

## Agenda!

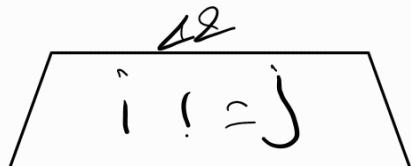
1. First Non Repeating Element
2. Pair Sum
3. Count of Pair Sum
4. check if there exists a subarray with  $\text{Sum} = 0$
5. check if there exists a subarray with  $\text{Sum} = K$ .

## 1. First non-Repeating Elements $O(n) \Theta(n)$

1. Count the element frequencies in freq map.
2. iterate over A again and check the element frequency if it is == 1

2.

Given int[] A and int k  
check if there exist a pair (i, j)  
such that  $\text{arr}[i] + \text{arr}[j] = k$



Brute force :  $O(n^2)$

1. lets create all possible pairs,  
using nested loop one for i one for  
j from i+1 to n.  $O(n^2)$   
and check if element exist or not

Optimized: 0 1 2 3 4 5 6 7 8

8	9	1	-2	4	5	11	-6	4
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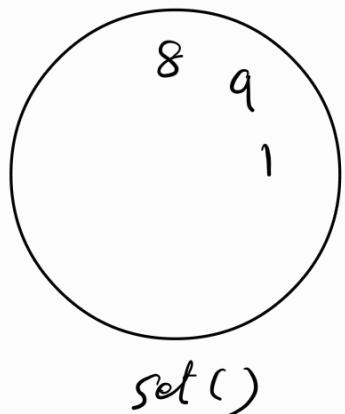
$n=7$

$$\begin{aligned}k &= 6 & 1 \\k &= 2 & 2 \\k &= 8 & 1\end{aligned}$$

$$\begin{array}{cccccc}6-8 & 6-9 & 6-1 & 6+2 \\-2 & -3 & 5 & 8 \\& & & \downarrow\end{array}$$

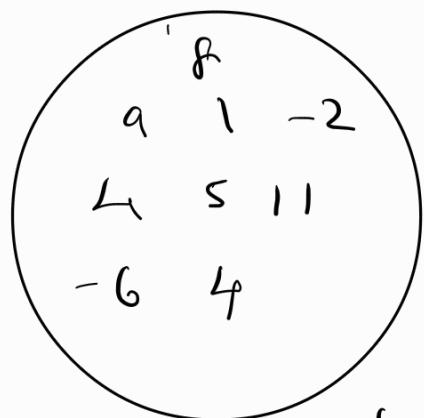
$$b = k - a$$

Return True



$$8 - b = k-a \Rightarrow -2$$

$\Rightarrow$  do we have it in set?



Create set out of A

1. # create st from A
2. # iterate over A and check if b is found in set.

TC:  $O(n)$

SC:  $O(n)$

issue : above approach will not work  $i \neq j$  condition

- use dictionary to store element and frequency. and keep  $i \neq j$

$$k = 18$$

$$18 - 9 = 9$$

we could keep check

if  $\text{fremap}[9] == A[i]$

if value == 1

↓ Pass

else:

↓ Consider

f:	1	11:	1
a:	1	-6:	1
l:	1	4:	1
-2:			
4:	1		
5:	1		

$T C: O(n)$

$SC: O(n)$

Count the number of pairs with sum = K

$$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \left[ \begin{matrix} 2 & 5 & 2 & 5 & 8 & 5 & 2 & 8 \\ \downarrow & \downarrow \\ 2 & ? & 1 & 1 & 0 & 1 & 0 \\ 2 & ? & 1 & 1 & 0 & 1 & 0 \end{matrix} \right] \quad k=10 \end{matrix}$$

$$\begin{array}{r}
 2 \quad 5 \quad 2 \quad 5 \quad 8 \quad 5 \quad 2 \quad 8 \\
 \downarrow \quad \downarrow \\
 \frac{10 - 2}{8 \sqrt{5}} \quad 8 \quad 5 \quad 2 \quad 5 \quad 8 \quad 2
 \end{array}
 \qquad
 \begin{array}{l}
 a+b=k \\
 b = k-a
 \end{array}$$

$$\begin{array}{l} 2 : 3 \\ 5 : 4 \\ 8 : 1 \end{array}$$

First create dictionary  
pairs = ifeat A and all all  
-frequency.

$$a+b-a = 12 - 9$$

$$b = k - a$$

f      from left to right = iterate A and create dictionary.  
on the fly and add if it finds.

