Department of Electrical
& Computer Engineering
Faculty of Engineering & Architectural Science

Course Title:	Signals & Systems I
Course Number:	ELE532
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Instructor:	Instructor: Professor Javad Alirezaie, TA: Brendan Wood

Assignment/Lab Number:	Lab 2
Assignment/Lab Title:	System Properties and Convolution

Submission Date:	Sunday, Oct 22, 2023
Due Date:	Sunday, Oct 22, 2023

Student LAST Name	Student FIRST Name	Student Number	Section	Signature*
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Lab 2 - System Properties and Convolution

By: Syed Maisam Abbas

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Introduction

In his lab, the objective is to implement M-files to generate graphs/plots, analyze data, compare data files/plots, and gain coding experience using MATLAB, the online programming software. These M-files provide a higher insight into the practice of using convolution and investigating system properties. These labs may prove to be beneficial in the future to analyze signals and different kinds of systems in the engineering field.

Lab Analysis

A. Impulse Response

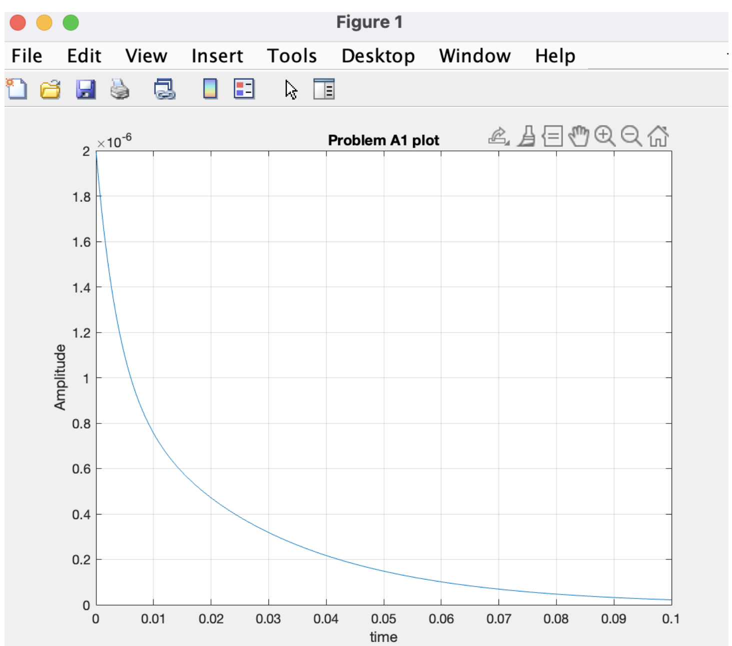
- **A.1**

By using the poly command to generate a random function specified by lambda, the roots (lambda values) are displayed below:

```
The lambda values for the poly function are:
 -261.8034
  -38.1966
```

- **A.2**

The impulse response of the system generated by problem A.1 is displayed below:



- **A.3**

The completed code can be found in the provided .m file and is displayed below:

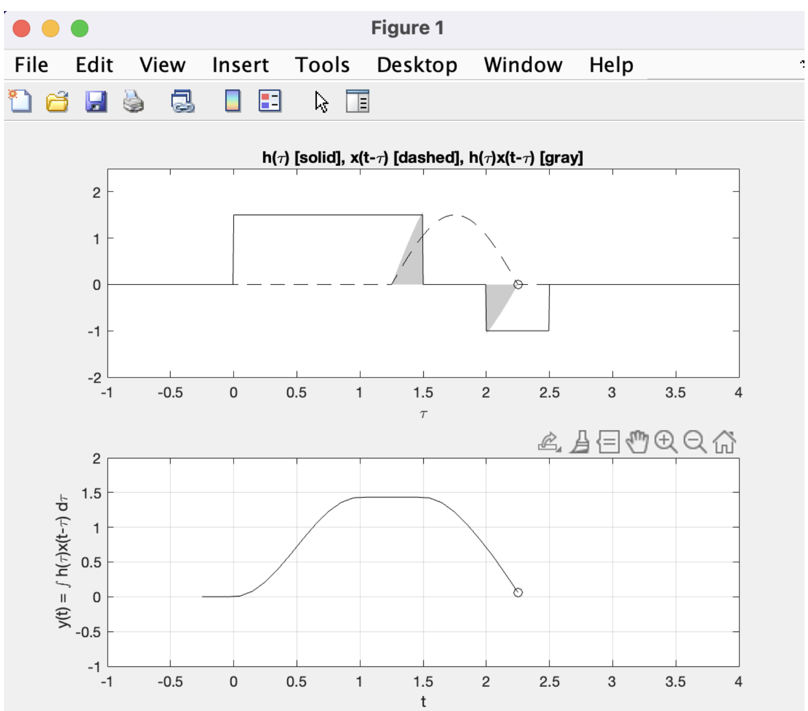
```
%%
%Problem A.3:
function [lambda] = CH2MP2(R,C)
% Function M-file finds characteristic roots of op-amp circuit.
% INPUTS:    R = length-3 vector of resistances
%            C = length-2 vector of capacitances
% OUTPUTS:   lambda = characteristic roots

% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);
end
```

B. Convolution

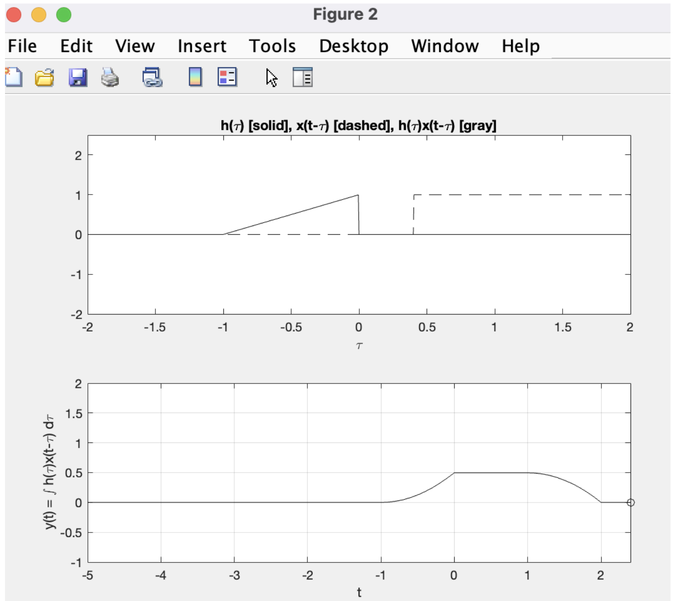
- **B.1**

The plot of y(t) at step t = 2.25 shown in Figure 2.28, page 219 is displayed below:



- **B.2**

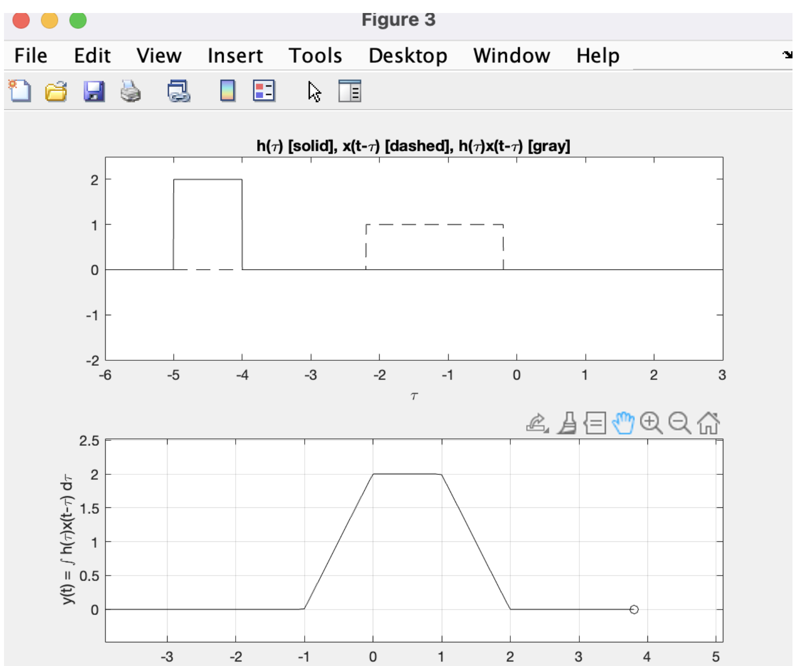
Image displaying the convolution of x(t) in figure 2.4-28 (a) on page 229 with h(t) in figure 2.4-30 on page 230. y(t) = x(t) * h(t):



- **B.3**

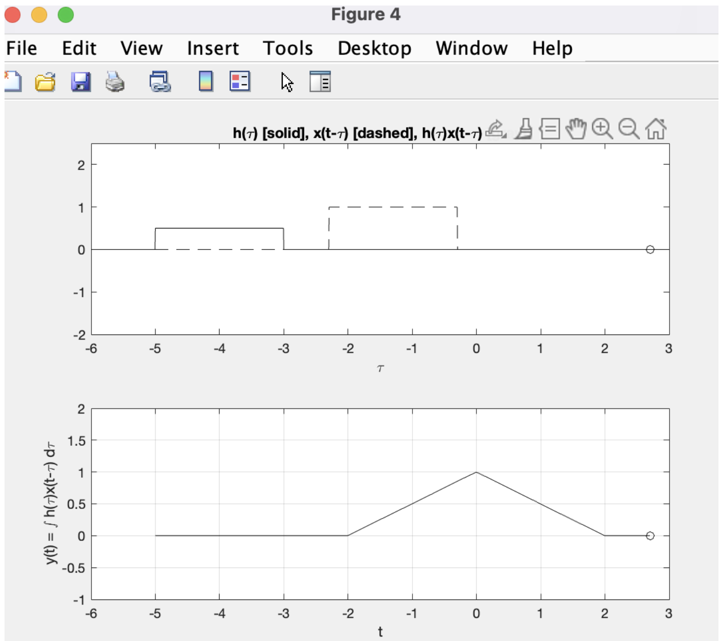
Part A: (Figure 2.4-27, part a)

Convolution of the two signals in part a: [y(t) = x1(t) * x2(t)]



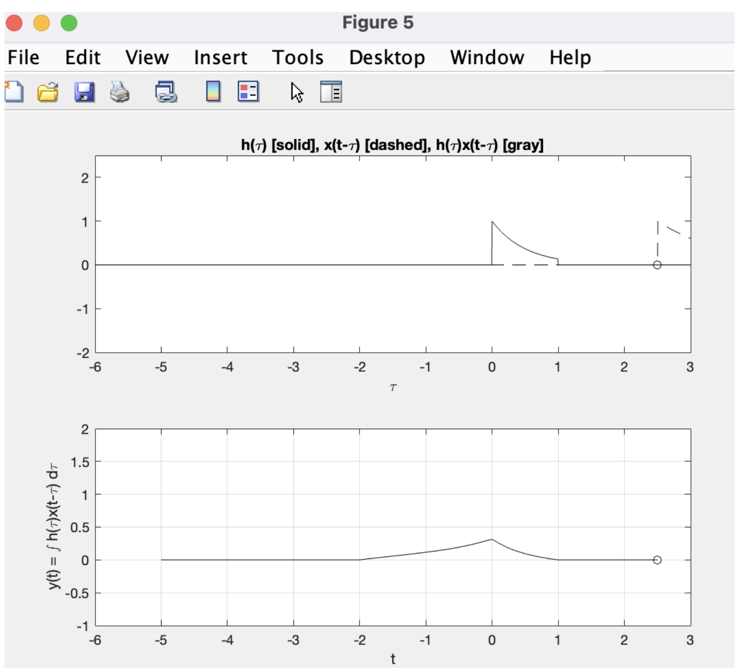
Part B: (Figure 2.4-27, part b)

Convolution of the two signals in part b: [y(t) = x1(t) * x2(t)]



Part C: (Figure 2.4-27, part h)

