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Faculty of Engineering & Architectural Science

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Instructor:	Instructor: Professor Javad Alirezaie, TA: Brendan Wood

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Lab 3 - Fourier Series Analysis using MATLAB

By: Syed Maisam Abbas

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Introduction

In this lab, the objective is to use the Fourier series in the analysis and synthesis of periodic signals while using MATLAB, the online programming software. Furthermore, this lab explores the characteristics of the Fourier series, investigates how the period of a signal affects the Fourier series coefficients, and studies the effects of series truncation on signal reconstruction. These labs may prove to be beneficial in the future to analyze signals and different kinds of systems in the engineering field.

Lab Analysis

A. Fourier Series Synthesis & Analysis

- **A.1 & A.2 calculations**

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ELE532 LAB3

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Problem A

1. $x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$

$$= \left[\frac{1}{2}e^{\frac{3\pi}{10}jt} + \frac{1}{2}e^{-\frac{3\pi}{10}jt}\right] + \frac{1}{2}\left[\frac{1}{2}e^{\frac{\pi}{10}jt} + \frac{1}{2}e^{-\frac{\pi}{10}jt}\right]$$

$$= \frac{1}{2}e^{\frac{3\pi}{10}jt} + \frac{1}{2}e^{-\frac{3\pi}{10}jt} + \frac{1}{4}e^{\frac{\pi}{10}jt} + \frac{1}{4}e^{-\frac{\pi}{10}jt}$$

$\left(\frac{3\pi}{10}\right)/\left(\frac{\pi}{10}\right) = 3 \Rightarrow$ rational, meaning the fundamental frequency is a multiple

$w_{0_1} = \frac{3\pi}{10}$, $w_{0_2} = \frac{\pi}{10}$, $w_0 = \frac{\pi}{10}$, $T_0 = \frac{2\pi}{(\pi/10)} = 20$

$$\sum_{n=-\infty}^{\infty} D_n \cdot e^{jn(\pi/10)t} \Rightarrow e^{\frac{3\pi}{10}jt} \Rightarrow n=3 \ \boxed{D_3 = \frac{1}{2}} \quad e^{-\frac{3\pi}{10}jt} \Rightarrow n=-3 \ \boxed{D_{-3} = \frac{1}{2}}$$

$$e^{\frac{\pi}{10}jt} \Rightarrow n=1 \ \boxed{D_1 = \frac{1}{4}} \quad e^{-\frac{\pi}{10}jt} \Rightarrow n=-1 \ \boxed{D_{-1} = \frac{1}{4}}$$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t) \cdot e^{-jnw_0t}\,dt$$

$$= \frac{1}{20}\int_{-10}^{10}\left[\frac{1}{2}e^{\frac{3\pi}{10}jt} + \frac{1}{2}e^{-\frac{3\pi}{10}jt} + \frac{1}{4}e^{\frac{\pi}{10}jt} + \frac{1}{4}e^{-\frac{\pi}{10}jt}\right]dt$$

$$= \frac{1}{20}\left[\frac{e^{(3-n)j\pi} - e^{-(3-n)j\pi}}{2j(3-n)\frac{\pi}{10}} + \frac{e^{(3+n)j\pi} - e^{-(3+n)j\pi}}{2j(3+n)\frac{\pi}{10}} + \frac{e^{(1+n)j\pi} - e^{-(1+n)j\pi}}{4j(1+n)\frac{\pi}{10}} + \dots\right.$$

$$\left.\dots + \frac{e^{(1-n)j\pi} - e^{-(1-n)j\pi}}{4j(1-n)\frac{\pi}{10}}\right]$$

②

$$D_n = \frac{1}{2}\left[\sin((3-n)\pi) + \sin((3+n)\pi) + \frac{1}{2}\sin(1+n)\pi + \frac{1}{2}\sin(1-n)\pi\right]$$

2.

- $x_2(t) \Rightarrow T_0 = 20,\ w_0 = \frac{2\pi}{20} = \frac{\pi}{10}$

$$D_n = \frac{1}{20}\int_{-5}^{5}(1)e^{-jn\frac{\pi}{10}t}\,dt = \frac{1}{20}\left[\frac{-1}{jn\frac{\pi}{10}}\cdot e^{-jn\frac{\pi}{10}t}\right]_{-5}^{5}$$

$$= \frac{1}{20}\left[\frac{-10}{jn\pi}\cdot e^{-jn\frac{\pi}{2}} + \frac{10}{jn\pi}\cdot e^{jn\frac{\pi}{2}}\right]$$

$$D_n = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)t$$

- $x_3(t) \Rightarrow T_0 = 40,\ w_0 = \frac{2\pi}{40} = \frac{\pi}{20}$

$$D_n = \frac{1}{40}\int_{-5}^{5}(1)e^{-jn\frac{\pi}{20}t}\,dt = \frac{1}{40}\left[\frac{-1}{jn\frac{\pi}{20}}\cdot e^{-jn\frac{\pi}{20}t}\right]_{-5}^{5}$$

