Numerical Analysis
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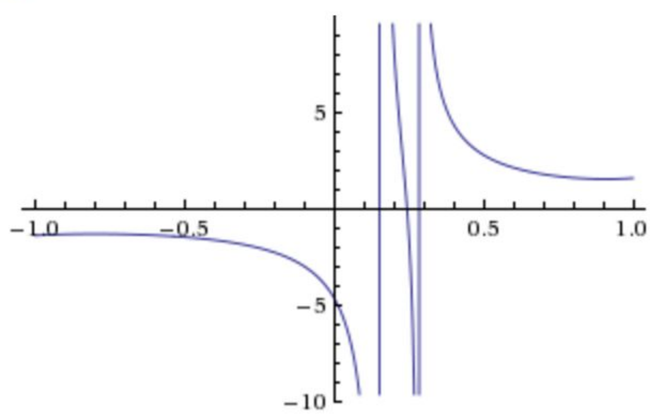
Homework 1

1. See code BreakMath.cpp
2. See code
3. See code (from: code source)
4. See code BinaryFloatPoint.cpp for decimal and binary notation. Floating Point representation from: http://www.h-schmidt.net/FloatConverter/IEEE754.html
    a. 01000001001010000000000000000000
    b. 00111110101010101010101010101011
    c. 01000000010010010010010010011110
5. See code
6. See code, Each function is stable and converges to at least one point. To find a function that works for all, we should make alpha in the third function some variable that is negative near the second rot but positive elsewhere. This would theoretically give a function that can find all three.
7.

Input interpretation:

plot	$f(x) = \dfrac{1}{1\,x^2 + 1.5\,x - 0.25} - \dfrac{1}{4\,x^2 - 6.4\,x + 1.5}$	$x = -1$ to 1

Plot:



The secant method relies on the intermediate value theorem which requires a continuous function to be applicable. This function , however is discontinuous near the roots of the quadratics in each denominator. To find a root with the given bracketing, we could use FPI with the first guess being the result of a single Secant method approximation. Or, simply choosing brackets that are within the bounds of the roots to the equations in the denominator.

8. Answers are still accurate although it obviously takes a few more steps to calculate more degrees of accuracy. It converges to a large positive number when given an input of 20. Similarly, it converges to a large negative number when given an input of -20, however this is unlikely to be accurate.
9. See code
10. See Next Page

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(i) $e^{h\nu/kT} = 1 + \frac{h\nu}{kT}$

$\therefore u(\nu, T) = \frac{dU}{d\nu} = \frac{8\pi h\nu^3}{c^3} \cdot \left(\frac{1}{1+\frac{h\nu}{kT} - 1}\right)$

$= \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\frac{kT + h\nu - kT}{kT}}$

$= \frac{8\pi \cancel{h}\nu^{\cancel{3}\,2}}{c^3} \cdot \frac{kT}{\cancel{h\nu}}$

$\cancel{= 8\pi h\nu^2} = \frac{8\pi kT\nu^2}{c^3}$

$$\boxed{\text{Rayleigh Jeans Law: } u(\nu, T) = \frac{8\pi kT\nu^2}{c^3}}$$

The next higher order polynomial term:

$$\frac{(h\nu/kT)^2}{2!} = \boxed{\frac{h^2\nu^2/k^2T^2}{2}}$$

$\because h\nu \ll kT$

$\cancel{k^2T^2 = h^2\nu^2(1-x)}$

Let us consider $\frac{1}{1-x} = \frac{h^2\nu^2}{k^2T^2}$

$\therefore h^2\nu^2(1-x) = k^2T^2$

$h^2\nu^2 - h^2\nu^2 x = k^2T^2$

$h^2\nu^2 x = h^2\nu^2 - k^2T^2$

$\therefore x = \frac{h^2\nu^2 - k^2T^2}{h^2\nu^2}$

given $h\nu \ll kT$

$\therefore (h\nu)^2 \lll (kT)^2$

$\therefore x \ll 1$

$\therefore x \ll 1, \frac{1}{1-x} \approx 1 + x$.

