Agenda:

1. Introduction & Properties of % wrt [+, -, × and Power]
   - ↳ mod on Power funtion
   - ↳ pair sum divisible by m
2. Find GCD (a, b)
   - ↳ Properties of GCD
   - ↳ GCD optimized code.

Modular operator: % (gives Remider)

A % B = will get the reminder

Range of A % B:

↳ is (0 to N-1)

Note: It is requird to restrict the ans with in 0 to N-1 range

- $0 \% 3 = \boxed{0}$
- $1 \% 3 = \boxed{1}$
- $2 \% 3 = \boxed{2}$
- $3 \% 3 = 0$
- $4 \% 3 = 1$
- $5 \% 3 = 2$
- $6 \% 3 = 0$

- $0 \% 5 = \boxed{0}$
- $1 \% 5 = \boxed{1}$
- $2 \% 5 = \boxed{2}$
- $3 \% 5 = \boxed{3}$
- $4 \% 5 = \boxed{4}$
- $5 \% 5 = 0$
- $6 \% 5 = 1$
- $7 \% 5 = 2$

1. Addition with mod

$(a+b) \% m$

$(9+8) \% 5$

$17 \% 5$

2

$((a \% m)+(b \% m)) \% m$

$(9 \% 5)+(8 \% 5) \% 5$

$4 + 3$

$7 \% 5$

2

2. multiplication with mod

$(a * b) \% m$

$(4 \times 5) \% 3$

$20 \% 3$

2

$((a \% m) * (b \% m)) \% m$

$(4 \% 3) \times (5 \% 3) \% 3$

1×2

$2 \% 3$

2

3. Note:

$(a+m) \% m$

$(9+5) \% 5$

$14 \% 5$

4

$a \% m$

$9 \% 5$

4

4. Subtraction with mod

$(a-b) \% m$

<u>ex</u>:

$a=10, b=8, m=9$

$(10-8) \% 9$

$2 \% 9$

2

$(a \% m - b \% m) \% m$

To mak negative to possitive

$((10 \% 9) - (8 \% 9) + m) \% m$

$1 - 8$

$-7 + 9$

$2 \% 9$

2

5. Power with mod

$a^b \% m$

$7^2 \% 5$

$49 \% 5$

4

$(a \times a \times a \dots b \text{ times}) \% m$

$((a \% m) \times (a \% m) \dots) \% m$

$(a \% m)^b \% m$

$(7 \% 5)^2 \% 5$

2^2

$4 \% 5$

4

$$
\begin{array}{r|l}
 & 8,1 \\
5 & 49 \\
 & 40 \\
\hline
 & 9 \\
 & 5 \\
\hline
 & 4
\end{array}
$$

Quiz : $(37^{103} - 1)$ % 12

$$((37 \% 12)^{103} - 1) \% 12$$

$$1^{103}$$

$$1 - 1$$

$$0 \% 12 = \underline{\underline{0}}$$

$$\begin{array}{r|l} & 3 \\ \hline 12 & 37 \\ & 36 \\ \hline & 1 \end{array}$$

Problem 1 : Fast Power $a^n \% m$

"idea" $(2^3) = 2 \times (2^2)$

$(2^n) = 2 \times (2^{n-1})$

$(2^{n/2}) \times (2^{n/2})$

$9/2 = 4$

def power(a, n, m):

if n==0: return 1

p = power(a, n//2, m)

if n %2 == 0

$\downarrow$ return $\left(\underbrace{(p \% m)}_{10^9} \times \underbrace{(p \% m)}_{10^9}\right) \% m$

else

$\downarrow$ return $\left(\left(\underbrace{(a \% m)}_{10^9} \times \underbrace{(p \% m)}_{10^9}\right) \% m\right) \times \underbrace{p(\% m)}_{10^9}) \% m$

Problem 2

⇒ Given array A.

⇒ Find the count of pairs (i, j), such that $(arr[i] + arr[j]) \% m = 0$

$A = [4 \quad 3 \quad 6 \quad 3 \quad 8 \quad 12] \quad m = 6$

Note:

Number of pair in array $\Big\} \Rightarrow \frac{N(N-1)}{2}$

$\Rightarrow \frac{\cancel{6}^{3}(6-1)}{\cancel{2}} = 15$

Brute force:

```
count = 0
for i in range(N)
    for j in range(i+1, N)
        (if ((A[i]%m) (A[j]%m)) % m == 0
            Print(A[i], A[i])
            Count += 1
return Count
```

TC: $O(N^2)$ due to the nested loops.