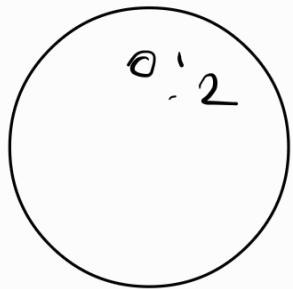
freqMap = {}, ans = 0, k = 4

for i in range(n)

$$b = k - A[i]$$

value = freqMap.get(b, 0)

ans += value



if  $A[i]$  in freqMap.

↓  
ans += value  
increase frequency also.

else:

freqMap[A[i]] = 1

[0 0 0]

ans = 85

## Subarray with sum zero

- Given int [] A
- Find if subarray with sum zero. return True

Approach 1: BF  $\Theta(n^3)$

- going through all the subarray and  $\Theta(n^2)$
- check if it's sum = 0  $\Theta(n)$

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 2 & 1 & -3 & 4 & 3 & 1 & -2 & -3 & 2 \end{bmatrix} \quad N=11$$

2  
2

$$\frac{N(N+1)}{2} = 66$$

2 2  
2 2 1  
2 2 1 -3

```
n = len(A)
for i in range(n)
    for j in range(i, n)
        temp = 0
        for k in range(i, j+1)
            temp += A[k]
        if temp == 0:
            return True
return False.
```

## approach 2 Prefix Sum $O(n^2)$ , $O(N)$ space

1. creating Prefix sum

2. for every subarray get sum from prefix sum

PrefixSum = [], PrefixSum.append(A[0])

for i in range(1, n): PrefixSum.append(PrefixSum[i-1] + A[i])

for i in range(n)

for j in range(i, n)

temp = 0

if i == 0: temp = PrefixSum[i]

else:

temp = PrefixSum[j] - PrefixSum[i-1]

if temp == 0: return True

return False

## approach 3: carry forward $O(n^2)$

for i in range(n)

temp = 0

for j in range(i, n)

temp += A[j]

if temp == 0

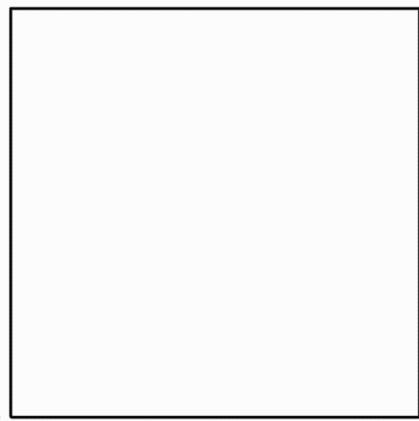
return True

Approach  $\leftarrow$  !

$$\text{Sum}(i, j) = 0$$

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 2 & 1 & -3 & 4 & 3 & 1 & -2 & -3 & 2 \end{bmatrix} \quad N=11$$

$i$   
 $j$



$$\text{PrefixSum} = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 5 & 2 & 6 & 9 & 10 & 8 & 5 & 7 \end{bmatrix}$$

$i$                    $j$

$$\begin{aligned} &= \text{Prefix}[i] - \text{Prefix}[i-1] \\ &= 6 - 2 \\ &\Rightarrow 4 \end{aligned}$$

Idea!

