Agenda:

1. Introduction & Properties of % wrt [+, -, × and Power]
   - ↳ mod on power funtion
   - ↳ pair sum divisible by m

2. Find GCD (a, b)
   - ↳ Properties of GCD
   - ↳ GCD optimized code.

Modular operator: % (gives Remider)

A % B = will get the reminder

Range of A % B:

↳ is (0 to N-1)

Note: It is requird to restrict the ans with in 0 to N-1 range

0 % 3 = $\boxed{0}$	0 % 5 = $\boxed{0}$
1 % 3 = $\boxed{1}$	1 % 5 = $\boxed{1}$
2 % 3 = $\boxed{2}$	2 % 5 = $\boxed{2}$
3 % 3 = 0	3 % 5 = $\boxed{3}$
4 % 3 = 1	4 % 5 = $\boxed{4}$
5 % 3 = 2	5 % 5 = 0
6 % 3 = 0	6 % 5 = 1
	7 % 5 = 2

1. Addition with mod

$(a+b) \% m$ | $((a \% m) + (b \% m)) \% m$

$(9+8) \% 5$

$17 \% 5$

2

$(9 \% 5) + (8 \% 5) \% 5$

$4 + 3$

$7 \% 5$

2

2. multiplication with mod

$(a * b) \% m$ | $((a \% m) * (b \% m)) \% m$

$(4 \times 5) \% 3$

$20 \% 3$

2

$(4 \% 3) \times (5 \% 3) \% 3$

1×2

$2 \% 3$

2

3. Note:

$(a+m) \% m$ | $a \% m$

$(9+5) \% 5$

$14 \% 5$

4

$9 \% 5$

4

4. Subtraction with mod

$(a-b) \% m$

ex:

$a=10, b=8, m=9$

$(10-8) \% 9$

$2 \% 9$

2

$(a \% m - b \% m) \% m$

To mak negative to possitive

$((10 \% 9) - (8 \% 9) + m) \% m$

$1 - 8$

$-7 + 9$

$2 \% 9$

2

5. Power with mod

$a^b \% m$

$(a \times a \times a \ldots b \text{ times}) \% m$

$((a \% m) \times (a \% m) \ldots) \% m$

$(a \% m)^b \% m$

$7^2 \% 5$

$49 \% 5$

4

$(7 \% 5)^2 \% 5$

2^2

$4 \% 5$

4

$$\begin{array}{r|l} & 8 \\ 5 & 49 \\ & 40 \\ \hline & 9 \\ & 5 \\ \hline & 4 \end{array}$$

Quiz : $(37^{103} - 1) \% 12$

$$((37 \% 12)^{103} - 1) \% 12$$

$$1^{103}$$

$$1 - 1$$

$$0 \% 12 = \underline{\underline{0}}$$

$$\begin{array}{r} 3 \\ 12 \overline{)\,37} \\ 36 \\ \hline 1 \end{array}$$

Problem 1: Fast power $a^n \% m$

"idea"

$$(2^3) = 2 \times (2^2)$$

$$(2^n) = 2 \times (2^{n-1})$$

$$(2^{n/2}) \times (2^{n/2})$$

$$9/2 = 4$$

def power(a, n, m):

  if n == 0: return 1

  p = power(a, n//2, m)

  if n % 2 == 0

  ↓ return $\left(\underbrace{(p \% m)}_{10^9} \times \underbrace{(p \% m)}_{10^9}\right) \% m$

  else

  ↓ return $\left(\left(\underbrace{(a \% m)}_{10^9} \times \underbrace{(p \% m)}_{10^9}\right) \% m \times \underbrace{p(\% m)}_{10^9}\right) \% m$

Problem 2

=> Given array A.

