MSE312

MECHANICAL DESIGN AND ANALYSIS REPORT

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MSE-312 / MECHATRONIC DESIGN - II

MECHATRONIC SYSTEMS ENGINEERING

SCHOOL OF APPLIED SCIENCE

SIMON FRASER UNIVERSITY

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Introduction

In this project, we're tasked with designing a one-degree of freedom arm that will produce a projectile motion using a regulation lacrosse ball to reach three targets. One-Degree of freedom robots are used in many different applications such as welding, manufacturing, pick and place etc. Based on our research, the most common solution to a one-degree of freedom (rotation) arm was a catapult with counterweight to produce the torque for the projectile motion. However, we don't need a counterweight for torque generation, instead we have the option of selecting our own motor. Another existing design solution we found was a slingshot used to create the projectile motion by stretching the elastic band using a motor - this design had position sensors and the control algorithm was too complex to implement in the given time frame for our course. Given the limited time and resources, the team chose three structures that we believed can be easily validated and manufactured.

Design Approach

The team unanimously agreed to follow the Human Centered Design theory. The general idea was to clearly identify our requirements, propose a solution that we believe would meet the requirements and analysis the solution using FEA and Simscape Multi-body. If the proposed design did not fail under the loading conditions applied, the proposed design was analyzed for its dynamic characteristics using Simscape Multi-body. If a proposed solution fails FEA analysis, the point of failure was improved and FEA was repeated. The improvements to the model were such that the model could still be manufactured within the given budget. This process was used by the team to design the mechanical structure that could produce the projectile motion while adhering to the given design constraints.

Design Specifications

The design specifications to meet the functional requirements of mechanical structure are as follows:

- The designed structure should not fail due to cyclic loading requirements for projectile motion.
- The designed structure must have a vertical plane operation of 30 cm and should fit in a dome 10 cm wide.
- The designed structure must have a passive mechanism to hold and release a regulation lacrosse ball of 145g and 63mm diameter.
- The designed structure must minimize radial and axial forces on the shaft of the motor.
- The designed structure should be stiff enough to minimize deflection due to rapid acceleration produced by the motor.
- The designed structure should be easily rotated by a motor.

Contributions

Team Member	Contributions
Syed Imad Azeem Rizvi	Worked on simscape and solidworks modelling of each design - including FEA
Abdullah Tariq	Worked on FEA and simscape modelling & analysis of each design
Junaid Jawed Khan	Worked on documentation, hand calculations of Design 1 and assisted with simscape troubleshooting.

Analytical Analysis

In this section we present our hand calculation results for static and dynamic characteristics of our three designs. These results are used for validating the FEA and Simscape simulation results.

Design 1

The first arm structure is a simple hollow beam pinned at one end and a ball-holder on the free end as shown in figure 1.

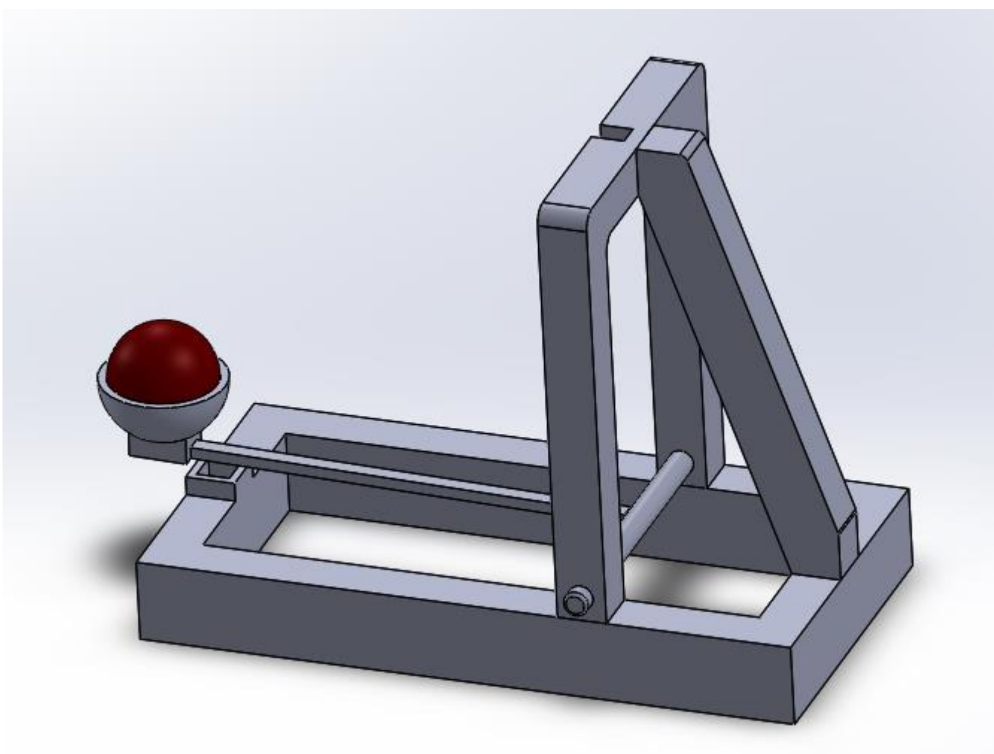


Figure 1: Design 1 SolidWorks Model

The loads considered in the static analysis include the weight of the regulation size lacrosse ball and gravity.

Statics

For our static analysis of Design 1, we were interested in minimizing deflection on the free end. The geometry of the beam and boundary conditions were fixed. Therefore, to analyze deflection, we used different materials to minimize the deflection on the free end, making it easier for controlling the arm.

The three different materials we used were 1023 Carbon Steel, Titanium alloy (Cp-Ti UNS R50400) and Brass. The justification for these materials is provided in the Material Selection section of this report.

Figure 2 is the Free Body diagram used for statics hand-calculations.

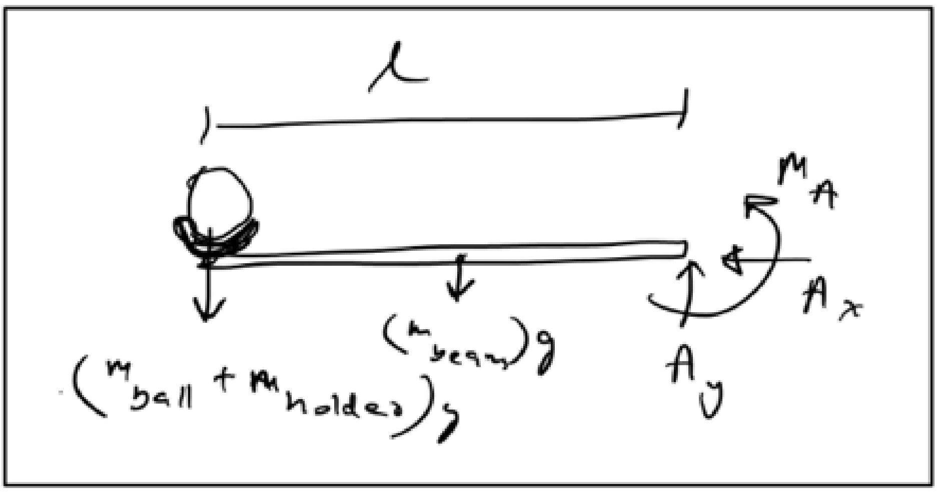


Figure 2: Statics Analysis Free Body Diagram.

The hand calculations are used for validation of FEA of Design 1. The equation used for populating the following table are included Appendix A under Statics. The results are tabulated as shown table 1.

Table 1: Static Analytical Results of Design 1

	Fatigue Stress Limit (N/mm^2)	Maximum Stress (N/mm2)	Deflection (mm)
1023 Carbon Steel	255	1.989e+01	0.980
Cp-Ti UNS R50400	291	1.245e+01	1.298
Brass	216	2.250e+01	1.879

Based on the analytical results shown in Table 1, using Brass as the material for the arm results in the greatest deflection. Moreover, using Brass results in a greater stress in comparison to the other materials.

Dynamics

For our kinematic analysis of Design 1, we were interested in minimizing the mass moment of inertia of the structure to reduce the torque needed from the motor to rotate the arm. Since the mass distribution and location of the axis of rotation is fixed for our structure, different choices of material were used to change the total mass moment of inertia of the structure.