$$= \frac{1}{40}\left[\frac{-20}{jn\pi}\cdot e^{-jn\frac{\pi}{4}} + \frac{20}{jn\pi}\cdot e^{jn\frac{\pi}{4}}\right]$$

$$D_n = \frac{1}{n\pi}\sin\left(\frac{\pi n}{4}\right)t$$

- **A.3**

The MATLAB function used to generate Dn for a user-specified range of n values is displayed below:

```
function [D] = ELE532_LAB3_(D,n)
D1 = (1/2) * [sin((3-n).*pi) + sin((3+n).*pi) + (1/2)*sin((1+n).*pi) + (1/2)*sin((1-n).*pi)];
D2 = (1/(n.*pi)) * sin ((n*pi)/2);
D3 = (1/(n.*pi)) * sin ((n*pi)/4);

if (d == 1)
    D = D1;
end

if (d == 2)
    D = D2;
end

if (d == 3)
    D = D3;
end
```

- **A.4**

<u>Part A - This part of this problem inspects the index range -5 ≤ n ≤ 5</u>

Figure 1: Magnitude and Phase spectra for x1(t) in correlation to D1:

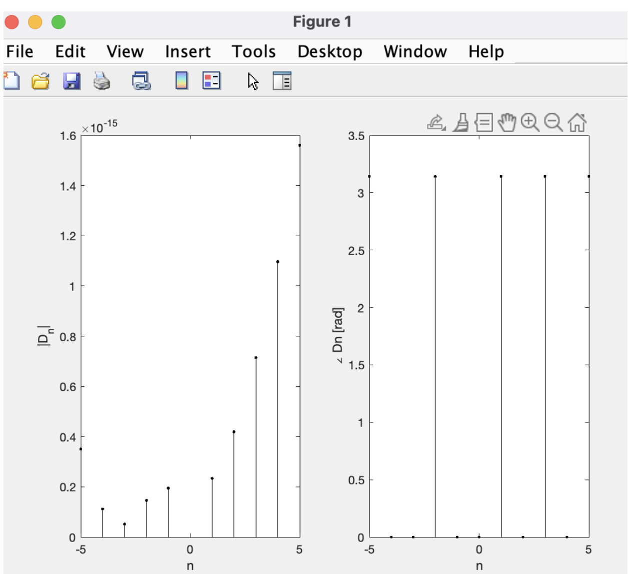


Figure 2: Magnitude and Phase spectra for x2(t) in correlation to D2:

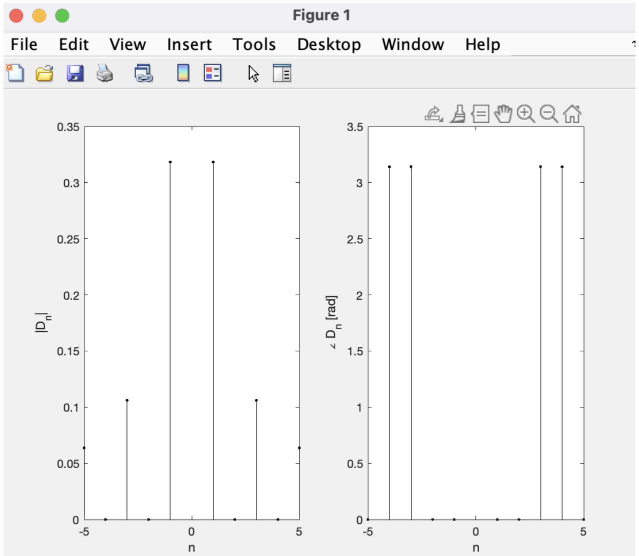
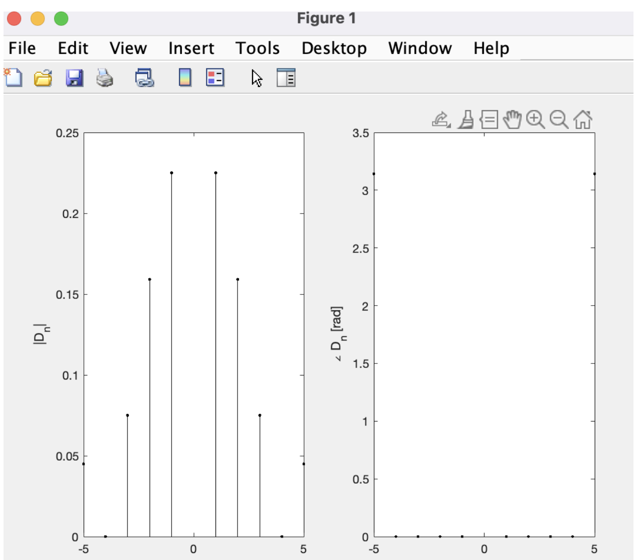


Figure 3: Magnitude and Phase spectra for x3(t) in correlation to D3:



Part B -This part of this problem inspects the index range -20 ≤ n ≤ 20

Figure 4: Magnitude and Phase spectra for x1(t) in correlation to D1:

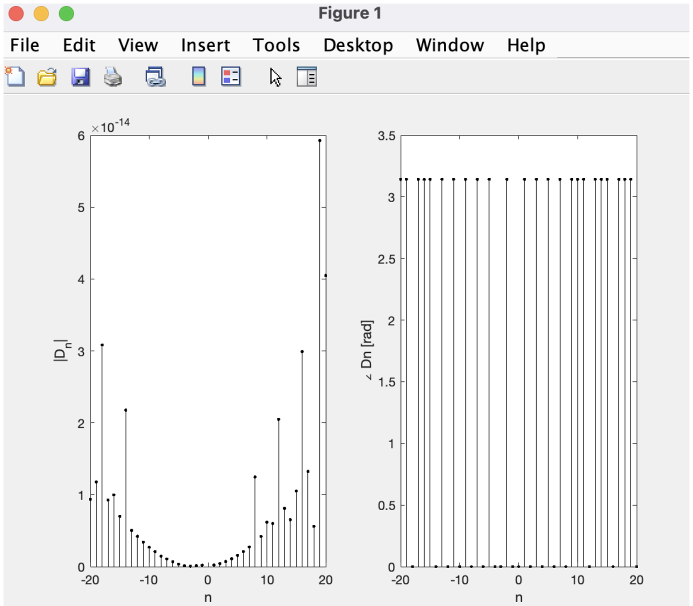


Figure 5: Magnitude and Phase spectra for x2(t) in correlation to D2:

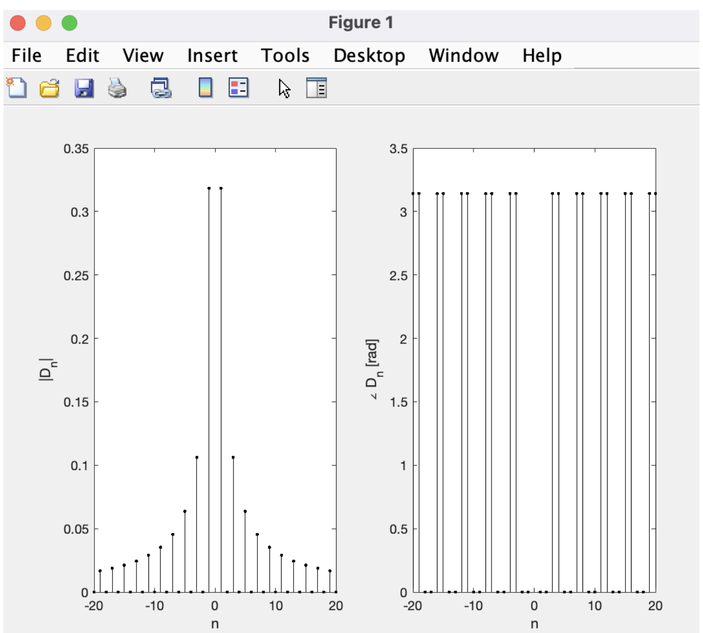
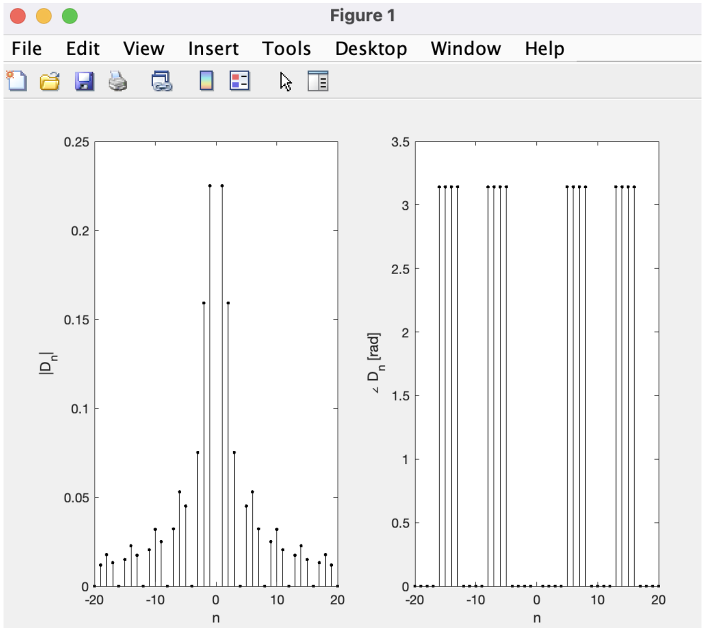


Figure 6: Magnitude and Phase spectra for x3(t) in correlation to D3:



<u>Part C -This part of this problem inspects the index range -50 ≤ n ≤ 50</u>

Figure 7: Magnitude and Phase spectra for x1(t) in correlation to D1:

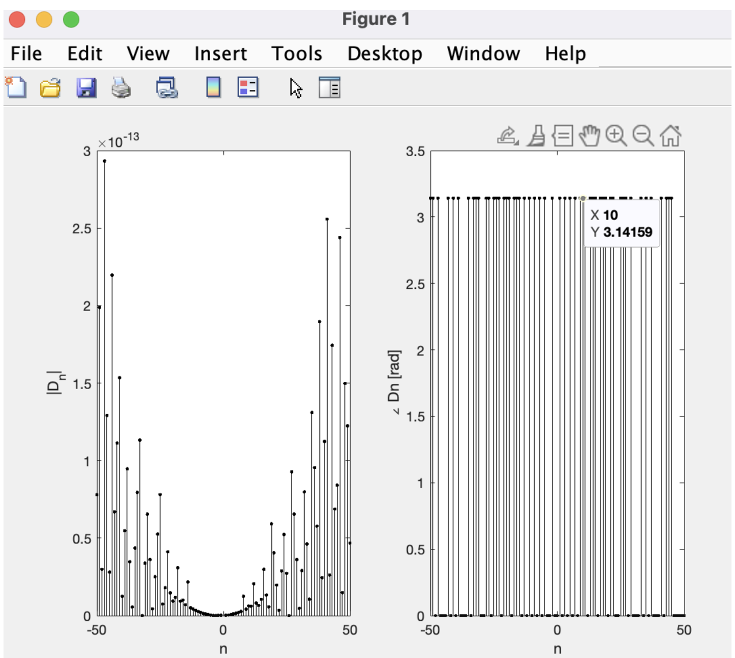


Figure 8: Magnitude and Phase spectra for x2(t) in correlation to D2:

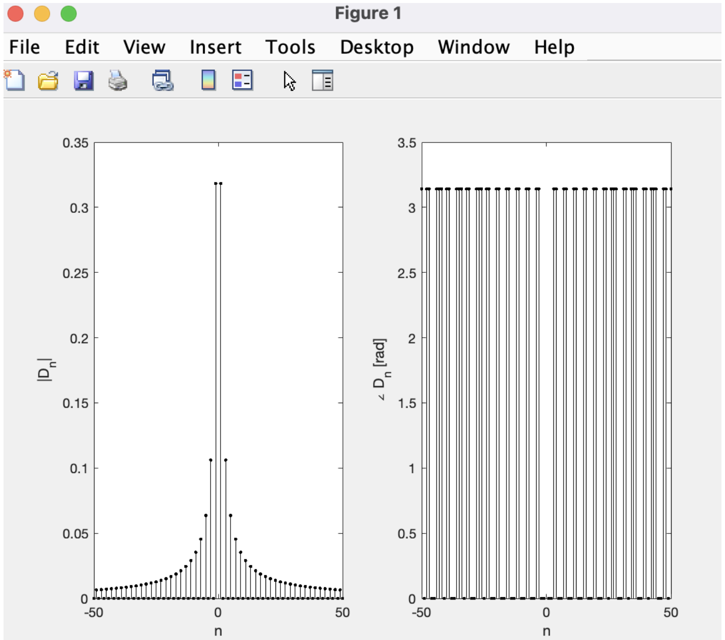
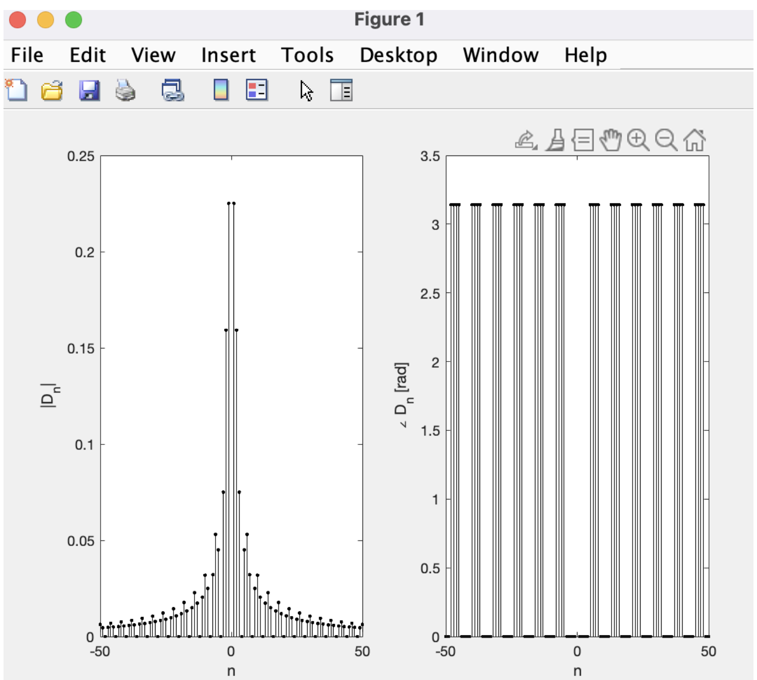


Figure 9: Magnitude and Phase spectra for x3(t) in correlation to D3:



Part D -This part of this problem inspects the index range -500 ≤ n ≤ 500

Figure 10: Magnitude and Phase spectra for x1(t) in correlation to D1:

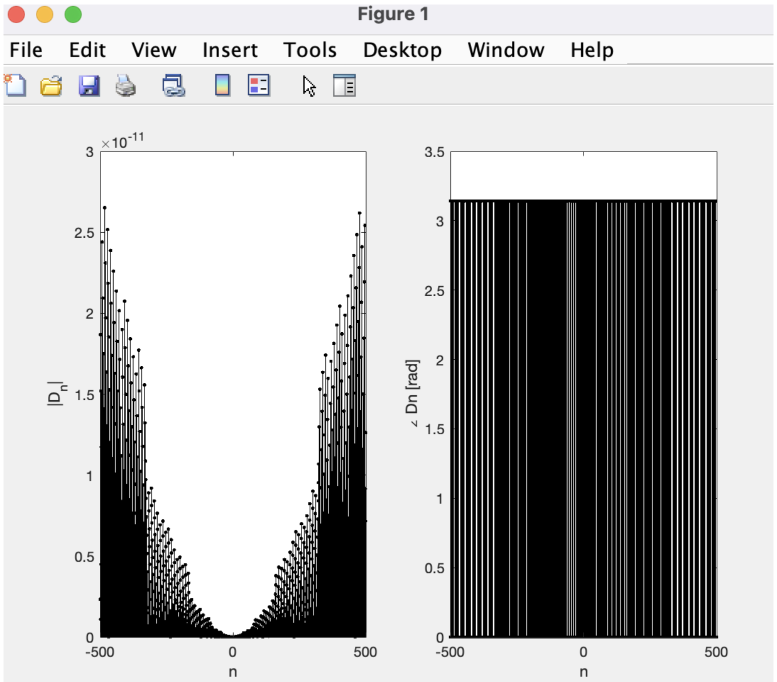


Figure 11: Magnitude and Phase spectra for x2(t) in correlation to D2:

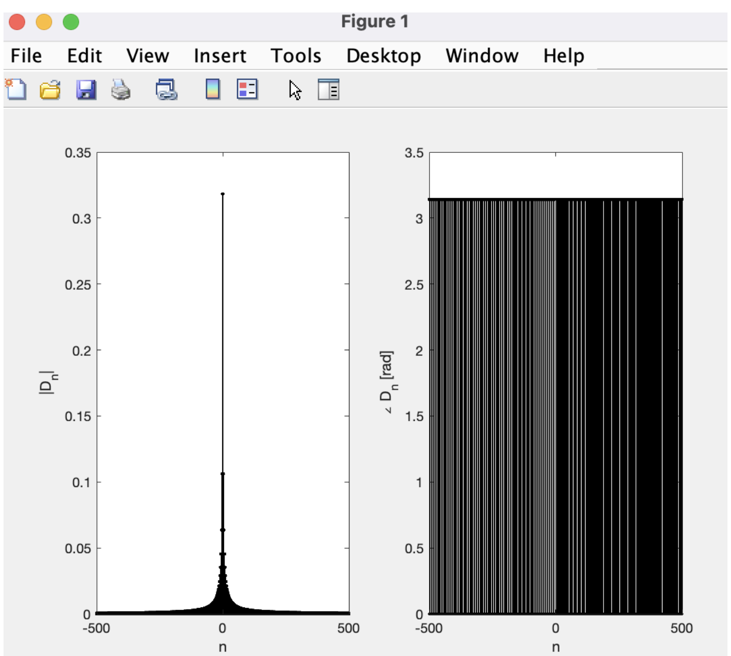
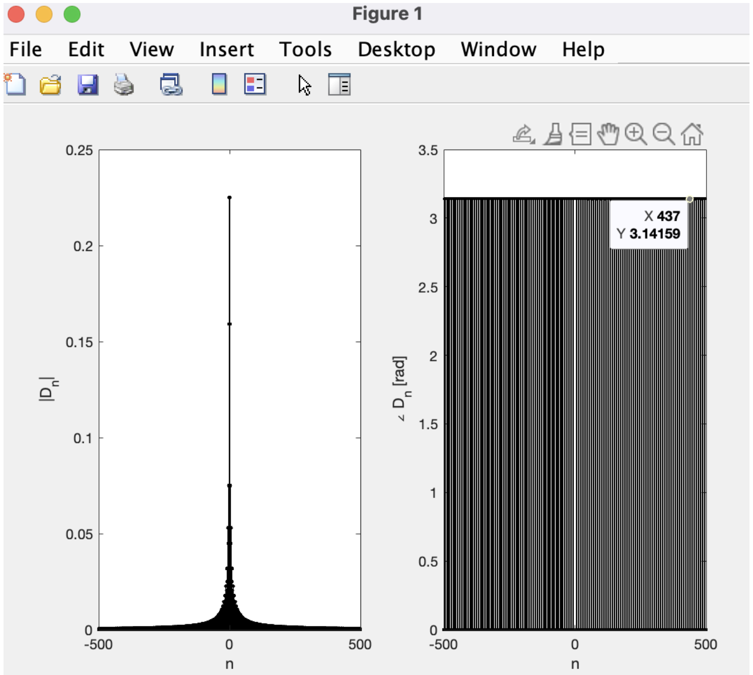