=> Find the count of pairs (i, j), such that $(arr[i] + arr[j]) \% m = 0$

$A = [4 \quad 3 \quad 6 \quad 3 \quad 8 \quad 12] \quad m = 6$

Note:

$$\boxed{\left.\begin{array}{l}\text{Number of} \\ \text{pair in array}\end{array}\right\} \Rightarrow \frac{N(N-1)}{2} \Rightarrow \frac{\cancel{6}^{3}(6-1)}{\cancel{2}} = 15}$$

Brute force:

```
count = 0
for i in range(N)
    for j in range(i+1, N)
        if ((A[i]%m) + (A[j]%m)) % m == 0
            Print(A[i], A[j])
            count += 1
return count
```

TC: $O(N^2)$ due to the nested loops.

SC: $O(1)$

$(A[i] \% m \quad A[j] \% m) \% m = 0$

$\dfrac{4 \% 6}{4} \quad \dfrac{8 \% 6}{2}$

$6 \% 6 = 0$

$\left(\dfrac{3 \% 3}{0} \quad \dfrac{3 \% 3}{0}\right) \% 6 = 0$

$\dfrac{6 \% 6}{0} \quad \dfrac{12 \% 6}{0} \% 6 = 0$

$(0 + 0) \% m = 0$

$(1 + (m-1)) \% 6 = 0$

should be multiplication of 6

$(2 + m-2) \% 6 = 0$

$(3 + m-3) \% m = 0$

$\vdots$

$(6 + (m-m)) \% m = 0$

$(i + (m-i)) \% m = 0$

6

12

18

24

$A = [4 \quad 3 \quad 6 \quad 3 \quad 8 \quad 12] \quad m = 6$

$\% 6$

$\text{modulo} = [4, 3, 0, 3, 2, 0]$

$m - i$

$b = [2, 3, 6, 3, 4, 6]$

```
array = [0] * N
for i in rang(N)

  a = A[i] % m   # 4
# b = m - a      # 2
  array[i] = a
  if array.has(b)
  ↓   Count += 1
  else
      arry.append(b)
```

```
array = [ ]
for i in range(N)
↓   arr.append( A[i] % m)

for i in range(n)
    a = A[i] % m
    b = m - a
    temp = arry[(i+1):]
    if temp.has(b)
    ↓   Count ++

return Count
```

A = [4 3 6 3 8 12]   m = 6

Create freq array of modulo range

freq arr =

0	1	2	3	4	5
2	0	1	2	1	0

calculate the pairs.

ans = ~~0~~ 2

handle zero $^nC_r = {^nC_2} = {^2C_2} = \frac{n(n-1)}{2} = \frac{2(2-1)}{2}$

ans += $\frac{freqar[0] \times (freqar[0]-1)}{2}$

=> 1

if m is even handle m/2

an += $\frac{freqarr(m/2) \times (freqarr(m/2)-1)}{2}$ => 1

handle othe indexes.

```
i = 1, j = m-1
while (i < j)
↓   ans += freqarr(i) × freqarr(j)
    i++, j--
```

= 1

a = 3

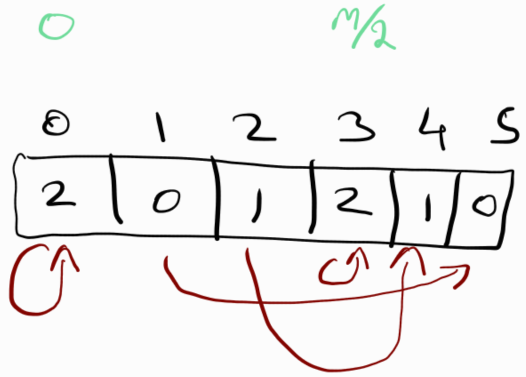
optimized approch

$A = [4 \quad 3 \quad 6 \quad 3 \quad 8 \quad 12]$ $\quad m = 6$

$a = [4, 3, 0, 3, 2, 0]$

step 1: create frequ map array with the frequency of modulo range of m wich is 0 to m-1

```
freqarr = [0] *(m-1)
for i in range(N)
    a = A[i] % m
    freqarr[a] += 1
```