Figure 3 is the Free Body diagram used for dynamics hand-calculations.

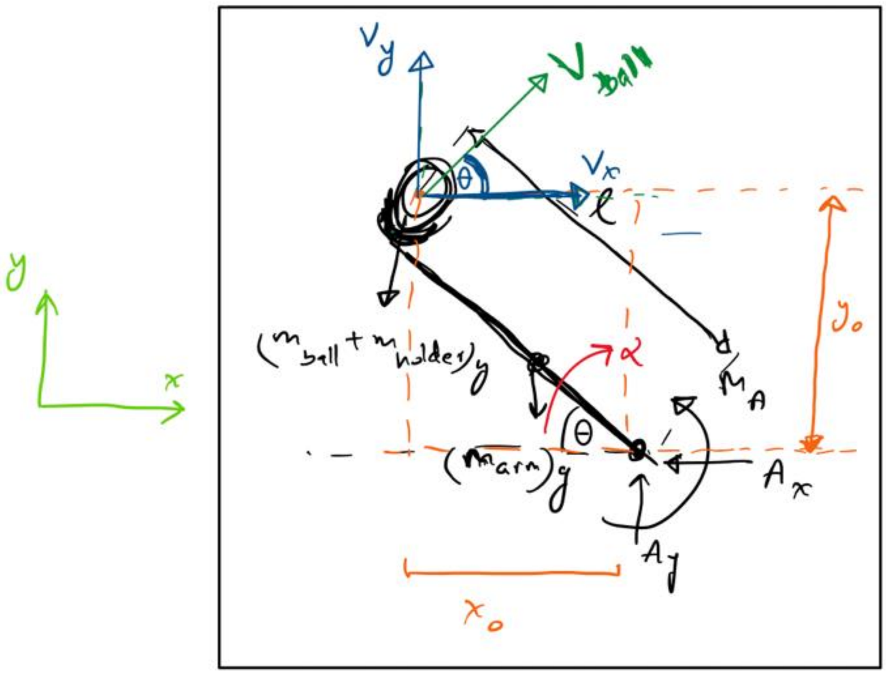


Figure 3: Kinematic Analysis Free Body Diagram

The hand calculations are used for validation of Simscape simulation of Design 1. The equation used for populating the following table are included Appendix A under Dynamics. The results are tabulated as shown in Table 2.

Table 2: Kinematic Analytical Results of Design 1

	Total Mass Moment of Inertia	Angular Velocity (rad/sec)	Angular Acceleration (rad/sec^2)
1023 Carbon Steel	0.03379290	14.89	98.78
Cp-Ti UNS R50400	0.00127061	18.69	143.43
Brass	0.03655379	13.27	94.14

Based on the analytical results shown in Table 2, using the Titanium alloy for the arm structure produced the largest angular velocity and acceleration. This was because the Titanium alloy has the lowest density ($\rho = 4510\ \frac{\text{kg}}{m^3}$) in comparison to the other materials (1023 Carbon Steel $\rho = 7858\ \frac{\text{kg}}{m^3}$, and Brass $\rho = 8500\ \frac{\text{kg}}{m^3}$) [1][2][3].

Design 2

The second design for creating a projectile motion is to launch a ball from an inclined base. The ball initially rests an elevation, followed by rolling down the track and getting projected by pair of flywheels connected to the motor. Figure 4 shows the SolidWorks model for this design.

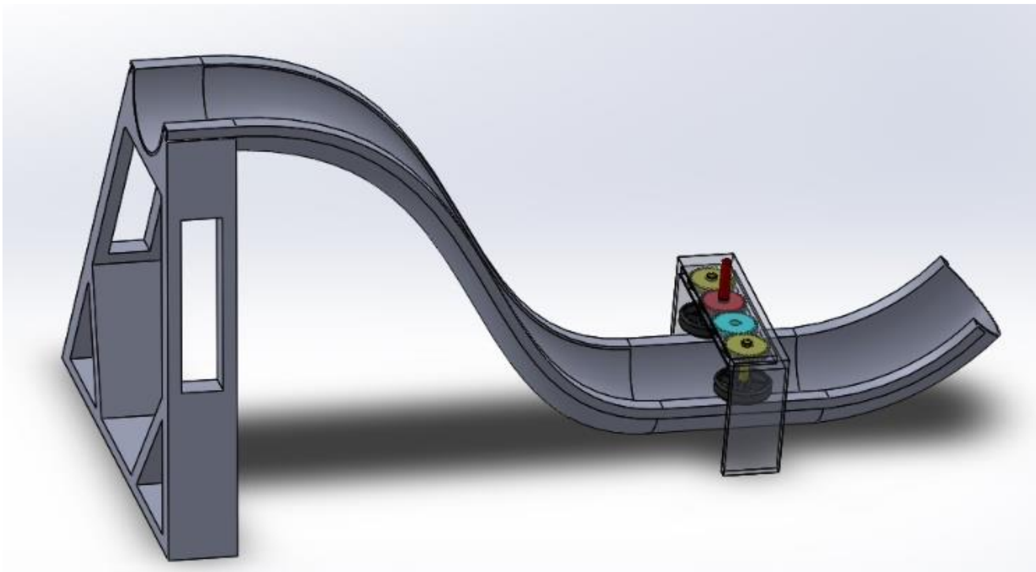


Figure 4: Design 2 – SolidWorks Model

Inspiration of this design is taken from the tennis ball launcher, that uses a similar mechanism of two combination of wheels that rotates opposite with respect to each other and launches the ball forward when passed through them. It can be observed from the figure that, the assembly constitutes of a combination of 4 gears required to produce a desired motion with the addition to the red color shaft that is driven by the motor. The gap between the two wheels is the optimum distance that is required to being in contact with the ball, compressing it a bit, but the ball remains in its elastic deformation.

Statics

The slide is fixed with a rigid support structure. The free body diagram in figure 5 shows the reaction forces, weight of the ball and bending moment that will be produced when the boll rolls down on the track. An assumption has been made, that the ball weight is evenly distributed on the slide when it is in contact with it. The slide is solid in its geometry, and its slope is assumed to be a straight line rather than being curved.

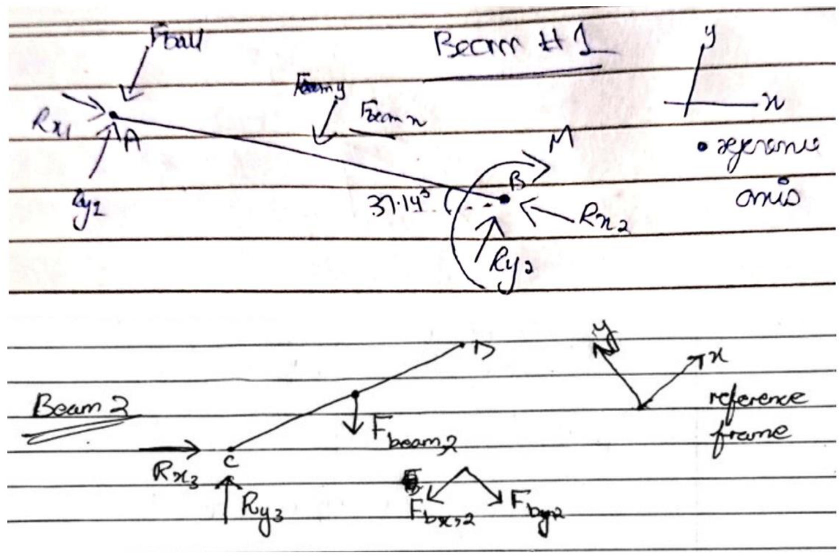


Figure 5: Free body diagram of track

The hand calculations are used for validation of FEA of Design 2. The equation used for populating the following table are included Appendix B under Statics. The results are tabulated as shown table 3.

Table 3: Statics Analytical Results of Design 2

	Fatigue Stress Limit (N/mm^2)	Maximum Stress (N/mm2)	Deflection (mm)
1023 Carbon Steel	255	1.436e-01	2.823e-04
Cp-Ti UNS R50400	291	0.926e-01	3.528e-04
Brass	216	1.362e-01	5.646e-04

Dynamics

Figure 6 shows the free body diagram of the ball when it is in motion.