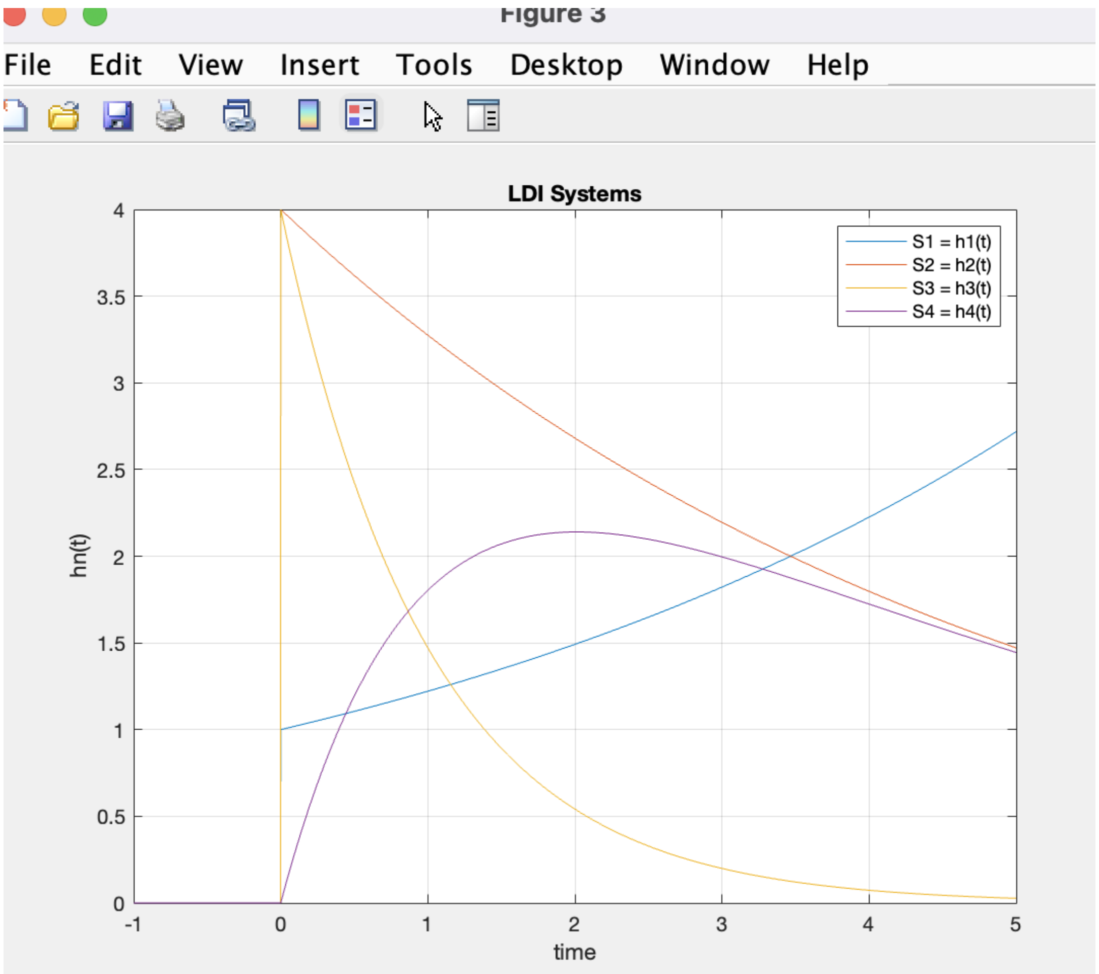
Convolution of the two signals in part h: [y(t) = x1(t) * x2(t)]



C. System Behavior and Stability

- **C.1**

Figure displaying the plots of the unit impulse response of the LTI systems: S1, S2, S3, and S4 represented by h1(t), h2(t), h3(t), and h4(t) respectively:



- **C.2**

The characteristic values (eigenvalues) of the systems S1-S4 are found using the exponential value of the function and are given as follows:

$\lambda_1 = \frac{1}{5}$ $\lambda_2 = -\frac{1}{5}$

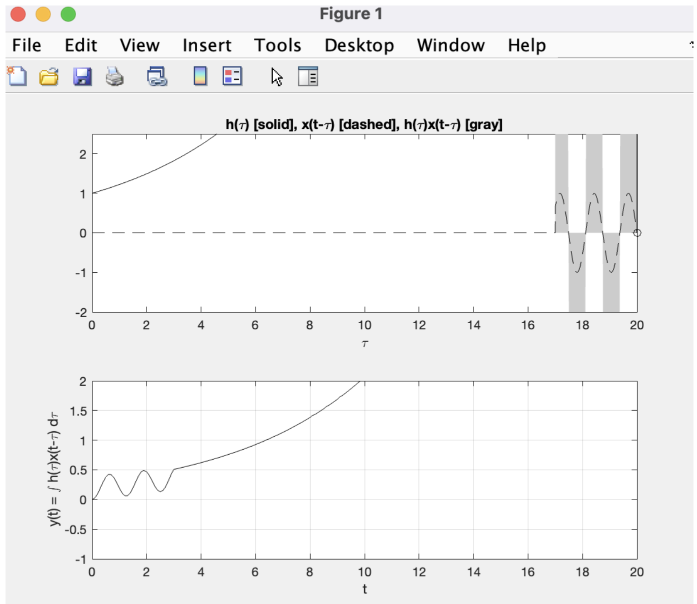
$\lambda_3 = -1$ $\lambda_4 = -\frac{1}{5}, -1$

- **C.3**

We are given x(t) = [u(t) - u(t-3)] sin(5t)