step 2

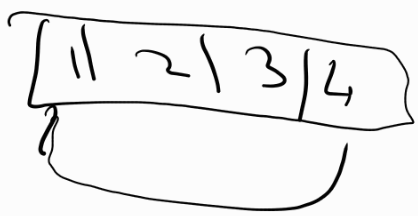
```
ans = 0
# Handle zero
ans += freqarr[0] x (freqarr[0]-1)
                    2
if m % 2 == 0
    # Handle m/2
    ans += freqarr[m/2] x (freqarr(m/2)-1)
                        2
# other elemets  i=1, j=m-1
while (i<j)
    ans += freqarr[i] x freqarr[j]
    i++
    j--
```

formula from below

$\Rightarrow \frac{n(n-1)}{2}$ $\quad {}^4C_2$

$\Rightarrow \frac{3 \times \cancel{2}}{\cancel{2}}$ $\quad \frac{{}^2\cancel{4}(4-1)}{\cancel{2}}$

$\Rightarrow 3$ $\quad \Rightarrow 6$



$$\boxed{{}^{n}C_r}$$

$r =$ the pair count (2)

$n =$ count

$${}^{n}C_r = \frac{n!}{r!\,(n-r)!}$$

$n! = n \times n-1 \times \underbrace{n-2}$

$${}^{n}C_2 = \frac{n!}{2!\,(n-2)!}$$

$n! = n \times (n-1)!$

$$\Rightarrow \frac{n\,(n-1)!}{2\,(n-2)!}$$

$2! = 2 \times 1$

$$\Rightarrow \frac{n\,(n-1)\cancel{(n-2)!}}{2\,\cancel{(n-2)!}}$$

$$\Rightarrow \frac{n\,(n-1)}{2}$$

GCD Greate Common Divisor /

HCF Highest Common Factor

$$GCD(a, b) = \underline{\underline{x}}$$

$\Rightarrow$ x is the highest factor that deride both a and b

$a \% x = 0$, $b \% x = 0$

$GCD(20, 65) = 5$

1	1
2	5
4	13
5	65
10	
20	

$GCD(-10, 20) =$

-10	1
-5	2
-2	4
-1	5
1	10
2	20
5	
10	

$GCD(7, 9)$

1	1
7	3
	9

$GCD(0, 8)$

1	1 ×
2	2
3	4
⋮	
8	8
⋮	
∞	

$GCD(0, -10)$

0	-10
1	-5
2	-2
⋮	-1
10	1
⋮	2
∞	5
	10

Note: GCD(0, any +ve)
↳ +ve value is ans

GCD(0, any -ve value)
↳ +ve value is ans

Properties of GCD:

1. $GCD(a, b) = GCD(b, a)$ [commutative property]

2. $GCD(0, a) = |a|$ (absolute value of a)

3. $GCD(a, b, c) = GCD(GCD(a, b), c)$ [associative property]

   $GCD(GCD(b, c), a)$

   $GCD(GCD(d, a), c)$

$$\boxed{\begin{array}{l} 4.\quad a, b > 0 \ \& \ a > b \\ gcd(a, b) = gcd(a-b, b) = gcd((a \% b), b) \end{array}}$$

ex: $gcd(23, 5)$

↳ $gcd(23-5, 5)$

$gcd(18, 5)$

↳ $gcd(18-5, 5)$

$gcd(13, 5)$

↳ $gcd(8, 5)$

↳ $gcd(3, 5)$ swaping $a, b \to b, a$

=> repeted subtraction

↳ is finding the reminder

Hence, applying mod

$$\begin{array}{r|l} & 5 \\ \cancel{5} & 23 \\ & 20 \\ \hline & 3 \end{array}$$

3,	5
1	1
3	5

another property

↳ $gcd(5, 3)$

$5 \% 3 = 2$

↳ $gcd(2, 3)$

swap

gcd(15, 6) 15 % 6 = 3

gcd(3, 6)