$$\text{sum}(i, j) = 0$$

$$\text{Prefix}[j] - \text{Prefix}[i-1] = 0$$

$$\text{Prefix}[j] = \text{Prefix}[i-1]$$

So, if  $\text{Prefix}[j]$  and  $\text{Prefix}[i-1]$  are same,  
they will mark "Zero"

get frequency of all element in PrefixSum

$2 : 2 \equiv \text{True}$

$4 : 1$

$S : 2$

$6 : 1$

$a : 1$

$c : 1$

$f : 1$

$T : 1$

\_\_\_\_\_ x \_\_\_\_\_

$$A = \begin{bmatrix} -2 & -1 & 3 & 5 \end{bmatrix}$$

$$PF = \begin{bmatrix} -2 & -3 & 0 & 5 \end{bmatrix}$$

if value = 0 :

return True

$-2 : 1$

$-3 : 1$

$0 : 1$

$5 : 1$

## Subarray with sum $\geq k$

Given int [] A, check if there is a subarray with sum  $\geq k$

$$1 \leq n \leq 10^8$$

$$arr = [2, 3, 9, -4, 1, 5, 6, 2, 5]$$

$\wedge$      $\wedge$      $\wedge$      $\wedge$      $\wedge$      $\wedge$      $\wedge$      $\wedge$      $\wedge$

$$k = 11$$

$$1 \leq i, j \leq n$$

$$\text{sum}(i, j) \geq k$$

$$\text{prefixSum}[j] - \text{prefixSum}[i-1] = k$$

$$PF = [2 \quad 5 \quad 14 \quad 10 \quad 11 \quad 16 \quad 22 \quad 24 \quad 29]$$

$\underline{\hspace{1cm}}$      $\underline{\hspace{1cm}}$

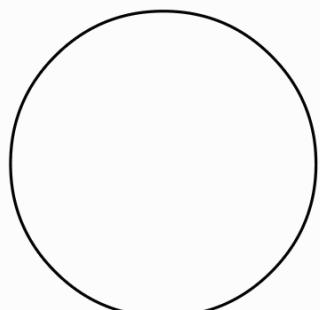
$$arr = [5 \quad \boxed{10} \quad 20 \quad 100 \quad 105]$$

$$PF = [5 \quad 15 \quad 35 \quad 135 \quad 240]$$

$$k = 10$$

$$PF[j] - PF[i-1] \geq k$$

$$15 - 5 = 10$$



## Approach 1: BF $\Theta(n^3)$

1. going through all subarray and check if
2. sum is  $\leq k$ .

## Approach 2:

$$\Rightarrow a - b = k$$

# add  $b$  both sides

$$\Rightarrow a - b + b = k + b$$

# simplify it.

$$\Rightarrow a = k + b$$

# subtract  $k$  from both sides

$$\Rightarrow a - k = k + b - k$$

# Simplify

$$a - k = b$$

$$\boxed{b = a - k}$$

$$\text{sum}(i, j) \leq k$$

$$\text{prefixSum}[j] - \text{prefixSum}[i-1] = k$$

(a) - (b)  $= k$

as per derived equation!

$$\text{prefix}[i-1] = \text{prefix}[j] - k$$

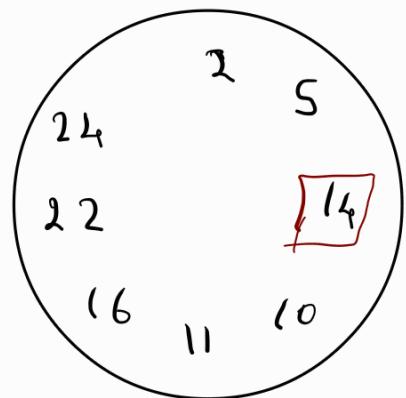
$$b = a - k$$

$$\text{prefix}[i-1] = \text{prefix}[j] - k$$

Dry run!

`arr = [2 3 9 -4 1 5 6 2 5]`

$$PF = \begin{bmatrix} 2 & 5 & 14 & 10 & 11 & 16 & 22 & 24 & 29 \\ \downarrow & \downarrow \\ -15 & -10 & -1 & -5 & -4 & 1 & 7 & 9 & 14 \end{bmatrix}$$



$$\text{arr} = \{2 \ 3 \ 9 \ -4 \ 1\}$$

$$PF = \begin{pmatrix} 2 & 5 & 14 & 10 & 11 \\ 8 & 5 & 4 & 0 & 1 \end{pmatrix} \quad L=10$$

if  $\text{PF}\{j\} = 1$   
return True

2nd way

one way

ad d  
o  
before hand

Code :