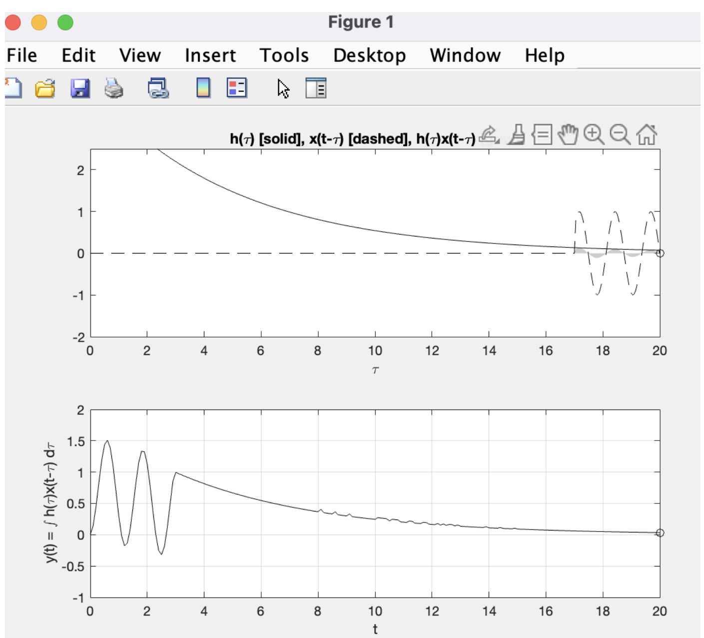
Part A: (Implementation of S1)

Image displaying the convolution of y(t) = x(t) * h1(t)



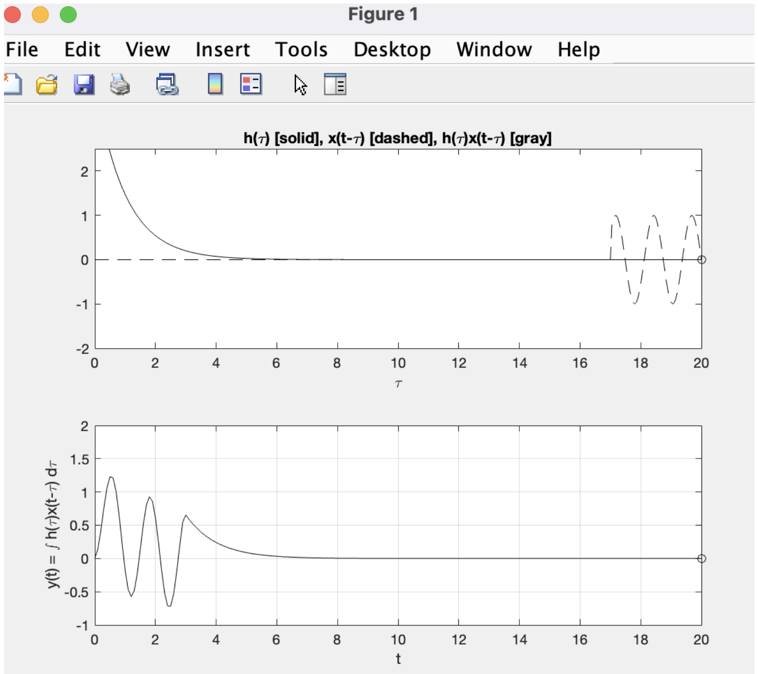
Part B: (Implementation of S2)

Image displaying the convolution of y(t) = x(t) * h2(t)



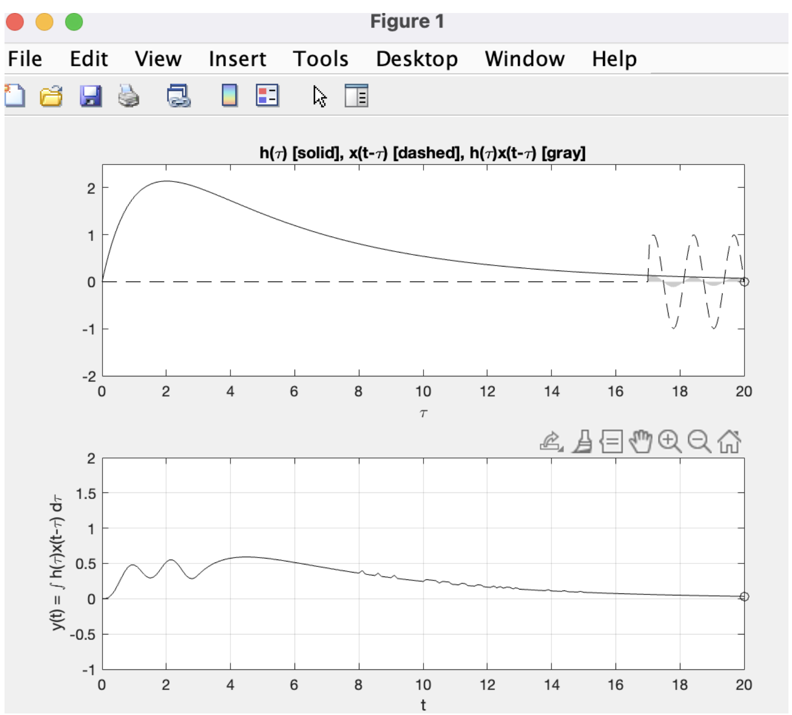
Part C: (Implementation of S3)

Image displaying the convolution of y(t) = x(t) * h3(t)



Part D: (Implementation of S4)

Image displaying the convolution of y(t) = x(t) * h4(t)