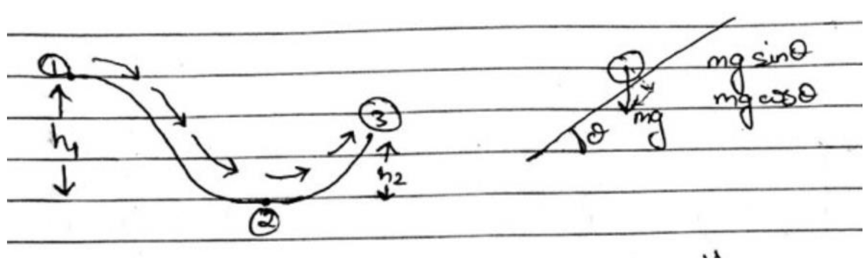


Figure 6: Free body diagram of ball's motion.

A few assumptions have been made to ease with the calculations and simulations. The ball rolls on the track throughout its motion, and it never slips or skids. The contact frictional forces have been considered for frictional losses and its coefficient changes with the material. The slide has been broken down into three parts to simplify the calculation. The ball has a fixed gravitational potential energy at the beginning, that is transferred into both rotational and translational kinetic energy with frictional losses just before reaching the rotating fly wheels. It is assumed that there is no deformation taking place when the ball passes through the wheels. The ball then launches at an angle of 45° degree to achieve maximum distance.

The hand calculations are used for validation of Simscape simulation of Design 2. The equation used for populating the following table are included Appendix B under Dynamics. The results are tabulated as shown in Table 4.

Table 4: Kinematics Analytical Results of Design 2

	Angular Velocity (rad/sec)	Angular Acceleration (rad/sec^2)
1023 Carbon Steel	165.671	2.420
Cp-Ti UNS R50400	162.14	2.613
Brass	144.83	2.1175

Design 3

The third design for creating a projectile motion is to launch the ball using a spring mechanism. Figure 5 shows the SolidWorks model for this design.

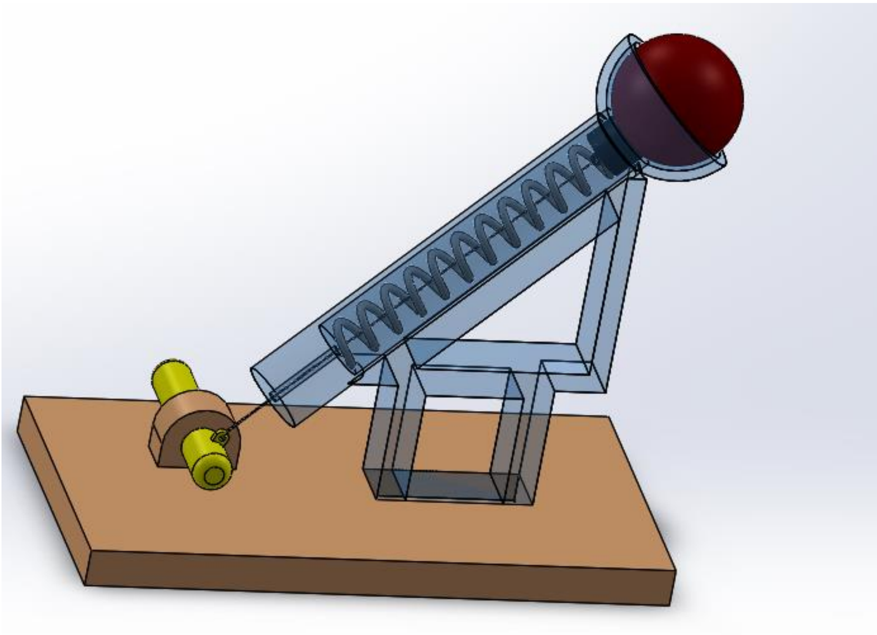


Figure 7: Design 3 SolidWorks Model

This design uses a similar idea of a gun firing a bullet. A cork shaped bullet is screwed down to a spring (fixed on one end of the gun), and the bullet is tied down to a shaft, by a non-flexible string as shown above.

Statics

Figure 8 shows the free body diagram for the gun.

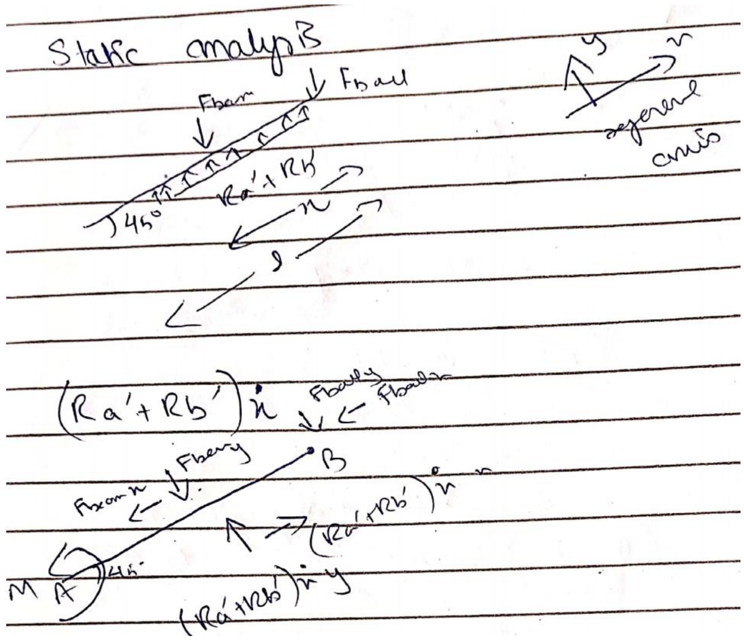


Figure 8: Free body diagram of gun.

As it can be observed in figure 5 there is a hemispherical opening for the ball that is placed on the gun elevated at a 45° angle. The ball is assumed to be exerting an evenly distributed force (i.e., its weight) at the lower half of the hemispherical opening of the gun. It is assumed that the gun has a uniform distribution, where its center of mass is concentrated at its geometrical center. The reaction forces then act on the gun at its pillars, and their sum is evenly distributed on its cylindrical part of contact. This helps balance the weight of the gun as well as the ball.

The hand calculations are used for validation of FEA of Design 3. The equation used for populating the following table are included Appendix C under Statics. The results are tabulated as shown table 5.

Table 5: Statics Analytical Results of Design 3

	Fatigue Stress Limit (N/mm^2)	Maximum Stress (N/mm2)	Deflection (mm)
1023 Carbon Steel	255	1.151e-01	1.983e-04
Cp-Ti UNS R50400	291	8.277e-02	2.783e-04
Brass	216	1.042e-01	3.868e-04

Dynamics

Figure 9 shows the free body diagram used for dynamic analysis of Design 3.

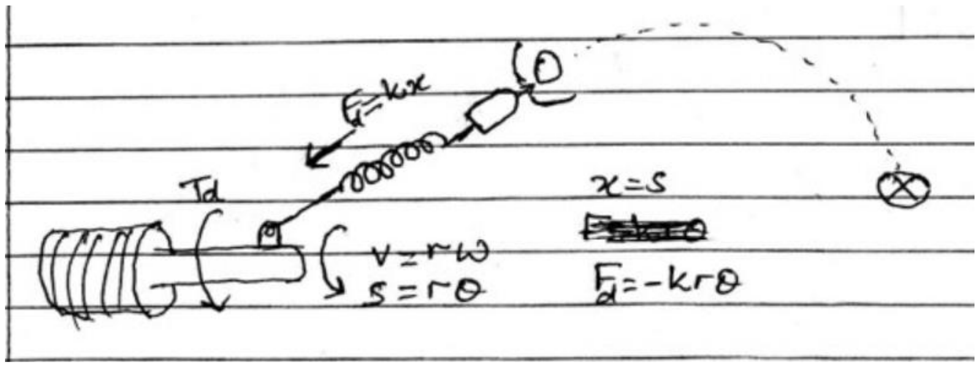


Figure 9: Free body diagram of ball's motion.