#swap

gcd(6, 3) 6 % 3 = 0

gcd(0, 3)

ans = 3

gcd(3,2) 3 % 2 = 1

gcd(1,2)

swap

gcd(2,1) 2 % 1 =

$$1 \overline{)\,2\,}\quad \begin{array}{r}2\\ \underline{2}\\ 0\end{array}$$

gcd(0,1)

swap

gcd(1,0) 1 % 0 = 0

if a (or) b = 0
absolute of another is ans

$\boxed{\text{ans} = 1}$

4. $a, b > 0$ & $a > b$

$$gcd(a,b) \Rightarrow gcd(a-b, b) = gcd(\underbrace{(a \% b)}_{[0-(b-1)]}, b)$$

so, $gcd(b, (a \% b))$

gcd(23, 5) 23 % 5 = 3

=> gcd(5, (23 % 5))

=> gcd(5, 3) 5 % 3 = 2

=> gcd(3, (5 % 3))

(3, 2)

$\Rightarrow$ gcd (2, (3 % 2)) 3 % 2 = 1

gcd (1, (2 % 1)) 2 % 1 = 0

gcd (1, 0)

if (b == 0) return a

```
def gcd(a, b)
    if b == 0: return a

    gcd(b, (a % b))
```

gcd (25, 5)

gcd (5, 5)

5 % 5

ans = 5

TC: $O(\log_2 \min(a, b))$

SC: "

4, 7

7, (4 % 7)

7, 4

4, (7/4)

4, 3

3, 4/3

3, 1

1, 3/1

1 0

ans = 1

$4 \overline{)\,7\,}\ 1$
$\underline{4}$
3

1

Given [ ], calculate GCD of entire array.

[18, 12, 24, 30]

Using associate property.

1. find first two ele gcd and find it for every element.

$$TC: O\left(N \times \left(\log_2 \underset{max}{\cancel{min}}(a, b)\right)\right)$$

arr = [6 , 3, 9] = False

=> Find if there is a subsequence of GCD '1'

6, 3 = 3

3, 9 = 3

arr = [6, 3, 9, 17] = True

6, 9 = 3

Implement Power function

idea = $fun(a,b,c) = func(a,b,c) \times func(a,b,c)$

=

Find $a^b \% c$

$a = -1$ $b = 1$ $c = 20$

$p = pow(-1, 1, 20)$ -1

$pow(-1, 0, 20)$

$-2 \% 20$

+ve value

```
def Power(a, b, c)
    if a == 0:
        return 0
    if b == 1:
        if a < 0:
            return (a+m) % m    # To go +ve value in output
        else
            return (a % m)
    if b == 0:
        if a < 0:
            return -1
        else:
            return 1
```

$b = 0$

$b = 1$

p = power (a, b//2, c)

if b % 2 == 0: even

return ((p % m) x (p % m)) % m

else:

return (((a % m) x (p % m)) % m x (p % m)) % m

1. if number itself zero return zero

2. if exponent is zero return 1

2. Greate Common Divisor (GCD)

Highest Common factor. (HCF)

1. gcd(a,b) = gcd(b,a)

2. if a > b & a,b > 0

3. gcd(0,a) = |a|

gcd(a,b)

gcd(4,6)

gcd(6,4)

gcd(4, 6%4)

gcd(4, 2)

2, 4%2)

(2, 0)

[0 - 3)

def gcd(a,b)

if b == 0: return a

return gc(b, a%b)

3. Pair Sum divisible by M

→ Given int [] A and int B.

→ return num of pairs in A whose sum is divisible by B.

→ return ans $\% (10^9+7)$

$1 <= |A| <= 10^5$

$1 <= A[i] <= 10^9$

$1 <= B <= 10^6$

$A = [1, 2, 3, 4, 5]$ $B = 2$

3

$\frac{5(5-1)}{2} \Rightarrow 15$

$\Rightarrow (A[i] + A[j]) \% m = 0$

$\Rightarrow (A[i] \% m) + (A[j] \% m)) \% m = 0$

$m = 5$

$(0 - m-1)$

$\boxed{(0-4)}$

$(0 + 0) \% 5 = 0$

$(0,4) + (0,4)$

$(1 + 4) \% 5 = 0$

$(2 + 3) \% 5 = 0$

$3 + 2 \% 5 = 0$

$4 + 1 \% 5 =$

$1 \quad 2$

1

freqarr =

0	1	2	3	4
2	3	0	0	0

$^{n}C_{r}$

def func(A,B):

frearr = [0]*m

for i in range(m)

a = A[i] % m

frearr[a] += 1

we have created frearr.