# create prefixsum O(n)

Pf = [ ]

Pf.append(A[0])

for i in range(1, n)

↓ Pf.append(Pf[i-1] + A[i])

# iterate through the array and O(n)

find if sum == 0 exists

hashset = {}

for i in range(n)

# b = a - k formula

b = Pf[j] - k

if b in hashset:

↓ return True

else:

↓ hashset.add(Pf[j])

Tc : O(n)

Sc : O(n)

# without using PF array

sum = 0

hashset =  $\emptyset$ ,

for j in range(n)

    sum += A[j]

    b = sum - k

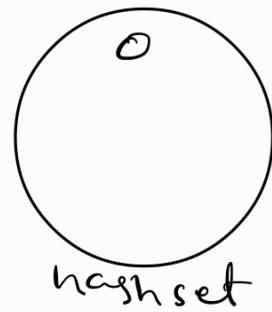
    if b in hashset:

        ↓ return True

    else:

        ↓ hashset.add(sum)

return False



TC:  $O(N)$

SC:  $O(N)$

# ①. check pair sum.

Given  $\inf B$ , int A as sum.

$\Rightarrow$  check if there exists a pair  $(i, j)$  such that

$$B_i + B_j = A \quad \text{and } i \neq j$$

$$\begin{aligned} a+b &= k \\ b &= k-a \quad B[i] \\ &\quad B[j] \end{aligned}$$

$$\begin{aligned} A &= 8 \\ B &= \left[ \begin{array}{cccccc} 8 & 5 & 1 & 2 & 1 & 2 \end{array} \right] \\ &\quad \downarrow \\ &\quad \frac{8+2}{0} \end{aligned}$$

$$1 \leq \text{length of } B \leq 10^5$$

$$0 \leq B_i \leq 10^9$$

$$0 \leq A \leq 10^9$$

code!

```

n = len(B), eleSet = set()
for i in range(n):
    b = A - B[i]
    if b in eleSet:
        return 1
    else
        eleSet.add(B[i])
return 0
    
```

②

## Count pair Difference

1. Given int [ ] A, int B
2. Count no. of pairs (i, j) such that  
 $A[i] - A[j] = B \quad \& \quad (i \neq j)$

$$B = 4$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & 12 \\ 3-4 & 5-4 \\ -1 & 1 \end{bmatrix}$$

$$1 \leq n \leq 10^5$$

$$1 \leq A[i] \leq 10^9$$

$$1 \leq B \leq 10^9$$

$$A[i] - A[j] = B$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1, 2, 1, 2 \end{bmatrix} \quad B = 1$$

$$\frac{1+1}{2} \quad \frac{2+1}{3} \quad \frac{4+1}{2}$$

$$a - b = k \quad \# \text{ find } a$$

$$1 : 2$$

$$2 : 1$$

$$a - b + b = k + b$$

$$\text{ans} = +2$$

$$a = k + b$$

$$A[i] = k + A[j]$$

$$A = \begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 \\ \frac{2+1}{3} & \frac{1+1}{2} & \frac{2+1}{3} & \frac{1+1}{2} & \frac{1+1}{2} & \frac{1+1}{2} \end{bmatrix} \quad B = 1$$

2 : 5

1 : 4

$$A[i] = A[i] + k$$

Code:

def periodSumB(A, B):

N = len(A)

ans = 0, freqMap = {}

for i in range(N):

    freqMap[A[i]] = freqMap.get(A[i], 0) + 1

for i in range(N)

    b = A[i] + B

    ans += freqMap.get(b, 0)

return ans.