SC: $O(1)$

$$(A[i] \% m \quad A[j] \% m) \% m = 0$$

$$\underbrace{\frac{4 \% 6}{4} \quad \frac{8 \% 6}{2}}_{6 \% 6 = 0}$$

$$\left(\frac{3 \% 3}{0} \quad \frac{3 \% 3}{0}\right) \% 6 = 0$$

$$\frac{6 \% 6}{0} \quad \frac{12 \% 6}{0} \% 6 = 0$$

6

12

18

24

$$(0 + 0) \% m = 0$$

$$(1 + (m-1)) \% 6 = 0$$

should be multiplication of 6

$$(2 + m-2) \% 6 = 0$$

$$(3 + m-3) \% m = 0$$

$$\vdots$$

$$(6 + (m-m)) \% m = 0$$

$$(i + (m-i)) \% m = 0$$

$$A = [4 \quad 3 \quad 6 \quad 3 \quad 8 \quad 12] \quad m = 6$$

$\% 6$

$$\text{modulo} = [4, 3, 0, 3, 2, 0]$$

$m - i$

$$b = [2, 3, \underbrace{6, 3, 4, 6}]$$

```
array = [0] * N
for i in rang(N)
    a = A[i] % m   # 4
    # b = m - a    # 2
    array[i] = a
    if array.has(b)
        Count += 1
    else
        arry.append(b)
```

```
array = [ ]
for i in range(N)
    arr.append( A[i] % m)

for i in range(n)
    a = A[i] % m
    b = m - a
    temp = arry[(i+1):]
    if temp.has(b)
        Count ++

return Count
```

A = [4 3 6 3 8 12]   m = 6

Create freq array of modulo range

0	1	2	3	4	5
2	0	1	2	1	0

fre arr =

calculate the pairs.

ans = ~~0~~ 2

handle zero $^nC_r = {^nC_2} = {^2C_2} = \frac{n(n-1)}{2} = \frac{2(2-1)}{2}$

ans += $\frac{freqar[0] \times (freqar[0]-1)}{2}$

=> 1

if m is even handle m/2

an += $\frac{freqarr(m/2) \times (freqarr(m/2)-1)}{2}$ => 1

handle othe indexes.

```
i = 1 , j = m-1
while ( i < j )
    ans += freqarr( i ) x freqarr( j )
    i++ , j--
```

= 1

a = 3

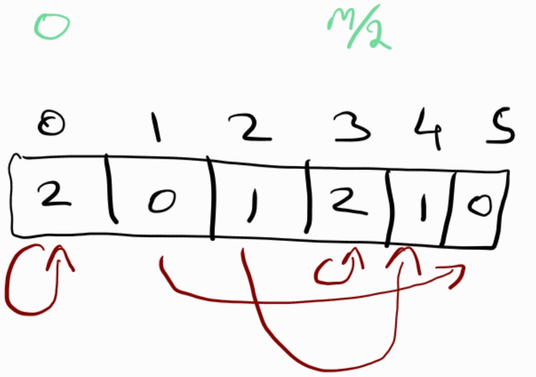
optimized approch

$A = [4 \quad 3 \quad 6 \quad 3 \quad 8 \quad 12] \qquad m = 6$

$a = [4, 3, 0, 3, 2, 0]$

step 1: create frequ map array with the frequency of modulo range of m wich is 0 to m-1

```
freqarr = [0] *(m-1)
for i in range(N)
    a = A[i] % m
    freqarr[a] += 1
```



step 2

ans = 0

\# Handle zero

$ans += \frac{freqarr[0] \times (freqarr[0]-1)}{2}$

if M % 2 == 0

\# Handle M/2

$ans += \frac{freqarr[M/2] \times (freqarr(M/2)-1)}{2}$

\# other elements $i = 1, j = m-1$

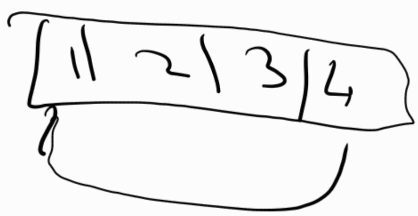
```
while (i<j)
    ans += freqarr[i] x freqarr[j]
    i++
    j--
```

formula from below

$\Rightarrow \frac{n(n-1)}{2} \qquad 4C_2$

$\Rightarrow \frac{3 \times \cancel{2}}{\cancel{2}} \qquad \frac{{}^{2}\cancel{4}(4-1)}{\cancel{2}}$