ans = 0

handle zero

$$ans \mathrel{+}= \frac{frearr[0]\,(frearr[0]-1)}{2}$$

if m is even

if m % 2 == 0:

$$ans \mathrel{+}= \frac{frearr[m/2]\,(frearr[m/2]-1)}{2}$$

i = 1, j = m-1

while (i<j): i++, j--

ans += frearr[i] × frearr[j]

4. Larger Coprime Divisor

$gcd(a, b) = x$

$a \% x = 0$ ✓

$b \% x = 0$ $\quad\quad$ $gcd(x, B) = 1$

$A = 30$ $\quad$ $B = 12$ $\quad\quad$ $gcd(30, 12)$

A = 30	B = 12
1	1
2	2
3	3
5 ✓	4
6	6 ✓
10	12
30	

$\downarrow$

ans = 6

5

30 % 12

12 % 6

$[1 \ 2 \ 3 \ 5 \ 6]$ $\leftarrow$ $\quad\quad$ $B = 12$

(X) 1 as $gcd(x, B) = 1$. x and B are co-prime

Co-Prime means a and b has ony one common factor. hence, x and B are Co-prim

$$A \% x = 0$$

$\Rightarrow$ as x can be factor of A,
and A will be multiple of x, it can be writen as

$$\Rightarrow A = kx$$

$$\Rightarrow x = A/k$$

$$A = 30 = \cancel{2}^1 \times \cancel{3}^1 \times \boxed{5}^1$$

$$B = 12 = \cancel{2}^2 \times \cancel{3}^1$$

12
6 2
3 2

$\Rightarrow$ ans diviser of A

$\Rightarrow$ that ans should be co-prime with B

$\Rightarrow$ if we eliminate common facter in $A \times B$
we left with '5'

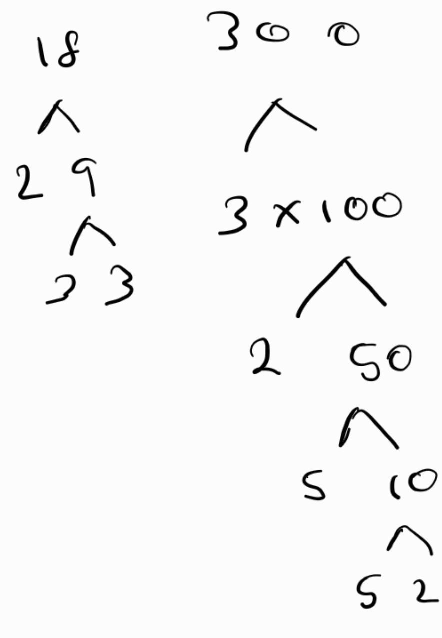
Prime factorization

Ex:

$A = 300 = \cancel{3} \times \cancel{2^2} \times \boxed{5^2}$

$B = 18 = \cancel{3^2} \times \cancel{2}$

$\boxed{5^2} = 25$



To achive this ans:

$\Rightarrow A \,/\, gcd(A,B)$

untill $gcd(A,B) == 1$

value = gcd(A,B) = 6   300,18

A = A/6 = 50

value = gcd(A,B) = 2   50,18

A = A/6 = 25   50/2

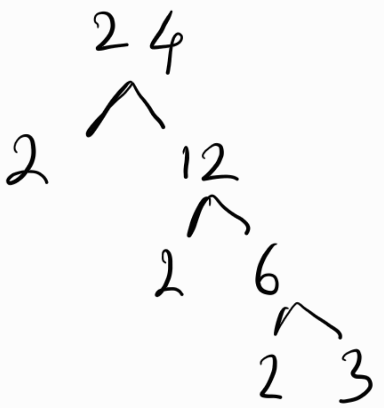
gcd(25, 18) = 1

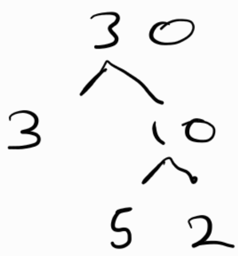
25 = Ans

```
def gcd(a,b)
    if b == 0:
        return a
    return gcd(b, a%b)
gcdvalue = gcd(A,B)
while ( gcdvalue != 1)
    A = A // gcdvalue
    gcdvalue = gcd(A,B)
```

Prime Factorization

Break a number down into its prime factors (all of the prime numbers that multiply together to equal the original number.