$\therefore$

$$\therefore \frac{1}{1-x} = 1 + x = 1 + \frac{(h\nu)^2 - (kT)^2}{(h\nu)^2}$$

$$= 1 + 1 - \frac{(kT)^2}{(h\nu)^2}$$

$$= 2 - \frac{(kT)^2}{(h\nu)^2}$$

Adding correction factor to numerator:

$$\therefore u(\nu, T) = \frac{8\pi kT\nu^2}{c^3} \cdot \frac{1}{2}\left(2 - \frac{(kT)^2}{(h\nu)^2}\right)$$

$$= \frac{8\pi kT\nu^2}{c^3} \cdot \left(1 - \frac{(kT)^2}{(h\nu)^2}\right)$$

$$= \frac{8\pi kT\nu^2}{c^3} - \frac{8\pi kT\cancel{\nu^2}}{c^3} \cdot \frac{(kT)^2}{h^2\cancel{\nu^2}}$$

$$= \boxed{\frac{8\pi kT}{c^3}\left(\nu^2 - \frac{(kT)^2}{h^2}\right)}$$

(ii) $\lambda = \frac{c}{\nu}$

$$\therefore \frac{d\lambda}{d\nu} = -\frac{c}{\nu^2}$$

from (i) $\frac{dU}{d\nu} = \frac{8\pi kT\nu^2}{c^3}$

$$\therefore \frac{dU}{d\lambda} = \frac{dU}{d\nu} \div \frac{d\lambda}{d\nu} = \frac{dU}{d\nu} \times \frac{d\nu}{d\lambda}$$

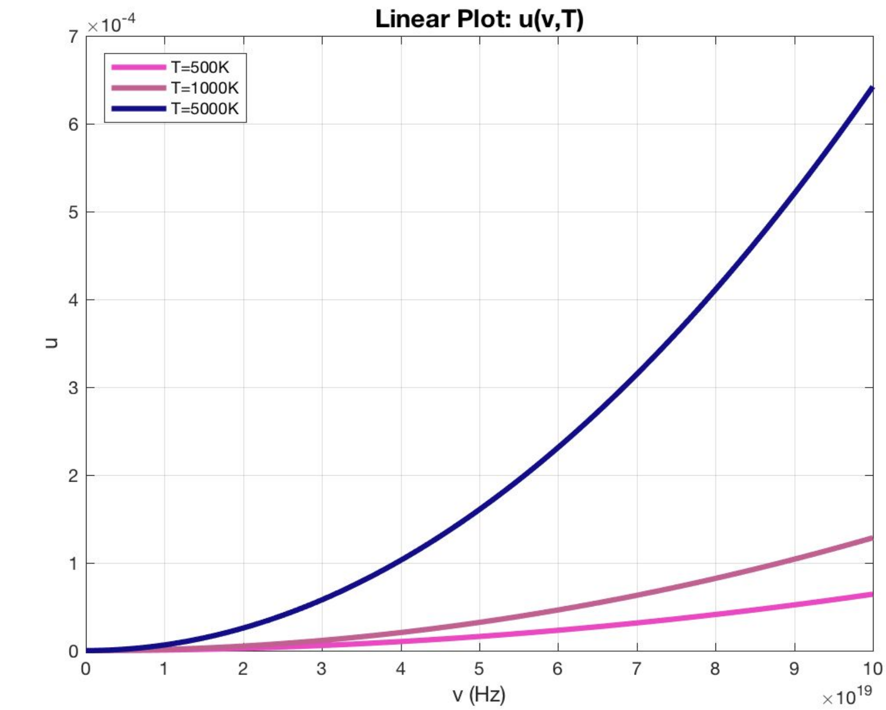
$$= \frac{8\pi kT\nu^2}{c^3} \times -\frac{\nu^2}{c}$$

