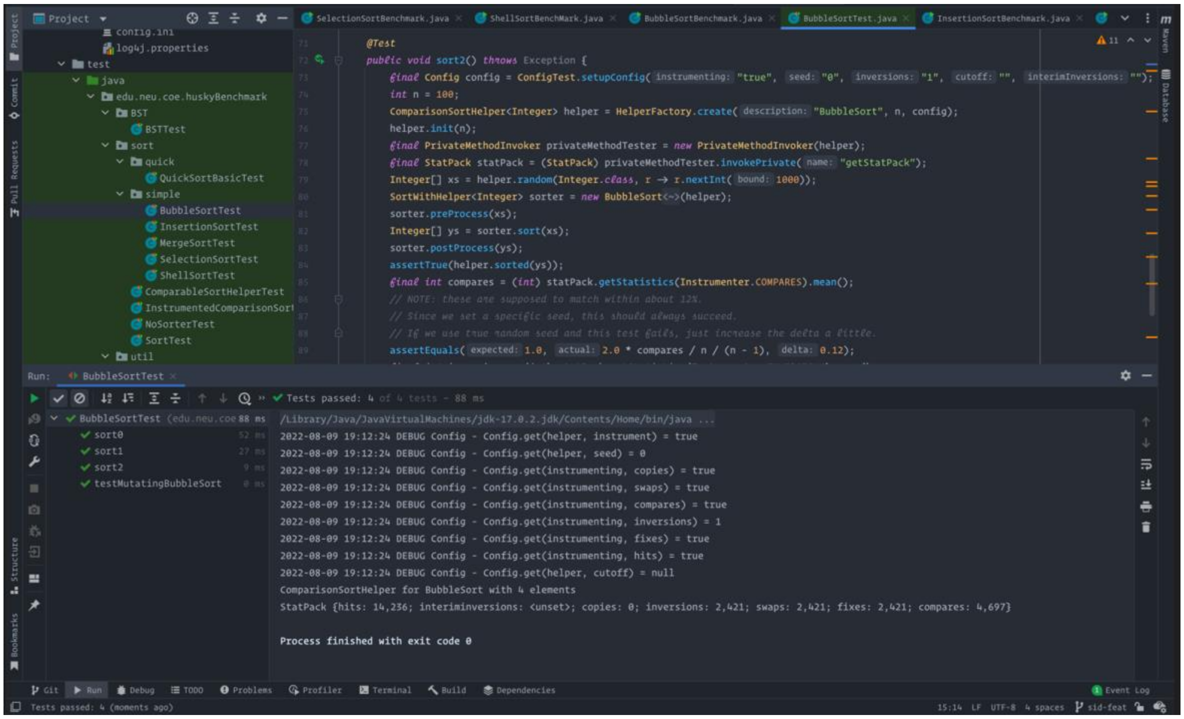
4.2.4 SelectionSort



4.2.5 MergeSort



4.2.6 BubbleSort



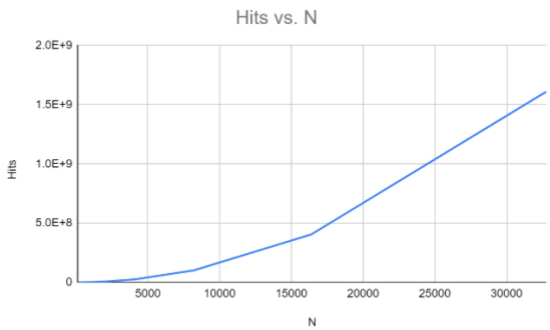
4.3 Output:

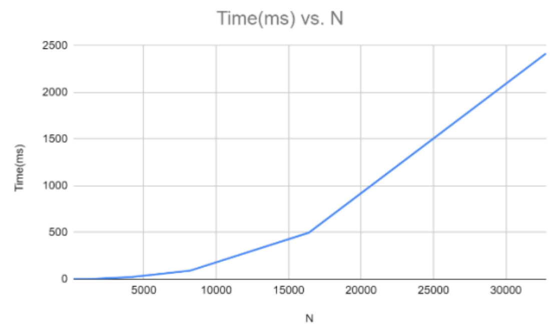
The output can be found here.