$30 = \cancel{2} \times \cancel{3} \times 5$

$12 = \overset{2}{\cancel{2}} \times \cancel{3}$

a, b $\quad x$

$a \% x = 0$

$\gcd(x, b) = 1$

Divisor game

=> int A, B, C

=> Find x, such that $x \% B = 0$, $x \% C = 0$

=> and that x is less than or equal to A

=> Find and return number of x fund.

$A = 12$

$B = 3$

$C = 2$

number of int found = 2

12 → 2, 6 → 3, 2   3 => 3^1 2 => 2^1 => 2

=> 3×2^2

---

6 → 3, 2   1 → 1   4 → 2, 2   => 1

$6 \% 3 = 0$

$6 \% 2 = 0$

multiple of 6

1	2	3	4	5	6	7	8	9	10	11	12
×	×	×	×	×	✓	×	×	×	×	×	✓

↓

lcm(3, 2)

ex: A = 100
B = 2
c = 4

lcm(2, 4) = 4

1. all the multiple of 4 lesstan equal to A are divisible by both A and B

Relation betwen GCD and LCM

$$\Rightarrow \text{GCD}(a,b) \times \text{LCM}(a,b) = a \times b$$

$$\Rightarrow \text{LCM}(a,b) = \frac{a \times b}{\text{GCD}(a,b)}$$

Steps:

1. Find GCD(a,b) $(\log \max(a,b))$
2. Find $\text{LCM} = \dfrac{a \times b}{\text{GCD}(a,b)}$
3. by simple math $\left\lfloor \dfrac{A}{\text{LCM}} \right\rfloor$
4. return the floor of $\dfrac{A}{\text{LCM}}$

TC: $(\log \max(a,b))$
SC: $(\log \max(a,b))$

Least Common multiplier

84, 126

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 7 & 21 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 126 \\ \hline 3 & 63 \\ \hline 7 & 21 \\ \hline & 3 \end{array}$$

$84 = 2 \times 2 \times 7 \times 3$

$126 = 2 \times 3 \times 7 \times 3$

Common factors are $= 2 \times 3 \times 7 = 42$

HCF / GCD

LCM $= 2 \times 3 \times 7 \times 2 \times 3$

$\Rightarrow 252$

Delete one element To get max gcd of array

⇒ Given an int[] A.