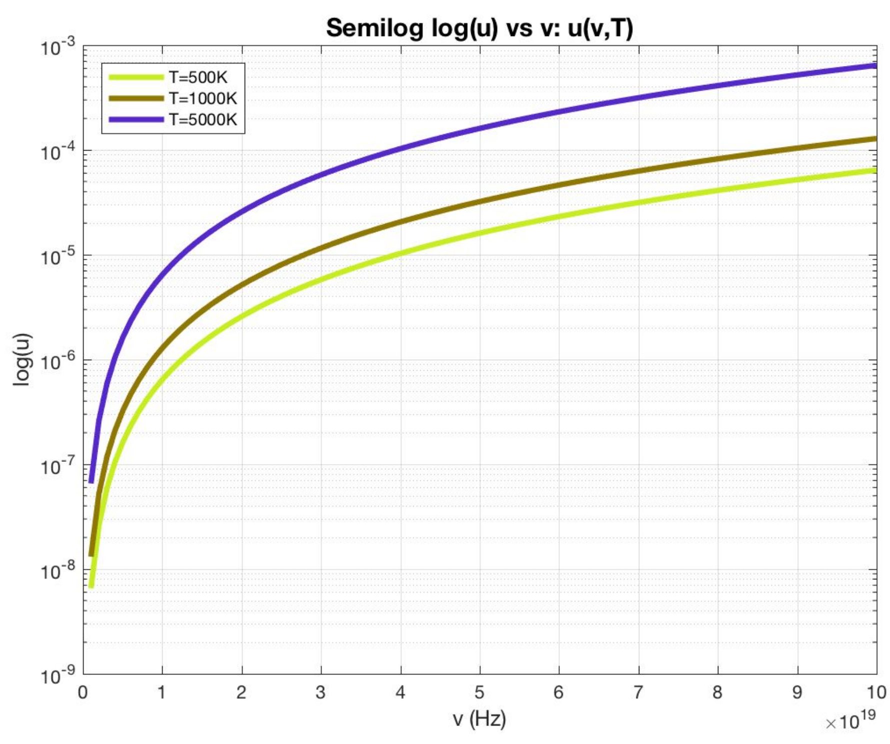
$$= -\frac{8\pi kT\nu^4}{c^4} = -\frac{8\pi kT\cancel{\nu^4}}{\lambda^4\cancel{\nu^4}}$$

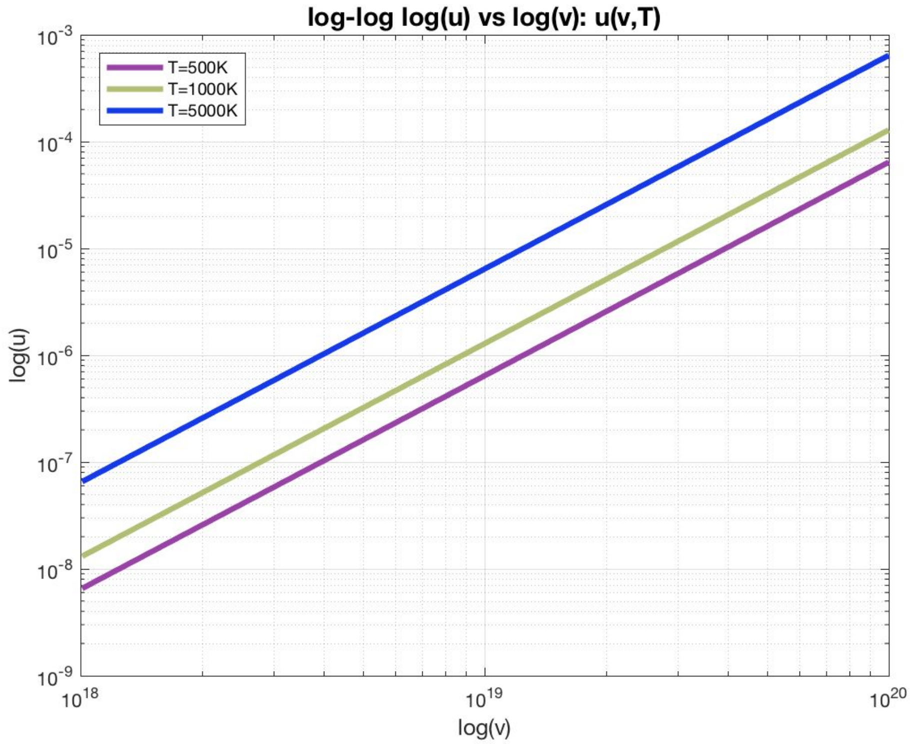
$$= -\frac{8\pi kT}{\lambda^4}$$

$$\therefore \boxed{u(\lambda, T) = \frac{dU}{d\lambda} = -\frac{8\pi kT}{\lambda^4}}$$

(iii)

