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2. Nth magic number
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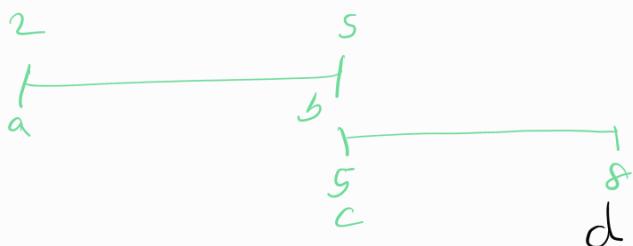
Flip:

You are given a binary string A(i.e., with characters 0 and 1) consisting of characters A₁, A₂, ..., A_N. In a single operation, you can choose two indices, L and R, such that $1 \leq L \leq R \leq N$ and flip the characters A_L, A_{L+1}, ..., A_R. By flipping, we mean changing character 0 to 1 and vice-versa.

Your aim is to perform ATMOST one operation such that in the final string number of 1s is maximized.

If you don't want to perform the operation, return an empty array. Else, return an array consisting of two elements denoting L and R. If there are multiple solutions, return the lexicographically smallest pair of L and R.

NOTE: Pair (a, b) is lexicographically smaller than pair (c, d) if $a < c$ or, if $a == c$ and $b < d$.



(a, b) ✓

if $a < c$

elif $a = c$ and $b < d$



$$\begin{array}{r}
 \textcircled{0} \quad | \quad \begin{matrix} 2 \\ 3 \end{matrix} \quad \textcircled{4} \\
 A = \begin{array}{r} 1 \quad 0 \quad 1 \quad \begin{array}{c} \textcircled{0} \quad \textcircled{0} \\ \hline 1 \quad 1 \end{array} \end{array} = 4 \\
 \hline
 \begin{array}{r} 1 \quad 0 \quad 1 \quad 1 \end{array} = 4 \\
 \hline
 \begin{array}{r} 1 \quad 4 \end{array}
 \end{array}$$

$(1 \ 4) \nearrow$ \rightarrow Lexicographically smaller
 $(3 \ 4)$

0 1 2 3 4 5 6
 $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$
 (0,4) " " 4 0s
 (0,6) subarray has 5 0s

BF: $TC: O(n^3)$ $SC: O(1)$

→ Create all subarray and calculate the max ones possible.

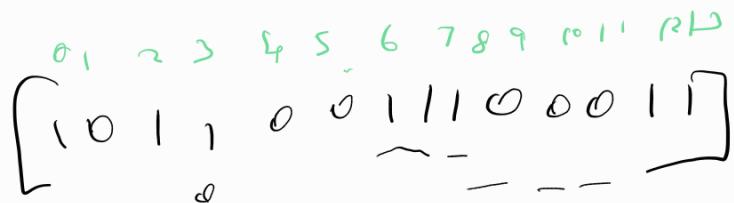
→ return the indexes.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

- ① No need to switch
 - ② return empty array

ans = []

~~Tip: flip subarray has max count of 0s~~



(1, 11) 9 1's

(1, 11) 11 1's

⇒ Flip subarray where we get net gain of 1's maximum



means

we need subarray with maximum sum



To solve this

Kadane's algorithm

↳ if the sum is positive keep the sum carry forwarded.

arr $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ -1 & +1 & -1 & +1 & +1 \end{array}$$

$$\text{sum} = \emptyset \neq \emptyset \neq 0$$

\rightarrow we can not carry forward negative value

keep it 0

Code:

```
def flip(A):
    n = len(A)
    csum, l, r, max_sum, nsi = 0, 0, 0, 0, 0  $\rightarrow$  integers
    for i in range(n):
        if A[i] == '0':
            csum += 1
        else:
            csum -= 1
        # update max_sum and l, r
        if csum > max_sum:
            max_sum = csum
            l = nsi
            r = i
        # reset csum if < 0
        if csum < 0:
            nsi = i + 1
            csum = 0
        if max_sum == 0:
            if we do not have possible subarray
            return []
        else:
            return [l, r]
```

TC: $O(N)$
SC: $O(1)$

Nth magic number.