## Sub array with sum zero

1. given int[] A,
2. return array contains subarray with sum zero

$$1 \leq |A| \leq 1000$$

$$A = [1, 2, 3, 4, 5] \quad \text{ans} = 0$$

$$Pf = [1 \ 3 \ 5 \ 9 \ 14]$$

$$\text{sum}(i, j) = 0$$

$$Pf[j] - P[i-1] = 0$$

$$a - b = 0$$

$$a - b \neq b = b$$

$$a = b$$

# create Pf array (optional)

$$Pf = []$$

# iterate through array and check if condition is metting.

# Handling zero

while creating set. add 0 before the hand

④ Subarray sum = k

1. Given int [] A, int B

2. Find Num subarray sum equal to B

$$PF = \begin{bmatrix} 1, 1, 0 \\ \cancel{1-1}, \cancel{1-1}, \cancel{0-1} \end{bmatrix}$$

$$\text{sum}(i, j) = k$$

$$PF[j] - PF[i-1] = k$$

$$a - b = k \quad \# \text{Find } b$$

$$a - b + b = k + b$$

$$a = k + b$$

$$a - k = k + b - k$$

$$b = a - k$$

$$PF[i-1] = PF[j] - k$$

$$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

Count =  $\emptyset \times 4$

Sum =  $\emptyset \times 2$

$$b = \overset{1}{a} - \underset{k}{k}$$

$$2 - 1 = 1$$

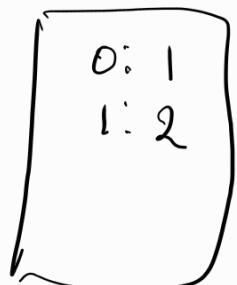
$$\text{Count} = \emptyset \times 3 + 3 = \boxed{6}$$

currentSum = 0

Prefix sum = { 0 : 3 }

$$b = a - k$$

0



steps: iterate over array, add the element to prefix sum.

$$\text{ans} = 0$$

$$\text{prefixSum} = 0$$

$$\text{PfMap} = \{0: 1\}$$

↓

because there might be subarray sum that is equal to K from the start.

for a in A:

$$\text{prefixSum} += a$$

#  $b = a - k$

$$b = \text{prefixSum} - B$$

# check if b present in PfMap.

if b in PfMap:

$$\downarrow \text{ans} += \text{PfMap}[b]$$

# add the prefixSum into PfMap:

$$\text{Value} = \text{PfMap.get(prefixSum, 0)} + 1$$

$$\text{PfMap}[prefixSum] = \text{Value}$$

return ans

## Count Pair Sum

1. Given int [] A, int B
2. Count the number of pairs (i, j) such that  $A[i] + A[j] = B$  and  $i \neq j$
3. return ans %  $10^9 + 7$

$$A = \begin{bmatrix} 3 & 5 & 1 & 2 \\ \cancel{8-3} & \cancel{8-5} & \cancel{8-1} & \cancel{8-2} \\ 5 & 3 & 1 & 6 \end{bmatrix} \quad B = 8 \quad ans = 1$$

$$A[i] + A[j] = B$$

$$a + b = k$$

$$a + b - a = k - a$$

$$b = k - a$$

3 : 1
5 : 1
1 : 1
2 : 1

$$ans = 1$$

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ \cancel{3-1} & \cancel{3-2} & \cancel{3-1} & \cancel{3-2} \\ 2 & 1 & 2 & 1 \end{bmatrix} \quad B = 3$$

1 : r2
2 : 1

1. iterate over array

$$2. b = k - A[i]$$

3. if b in dictionary

$$ans += value$$

4. dictionary [a] += 1

$$ans = r2 + 2 = 4$$

```
freqMap = {}  
ans = 0  
for a in A:  
    b = B - a  
    if b in freqMap:  
        ans += freqMap[b]  
    # adding count of a in dictionary.  
    freqMap[a] = freqMap.get(a, 0) + 1  
return ans
```

## Count Subarray Zero Sum

1. Given int [] A,
2. Find count of Subarray sum = zero
3. return ans  $\leq (10^9 + 1)$

ex: 1  $A = [1, -1, -2, 2]$

$$PF = [1, 0, \underline{-2}, \underline{0}]$$

equations:

$$\text{sum}(i, j) = 0$$

$$PF[j] - PF[i-1] = 0$$

$$\boxed{PF[j] = PF[i-1]}$$

1: 1
0: 2
-2: 1

$\Rightarrow$  if element repeating more than 1  
they make zero.

$$\text{ans} = 1 + 2 = 3$$

ex: 2  $A = [-1, 2, -1]$

$$PF = [-1, 1, \underline{0}]$$

0: 1
-1: 1
1: 1



Keep (0:1) in the beginning.  
as there can be subarray = zero from starting.