4.4 Observations:

4.4.1 Bubble Sort:

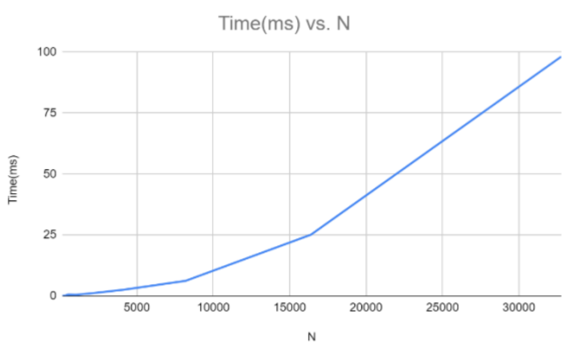
N	Hits	Time(ms)
128	24022	0.03
256	99052	0.09
512	395410	0.35
1024	1572996	1.3
2048	6278336	6.73
4096	25055602	22.47
8192	100523892	90.99
16384	403668890	496.96
32768	1609361800	2416.25

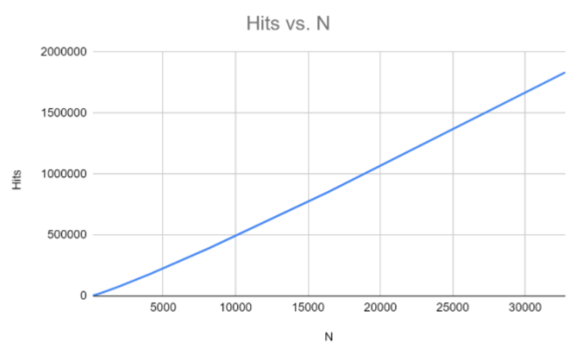




4.4.2 Merge Sort:

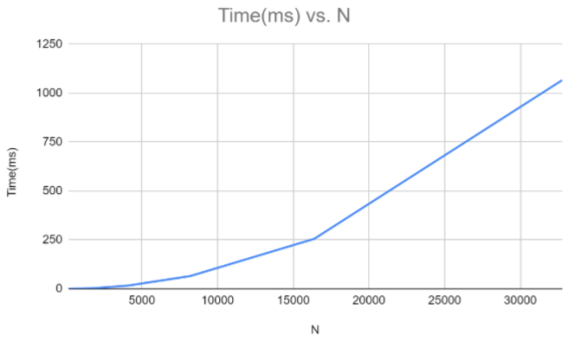
N	Hits	Time(ms)
128	3064	0.1
256	7206	0.07
512	16440	0.59
1024	36922	0.48
2048	81708	1.08
4096	180164	2.52
8192	393084	6.19
16384	851296	25.09
32768	1833784	98.12

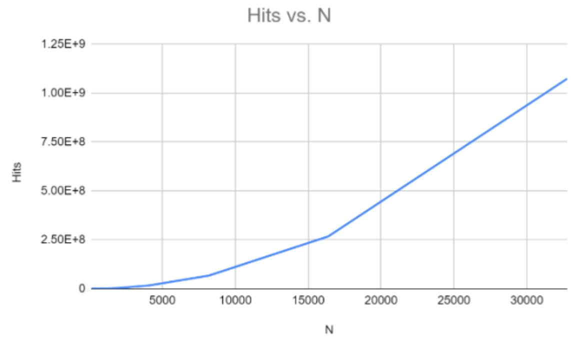




4.4.3 Insertion Sort:

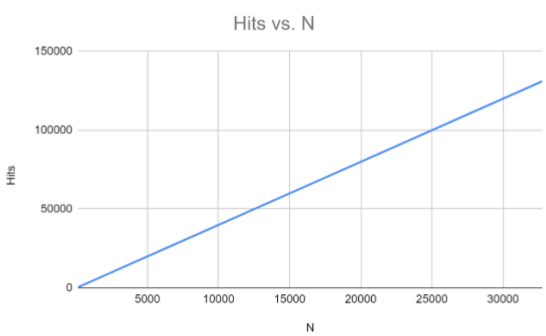
N	Hits	Time(ms)
128	16848	0.03
256	68910	0.06
512	260288	0.22
1024	1046364	0.9
2048	4135626	4.3
4096	16895392	16.38
8192	67388144	64.61
16384	267453436	254.77
32768	1073606938	1066.19