Nth Magic Number -

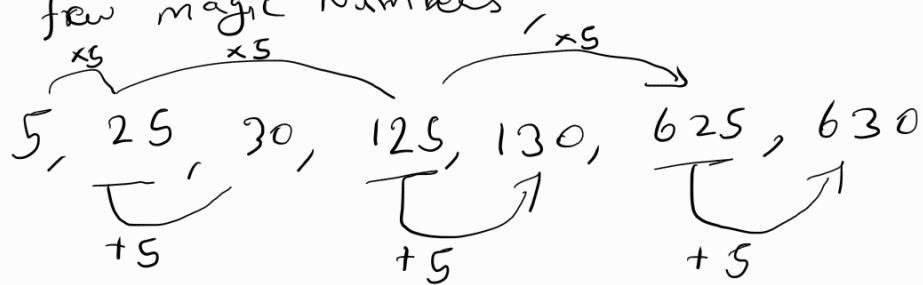
Given an integer A, find and return the Ath magic number.

A magic number is defined as a number that can be expressed as a power of 5 or a sum of unique powers of 5.

First few magic numbers are 5, 25, 30(5 + 25), 125, 130(125 + 5),

- int A
- Find Ath magic number-
- magic number is, number that can be expressed as a power of 5 (or)
 - ↳ sum of unique powers of 5

→ first few magic numbers



| | | | | | | |
|---|-----|-------------|---------------|---|---|------|
| 1 | - 5 | 5^1 | \rightarrow | 1×5^1 | = | 1 |
| 2 | 25 | 5^2 | \rightarrow | $1 \times 5^2 + 0 \times 5^1$ | = | 10 |
| 3 | 30 | $5^2 + 5^1$ | \rightarrow | $1 \times 5^2 + 1 \times 5^1$ | = | 11 |
| 4 | 125 | 5^3 | \rightarrow | $1 \times 5^3 + 0 \times 5^2 + 0 \times 5^1$ | = | 100 |
| 5 | 130 | $5^3 + 5^1$ | \rightarrow | $1 \times 5^3 + 0 \times 5^2 + 1 \times 5^1$ | = | 101 |
| 6 | 150 | 5^4 | \rightarrow | $1 \times 5^3 + 1 \times 5^2 + 0 \times 5^1$ | = | 110 |
| 7 | 155 | $5^4 + 5^1$ | \rightarrow | $1 \times 5^3 + 1 \times 5^2 + 1 \times 5^1$ | = | 111 |
| 8 | 625 | | | $\rightarrow 1 \times 5^4 + 0 \times 5^3 + 0 \times 5^2 + 0 \times 5^1$ | = | 1000 |
| 9 | 630 | | | | | |

$10 = 650$

\Rightarrow binary number of 1, 2, 3...

\Rightarrow instead of multiplying with powers of 2
multiply with powers of 5

$$9 \rightarrow (4 \quad 3 \quad 2 \quad 1) \rightarrow 2 \text{ based indexing}$$

$$\begin{matrix} 5^4 & 5^3 & 5^2 & 5^1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 625 & 0 & 0 & 5 \end{matrix} \Rightarrow 630$$

BF?

Find A^{th} magical number!

ans = 0, power = 5, i = 1

while ($A > 0$):

find last bit is set or not

if ($(A \& 1) > 0$):

bit is set

ans += ($5 \times i$)

$A = (A \gg 1) = A/2$

i += 1

return ans

32
A bits

1 0 0 0

0 1 0 0

0 0 1 0

$A = 10$

4 3 2 1
1 φ X φ

$i = 4$

$ans = 25 \times 650$

$$\begin{array}{r}
 (0\ 1\ 0 \\
 0\ 0\ 0\ 1 \\
 \hline
 0\ 0\ 0\ 0
 \end{array}$$

$$\begin{array}{r}
 25 \times 5 \\
 \hline
 125 \times 5 \\
 \hline
 625
 \end{array}$$

$A = 1$



Subarray OR

Subarray OR .

You are given an array of integers A of size N .

The value of a subarray is defined as BITWISE OR of all elements in it.

Return the sum of value of all subarrays of $A \% 10^9 + 7$.

→ Given int [] A

→ find value for each subarray

→ and return sum of all values.

$$\text{value}(i, j) = A[i] | A[j]$$

BF:

1. going through all subarray of A. $\rightarrow N^2$
2. find value of each subarray $\rightarrow N$
3. maintaining ans, which is sum of all values.