Figure 12: Magnitude and Phase spectra for x3(t) in correlation to D3:



- **A.5**

The MATLAB function used to reconstruct the time domain signal using equation 1 (from the lab manual) is displayed below:

```
function x = a5(d ,Dn)
if(d == 1)
    w = pi/10;
elseif (d == 2)
    w = pi/10;
elseif (d == 3)
    w = pi/20;
end

t = -300:1:300;
x = zeros(size(t));
for i = 1:length(x)
    total = 0;
    j = 1;
    for n = -500:500
        total = total + Dn(j) * exp(1i* n * w * t(i));
        j = j+1;
    end
```

```
    x(i) = total;
end

figure(1);
plot(t, x, 'b')
xlabel('t (s)');
ylabel('x(t)');
if(d ~= 1)
    axis([-300 300 -1 2]);
end

title('Reconstructed Fourier Coefficients');
grid;
```

- **A.6**

<u>Part A - This part of this problem inspects the reconstructed plots for x1(t)</u>

Figure 13: For ranges -5 < n < 5

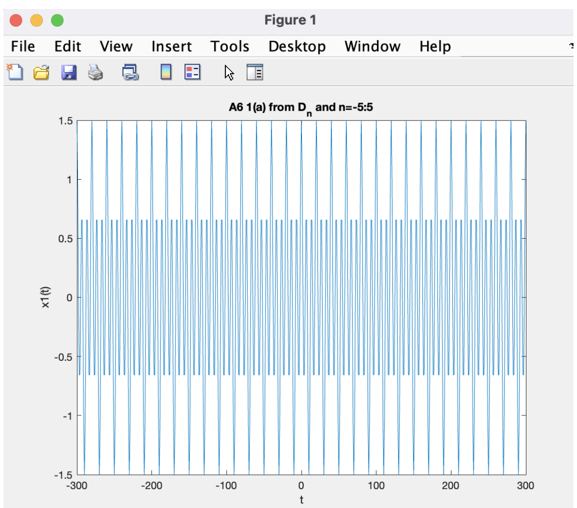


Figure 14: For ranges -20 < n < 20

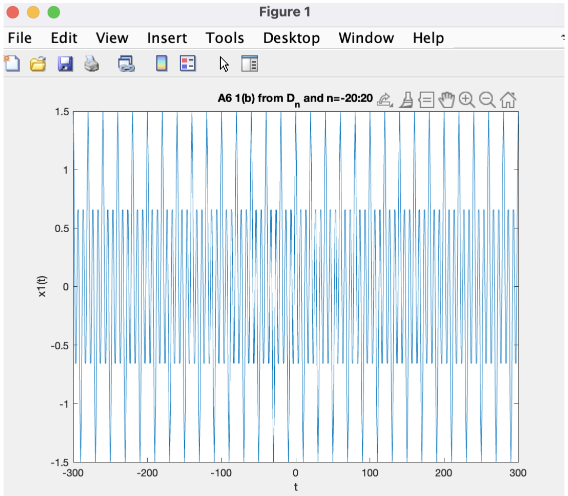


Figure 15: For ranges -50 < n < 50

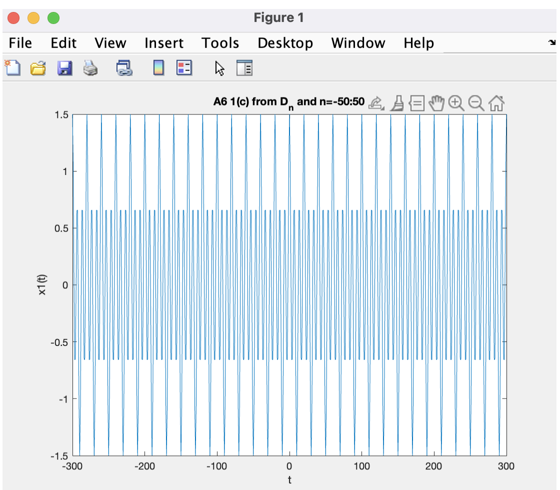
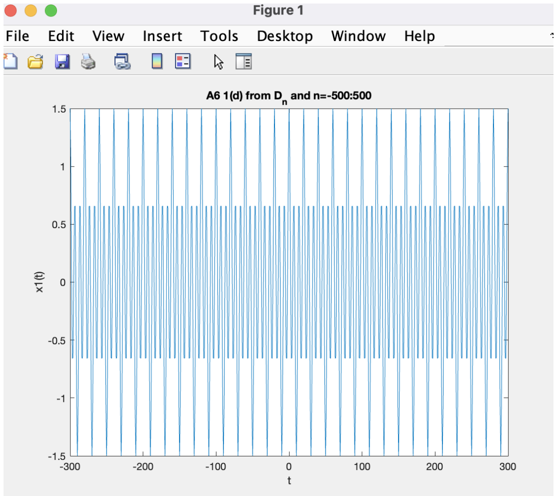