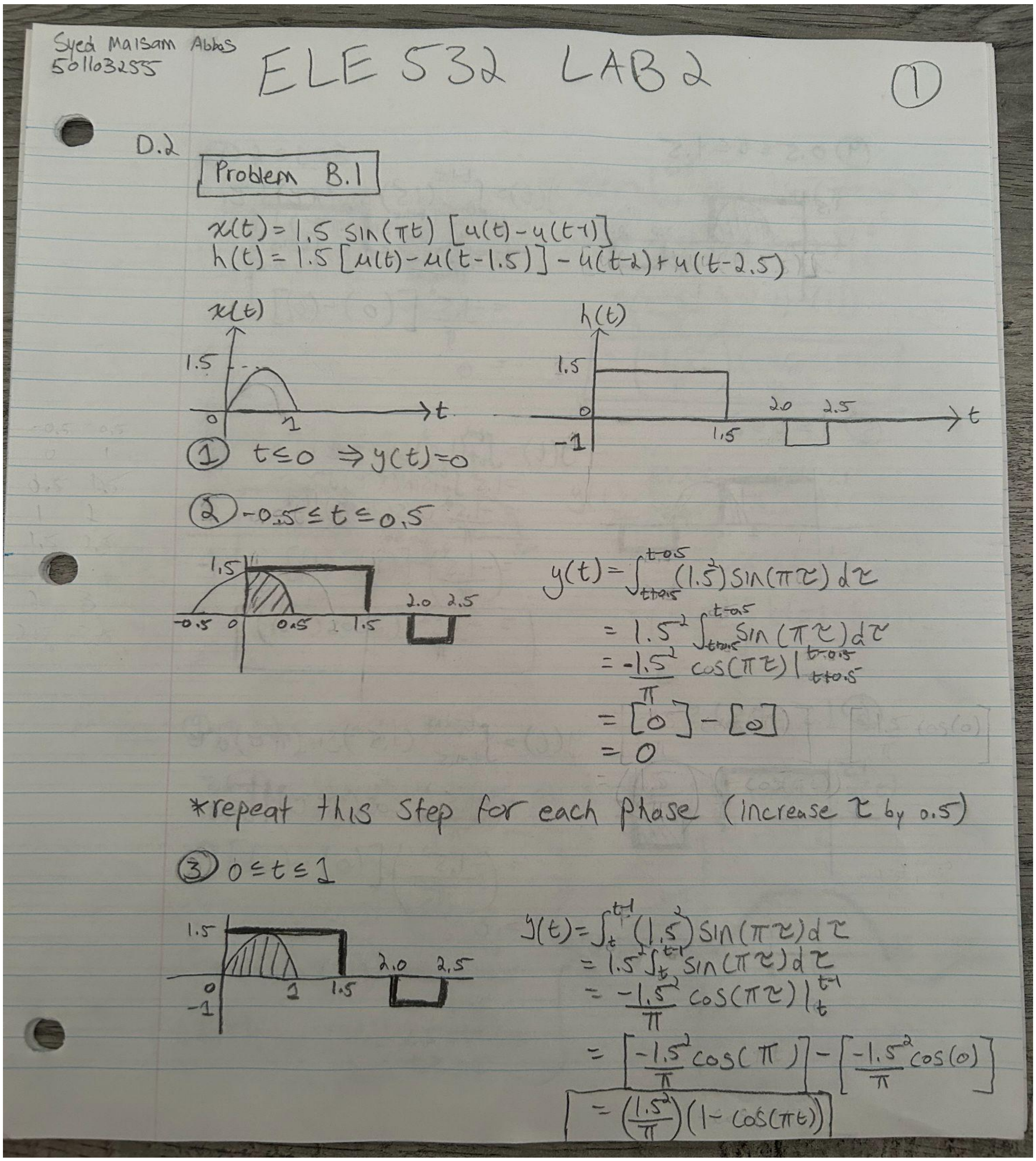


The plots for S2, S3, and S4 all produce similar waveforms in the convolution with h(t). These waveforms all have a similar structure as it starts off with some sort of sin wave and saturates into a straight line.

D. Discussion

- **D.1**

Calculations of problems B.1, B.2, and B.3:



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D.2

Problem B.1

$x(t) = 1.5\sin(\pi t)[u(t) - u(t-1)]$
$h(t) = 1.5[u(t) - u(t-1.5)] - u(t-2) + u(t-2.5)$

① $t \le 0 \Rightarrow y(t) = 0$

② $-0.5 \le t \le 0.5$

$$y(t) = \int_{t+0.5}^{t-0.5} (1.5^2)\sin(\pi\tau)\,d\tau$$
$$= 1.5^2 \int_{t+0.5}^{t-0.5} \sin(\pi\tau)\,d\tau$$
$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t+0.5}^{t-0.5}$$
$$= [0] - [0]$$
$$= 0$$

*repeat this step for each phase (increase τ by 0.5)

③ $0 \le t \le 1$

$$y(t) = \int_{t}^{t-1} (1.5^2)\sin(\pi\tau)\,d\tau$$
$$= 1.5^2 \int_{t}^{t-1} \sin(\pi\tau)\,d\tau$$
$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t}^{t-1}$$
$$= \left[\frac{-1.5^2}{\pi}\cos(\pi)\right] - \left[\frac{-1.5^2}{\pi}\cos(0)\right]$$
$$\boxed{= \left(\frac{1.5^2}{\pi}\right)(1 - \cos(\pi t))}$$

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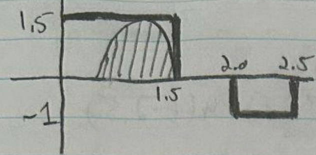
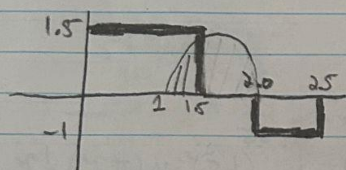
4) $0.5 \le t \le 1.5$



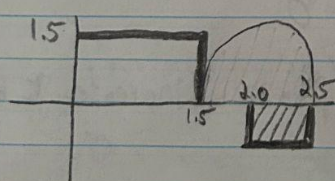
$$y(t) = \int_{t-0.5}^{t-1.5} (1.5^2)\sin(\pi\tau)\,d\tau$$
$$= 1.5^2 \int_{t-0.5}^{t-1.5} \sin(\pi\tau)\,d\tau$$
$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t-0.5}^{t-1.5}$$
$$= \frac{-1.5^2}{\pi}\left[(0)-(0)\right]$$
$$= 0$$

5) $1 \le t \le 2$



$$y(t) = \int_{t-1}^{t-2} (1.5^2)\sin(\pi\tau)\,d\tau$$
$$= 1.5^2 \int_{t-1}^{t-2} \sin(\pi\tau)\,d\tau$$
$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t-1}^{t-2}$$
$$= \left(\frac{-1.5^2}{\pi}\right)\left[(1)-(-1)\right]$$
$$\boxed{= \frac{-1.5^2}{\pi}\left(2\cos(\pi t)\right)}$$

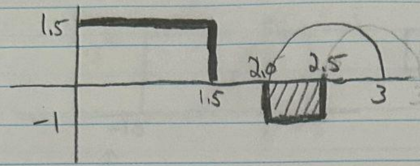
6) $1.5 \le t \le 2.5$



$$y(t) = \int_{t-1.5}^{t-2.5} (1.5^2)\sin(\pi\tau)\,d\tau$$
$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t-1.5}^{t-2.5}$$
$$= \left(\frac{-1.5^2}{\pi}\right)\left[(0)-(0)\right]$$
$$= 0$$