From figure 9 it can be observed that when the shafts rotate, its angular displacement (i.e., the arc length) is equal to the linear displacement of the string when the string wraps around the shaft. We assume that the string compresses the spring sufficiently to store the elastic potential energy in it. When the input from the motor shaft is stopped, the shafts rotate freely, and the spring is relaxed. The spring when released generates a force required to hit the ball giving it the desired initial velocity to reach the given target.

The hand calculations are used for validation of Simscape simulation of Design 3. The equation used for populating the following table are included Appendix C under Dynamics. The results are tabulated as shown in Table 6.

Table 6: Kinematics Analytical Results of Design 3

	Angular Velocity (rad/sec)	Angular Acceleration (rad/sec^2)
1023 Carbon Steel	1049.22	18.426
Cp-Ti UNS R50400	775.62	20.896
Brass	916.11	16.206

Qualitative Analysis

In this section the results of FEA and Simscape simulation for each design are presented. The sensitivity of each design to different materials and mesh size was observed. Moreover, the dynamic sensitivity of each design was studied by creating a simulation in Simscape.

The three different materials used for FEA study and Simscape simulation have different densities. One of our design objectives is to ensure the arm structure does not fail under cyclic loading and has minimal deflection. The three materials studied are 1023 Carbon Steel ($\rho = 7858 \frac{\text{kg}}{m^3}$), Cp-Ti UNS R50400 ($\rho = 4510 \frac{\text{kg}}{m^3}$), and Brass ($\rho = 8500 \frac{\text{kg}}{m^3}$) [1][2][3].

FEA Results

In this section we have tabulated all our FEA results for each design. The results are compared with the analytical values in the Validation section.

Design 1

Table 7: FEA of Design 1

	Coarse Mesh			Fine Mesh		
	Deflection (mm)	Maximum Stress (N/mm2)	Time Taken (s)	Deflection (mm)	Maximum Stress (N/mm2)	Time Taken (s)
1023 Carbon Steel	1.067	2.459e+01	01	7.961e-01	2.012e+01	08
Cp-Ti UNS R50400	1.418	1.708e+01	01	8.908e-01	1.212e+01	08
Brass	2.317	2.634e+01	01	1.764	2.226e+01	08

Design 2

Table 8: FEA of Design 2

	Coarse Mesh			Fine Mesh		
	Deflection (mm)	Maximum Stress (N/mm2)	Time Taken (s)	Deflection (mm)	Maximum Stress (N/mm2)	Time Taken (s)
1023 Carbon Steel	3.257e-04	1.62e-01	01	4.186e-04	2.564e-01	05
Cp-Ti UNS R50400	3.878e-04	1.003e-01	01	5.086e-04	1.616e-01	06
Brass	7.139e-04	1.723e-01	01	9.232e-04	2.751e-01	06

Design 3

Table 9: FEA of Design 3

	Coarse Mesh			Fine Mesh		
	Deflection (mm)	Maximum Stress (N/mm2)	Time Taken (s)	Deflection (mm)	Maximum Stress (N/mm2)	Time Taken (s)
1023 Carbon Steel	2.359e-04	1.318e-01	01	2.624e-04	2.216e-01	03
Cp-Ti UNS R50400	2.999e-04	8.587e-02	01	3.374e-04	1.413e-01	03
Brass	5.084e-04	1.393e-01	01	5.672e-04	2.351e-01	04

Simscape Results

In this section we studied the dynamic characteristics of each design. For each design we started the simulation with a base model, followed by changing each base model property and observing its effect on the angular velocity and acceleration.

Design 1

The base model configuration for design 1 is as follows. The arm material was set to 1023 Carbon Steel ($\rho = 7858 \frac{\text{kg}}{m^3}$), initial torque from a virtual motor was set to 5 Nm, the damping coefficient was set to 1e-4, and we used auto ode45 solver to carry out the base model simulation [2].

Table 10 shows the dynamic sensitivity analysis results of Design 1.

Table 10: Simulation Results of Design 1

	Angular Velocity (rad/sec)	Angular Acceleration (rad/sec^2)
Base model	15.46	101
Material Titanium	19.562	146
Material Brass	15.14	96.3
Solver (ode15s / step time 0.001s)	15.793	102
Damping (1e-1)	0.589	0.152
Torque (50 Nm)	54.088	1008.933

Based on the results shown in Table 10, the angular acceleration for the base model was high because the base material required more torque to rotate the arm from the rest position. The change in material of the rotating increased the angular velocity achieved for Titanium because its density is almost half that of both Carbon Steel and Brass. Therefore, for the same torque it achieved a higher angular velocity. Using a different solver for the base model did not have any significantly effect on the angular velocity and acceleration. Damping the base model had a significant impact on Design 1 as the initial torque wasn't enough to lift the shaft from its resting position at 0 degrees. Increasing the torque by ten times provided a greater moment and this led an increase in angular velocity and acceleration for the base model.

Design 2

The base model configuration for design 2 is as follows. The slide material was set to 1023 Carbon Steel ($\rho = 7858 \frac{\text{kg}}{m^3}$) and the gears were given a fixed density of 500 $\frac{\text{kg}}{m^3}$, initial torque from a virtual motor was set to 1 Nm, the damping coefficient was set to 1e-4, and we used auto ode45 solver to carry out the base model simulation [2].

Table 11 shows the dynamic sensitivity analysis results of Design 1.

Table 11: Simulation Results of Design 2

	Angular Velocity (rad/sec)	Angular Acceleration (rad/sec^2)
Base model	174.391	1206
Material Titanium	174.34	834
Material Brass	174.498	1131
Solver (ode15s / step time 0.001s)	174.533	-1.3e-1
Damping (4e-5)	436	1000
Torque (0.01 Nm)	1.744	12.8

Based on the results shown in Table 11, we noticed that the angular velocity didn't change significantly when changing the materials of the base model. As we know from previous design analysis that densities of Carbon Steel, and Brass are close, which explains the similarity in angular velocity and acceleration. The angular acceleration needed to achieve the same angular velocity with Titanium was lower in comparison to base model because in comparison to Carbon Steel its density is almost half. Hence, titanium structure would require less torque to achieve the same angular velocity in comparison to a denser material. Reducing damping of the base model increased the angular velocity achieved as less input energy was dissipated.

Design 3

The base model configuration for design 3 is as follows. The spring material was set to 1023 Carbon Steel ($\rho = 7858 \frac{\text{kg}}{m^3}$) and the bullet was given a fixed density of $500 \frac{\text{kg}}{m^3}$, initial force derived from a virtual motor torque was set to 4 N, the damping coefficient was set to 1e-8, and we used auto ode45 solver to carry out the base model simulation [2].

Table 12 shows the dynamic sensitivity analysis results of Design 3.

Table 12: Simulation Results of Design 3

	Angular Velocity (rad/sec)	Angular Acceleration (rad/sec^2)
Base model	2.751	22.2
Material Titanium	2.751	22.2
Material Brass	2.751	22.2
Solver (ode15s / step time 0.001s)	2.751	22.2
Damping (1e-1)	2.634	20.17
Force (40 N)	33.92	271.3

Based on the results shown in Table 12, simulating the model with the initial configuration produced the shown angular velocity and acceleration. As our third design mechanism is strongly depended on spring contraction and expansion, we provided planar joint with a certain force to attain our desired torque in Simscape. As the joint is located after spring and bullet, changing their materials did not have any effect on the simulation result. Same conclusions were drawn when using two different solvers. Damping affect was also minimal in this design as after increasing damping coefficient to 1e-1 from 1e-8 angular velocity value only decreased by 0.12 while angular acceleration drooped to 20.17. This minimal affect was seen because force was acting on the joint just behind the ball so damping property for the parts connected behind that point did not have any effect. Lastly simulation results were obtained by increasing the torque to 40Nm. As expected, angular velocity and angular acceleration increased significantly in comparison to the base model.