Figure 16: For ranges -500 < n < 500



Part B - This part of this problem inspects the reconstructed plots for x2(t)

Figure 17: For ranges -5 < n < 5

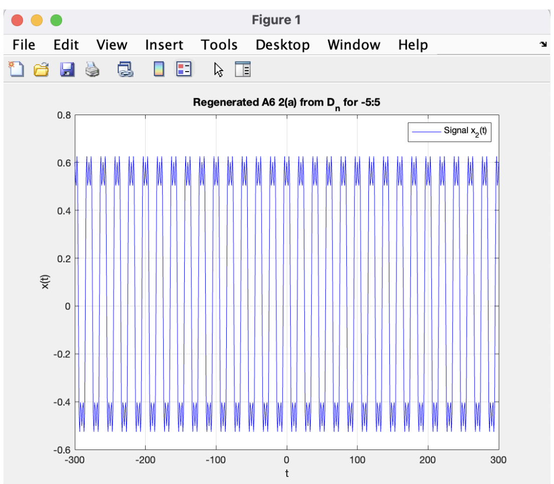


Figure 18: For ranges -20 < n < 20

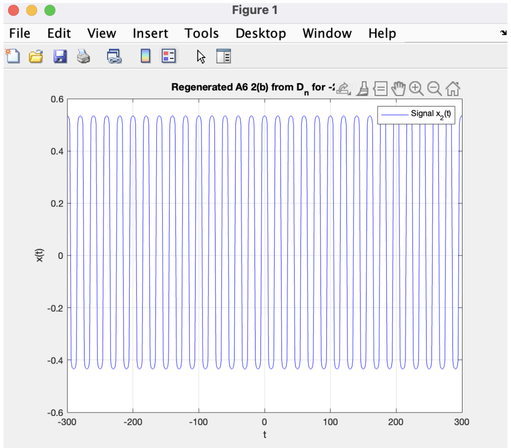


Figure 19: For ranges -50 < n < 50

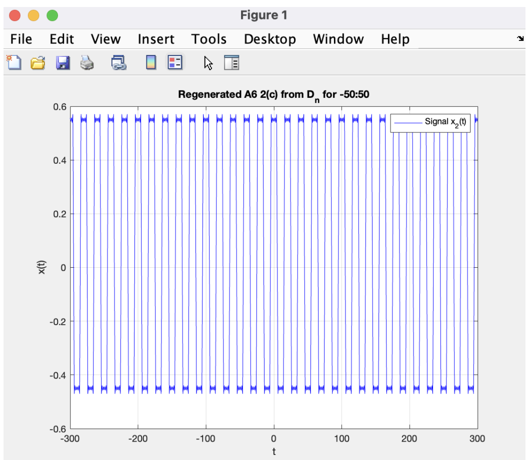
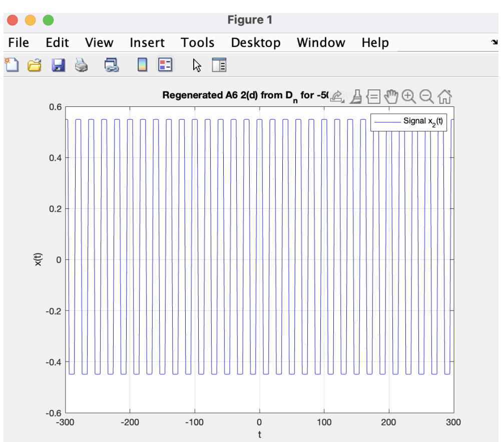


Figure 20: For ranges -500 < n < 500



Part C - This part of this problem inspects the reconstructed plots for x3(t)