3

⑦ $2 \le t \le 3$



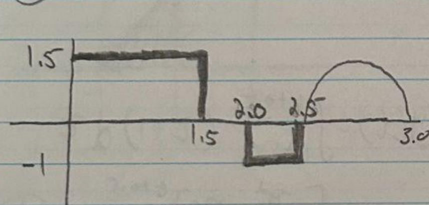
$$y(t) = \int_{t-2}^{t-3} (1.5^2)\sin(\pi\tau)\,d\tau$$

$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t-2}^{t-3}$$

$$= \left(\frac{-1.5^2}{\pi}\right)[(-1)-(1)]$$

$$\boxed{= \left(\frac{-1.5^2}{\pi}\right)(-2\cos(\pi t))}$$

⑧ $2.5 \le t \le 3.5$



$$y(t) = \int_{t-2.5}^{t-3.5} (1.5^2)\sin(\pi\tau)\,d\tau$$

$$= \frac{-1.5^2}{\pi}\cos(\pi\tau)\Big|_{t-2.5}^{t-3.5}$$

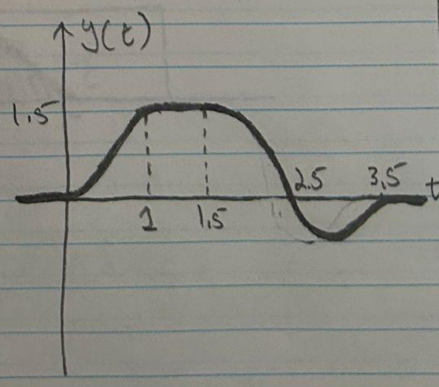
$$= \left(\frac{-1.5^2}{\pi}\right)[(0)-(0)]$$

$$= 0$$

⑨ $t \ge 2.5 \Rightarrow y(t) = 0$

$\therefore$ the individual regions of t form the function, $y(t)$

$$y(t) = \begin{cases} 0 & t \le 0 \\ 0 & 0 \le t \le 1 \\ (1.5^2/\pi)(1-\cos(\pi t)) & 0.5 \le t \le 2 \\ 0 & 1 \le t \le 1.5 \\ (-1.5^2/\pi)(2\cos(\pi t)) & 1.5 \le t \le 2.5 \\ 0 & 2 \le t \le 3 \\ (-1.5^2/\pi)(-2\cos(\pi t)) & 2.5 \le t \le 3.5 \\ 0 & t \ge 3.5 \end{cases}$$



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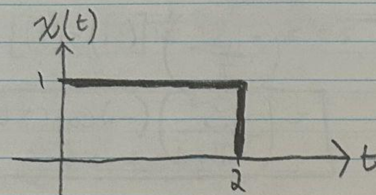
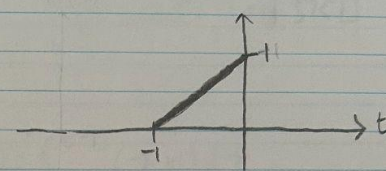
Problem B.2

$x(t) = u(t) - u(t-2)$

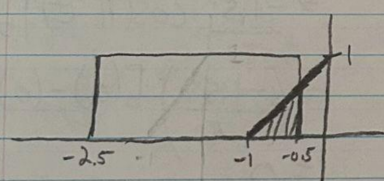
$h(t) = (t+1)\,[u(t+1) - u(t)]$





① $t \leq -1 \Rightarrow y(t) = 0$

② $-2.5 \leq t \leq -0.5$



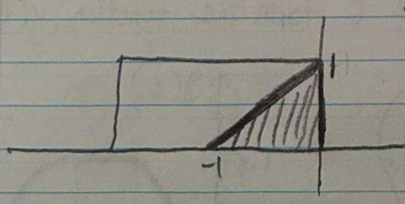
$$y(t) = \int_{t+2.5}^{t+0.5} (\tau+1)\,d\tau$$

$$= \left[\frac{\tau^2}{2} + \tau\right]_{t}^{t+0.5}$$

$$= \left[\left(\frac{1}{8} - \frac{1}{2}\right) - \left(\frac{28}{5} + \frac{5}{2}\right)\right]$$

$$= \frac{t^2}{2} + t + 0.5$$

③ $-2 \leq t \leq 0$



$$y(t) = \int_{t+2}^{t} (\tau+1)\,d\tau$$

$$= \left[\frac{\tau^2}{2} + \tau\right]_{t+2}^{t}$$

$$= [(0) - (0)]$$

$$= 0$$

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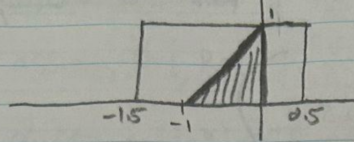
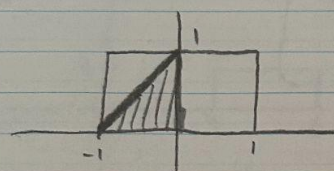
④ $-1.5 \le t \le 0.5$



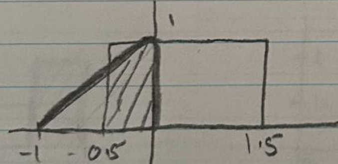
$$y(t) = \int_{t+1.5}^{t-0.5} (\tau+1)\,d\tau$$
$$= \left[\frac{\tau^2}{2} + \tau\right]_{t+1.5}^{t-0.5}$$
$$= \left[(0.625) - (-0.375)\right]$$
$$= 1$$

⑤ $-1 \le t \le 1$



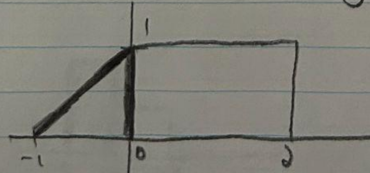
$$y(t) = \int_{t+1}^{t-1} (\tau+1)\,d\tau$$
$$= \left[\frac{\tau^2}{2} + \tau\right]_{t+1}^{t-1}$$
$$= \left[(1.5) - (-0.5)\right]$$
$$= 2$$