TC: $O(N^3)$

SC: Constant

$A = [1, 2, 3]$ $(3, 5)$

 def orvalue(A, i, j):

```

    ans = A[k]
    for k in range(i+1, (j-i+1)+1):
        ↓
        ans = ans | A[1:k]
    return ans
  
```

$n = \text{len}(A)$, $ans = 0$

```

for i in range(n)
    for j in range(i, n)
        # find value for subarray(i, j)
        value = orvalue(A, i, j)      TC:  $O(N^3)$ 
        # adding it to sum          SC:  $O(1)$ 
        ans += value
    return ans
  
```

optimized using carry forward $O(N^2)$

1 →

1 2

1 2 3

2 →

2 3

3 →

$$\begin{array}{r}
 0\ 0\ 0\ 1 \\
 0\ 0\ 1\ 0 \\
 \hline
 0\ 0\ 1\ 1 \\
 0\ 0\ 1\ 1 \\
 \hline
 0\ 0\ 1\ 1 \\
 0\ 1\ 0\ 0 \\
 \hline
 0\ 1\ 1\ 1
 \end{array}$$

1 0 0 0 1 OR

2 0 0 1 0 | 0
 3 0 0 1 1 | 1
 4 0 1 0 0 |
 5 0 1 0 1 |

$$\underbrace{n(n+1)}_{\cancel{2}} = \frac{3(3+1)}{\cancel{2}} = 6$$

optimized

OR's Property

if any bit is 1 output is 1

0
0
1
0

output = 1

0th bit → Count the subarray that has
 $A[0] \rightarrow 0$ 0 in 0th bit
 ↙
 $A[1] \rightarrow 1$ 1
 $A[2] \rightarrow 0$
 $A[3] \rightarrow 0$
 $A[4] \rightarrow 0$
 ↗ $\frac{3 \times (3+1)}{2} = 6$ Subarray has 0th bit

 $A[5] \rightarrow 1$

Num of subarray that has 1 in 0th bit = num of total subarray - num of subarray that has 0 in ith bit.

$$= 21 - 7$$

$$0\text{th bit} = 14$$

$$= 14 \times 2^0$$

$$A[0] \rightarrow (\textcircled{5}) \backslash \backslash \textcircled{0} \rightarrow 1 \text{ subarray}$$

$$A[1] \rightarrow (\textcircled{0}) \backslash \backslash \textcircled{1} \backslash \backslash \textcircled{1} \rightarrow 2 \text{ subarray}$$

$$A[2] \rightarrow (\textcircled{0}) \textcircled{0} \backslash \backslash \textcircled{0} \backslash \backslash \textcircled{0} \rightarrow 3 = 6 \text{ subarray}$$

$$A[3] \rightarrow (\textcircled{0}) \textcircled{1} \textcircled{0} \backslash \backslash \textcircled{0} \backslash \backslash \textcircled{0} \rightarrow 4 = 12 \text{ subarray}$$

$$A[4] \rightarrow (\textcircled{0}) \textcircled{1} \textcircled{0} \textcircled{0} \backslash \backslash \textcircled{0} \backslash \backslash \textcircled{0} \rightarrow 5 = 24 \text{ subarray}$$

$$A[5] \rightarrow (\textcircled{0}) \textcircled{1} \textcircled{0} \textcircled{0} \textcircled{1} \textcircled{1} \rightarrow 6 = 48 \text{ subarray}$$

$$\begin{array}{cccccc} 15 & 18 & 18 & 11 & 14 \\ \times & & \times & \times & \times \\ 2^4 & & 2^3 & 2^2 & 2^1 & 2^0 \\ \hline + & + & + & + & + & 14 \end{array}$$

$$\frac{21-7}{14}$$

$$\begin{array}{c} 2 \\ \hline 4 \times (4+) \\ \hline \cancel{2} \\ \hline 21-10 \\ \hline 11 \end{array}$$

Code:

$n, ans, modulo = len(A), 0, (10^{10} \times 9 + 7)$

for i in range(32):

Count = 0, subarrwithzeros = 0

for j in range(n)

for every element find num of subarra that has
ith bit zero.

if $(A[j] \& (1 \ll i)) == 0:$

ith bit un-set

Count += 1

else:

update subarr with zeros

subarrwithzeros = $(Count \times (Count + 1)) // 2$

reset zeros Count

Count = 0

update subarr with zeros after the loop complete

subarrwithzeros = $(Count \times (Count + 1)) // 2$

num of subarray - Count

value = subArrCount - subarrwithzeros

ans += (value * $(1 \ll i)$) % modulo

return ans % modulo

TC: $(32 \times n)$

SC: (1)

strange Equality:

Strange Equality ↴

Given an integer A.

Two numbers, X and Y, are defined as follows:

X is the greatest number smaller than A such that the XOR sum of X and A is the same as the sum of X and A.

Y is the smallest number greater than A, such that the XOR sum of Y and A is the same as the sum of Y and A.

Find and return the XOR of X and Y.