$\Rightarrow 3 \qquad \Rightarrow 6$



$$\boxed{{}^{n}C_{r}}$$

r = the pair count (2)

n = count

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

n! = n × n-1 × n-2

$${}^{n}C_{2} = \frac{n!}{2!\,(n-2)!}$$

$$n! = n \times (n-1)!$$

$$\Rightarrow \frac{n\,(n-1)!}{2\,(n-2)!}$$

$$2! = 2 \times 1$$

$$\Rightarrow \frac{n\,(n-1)\,\cancel{(n-2)!}}{2\,\cancel{(n-2)!}}$$

$$\Rightarrow \frac{n\,(n-1)}{2}$$

GCD Greate Common Divisor / HCF Highest Common Factor

$GCD(a, b) = \underline{\underline{x}}$

⇒ x is the highest factor that deride both a and b

$a \% x = 0$, $b \% x = 0$

$GCD(20, 65) = 5$

20	65
1	1
2	5
4	13
5	65
10	
20	

$GCD(-10, 20) =$

-10	20
-10	1
-5	2
-2	4
-1	5
1	10
2	20
5	
10	

$GCD(7, 9)$

7	9
$\boxed{1}$	$\boxed{1}$
7	3
	9

$GCD(0, 8)$

0	8
1	1 ×
2	2
3	4
⋮	
8	8
⋮	
∞	

$GCD(0, -10)$

0	-10
0	-10
1	-5
2	-2
⋮	-1
10	1
⋮	2
∞	5
	10

Note: GCD(0, any +ve)
↳ +ve value is ans

GCD(0, any -ve value)
↳ +ve value is ans

Properties of GCD:

1. $GCD(a,b) = GCD(b,a)$ [commutative property]

2. $GCD(0,a) = |a|$ (absolute value of a)

3. $GCD(a,b,c) = GCD(GCD(a,b),c)$ [associative property]

$GCD(GCD(b,c),a)$

$GCD(GCD(d,a),c)$

4. $a,b > 0$ & $a > b$

$\boxed{\gcd(a,b) = \gcd(a-b,b) = \gcd((a\%b),b)}$

ex: $\gcd(23,5)$

↳ $\gcd(23-5,5)$

$\gcd(18,5)$

↳ $\gcd(18-5,5)$

$\gcd(13,5)$

↳ $\gcd(8,5)$

↳ $\gcd(3,5)$ swaping $a,b \to b,a$

$$5 \overline{)23}$$
$$\underline{20}$$
$$3$$

$$\begin{array}{cc} 3, & 5 \\ \hline 1 & 1 \\ 3 & 5 \end{array}$$

=> repeted subtraction

↳ is finding the reminder

Hence, applying mod

another property

↳ $\gcd(5,3)$ $\quad 5\%3 = 2$

↳ $\gcd(2,3)$

swap

$gcd(15, 6)$ $15 \% 6 = 3$

$gcd(3, 6)$

#swap

$gcd(6, 3)$ $6 \% 3 = 0$

$gcd(0, 3)$

ans = 3

$gcd(3, 2)$ $3 \% 2 = 1$

$gcd(1, 2)$ swap

$gcd(2, 1)$ $2 \% 1 =$

$$1 \overline{)2}$$
$$\underline{2}$$
$$0$$

$gcd(0, 1)$ swap

$gcd(1, 0)$ $1 \% 0 = 0$

#if a (or) b = 0
absolute of another is ans

$\boxed{ans = 1}$

4. $a, b > 0$ & $a > b$

$$gcd(a, b) \Rightarrow gcd(a-b, b) = gcd(\underbrace{(a \% b)}_{[0-(b-1)]}, b)$$

so, $gcd(b, (a \% b))$

$gcd(23, 5)$ $23 \% 5 = 3$

$\Rightarrow gcd(5, (23 \% 5))$

$\Rightarrow gcd(5, 3)$ $5 \% 3 = 2$

$\Rightarrow gcd(3, (5 \% 3))$

$(3, 2)$

$\Rightarrow$ gcd (2, (3 % 2))

gcd (1, (2 % 1))

gcd (1, 0)

if (b == 0) return a

$3 \% 2 = 1$

$2 \% 1 = 0$

```
def    gcd (a, b)
       if b == 0 : return a

       gcd( b, (a % b))
```

gcd (25, 5)

gcd (5, 5)

5 % 5

ans = 5

TC: $O(\log_2 \min(a, b))$

SC: "

4, 7

7, (4 % 7)

7, 4

4, (7/4)

4, 3

3, 4/3

3, 1

1, 3/1

1 0

ans = 1

$$\begin{array}{r} 1 \\ 4 \,\overline{)\, 7} \\ \underline{4} \\ 3 \end{array}$$

1

Given [ ], calculate GCD of entire array.

[18, 12, 24, 30]

Using associate property.

1. find first two ele gcd and find it for every element.

$$TC: O\left(N \times \left(\log_2 \underset{max}{\cancel{min}}(a,b)\right)\right)$$

arr = [6 , 3, 9] = False

=> Find if there is a subsequence of GCD '1'

6, 3 = 3

3, 9 = 3

arr = [6, 3, 9, 17] = True

6, 9 = 3