⑥ $-0.5 \le t \le 1.5$



$$y(t) = \int_{t+0.5}^{t-1.5} (\tau+1)\,d\tau$$
$$= \left[\frac{\tau^2}{2} + \tau\right]_{t+0.5}^{t-1.5}$$
$$= \left[(2.625) - (-0.375)\right]$$
$$= 3$$

⑦ $0 \le t \le 2$



$$y(t) = \int_{t}^{t-2} (\tau+1)\,d\tau$$
$$= \left[\frac{\tau^2}{2} + \tau\right]_{t}^{t-2}$$
$$= \left[\frac{(t-2)^2}{2} + t - 2\right]$$

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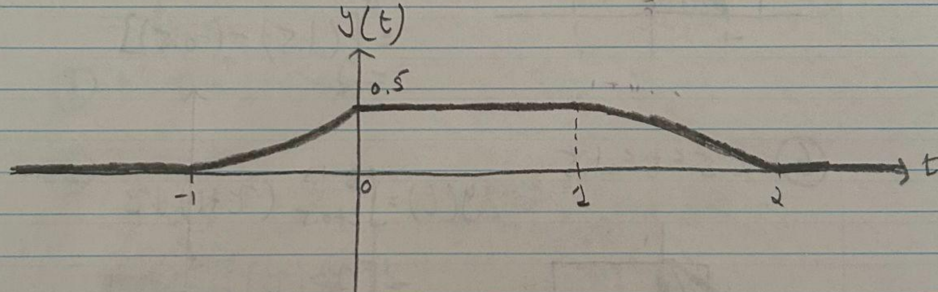
8 $t \geq 2 \Rightarrow y(t) = 0$

∴ the Individual regions of t form the function, y(t)

$$\left.\begin{array}{ll} 0 & t \leq -1 \\ \left[(t^2/2) + t + 0.5\right] & -2.5 \leq t \leq -0.5 \\ 0 & -2 \leq t \leq 0 \\ 1 & -1.5 \leq t \leq 0.5 \\ 2 & -1 \leq t \leq 1 \\ 3 & -0.5 \leq t \leq 1.5 \\ \left[-((t-2)^2/2) + t - 2\right] & 0 \leq t \leq 2 \\ 0 & t \geq 2 \end{array}\right\} y(t)$$



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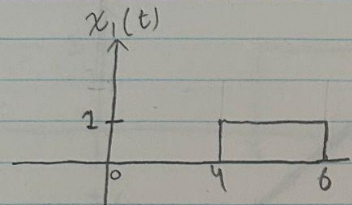
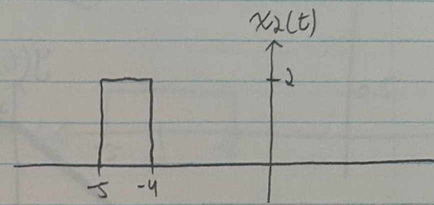
Problem B.3
Part a

assume $A=1, B=2$

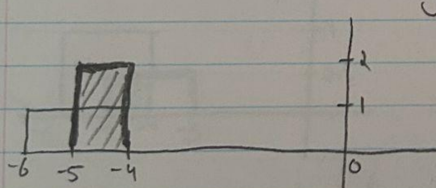
$x_1(t) = u(t-4) - u(t-6)$
$x_2(t) = 2[u(t+5) - u(t+4)]$





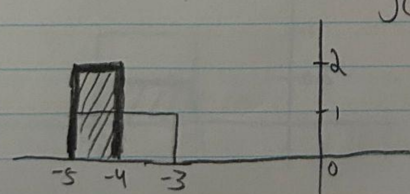
① $t \le -5 \Rightarrow y(t) = 0$

② $-5 \le t \le -4$



$$y(t) = \int_{t+5}^{t+4} 2\, d\tau$$
$$= [2\tau]_{t+5}^{t+4}$$
$$= [(-8)-(-10)]$$
$$= \boxed{2t+2}$$

③ $-4 \le t \le -3$



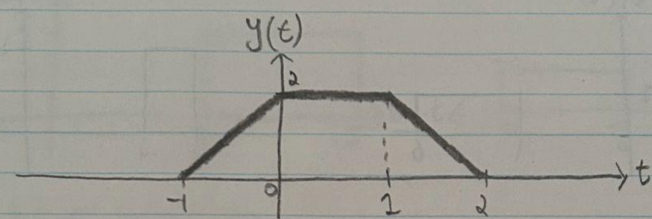
$$y(t) = \int_{t+4}^{t+3} 2\, d\tau$$
$$= [2\tau]_{t+4}^{t+3}$$
$$= [(-6)-(-8)]$$
$$= \boxed{-2t+4}$$

8

(4) $t \geq -3 \Rightarrow y(t) = 0$

$\therefore$ the individual regions of t form the function, $y(t)$

$$y(t) = \begin{cases} 0 & t \leq -5 \\ 2t+2 & -5 \leq t \leq -4 \\ -2t+4 & -4 \leq t \leq -3 \\ 0 & t \geq -3 \end{cases}$$



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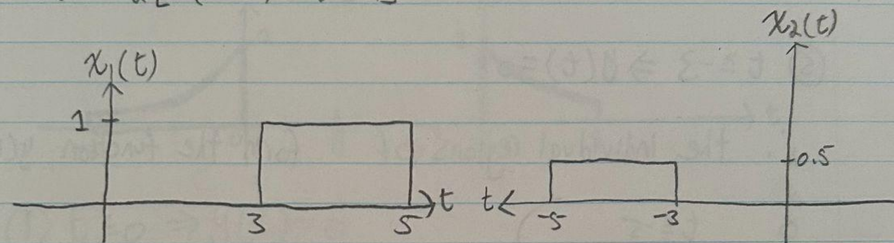
Problem B.3
Part b