NOTE 1: XOR of X and Y is defined as $X \wedge Y$ where ' \wedge ' is the BITWISE XOR operator.

NOTE 2: Your code will be run against a maximum of 100000 Test Cases. ↴

- given int A
- x greatest < A,
$$X \wedge A = X + A$$
- y smallest number > A, such that
$$Y \wedge A = Y + A$$
- Find and return XOR of x and y

| XOR (\wedge) | | Output |
|------------------|---|--------|
| a | b | |
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

$\rightarrow x$ greatest $< A$, $x \wedge A = x + A$

$$\begin{array}{r}
 & 5 & 4 & 3 & 2 & 1 & 0 \\
 A = & 1 & 0 & 1 & 1 & 0 & 0 = & 2^5 + 2^4 + 2^3 + 2^0 = 44 \\
 x = & 0 & 1 & 0 & 0 & 1 & 1 = & 2^4 + 2^1 + 2^0 = & 19 \\
 & \hline & 1 & 1 & 1 & 1 & 1 = & 2^6 + 2^5 + 2^4 + 2^3 + 2^2 = & 63 \\
 & & & & & & 1 & 2 & 4 & 8 & 16 & 32 & \hline
 \end{array}$$

1. x can not be same as A ,
2. and it has to be smaller than A .

Code:

$$x = 0, i = 0$$

while($A > 0$):

if ($(A \& 1) == 0$): # bit is un-set

$$\downarrow x = \boxed{x + (1 \ll i)} \quad (x | (1 \ll i))$$

$$i += 1$$

$$A = \boxed{A \gg 1} \Rightarrow A/2$$

i currently holding net bit index of A

$$y = (1 \ll i)$$

TC: $\log_2 A$

SC: O(1)

$$\begin{array}{r}
 001 = 10 \\
 10000 = 16 \\
 \hline
 11001
 \end{array}$$

\Rightarrow γ smallest number $\boxed{> A}$, such that

$$\gamma \wedge A = \gamma + A$$

$$A = \begin{matrix} 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & \end{matrix} = 19$$

$$\gamma = \frac{1000000}{1101100} = 2^6 = 64$$

$$= 2^6 + 2^5 + 2^3 + 2^2 = 108$$

$$64 \ 32 \ 8 \ 4$$

$$\begin{array}{cccccccccc}
 & 6 & & 5 & & 4 & & 3 & & 2 & \\
 2 & & 2 & & 2 & & 2 & & 2 & & 1 \\
 64 & 32 & 16 & 8 & 4 & 2 & 1 & 2 & 1 & 2^0 \\
 & \swarrow & \curvearrowleft \\
 & 604 & 400 & 108 & & & & & &
 \end{array}$$

Tower of Hanoi (classical example of Recursion)

Tower of Hanoi

In the classic problem of the Towers of Hanoi, you have 3 towers numbered from 1 to 3 (left to right) and A disks numbered from 1 to A (top to bottom) of different sizes which can slide onto any tower. The puzzle starts with disks sorted in ascending order of size from top to bottom (i.e., each disk sits on top of an even larger one).

You have the following constraints:

Only one disk can be moved at a time.

A disk is slid off the top of one tower onto another tower.

A disk cannot be placed on top of a smaller disk.

You have to find the solution to the Tower of Hanoi problem.

You have to return a 2D array of dimensions $M \times 3$, where M is the minimum number of moves needed to solve the problem.

In each row, there should be 3 integers (disk, start, end), where:

disk - number of the disk being moved

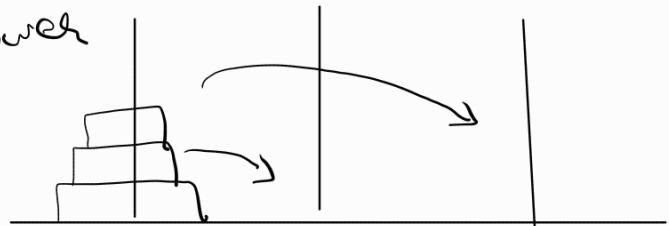
start - number of the tower from which the disk is being moved

end - number of the tower to which the disk is being moved

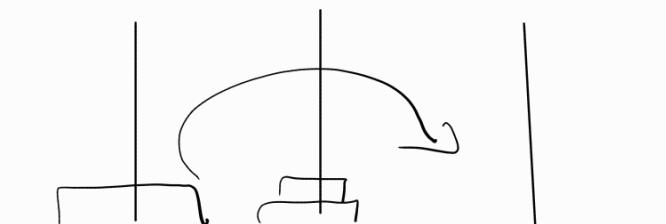
1. N num of disc placed on a tower

return [disk , from , to]

ex: $N = 3$



A B C
S h d



A B C
S h d

1. move $n-1$ disk to helper disk
 2. move n^{th} disk to destination tower
 3. move $n-2$ disk from helper disk to source tower
 4. move $(n-1)^{\text{th}}$ disk to destination.
- ⋮

move (n) disks.