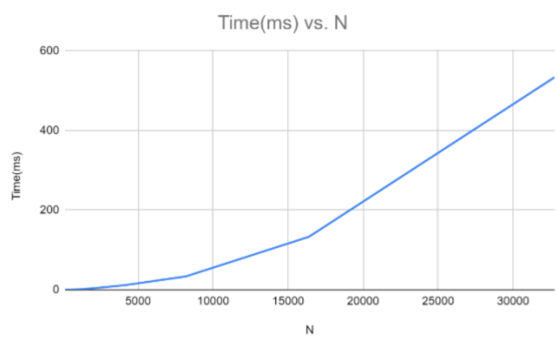




4.4.4 Selection Sort:

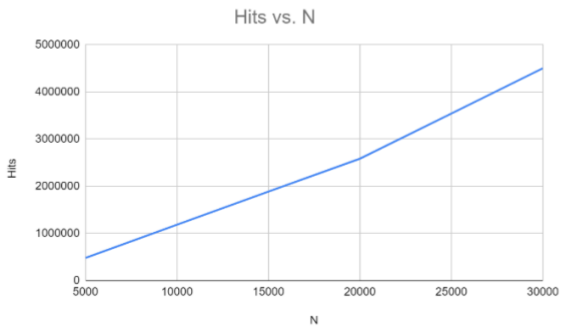
N	Hits	Time(ms)
128	492	0.06
256	996	0.07
512	2020	0.27
1024	4068	1.05
2048	8160	3.96
4096	16356	11.38
8192	32716	33.5
16384	65484	133.12
32768	131032	533.77

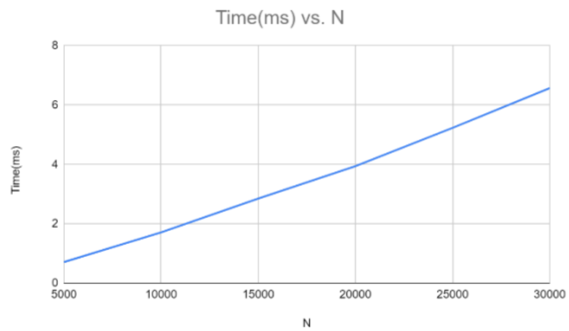




4.4.5 Shell Sort

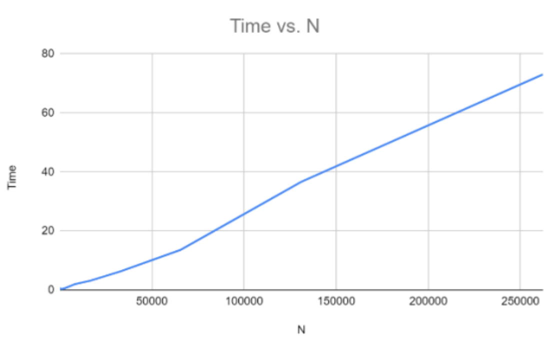
N	Hits	Time(ms)
5000	479728	0.71
10000	1184372	1.71
15000	1886570	2.85
20000	2583810	3.94
25000	3541344	5.23
30000	4501166	6.57

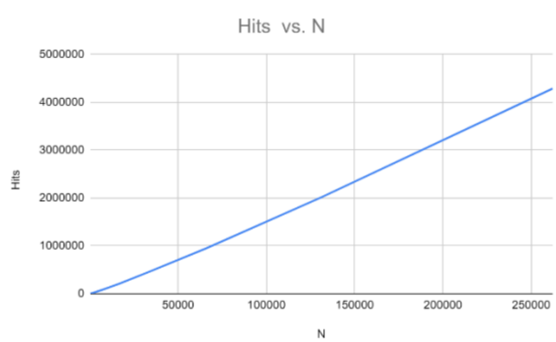




4.4.6 Quick Sort

N	Time(ms)	Hits
128	0.18	772
256	0.49	1748
512	0.35	4132
1024	0.37	9184
2048	0.43	19888
4096	0.92	44356
8192	1.98	95820
16384	3.12	203868
32768	6.28	445748
65536	13.59	942772
131072	36.64	2013752
262144	73	4287604





4.5 Conclusion:

In the quick sort algorithm, we vary the array size from 128 to 262144 and for shell sort, from 5000 to 30000. For other sorting algorithms, the size varies from 128 to 32768.

From the above graphs we can observe that the number of hits and the time taken to execute the algorithm increases linearly with respect to the varying array sizes.

5.0 CI/CD Pipeline-

We have implemented CircleCI for continuous integration that checks all the unit tests before merging any code into the master branch. The dashboard can be found here.