Validation

Statics (SolidWorks)

Table 13: Qualitative vs Analytical Comparison of Statics

	Maximum Stress					
	Design 1		Design 2		Design 3	
Material	Analytical	FEA	Analytical	FEA	Analytical	FEA
Carbon Steel	1.989e+01	2.459e+01	1.436e-01	1.62e-01	1.151e-01	1.318e-01
Titanium	1.245e+01	1.708e+01	0.926e-01	1.003e-01	8.277e-02	8.587e-02
Brass	2.250e+01	2.634e+01	1.362e-01	1.723e-01	1.042e-01	1.393e-01
	Deflection					
	Design 1		Design 2		Design 3	
Material	Analytical	FEA	Analytical	FEA	Analytical	FEA
Carbon Steel	0.980	1.067	2.823e-04	3.257e-04	1.983e-04	2.359e-04
Titanium	1.298	1.418	3.528e-04	3.878e-04	2.783e-04	2.999e-04
Brass	1.879	2.317	5.646e-04	7.139e-04	3.868e-04	5.084e-04

Based on the qualitative and analytical comparisons shown in Table 13, we can note that the maximum stress and deflection of all each design's hand calculations are not exact but are within an acceptable degree of error. So, we're certain that based on our assumptions mentioned in the analytical section FEA studies are acceptable.

Kinematics (Simscape)

Table 14: Qualitative vs Analytical Comparison of Kinematics

	Angular Velocity					
	Design 1		Design 2		Design 3	
Material	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
Carbon Steel	14.89	15.46	165.671	174.391	1049.22	1206
Titanium	18.69	19.562	162.14	174.34	775.62	834
Brass	13.27	15.14	144.83	174.498	916.11	1131
	Angular Acceleration					
	Design 1		Design 2		Design 3	
Material	Analytical	Simulation	Analytical	Simulation	Analytical	Simulation
Carbon Steel	98.78	101	2.420	2.751	18.426	22.2

Titanium	143.43	146	2.613	2.751	20.896	22.2
Brass	94.14	96.3	2.1175	2.751	16.206	22.2

Based on the qualitative and analytical comparisons shown in Table 14, we can note that the angular velocity and acceleration of all each design's hand calculations are not exact but are within an acceptable degree of error. So, we're certain that based on our assumptions mentioned in the analytical section we've modelled our designs correctly.

Design Evaluations

This section evaluates the three proposed designs for the mechanical structure based on the results presented in the analysis sections. The evaluation was carried out using a decision matrix. The template of the decision matrix is as shown below.

Structure		Design #	
Objective	Weight	Score	Weight*Score

The objectives of the mechanical structure were assigned a weight between 1-5 and based on the results found in the analysis sections, a score was assigned to each design. The largest overall score is the optimal design to meet the design specifications.

Decision Matrix

Table 15 is the decision matrix constructed to select the optimal design.

Table 15: Design Evaluation Matrix

Structure		Design 1		Design 2		Design 3	
Moment of Inertia	5	3	15	4	20	5	25
Volume (Cost)	5	1	5	5	25	3	15
Deflection	4	1	4	3	15	4	16
Maximum Stress	4	1	4	3	12	5	20
Overall Value		28		72		76	

Based on Table 14, Design 3 provided the highest angular velocity for each material. Therefore, Design 3 was given the highest score as it provided the minimal moment of inertia - a design requirement.

To minimize the cost of manufacturing the selected design, volume was taken into consideration in the decision matrix. The volumes of Design 1, 2 and 3 are 0.00277246603, 0.00112849786, 0.0014241914 cubic meters, respectively. Design 1 would cost the most and Design 2 would cost the least, which is reflected by the scores given.

Based on Table 13, Design 1 has the highest deflection and stress, and Design 3 has the lowest. Therefore, Design 3 was given the highest score and Design 1 was given the lowest score.

The overall score shows that Design 3 is the optimal design that meets the design specifications that were laid out at the start of this documentation.

Material Selection

In this section we compare the stiffness, strength, processability and availability properties of the three materials used in the analysis sections of this report to select the optimal material for our final design. The three properties are used for comparison because they help us achieve our design specifications.

The following decision matrix was constructed to assist the optimal choice. Each material is scored from 0-5 for each property.

Decision Matrix

Table 16: Material Selection Decision Matrix

Material Property	Cp-Ti UNS R50400	1023 Carbon Steel	Brass
Stiffness	3	5	2
Strength	5	3	1
Processibility	2	4	2
Availability	3	0	5
Overall Value	13	12	10

The scores assigned to each material are justified as shown below:

- The Elastic Modulus of Cp-Ti UNS R50400, 1023 Carbon Steel, Brass is 120 GPa, 210 GPa, and 117 GPa respectively [1][2][3]. 1023 Carbon Steel can help us reduce deflection as it is relatively stiffer.
- The Tensile Strength of Cp-Ti UNS R50400, 1023 Carbon Steel, Brass is 485 MPa, 425 GPa, and 360 MPa respectively [1][2][3]. A stronger material will provide a greater safety factor for our cyclic loading application. Hence, Cp-Ti UNS R50400 is desired as it relatively stronger.
- The Machinability Index of Cp-Ti UNS R50400, 1023 Carbon Steel, Brass is 40, 65 and 40 respectively [3][4][5]. A lower machinability index means the cost of processing the material would be high as it would require more effort. Therefore, 1023 Carbon Steel is the best option in terms of processability.

- The availability score was given by taking our \$550 budget into consideration. The 1023 Carbon Steel would require importing a ton from China which would cost us more than the available budget [6]. The Titanium alloy and brass are available within Canada from local vendors [7][8].

Based on the overall score, we have concluded that the Titanium alloy is the optimal choice of material for our final design as it will help us minimize deflection in our final design and is affordable.

Bill of Materials

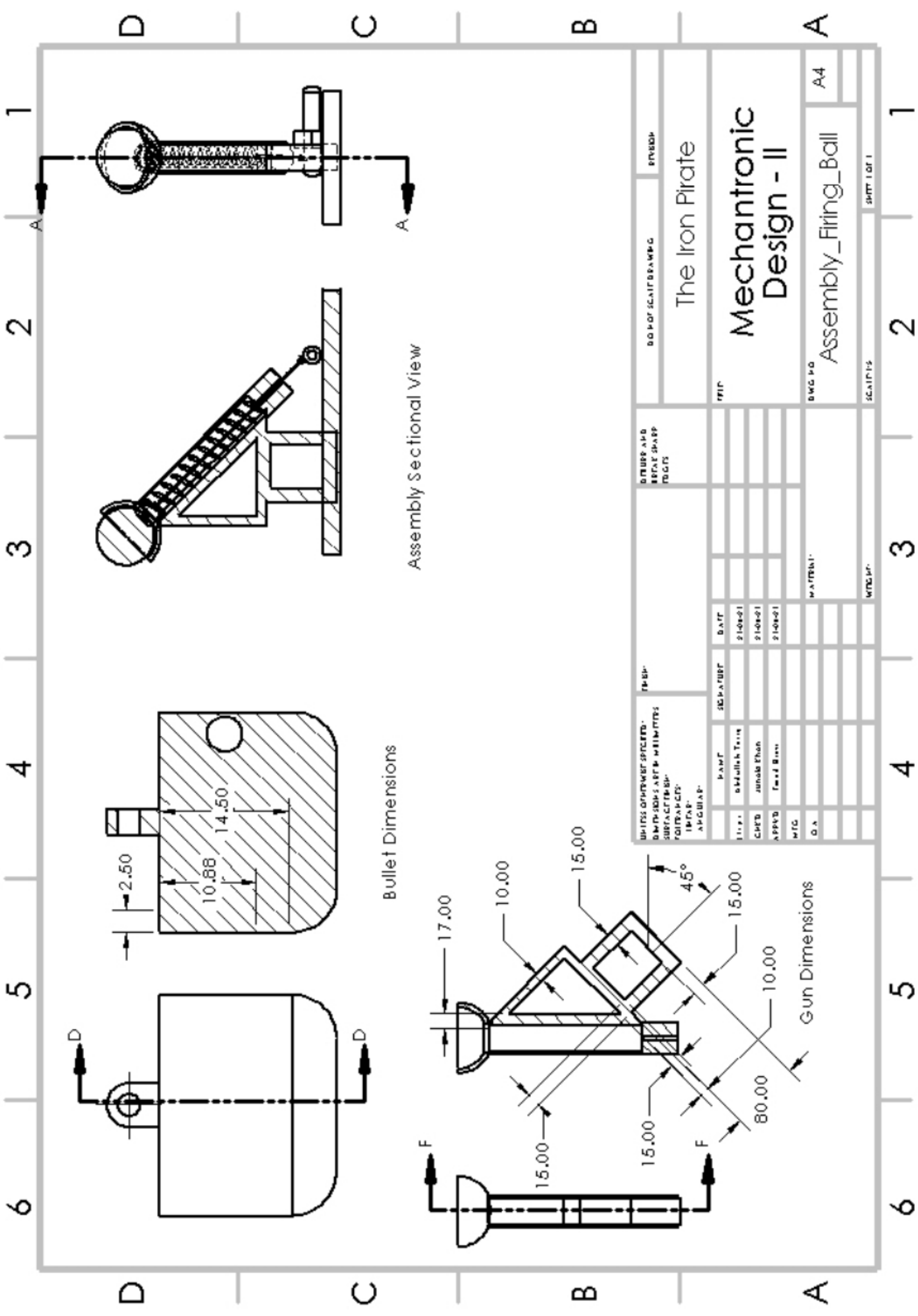
The allocated budget for this project is \$550 CAD. For processing our Design 3, we will be taking advantage of MSE's Lab Services. The following table lists all the expense of designing our selected design using the Titanium alloy.