- ① move $(n-1)$ disks to helper
- ② move n^{th} disk to destination.
- ③ move $(n-1)$ disk to destination.

def moveDisks(N, S, h, d, ans)
 if $N = 0$: return ans # base case

moveDisks($N-1, S, d, h$)

$\text{temp} = []$

$\text{temp.append}(N)$

$\text{temp.append}(S)$

$\text{temp.append}(d)$

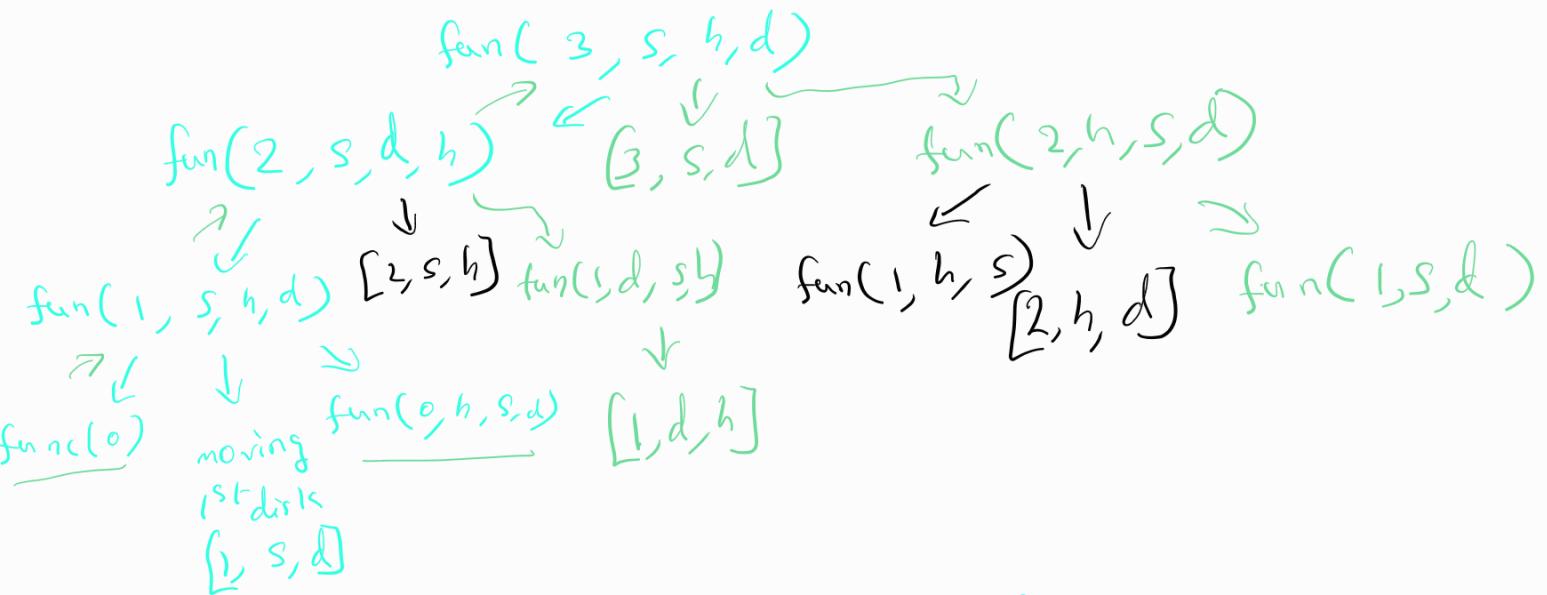
ans.append(temp)

moveDisks($n-1, h, S, d$)

return ans

moving n^{th} disk.

$n=3$



$[1, s, d]$
 $2, s, h$
 $1, d, h$
 $3, s, d$
 $1, h, s$
 $2, h, d$
 $[1, s, d]$

TC: (2^n)

func calls $\Rightarrow 2^0, 2^1, 2^2, 2^3, \dots, 2^n$
 $\Rightarrow 2^n$

Time per function call \Rightarrow constant

SC: $O(n)$

Def of func calls = n

$1, 2, 3, \dots, n$

Space per func call = 3