$$\text{ans} = \emptyset 1$$

Code:

freqMap = [0:1]

currentSum = 0

ans = 0

for a in A:

    currentSum += a

    if currentSum in freqMap:

        ↓  
        ans += freqMap[currentSum]

        freqMap[currentSum] = freqMap.get(currentSum) + 1

return ans

## Subarray with given Sum = K

1. Given int [] A, and int B
2. Find and return first continuous subarray which adds to B
3. else return [-1]

$$\begin{aligned}
 & \text{A: } [1, 2, 3, 4, 5] \quad B=5 \\
 & \text{PF: } [1, 3, 6, 10, 15] \\
 & \quad \frac{1}{4}, \frac{3}{2}, \frac{6}{1}, \frac{10}{4}, \frac{15}{9} \\
 & \quad 6 - 5 = 1 \\
 & \quad 6 - 1 = 5
 \end{aligned}$$

$$\begin{aligned}
 & \text{sum}(i, j) = k \\
 & \text{PF}[j] - \text{PF}[i-1] = k \\
 & \text{PF}[j] = k + [\text{PF}[i-1]] \\
 & \boxed{\text{PF}[j] - k = \text{PF}[i-1]}
 \end{aligned}$$

0: 1
1: 1
2: 1
3: 1

current\\_sum = 0

```

for i in range(n)
    current_sum += A[i]
    b = current_sum - k
    if b in freqmap:
        break.
    freqmap[current_sum] = 1

```

0: 1
1: 0
2: 1
3: 2

$\text{curr\_sum} = 0 \times 3$

$$3 - 5 = -2$$

$$6 - 5 = 1$$

$$j = 2$$

$$i =$$

$$\text{elem} = 1$$

Sum-B

AC?

Sum

$\text{pfdefreq} = \{0: -1\}$ ,  $\text{curr\_sum} = 0$

AC[i]

$$s = -1$$

$$e = -1$$

for  $i$  in range( $\text{len}(A)$ )  $O(n)$

$$\text{curr\_sum} += A[i]$$

$$b = \text{curr\_sum} - B$$

if  $b$  in  $\text{pfdefreq}$ :

$$s = \text{pfdefreq}[b] + 1$$

$$e = i$$

break

$\text{pfdefreq}[\text{curr\_sum}] = i$

if  $s \neq -1$

$\text{ans} = []$

for  $i$  in range( $s, e+1$ )

$\text{ans.append}(A[i])$

return  $\text{ans}$

else:

return  $[-1]$

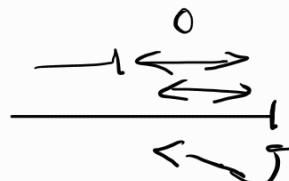
TC: O(n)

SC: O(n)

## Largest Subarray zero sum

1. Given int[] A
2. Find the length of longest subarray
3. if there is no subarray sum zero return 0.

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1, -2, 1, 2 \end{bmatrix}$$



$$\text{sum} = \begin{bmatrix} 1, -1, 0, 2 \end{bmatrix}$$

$$Pf\{S\} - P\{i-j\} = 0$$

$$Pf\{S\} = P\{i-j\}$$

A rectangular box containing the equation  $Pf\{S\} = P\{i-j\}$ . A double-headed arrow is positioned below the equals sign.

0: 2
1: 0
-1: 1



ans: 3



$\text{ans} = 0$

$\text{sum} = 0$

$\text{hm} = \{0: 1\}$  # if any subarray sum <= 0

$s, e = -1, -1$

for  $i$  in range(len(A)):

$\text{sum} += A[i]$

if  $\text{sum}$  in  $\text{hm}$ :

$s = \text{hm}[\text{sum}] + 1$

$e = i$

$\text{length} = e - s + 1$

if  $\text{ans} < \text{length}$

$\downarrow \quad \text{ans} = \text{length}$

else: # update the sum occurrence into hm if not seen ahead  
 $\text{hm}[\text{sum}] = i$

return  $\text{ans}$