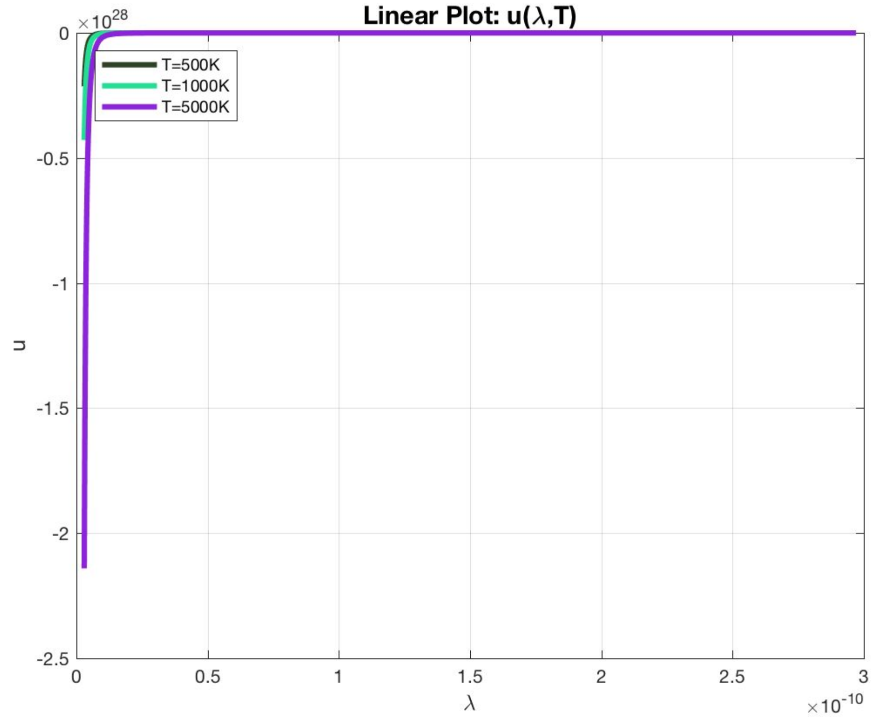

Linear Plot: u(v,T)
T=500K
T=1000K
T=5000K
u
v (Hz)
×10^-4
×10^19

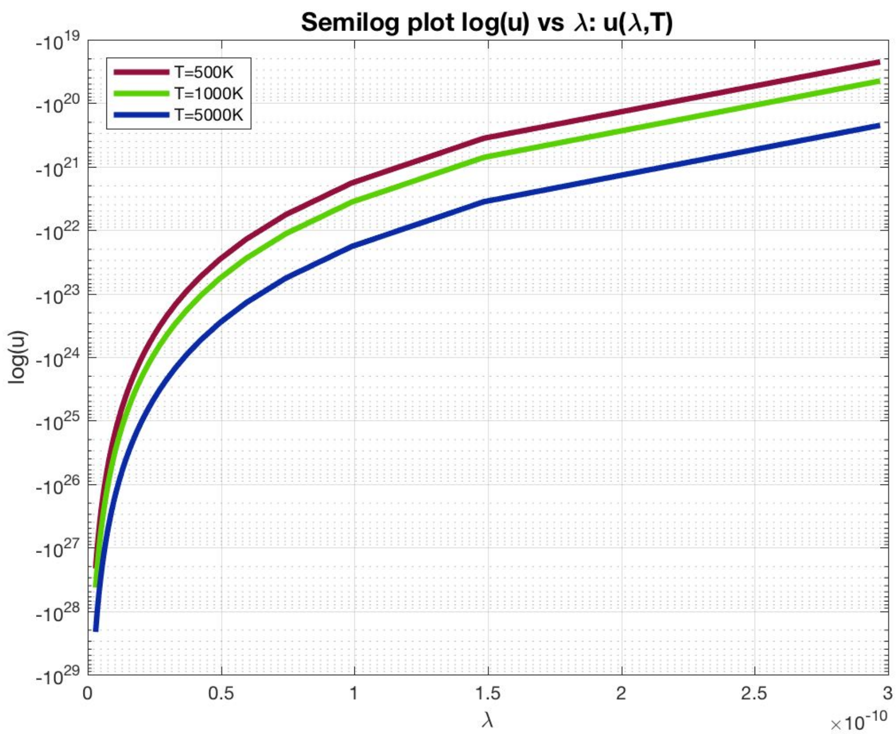

Semilog log(u) vs v: u(v,T)
T=500K
T=1000K
T=5000K
log(u)
v (Hz)
×10^19

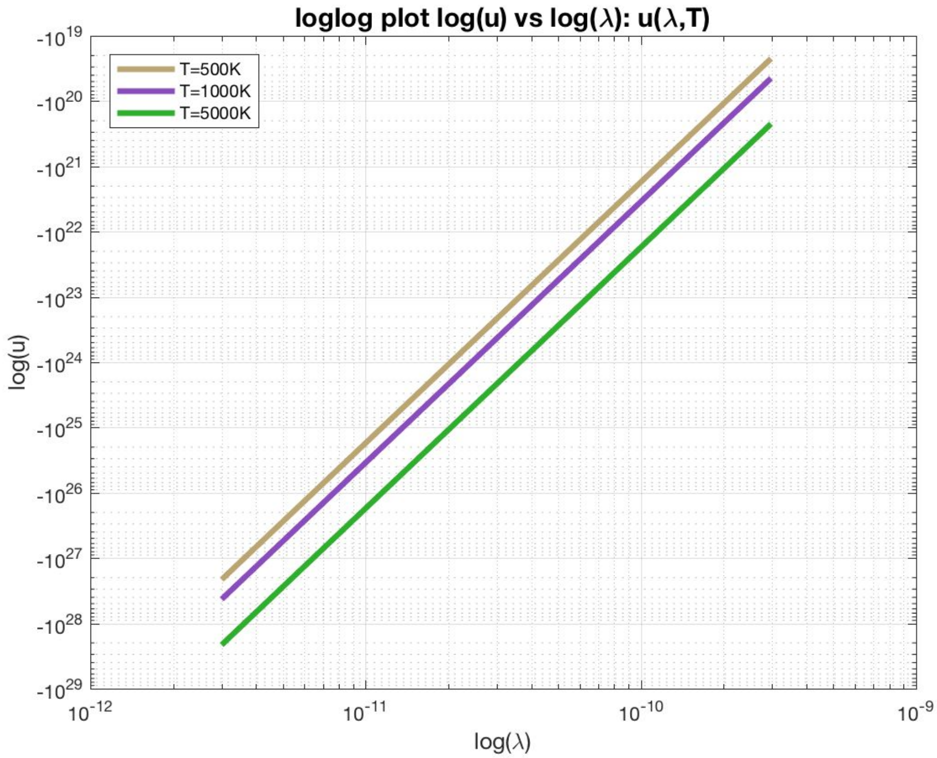


<u>Matlab Code:</u>

```
v = linspace(0,1e20);
h = 6.625e-34;
k = 1.380e-23
T = [500 1000 5000]
c = 3.00e8
figure
for i=1:length(T)
   u = (8*pi*k*T(i)*v.^2)/c.^3;
   loglog(v, u, '-', 'color',rand(1,3), 'LineWidth',3); % Plot the data
   hold on
end
title('log-log log(u) vs log(v): u(v,T)', 'FontSize', 14);
grid on
legend('T=500K','T=1000K','T=5000K', 'Location','northwest')
xlabel('log(v)','FontSize', 12); % Set the x axis label
ylabel('log(u)','FontSize', 12); % Set the y axis label
print -dpng rayleigh2.png
```


Linear Plot: u(λ,T)
×10^28
T=500K
T=1000K
T=5000K
0
-0.5
-1
-1.5
-2
-2.5
u
0
0.5
1
1.5
2
2.5
3
λ
×10^-10


Semilog plot log(u) vs λ: u(λ,T)
T=500K
T=1000K
T=5000K
-10^19
-10^20
-10^21
-10^22
-10^23
-10^24
-10^25
-10^26
-10^27
-10^28
-10^29
log(u)
0
0.5
1
1.5
2
2.5
3
λ
×10^-10



<u>Matlab code:</u>

```
v = linspace(0,1e20);
h = 6.625e-34;
k = 1.380e-23
T = [500 1000 5000]
c = 3.00e8
lambda = c./v;
figure
for i=1:length(T)
   u = -(8*pi*k*T(i))./lambda.^4;
   loglog(lambda, u, '-', 'color',rand(1,3), 'LineWidth',3);
   hold on
end
title('loglog plot log(u) vs log({\lambda}): u({\lambda},T)', 'FontSize', 14);
grid on
legend('T=500K','T=1000K','T=5000K', 'Location','northwest')
xlabel('log({\lambda})','FontSize', 12); % Set the x axis label
ylabel('log(u)','FontSize', 12); % Set the y axis label
print -dpng rayleigh5.png
```

From the graphs it can be seen the log-log graphs are the most useful as a linear correlation among the independent and dependent variable is visible.