assume $A=1$, $B=0.5$

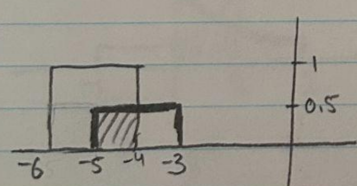
$x_1(t)=u(t-3)-u(t-5)$

$x_2(t)=\frac{1}{2}\left[u(t+5)-u(t+3)\right]$



① $t\le -5 \Rightarrow y(t)=0$

② $-6\le t\le -4$



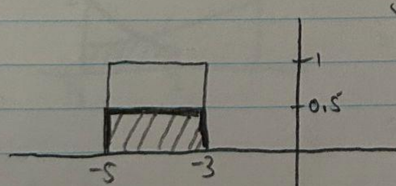
$$y(t)=\int_{t+6}^{t+4}\frac{1}{2}\,d\tau$$

$$=\left[\frac{\tau}{2}\right]_{t+6}^{t+4}$$

$$=\left[(-2)-(-3)\right]=1$$

$$=\boxed{\frac{t}{2}+1}$$

③ $-5\le t\le -3$



$$y(t)=\int_{t+5}^{t+3}\frac{1}{2}\,d\tau$$

$$=\left[\frac{\tau}{2}\right]_{t+5}^{t+3}$$

$$=\left[(-1.5)-(-2.5)\right]=1$$

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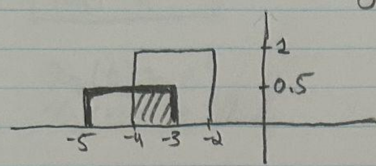
④ $-4 \le t \le -2$



$$y(t) = \int_{t+4}^{t+2} \frac{1}{2}\, d\tau$$

$$= \left[\frac{\tau}{2}\right]_{t+4}^{t+2}$$

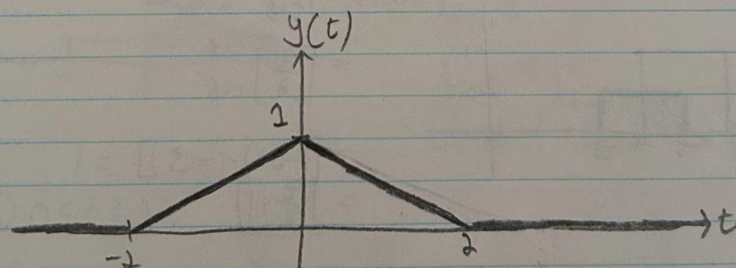
$$= \left[(-1)-(-2)\right] = 1$$

$$= -\frac{t}{2}+1$$

⑤ $t \ge -3 \Rightarrow y(t) = 0$

$\therefore$ the individual regions of t form the function, $y(t)$

$$y(t) = \begin{cases} 0 & t \le -5 \\ \frac{t}{2}+1 & -6 \le t \le -4 \\ 1 & -5 \le t \le -3 \\ -\frac{t}{2}+1 & -4 \le t \le -2 \\ 0 & t \ge -3 \end{cases}$$



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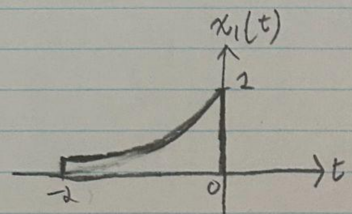
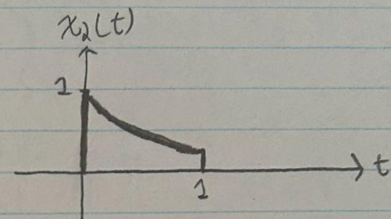
Problem B.3
Part h

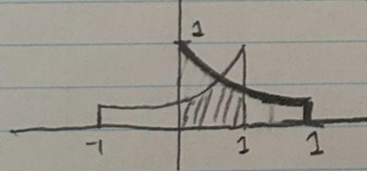
$x_1(t) = e^{t}$
$x_2(t) = e^{-2t}$





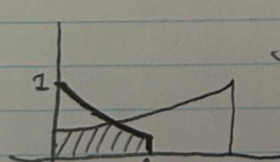
① $t \leq 0 \Rightarrow y(t) = 0$

② $-1 \leq t \leq 1$



$$y(t) = \int_{t+1}^{t-1} e^{\tau} \cdot e^{-2\tau}\, d\tau$$
$$= \int_{t+1}^{t-1} e^{-\tau}\, d\tau$$
$$= \left[-e^{-\tau}\right]_{t+1}^{t-1}$$
$$= \left[(-0.37) - (-e)\right] = -2.35$$

③ $0 \leq t \leq 2$



$$y(t) = \int_{t}^{t-2} e^{\tau} \cdot e^{-2\tau}\, d\tau$$
$$= \left[-e^{-\tau}\right]_{t}^{t-2}$$
$$= \left[(-0.14) - (-1)\right] = 0.86$$

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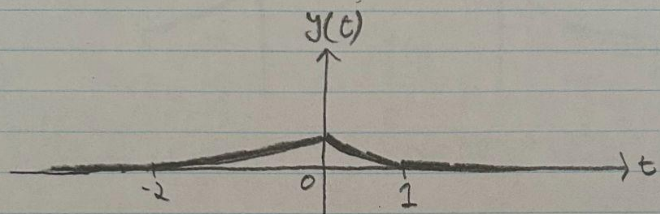
(4) $t \geq 1 \Rightarrow y(t) = 0$

$\therefore$ the individual regions of t form the function, $y(t)$

$$
\left.
\begin{array}{ll}
0 & t \leq 0 \\
-e^{-0.3x} + 1.35 & -1 \leq t \leq 1 \\
e^{-0.4x} - 0.65 & 0 \leq t \leq 2 \\
0 & t \geq 1
\end{array}
\right\} y(t)
$$



- **D.2**

After analyzing/reviewing the online simulations and the theoretical calculations of the resulting convolution graphs, it is clear that the width/duration of the resulting signal is the sum of the individual function durations. Mathematically, $T_a = T_{x1} + T_{y1}$ & $T_b = T_{x2} + T_{y2}$

Conclusions

In conclusion, this lab proved to be very beneficial as it provided greater insight into MATLAB commands, graphing tools, data analysis, convolution, and system properties. These commands and operations will come of use in future applications of MATLAB and signal analysis.