Table 17: Bill of Materials and Services [7][9].

Component	Cost (\$ CAD)	Supplier
Machining Services (Drilling & Cutting)	30.00/ hour	SFU Services & Training
Cost of Titanium alloy	22.34 /Kg	Aesteiron Steels

The selected design has a volume of 0.0014241914 cubic meters. Using the titanium alloy ($\rho = 4510\ \frac{\text{kg}}{m^3}$) as our choice of material, the total mass is 6.42 Kg. Therefore, the total cost of the material is \$143 CAD. According to our experience with the Machining services of SFU, we believe our design can be manufactured in three hours. Therefore, the total cost of manufacturing our selected design is \$223 CAD. We still have \$327 CAD to select our motor and circuit components to build a working prototype.

Final Design Drawing



References

[1]"Grade 2 Unalloyed Ti ("Pure") 50A (UNS R50400)", AZoM.com, 2021. [Online]. Available: https://www.azom.com/article.aspx?ArticleID=9413. [Accessed: 21- Jun- 2021].

[2]"AISI 1023 Carbon Steel (UNS G10230)", *AZoM.com*, 2021. [Online]. Available: https://www.azom.com/article.aspx?ArticleID=6524. [Accessed: 21- Jun- 2021].

[3]"Brass UNS C28000", *AZoM.com*, 2021. [Online]. Available: https://www.azom.com/article.aspx?ArticleID=6380. [Accessed: 21- Jun- 2021].

[4]"Machinability of Titanium - Machining Doctor", *Machining Doctor*, 2021. [Online]. Available: https://www.machiningdoctor.com/machinability/titanium/. [Accessed: 21- Jun- 2021].

[5]1. Low-Carbon Steels 1022, "Low-Carbon Steels 1022, 1023, 1025 and 1026", *All Metals & Forge Group*, 2021. [Online]. Available: https://steelforge.com/carbon-steel-1022/. [Accessed: 21- Jun- 2021].

[6]"1023 Carbon Mild Steel Sheet and Plates - Buy Hot Rolled Steel Plate, Ar500 Steel Plate, Carbon Steel Plate Product on Alibaba.com", *Alibaba.com*, 2021. [Online]. Available: https://www.alibaba.com/product-detail/1023-Carbon-Steel-Per-Sheet-1023_1600137686909.html?spm=a2700.7724857.normal_offer.d_title.a18d4fb7Ob8Qii&s=p. [Accessed: 21- Jun- 2021].

[7]"Titanium distributor and supplier/ Grade 2/ 5 Price per kg", *Aesteiron.com*, 2021. [Online]. Available: https://www.aesteiron.com/titanium.html. [Accessed: 21- Jun- 2021].

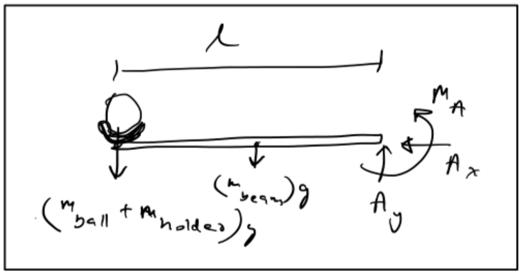
[8]"Scrap Metal Prices | Canada – All of the latest scrap metal prices in Canada.", *Scrapmetalpricer.ca*, 2021. [Online]. Available: https://www.scrapmetalpricer.ca/. [Accessed: 21- Jun- 2021].

[9]"Rates & Equipment List - Mechatronic Systems Engineering - Simon Fraser University", *Sfu.ca*, 2021. [Online]. Available: https://www.sfu.ca/mechatronics/services-and-training/rates-and-equipment-list.html. [Accessed: 21- Jun- 2021].

Appendix A

Design 1

Statics



$\therefore$ Static Equilibrium: $\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M_A = 0$

$(+) \rightarrow \quad \Sigma F_x = 0 \ \therefore \quad \boxed{A_x = 0}$

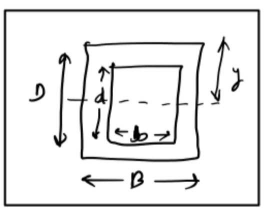
$(+) \uparrow \quad \Sigma F_y = 0 \ : \quad A_y - (m_{ball} + m_{holder})g - m_{beam}\, g = 0$

$$\boxed{A_y = (m_{ball} + m_{holder} + m_{beam})\, g}$$

$(+) \circlearrowleft \quad \Sigma M_A = 0 \qquad M_A + (m_{beam}\, g)\frac{l}{2} + (m_{ball} + m_{holder})g\, l = 0$

$$\therefore \boxed{M_A = -\left[(m_{beam}\, g)\frac{l}{2} + (m_{ball} + m_{holder})g\, l\right]}$$

$\sigma = -\dfrac{M y}{I}$;

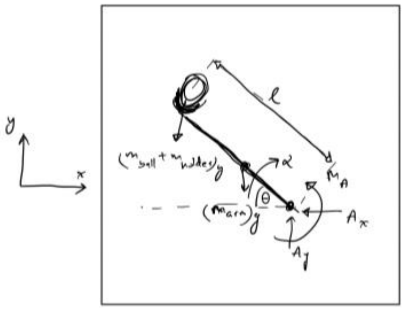


$\boxed{\sigma_{max} = -\dfrac{M_A\, y}{I}}$; $\boxed{I = \dfrac{BD^3}{12} - \dfrac{bd^3}{12}}$

max stress in the arm

$\boxed{\sigma_{fatigue} = 60\%\ UTS_{material}}$ $\qquad$ $\boxed{\text{Deflection} = \dfrac{\text{load} \times L^3}{3EI}}$

Kinematics



$\boxed{\text{Angular Acceleration}}$

$$\Sigma M_A = -\bar{y}\, m a_{G,x} + \bar{x}\, m a_{G,y} + I_{G,x}$$

Since, motion is purely rotational:

$$\boxed{\Sigma M_A = J_A \alpha} \; ; \; J_A = J_{ball} + J_{holder} + J_{arm}$$

$$\left.\begin{aligned}
J_{Ball,A} &= \tfrac{2}{3} m_{ball} r^2 + m_{ball} \ell^2 \\
J_{holder,A} &= \tfrac{2}{7} m_{holder} r^2 + m_{holder} \ell^2 \\
J_{arm,A} &= \tfrac{1}{3} m_{arm} \ell^2
\end{aligned}\right\} \text{Sum} = J_A$$

$$(\curvearrowleft +)\; \Sigma M_A = 0 : \quad M_A + (m_{arm}\, g)\cos(\theta)\frac{\ell}{2} + (m_{ball} + m_{holder}) g \cos(\theta)\, \ell = 0$$

$$\therefore \boxed{M_A = -\ell \left[(m_{arm}\, g)\cos(\theta) \times \frac{1}{2} + (m_{ball} + m_{holder}) g \cos\theta \right]}$$

$$\therefore \boxed{\alpha = \frac{M_A}{J_A}}$$

$\boxed{\text{Angular Velocity}}$

Assumption: Motor provides constant acceleration, starting at $\theta_1 = 0^\circ$, $\therefore \omega_1 = 0$