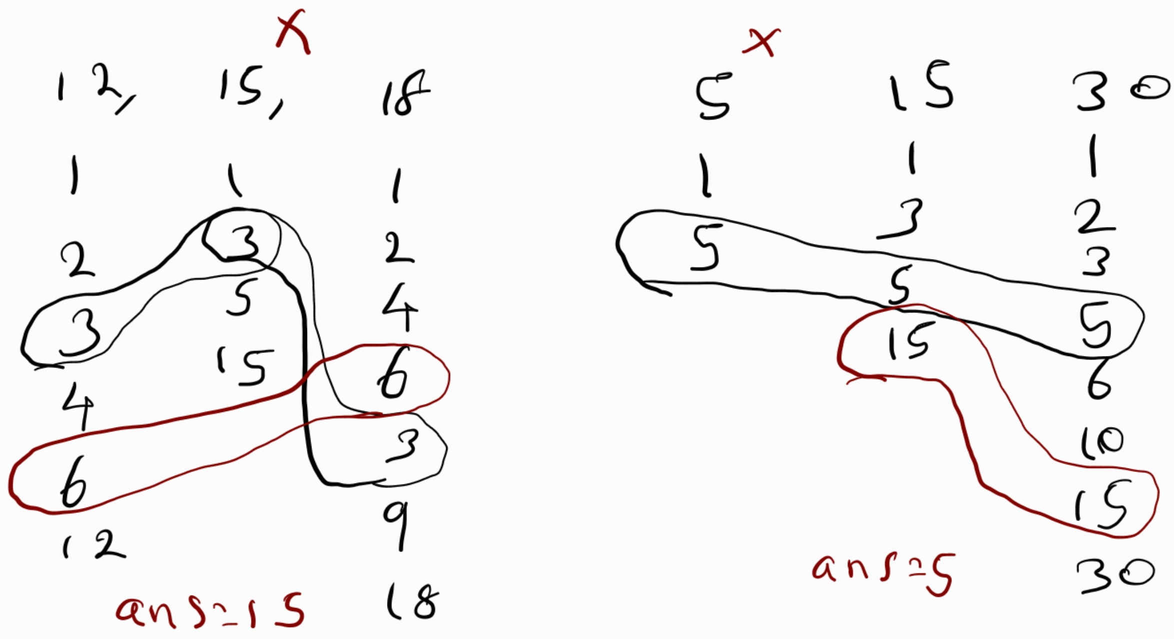
⇒ you have to delete one element such that the GCD of the remaining array is maximum.

A = [12 15 18]

$2 \le N \le 10^5$

$1 \le A[i] \le 10^9$

GCD of the array:



Brute force:

1. delting one by one element and
2. find the max GCD of the array
3. maintaining max GCD.

[12, 15, 18]

12 → 2, 6; 6 → 3, 2

15 → 5, 3

18 → 2, 9; 9 → 3, 3

$= 2^2 \times 3$ $\qquad 5 \times 3$ $\qquad 2 \times 3^2$

if we remove :

12 → gcd(18, 15) = 3

15 → gcd(12, 18) = 6 ✓

18 → gcd(12, 15) = 3

ex:

12	18	24	15	30
1	1	1	1	1
2	2	2	3	2
6	9	12	5	15
3	6	3	15	3
4	3	8		10
12	18	24		5
				6

6 6

$12 = 2 \times 2 \times 3 \Rightarrow 2^2 \times 3$

$18 = 2 \times 3 \times 3 \Rightarrow 2 \times 3^2$

$24 = 2 \times 2 \times 2 \times 3 \Rightarrow 2^3 \times 3$

$15 = 3 \times 5 \Rightarrow \times 3 \times 5$

$30 = 3 \times 2 \times 5 \Rightarrow 2 \times 3 \times 5$

} prime factorization

PrefArr = { 12, 6, 6, 3, 3 }

sufArr = { 3, 3, 3, 15, 30 }

$$A = \{ \overset{0}{12} \quad \overset{1}{18} \quad \overset{2}{24} \quad \overset{3}{15} \quad \overset{4}{30} \}$$

12 → i = 0, sufArr[i+1] = 3

18 ⇒ i = 1, PrefArr[i-1], SufArr[i+1]

0 2

gcd(12, 3) = 3

24 ⇒ i = 2, PrefArr[i-1], SufArr[i+1]

1 3

gcd(6, 15) = 3

15 ⇒ i = 3, PrefArr[i-1], sufArr[i+1]

(6, 30) = 6 ✓

30 ⇒ i = 4, i = N-1, PrefArr[i-1]

3

3 = 3

steps:

1. calculate gcd PrefixArr, SuffArray of A
2. Iterate over A and, maintain max(ans, output)

output = gcd(PrefArr[i-1], SufArr[i+1])

