

Agenda!

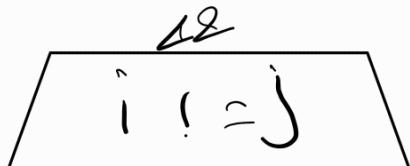
1. First Non Repeating Element
2. Pair Sum
3. Count of Pair Sum
4. check if there exists a subarray with $\text{Sum} = 0$
5. check if there exists a subarray with $\text{Sum} = K$.

1. First non-Repeating Elements $O(n) \Theta(n)$

1. Count the element frequencies in freq map.
2. iterate over A again and check the element frequency if it is == 1

2.

Given int[] A and int k
check if there exist a pair (i, j)
such that $\text{arr}[i] + \text{arr}[j] = k$



Brute force : $O(n^2)$

1. lets create all possible pairs,
using nested loop one for i one for
j from i+1 to n. $O(n^2)$
and check if element exist or not

Optimized: 0 1 2 3 4 5 6 7 8

8	9	1	-2	4	5	11	-6	4
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$n=7$

$k=6$ ✓
 $k=2$ ✗
 $k=8$ ✗

$6-8$
 -2

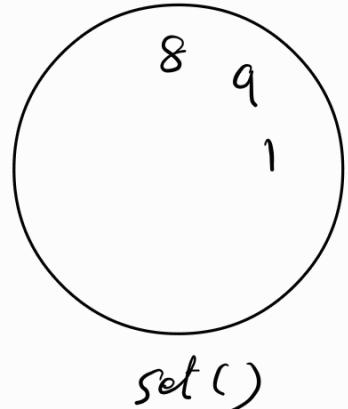
$6-9$
 -3

$6-1$
 5

$6+2$
 8

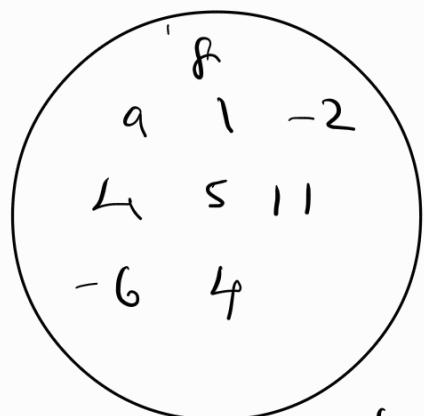
$b = k - a$

Return True



$$8 - b = k-a \Rightarrow -2$$

\Rightarrow do we have it in set?



Create set out of A

1. # create st from A
2. # iterate over A and check if b is found in set.

TC: $O(n)$

SC: $O(n)$

issue : above approach will not work $i \neq j$ condition

- use dictionary to store element and frequency. and keep $i \neq j$

$$k = 18$$

$$18 - 9 = 9$$

we could keep check

if $fremap[9] == A[i]$

if value == 1

↓ Pass

else:

↓ consider

8: 1	11: 1
a: 1	-6: 1
1: 1	4: 1
-2: 1	
4: 1	
5: 1	

$\overbrace{TC: O(n)}$

$SC: O(n)$

Count the number of pairs with sum = k

0	1	2	3	4	5	6	7
2	5	2	5	8	5	2	8
↓	↓	↓	↓	↓	↓	↓	↓
2	2	2	1	1	0	1	0
2	8						

$k = 10$

$$\begin{array}{ccccccccc} 2 & 5 & 2 & 5 & 8 & 5 & 2 & 8 \\ \downarrow & \downarrow \\ \frac{10-2}{8 \text{ or } 5} & 8 & 5 & 2 & 5 & 8 & 2 & 2 \end{array}$$

$a+b=k$
 $b = k-a$

$$\text{ans} = 2 + 3 + 2 + 3 + 3 + 3 + 2 + 3$$

~~2 : 3
5 : 4
8 : 1~~

10 11
21

all the possible pairs

2	:	3
5	:	4
8	:	1

$\text{pairs} = \text{First create dictionary iffeat A and all all frequency.}$

— X —

from left to right = iterate A and create dictionary on the fly and add if it finds.

freqMap = {}, ans = 0, k = 4

for i in range(n)

$$b = k - A[i]$$

value = freqMap.get(b, 0)

ans += value

if $A[i]$ in freqMap.