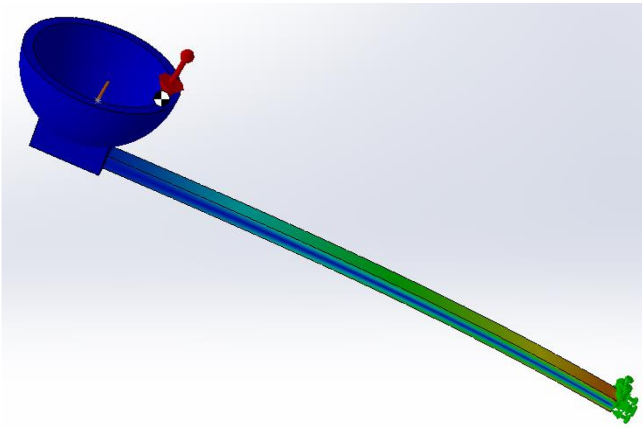
$$\omega_2^2 - \omega_1^2 = 2\alpha(\theta_2 - \theta_1)$$

$\therefore$ Based on assumption

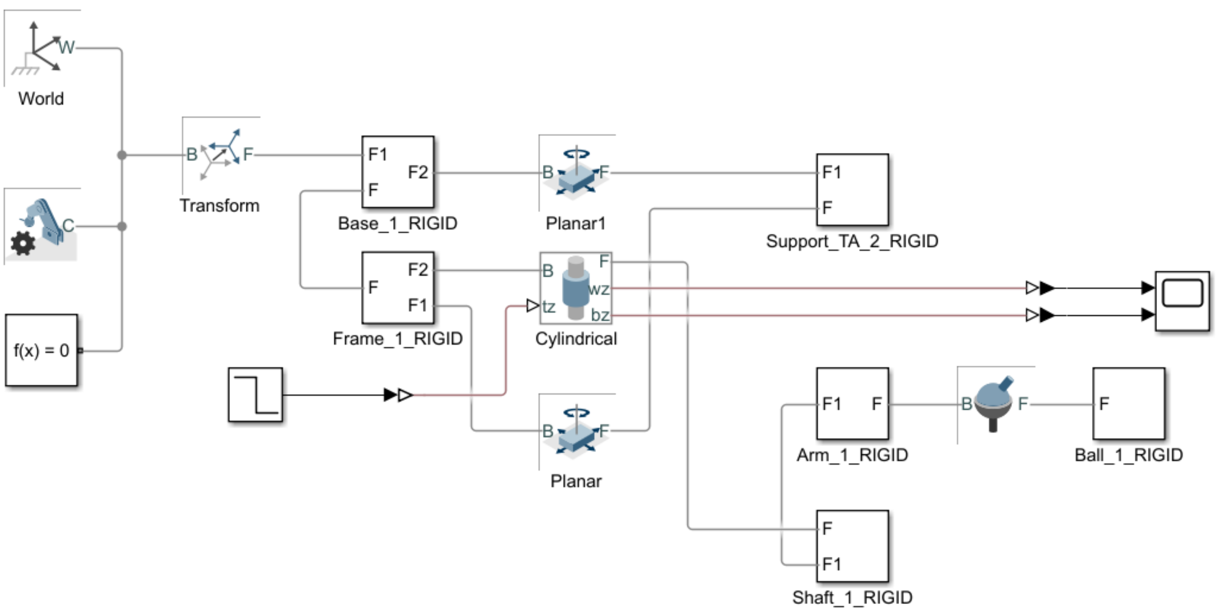
$$\hookrightarrow \boxed{\omega_2 = 2\alpha(\theta_2 - \theta_1)}$$

angular velocity provided by the motor

FEA Study Sample



Simscape Model

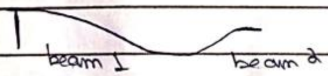


Appendix B

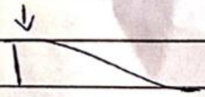
Design 2

Statics

~~assuming~~ Static analysis

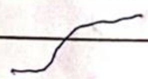


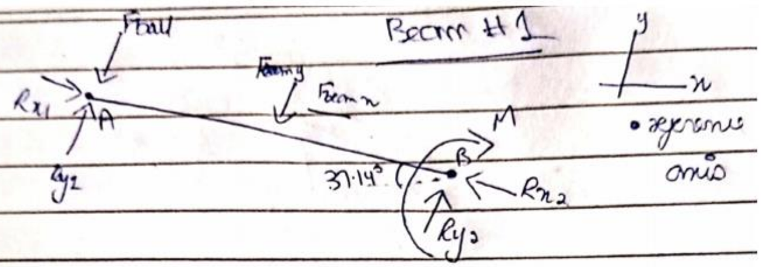
beam #1



assuming a straight beam
at an angle

beam #2





$\Sigma F_y = 0$

$F_{ball} + F_{beam\,y} - R_{y1} - R_{y2} = 0$ — (i)

$\Sigma F_x = 0$

$R_{x1} + F_{beam\,x} - R_{x2} = 0$ — (ii)

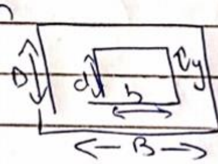
$\Sigma M = 0$ moment @ B

$-M + F_{beam\,y}\, l/2 + F_{ball}\, l = 0$

$M = F_{beam\,y}\dfrac{l}{2} + F_{ball}\, l$ — (iii)

(·) assuming a rectangular beam

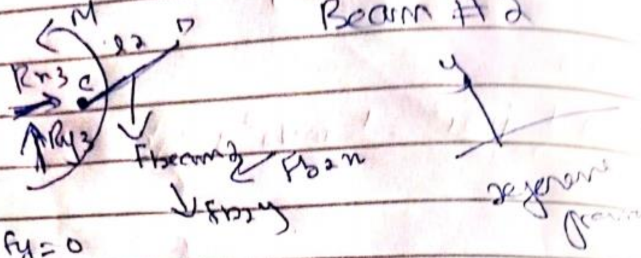
$I = \dfrac{bd^3}{12} - \dfrac{bd^3}{12}$



$\sigma = -\dfrac{My}{I}$

Σf_n

Beam #2



$\Sigma f_y = 0$

$F_{b2y} - R_{y3} = 0$

$\Sigma f_n = 0$

$F_{b2n} - R_{n3} = 0$

ΣM_D moment @ D

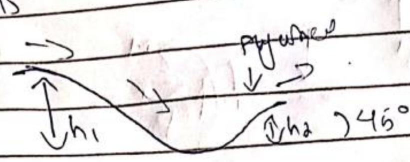
$-M + F_{bny}\frac{l_2}{2} + R_{y3} l_2 = 0$