TC: $O(N \times \log \max(a,b))$

SC: $O(N)$

calculate prefixArray of gcd of A

```
N = len(A)
prefArr = [0] * N
prefArr[0] = A[0]
for i in range(1, N)
    value = gcd( prefixArr[i-1], A[i])
    prefArr.append(value)
```

suffix Array with gcd of A in reverse

```
sufArr = [0] x N
sufArr[N-1] = A[N-1]
for i in range(N-2, -1, -1): # in reverse.
    value = gcd(sufArr[i+1], A[i])
    sufArr[i] = value
```

iterate over A and calculate left and right side of gcd output

```
ans = 0          # max gcd output
element = -1     # the element to be removed
```

```
for i in range(n):
    if i == 0:
        value = sufArr[i+1]

    elif i == N-1:
        value = prefArr[i-1]

    else
        value = gcd( prefArr[i-1], sufArr[i+1])
    # update ans
    if value > ans:
        ans = value
        element = A[i]

return ans.
```

A, B and Modulo

→ given int A, B.
→ Find greatest possible positive integer M
→ such that $A \% M = B \% M$

Constraint: $1 \le A, B \le 10^9$
$A \, != B$

ex1. $A = 1$
$B = 2$ ans = 1

$1 \% 1 = 2 \% 1$
$0 = 0$

$A = 5$
$B = 10$ ans = 5

ex2
$A = 12 = 2 \times 2 \times 3 = 2^2 \times 3$
$B = 18 = 2 \times 3 \times 3 = 2 \times 3^2$
$= 2 \times 3$
$\Rightarrow 6$

$$\begin{array}{r} 11 \\ 81 \\ 25 \\ \hline 56 \end{array}$$

ex3
$A = 25 = 5 \times 5 = 5^2$
$B = 81 = 3 \times 3 \times 3 \times 3 = 3^4$

ans $= 56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$

1. min(A, B)
2. from 25 to .... find $(A \% M == B \% M)$

Brute force:

1. getting the max(A, B)
2. iterat from 1 to max, if condition is meeting. maintaing the max value.

TC: $O(max(A,B)) = O(N)$

SC: Constant.

optimized:

→ given int A, B.
→ Find greatest possible positive integer m
→ such that $A \% m = B \% m$

possible A & B:

$A = B \Rightarrow A = m$

$$\boxed{\begin{matrix} A > B \\ A < B \end{matrix}}$$

as per our constrains.

1. $A > B$

$\Rightarrow B = B$

$\Rightarrow B = A - A + B$ # Adding A - A

$\Rightarrow B = A - (A - B)$

$\Rightarrow B \% (A-B) = A - (A-B) \% (A-B)$ # adding %(A-B) in both side

$= A \% (A-B) - \cancel{(A-B) \% (A-B)}$ # it gets distributed

$$B\%(A-B) = A\%(A-B)$$

$$B\% m = A\% m$$

$$\boxed{m = A - B}$$

2. $A < B$

$$A = A$$

$$A = B - B + A$$

$$= B - (B - A)$$

$$A\%(B-A) = B\%(B-A) - \underbrace{\cancel{(B-A)\%(B-A)}}_{0}$$

$$A\%(B-A) = B\%(B-A)$$

$$A\% m = B\% m$$

$$\boxed{m = B - A}$$

Ans is going to b $\boxed{m = abs(A-B)}$

Mod Sum

=> Given int [] A
=> calculate sum of $A[i] \% A[j]$ for all possible i, j pairs.
=> return sum % (10^9+7)

$10^3 = 1000$

Constrains: $1 \leq |A| \leq 10^5$
$1 \leq A[i] \leq 10^3$

ex: [1, 2, 3]

$1\%1=0$ $2\%2=0$ $3\%3=0$
$1\%2=1$ $2\%3=1$ $3\%1=0$ Ans = 4
$1\%3=1$ $2\%1=0$ $3\%2=1$.

Brute force: O(n^2)

=> iteratin A and nested loop to calculate the sum.

optimization

1. Create freq arr for 1001
2. iterat through given array, store the frequency of element.

frqarr =

0	1	2	3	4	.....	1001
0	1	1	1	0	......	0

```
for i in range(1, 1001)   # No need to include 0
    for j in range(1, 1001)   # no need have 0
        value = i % j
        temp = value × Freqarr[i] × Freqar[j]
```

i = 1

j = 2

value = i % j = 0

3 % 2 = 1

an = 4

frqarr[i] = 1

reqarr[j] = 1