↓ ans += value
increase frequency also.

else:

freqMap[A[i]] = 1

Subarray with sum zero

- Given int [] A
- Find if subarray with sum zero. return True

Approach 1: BF $\Theta(n^3)$

- going through all the subarray and $\Theta(n^2)$
- check if it's sum = 0 $\Theta(n)$

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 2 & 1 & -3 & 4 & 3 & 1 & -2 & -3 & 2 \end{bmatrix} \quad N=11$$

2
2

$$\frac{N(N+1)}{2} = 66$$

2 2
2 2 1
2 2 1 -3

```
n = len(A)
for i in range(n)
    for j in range(i, n)
        temp = 0
        for k in range(i, j+1)
            temp += A[k]
        if temp == 0:
            return True
return False.
```

approach 2 Prefix Sum $O(n^2)$, $O(N)$ space

1. creating Prefix sum

2. for every subarray get sum from prefix sum

PrefixSum = [], PrefixSum.append(A[0])

for i in range(1, n): PrefixSum.append(PrefixSum[i-1] + A[i])

for i in range(n)

for j in range(i, n)

temp = 0

if i == 0: temp = PrefixSum[i]

else:

temp = PrefixSum[j] - PrefixSum[i-1]

if temp == 0: return True

return False

approach 3: carry forward $O(n^2)$

for i in range(n)

temp = 0

for j in range(i, n)

temp += A[j]

if temp == 0

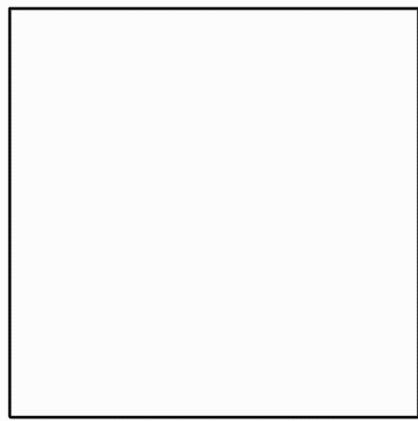
return True

Approach \rightarrow !

$$\text{Sum}(i, j) = 0$$

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 2 & 1 & -3 & 4 & 3 & 1 & -2 & -3 & 2 \end{bmatrix} \quad N=11$$

i
 j



$$\text{PrefixSum} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 5 & 2 & 6 & 9 & 10 & 8 & 5 & 7 \end{bmatrix}$$

i j

$$\begin{aligned} &= \text{Prefix}[j] - \text{Prefix}[i-1] \\ &= 6 - 2 \\ &\Rightarrow 4 \end{aligned}$$

Idea!

$$\text{sum}(i, j) = 0$$

$$\text{Prefix}[j] - \text{Prefix}[i-1] = 0$$

$$\text{Prefix}[j] = \text{Prefix}[i-1]$$

So, if $\text{Prefix}[j]$ and $\text{Prefix}[i-1]$ are same,
they will mark "Zero"

get frequency of all element in PrefixSum

$2 : 2 \equiv \text{True}$

$4 : 1$

$S : 2$

$6 : 1$

$a : 1$

$c : 1$

$f : 1$

$T : 1$

_____ x _____

$$A = \begin{bmatrix} -2 & -1 & 3 & 5 \end{bmatrix}$$

$$PF = \begin{bmatrix} -2 & -3 & 0 & 5 \end{bmatrix}$$

if value = 0 :

return True

$-2 : 1$

$-3 : 1$

$0 : 1$

$5 : 1$

Subarray with sum $\geq k$

Given int [] A, check if there is a subarray with sum $\geq k$

$$1 \leq n \leq 10^8$$

$$arr = [2, 3, 9, -4, 1, 5, 6, 2, 5]$$

$$k = 11$$

$$1 \leq i, j \leq n$$

$$\text{sum}(i, j) \geq k$$

$$\text{prefixSum}[j] - \text{prefixSum}[i-1] = k$$

$$PF = [2 \quad 5 \quad 14 \quad 10 \quad 11 \quad 16 \quad 22 \quad 24 \quad 29]$$