Figure 21: For ranges -5 < n < 5

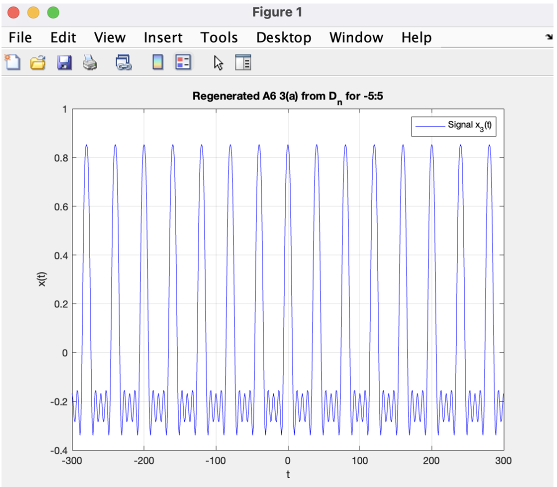


Figure 22: For ranges -20 < n < 20

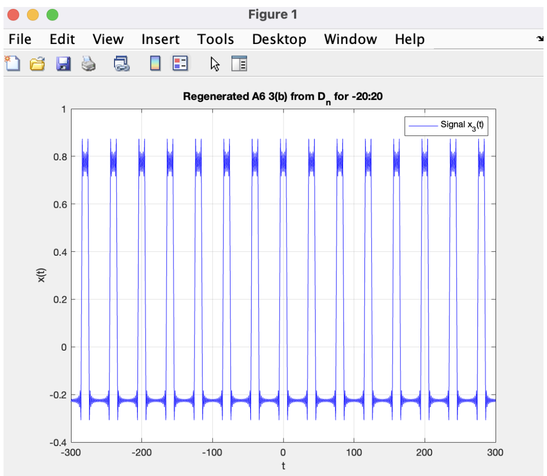


Figure 23: For ranges -50 < n < 50

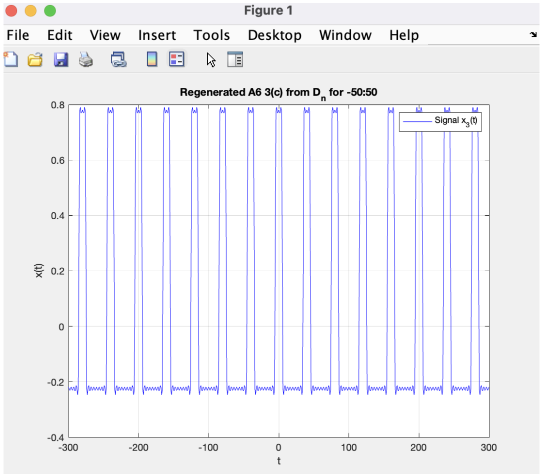
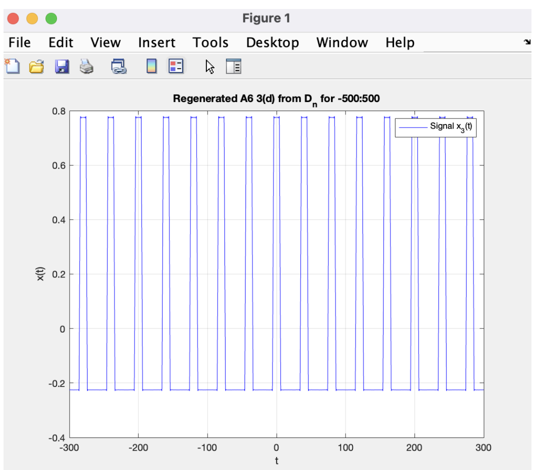


Figure 24: For ranges -500 < n < 500



B. Discussion

3

Problem B

1.

- $x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$

$$w_{o_1} = \frac{3\pi}{10}, \quad w_{o_2} = \frac{\pi}{10} \Rightarrow w_o = \frac{GCF}{LCF} = \boxed{\frac{\pi}{10}}$$

- $x_2(t) \Rightarrow T_0 = 20t$

$$w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{20} = \boxed{\frac{\pi}{10}}$$

- $x_3(t) \Rightarrow T_0 = 40t$

$$w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{40} = \boxed{\frac{\pi}{20}}$$

2. The main difference between the Fourier coefficients of $x_1(t)$ and $x_2(t)$ is that one is derived from an expression, $x_1(t)$, and the other is derived from the provided graph in figure 1 of the lab manual. Therefore, the D_n expressions are significantly different

3. Since $x_2(t)$ and $x_3(t)$ have different periods, they have different fundamental frequencies although they both have the rectangular pulse shape. $x_3(t)$ has a smaller fundamental frequency compared to $x_2(t)$ and therefore this difference is noted in the Fourier coefficient

4. $D_{n_2} = \frac{1}{n\pi} \sin\left(\frac{\pi n}{4}\right) t$

Since $x_2(t)$ from figure 1 and $x_4(t)$ from figure 2 are very similar, Do can be represented as 0.5 as seen in figure 2.

$\therefore D_0 = 0.5$

5. $x_1(t)$ has a fundamental frequency larger than $x_2(t)$. As a result the reconstructed time-domain for $x_1(t)$ has a larger frequency, making the signal more accurate. This is due to the increasing number of fourier coefficients as seen in the displayed graphs.

6. To perfectly reconstruct a periodic signal in this lab experiment, you would need an infinite amount of fourier coefficients, D_n.

7. The scenario explained in problem B.7 is not viable because a periodic signal has an infinite amount of D_n values. Therefore, storing an infinite amount of values on a hard drive is not optimal. Likewise, if D_n is assumed to be finite, there would be an arbitrarily large amount of values to store, thus reducing the run-time speed of the program. Therefore, this method is not viable.

Conclusion

In conclusion, this lab proved to be very beneficial as it provided greater insight into MATLAB commands, graphing tools, data analysis, system properties, and Fourier series analysis (including the characteristics, coefficients, and signal reconstruction) These commands and operations will come of use in future applications of MATLAB and signal analysis.