$M = F_{bny}\frac{l_2}{2} + R_{y3} l_2 = 0$

Dynamics

Kinematic analysis

Design #2



Stage 1

mgh_1

Stage 2

$mgh_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}J\omega_1^2$

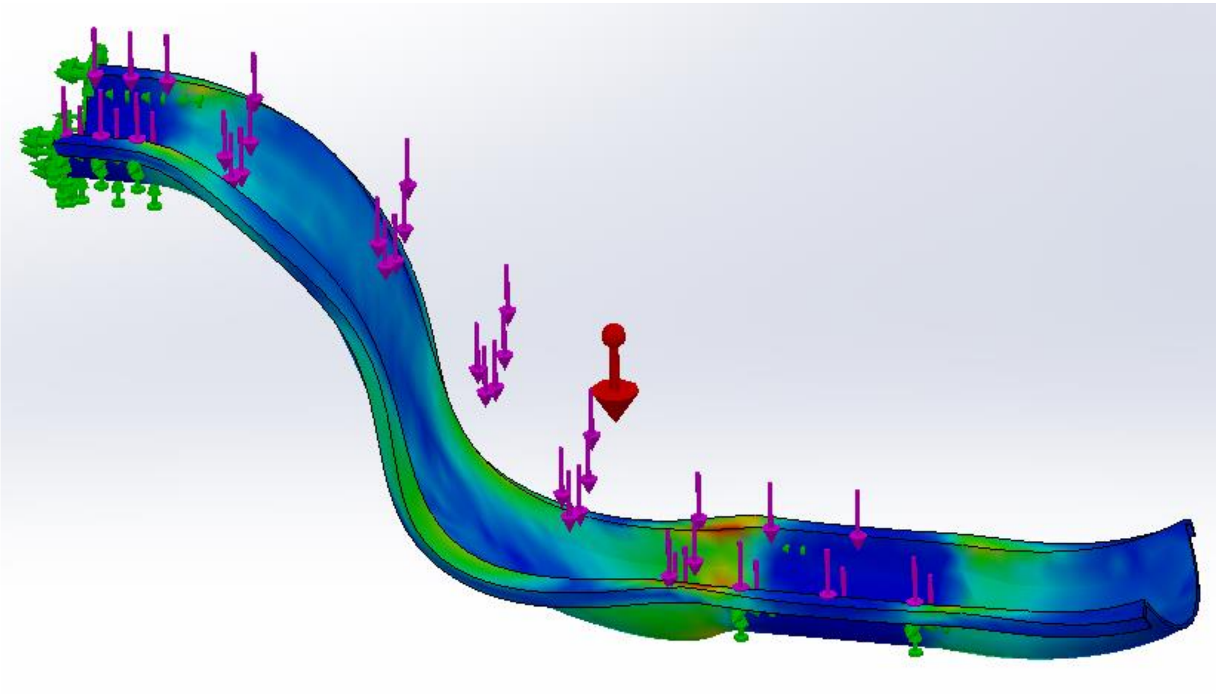
Stage #3

$\frac{1}{2}mv_1^2 + \frac{1}{2}J\omega_1^2 = \frac{1}{2}mv_2^2 + mgh_2 + \frac{1}{2}J\omega_2^2$

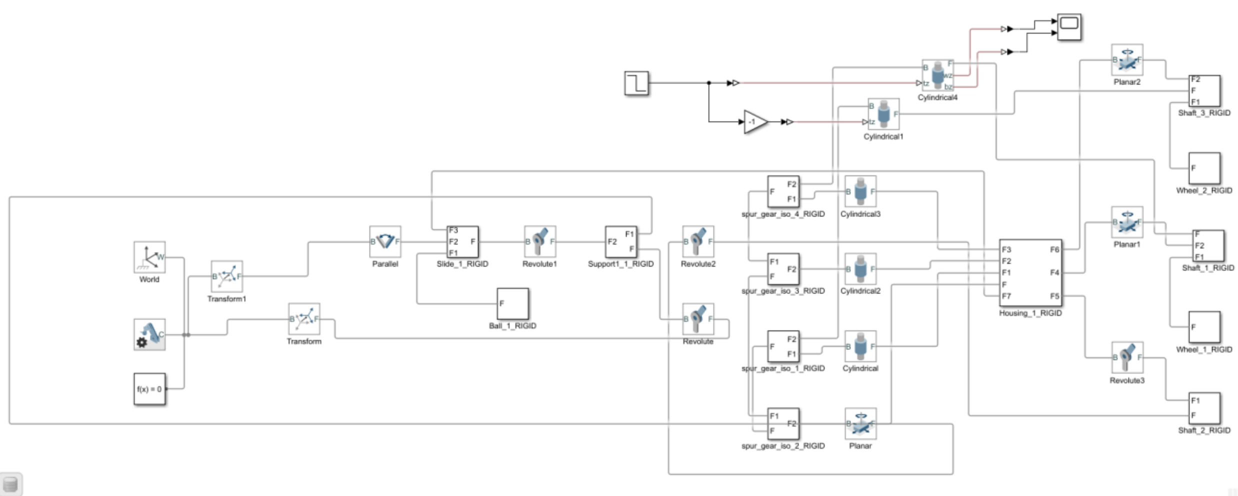
Stage #4

$\frac{1}{2}mv_2^2 + mgh_2 + \frac{1}{2}J\omega_2^2 + WD(\text{flywheel}) = \frac{1}{2}mV_3^2 + \frac{1}{2}m\omega_3^2$

FEA Study Sample



Simscape Model

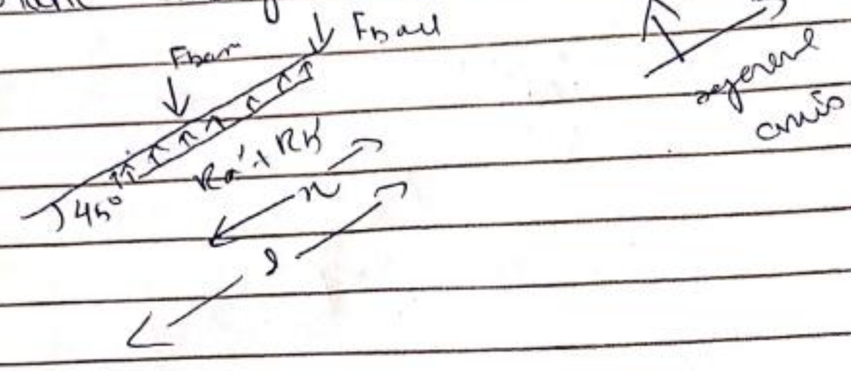


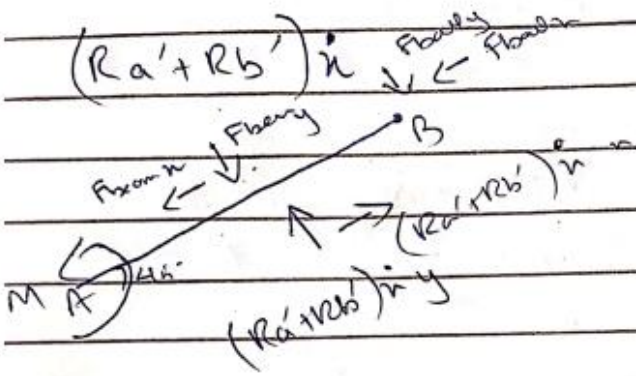
Appendix C

Design 3

Statics

Static analysis





$\Sigma F_y = 0$

$F_{beam\,y} + F_{ball\,y} - (Ra' + Rb')\ddot{x}_y = 0$

$\Sigma F_x = 0$

$(Ra' + Rb')\ddot{x}_x - F_{beam\,x} - F_{ball\,x} = 0$

$\Sigma M = 0 \quad @ \; A$

$F_{ball\,y}\, l + F_{beam\,y}\,\frac{l}{2} - (Ra' + Rb')\ddot{x}_y \left(l - \; + x/2\right) = 0$

Dynamics

Date

Design #3

$V = \omega r$

Kinematic analysis

(i) Stage #1 @ Rest

$mgh_1 \sin\theta =$

(ii) Stage #2 @ compression

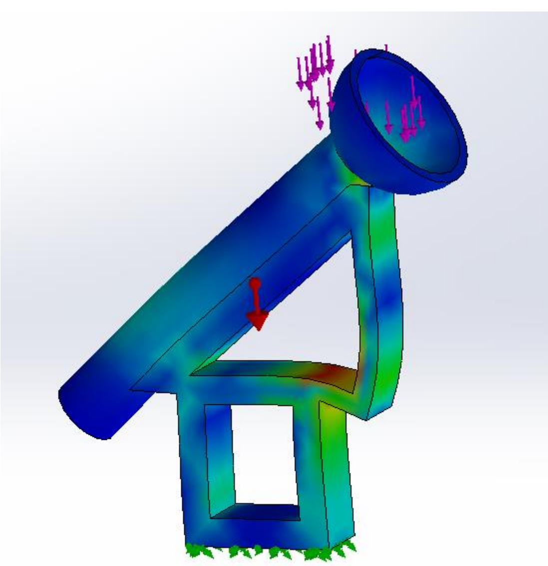
$\frac{1}{2} kx^2 + mgh_2 \sin\theta = mgh_1 \sin\theta$

(iii) Stage #3 @ release

$\frac{1}{2} mv^2 + \frac{1}{2} J\omega^2 = \frac{1}{2} kx^2 + mgh_2 \sin\theta = mgh_1 \sin\theta$

$$\frac{\left(\frac{1}{2} kx^2 + mgh_2 \sin\theta\right)^2}{\left(m + J/r^2\right)} = v^2$$

FEA Study Sample



Simscape Model