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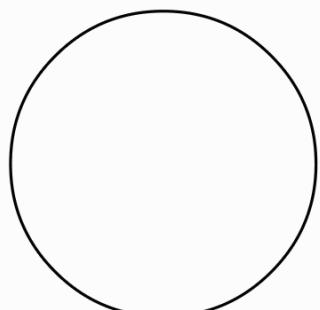
$$arr = [5 \quad \boxed{10} \quad 20 \quad 100 \quad 105]$$

$$PF = [5 \quad 15 \quad 35 \quad 135 \quad 240]$$

$$k = 10$$

$$PF[j] - PF[i-1] \geq k$$

$$15 - 5 = 10$$



Approach 1: BF $\Theta(n^3)$

1. going through all subarray and check if
2. sum is $\leq k$.

Approach 2:

$$\Rightarrow a - b = k$$

add b both sides

$$\Rightarrow a - b + b = k + b$$

simplify it.

$$\Rightarrow a = k + b$$

subtract k from both sides

$$\Rightarrow a - k = k + b - k$$

Simplify

$$a - k = b$$

$$\boxed{b = a - k}$$

$$\text{sum}(i, j) \leq k$$

$$\text{prefixSum}[j] - \text{prefixSum}[i-1] = k$$

(a) - (b) $= k$

as per derived equation!

$$\text{prefix}[i-1] = \text{prefix}[j] - k$$

$$b = a - k$$

$$\text{prefix}[i-1] = \text{prefix}[j] - k$$

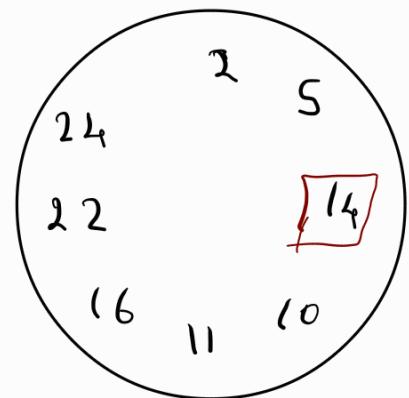
Dry run!

$$arr = \begin{bmatrix} j & 2 & 3 & 9 & -4 & 1 & 5 & 6 & 2 & 5 \\ Subarr = 15 & \boxed{-4} & \boxed{1} & \boxed{5} & \boxed{6} & \boxed{2} & \boxed{5} \end{bmatrix}$$

$$k = 15$$

$$PF = \begin{bmatrix} 2 & 5 & 14 & 10 & 11 & 16 & 22 & 24 & 29 \\ 2-15 & \downarrow \\ -13 & -10 & -1 & -5 & -4 & 1 & 7 & 9 & \boxed{14} \end{bmatrix}$$

29-15



$$arr = \{2, 3, 9, -4, 1\}$$

$$PF = \begin{bmatrix} 2 & 5 & 14 & 10 & 1 \\ 8 & 5 & 4 & 0 & 1 \end{bmatrix} \quad k = 10$$

if $PF[j] == k$
return True

2nd way



add
0
before hand

Code :

create prefixsum O(n)

Pf = []

Pf.append(A[0])

for i in range(1, n)

↓ Pf.append(Pf[i-1] + A[i])

iterate through the array and O(n)

find if sum == 0 exists

hashset = {}

for j in range(n)

b = a - k formula

b = Pf[j] - k

if b in hashset:

↓ return True

else:

↓ hashset.add(Pf[j])

Tc : O(n)

Sc : O(n)

without using PF array

sum = 0

hashset = \emptyset ,

for j in range(n)

 sum += A[j]

 b = sum - k

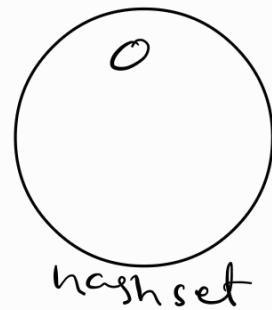
 if b in hashset:

 ↓ return True

 else:

 ↓ hashset.add(sum)

return False



TC: $O(N)$

SC: $O(N)$

